Recent results in black hole physics

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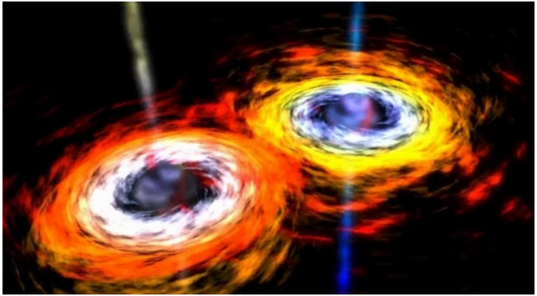
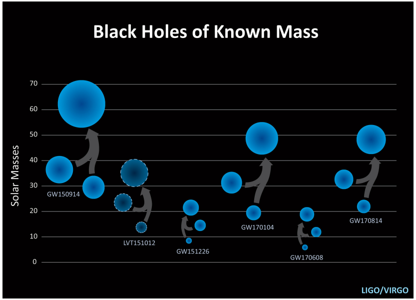
ICTP

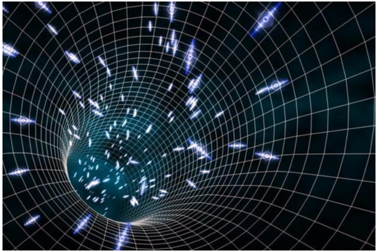
Annual Theory Meeting, Durham, December 2018

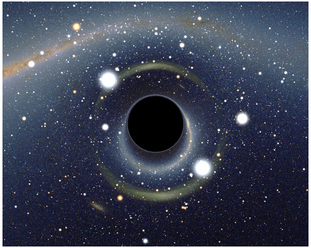


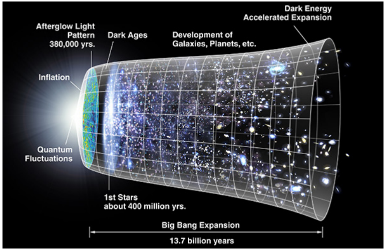


Black Holes of Known Mass
Solar Masses
70
60
50
40
30
20
10
0
GW150914
LVT151012
GW151226
GW170104
GW170608
GW170814
LIGO/VIRGO

Quantum aspects of black holes







Black Holes and Quantum Information

Central puzzle: Quantum Entanglement and space-time at the horizon

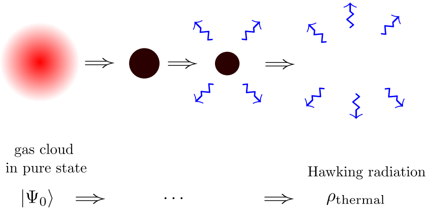
Principles of Quantum Gravity: limitations of locality, quantum mechanics behind the horizon?

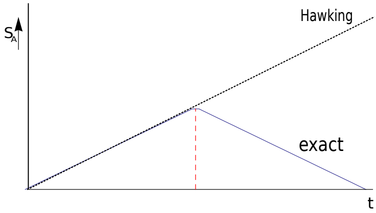
Possible implications for de Sitter horizon?

Outline

- Black hole information paradox and its recent re-formulation as the firewall paradox
- Some proposals for its resolution
- Traversable wormholes, novel methods of probing black hole interior
- Escaping the horizon of a black hole

The black hole information paradox

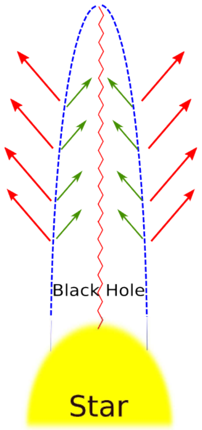




General expectation: small corrections to Hawking's computation may lead to small correlations between outgoing particles. These may encode information of the initial state and restore unitarity.

Entanglement between small number of Hawking particles need to be $O(e^{-S})$, but for large numbers it must be significant.

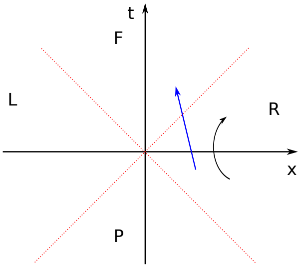
Entanglement near the horizon

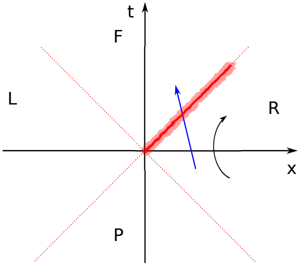


Hawking particles are produced in **entangled pairs**

This entanglement is **necessary** for the smoothness of spacetime near the horizon

Example: flat space, Unruh effect

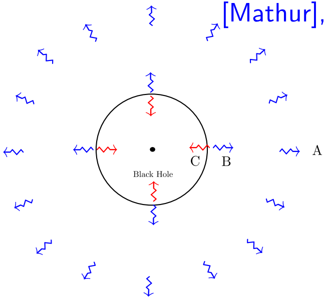


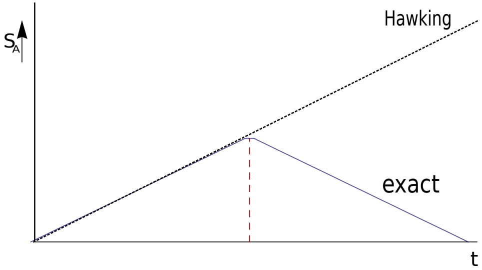


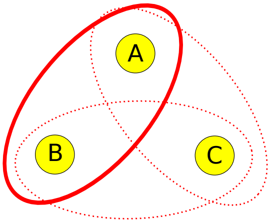
$$|0\rangle_M = \sum_{n=0}^{\infty} e^{-\pi\omega n} |n\rangle_L \otimes |n\rangle_R \qquad |\Psi\rangle = |0\rangle_L \otimes |0\rangle_R \to \langle T_{\mu\nu}\rangle \neq 0$$

The firewall paradox

[Mathur],[Almheiri, Marolf, Polchinski, Sully]





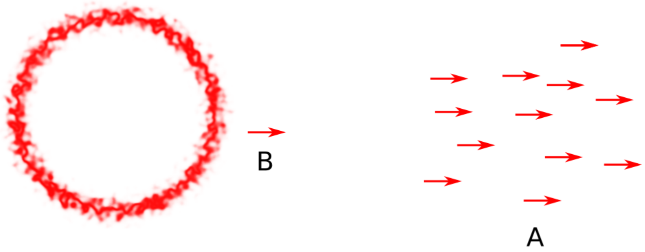


For information to escape black hole: B must be entangled with A.

For horizon to be smooth: B must be entangled with C.

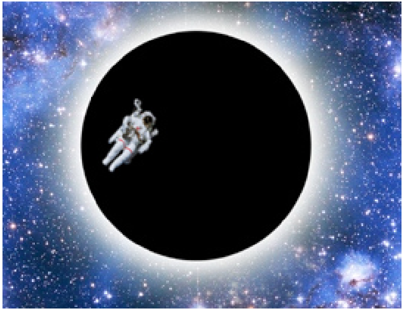
This violates the **monogamy of entanglement** for the particle B.

Is the horizon smooth?



Breaking the B-C entanglement near the horizon creates a huge energy density creating a "**firewall**" on the horizon, which would burn up an infalling observer.

General Relativity vs Quantum Mechanics



These proposals would be able to solve the information paradox, however they lead to massive violations of general relativity.

The curvature of spacetime near the black hole horizon is

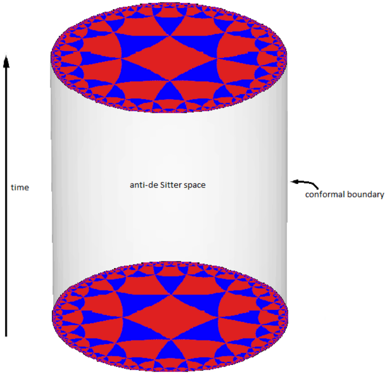
$$R_{ijkl}R^{ijkl} \sim \frac{1}{(GM)^4}$$

For a large black hole this curvature is very low and we expect standard general relativity to hold.

Modern info paradox: Can we reconcile unitarity with smoothness of the horizon?

The AdS/CFT correspondence
[Maldacena]

A $d+1$-dimensional theory of gravity with negative cosmological constant, is equivalent to a d-dimensional large N $SU(N)$ gauge theory without gravity



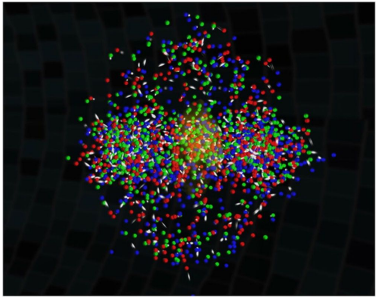
Questions about quantum gravity can be translated in the QFT

QFT is strongly coupled

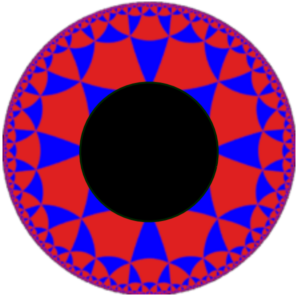
Emergence of extra AdS-dimension from QFT remains mysterious

Black Holes in AdS/CFT

Quark gluon plasma



Black Hole in AdS



Understanding of black hole entropy

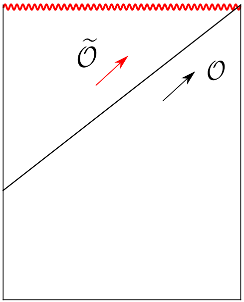
AdS/CFT settles that information is not lost

What about space-time behind the horizon?

Paradox in AdS/CFT

- Large black holes in AdS are holographically dual to QGP states of $\mathcal{N} = 4$ SYM in deconfined phase
- These black holes are in equilibrium with their Hawking radiation and do not evaporate
- Nevertheless the analogue of the firewall paradox has been formulated even for these stable black holes [Almheiri, Marolf, Polchinski, Stanford, Sully], [Marolf, Polchinski]
- It suggests that big AdS black holes may have a singular horizon and no geometric interior.
- Most precise formulation of the paradox.

Firewall paradox for large AdS black holes



$$[\mathcal{O}, \mathcal{O}^\dagger] = 1$$
$$[H, \mathcal{O}^\dagger] = \omega \mathcal{O}^\dagger$$

$$[\widetilde{\mathcal{O}}, \widetilde{\mathcal{O}}^\dagger] = 1$$
$$[H, \widetilde{\mathcal{O}}^\dagger] = -\omega \widetilde{\mathcal{O}}^\dagger$$

- [AMPSS, MP] paradox: if typical black hole states have smooth horizon, using $[H, \widetilde{\mathcal{O}}^\dagger] = -\omega \widetilde{\mathcal{O}}^\dagger$ we find
$$Tr[e^{-\beta H} \widetilde{\mathcal{O}}^\dagger \widetilde{\mathcal{O}}] < 0$$
which is inconsistent.
- This suggests that there are no operators $\widetilde{\mathcal{O}}$ in the CFT with the desired properties, hence the BH has no interior and horizon is singular. ???

Spacetime behind the horizon

- In work with S. Raju

  based on JHEP 1310 (2013) 212, PRL 112 (2014) 5, Phys.Rev. D89 (2014), PRL 115 (2015)

  we identified candidate CFT operators $\widetilde{\mathcal{O}}$ for describing the black hole interior

- This suggests some new possible insights for the modern version of the information paradox

Tomita-Takesaki modular theory

based on earlier work with S. Raju

Introduce a "small algebra" $\mathcal{A}$ of simple operators (single trace + small products).

It probes the typical pure state as a thermal state

$$\langle\Psi|\mathcal{O}(x_1)...\mathcal{O}(x_n)|\Psi\rangle = Z^{-1}\mathrm{Tr}[e^{-\beta H}\mathcal{O}(x_1)...\mathcal{O}(x_n)] + O(1/N)$$

We define the small Hilbert space (also called "code-subspace")

$$\mathcal{H}_\Psi = \mathcal{A}|\Psi\rangle$$

No annihiliation operators in $\mathcal{A} \Rightarrow |\Psi_0\rangle$ is a *cyclic* and *separating* vector.

Tomita-Takesaki theorem: *The representation of the algebra $\mathcal{A}$ on $\mathcal{H}_\Psi$ is reducible, and the algebra has a non-trivial commutant $\mathcal{A}'$ also acting on $\mathcal{H}_\Psi$.* Moreover $\mathcal{A}'$ is isomorphic to $\mathcal{A}$.

Tomita-Takesaki modular theory

Define an antilinear map acting on $\mathcal{H}_\Psi$ by

$$SA|\Psi\rangle = A^\dagger|\Psi\rangle \qquad A \in \mathcal{A}$$

we then define

$$\Delta = S^\dagger S, \qquad , \qquad J = S\Delta^{-1/2}$$

where J is (anti)-unitary. Then the operators in the commutant are

$$\boxed{\widetilde{O} = JOJ}$$

The operator Δ is a positive, hermitian operator and can be written as

$$\Delta = e^{-K}$$

where

$$K = \text{modular Hamiltonian}$$

For entangled bipartite system $A \times B$ this construction would give $K_A \sim \log(\rho_A)$ i.e. the usual modular Hamiltonian for A.

The mirror operators

Using the Tomita-Takesaki construction we define the "mirror operators" for $\omega < \omega_*$

$$\widetilde{\mathcal{O}}_\omega|\Psi\rangle = e^{-\frac{\beta H}{2}}\mathcal{O}^\dagger_\omega e^{\frac{\beta H}{2}}|\Psi\rangle$$

$$\widetilde{\mathcal{O}}_\omega\mathcal{O}....\mathcal{O}|\Psi\rangle = \mathcal{O}...\mathcal{O}\widetilde{\mathcal{O}}_\omega|\Psi\rangle$$

$$[H,\widetilde{\mathcal{O}}_\omega]\mathcal{O}....\mathcal{O}|\Psi\rangle = \omega\widetilde{\mathcal{O}}_\omega\mathcal{O}....\mathcal{O}|\Psi\rangle$$

These equations define the operators $\widetilde{\mathcal{O}}$ on the code-subspace $\mathcal{H}_\Psi \subset \mathcal{H}_{\rm CFT}$, which is relevant for EFT around BH microstate $|\Psi\rangle$

- Operators defined only on $\mathcal{H}_\Psi$, not on full CFT Hilbert space
- $[\mathcal{O},\widetilde{\mathcal{O}}] = 0$ only inside $\mathcal{H}_\Psi$, not as operator equation

Reconstructing the black hole interior

Using these operators in AdS/CFT we showed that the horizon is a smooth region of space-time as predicted by general relativity.

Quantum field inside the black hole

$$\phi(t, r, \Omega) = \int_0^{\omega*} d\omega \Big[\mathcal{O}_\omega\, f_\omega(t, \Omega, r) + \widetilde{\mathcal{O}}_\omega g_\omega(t, \Omega, r) + \text{h.c.}\Big]$$

Correlation functions of these operators

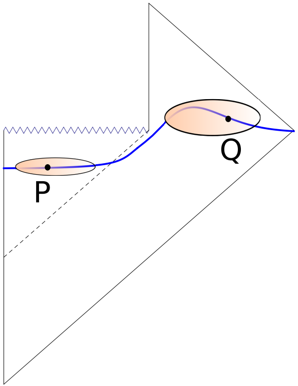
$$\langle\Psi|\phi(t_1, r_1, \Omega_1)...\phi(t_n, r_n, \Omega_n)|\Psi\rangle$$

reproduce those of effective field theory in the exterior/interior of the black hole, without any indication for a firewall or fuzzball.

At the same time, the entire framework is unitary.

How have we been able to avoid the previous paradox?

Non-locality in Quantum Gravity



$[\mathcal{O}, \widetilde{\mathcal{O}}] \approx 0$ in simple correlators, not as exact operator equation

$$[\phi(P), \phi(Q)] = O(e^{-S})$$

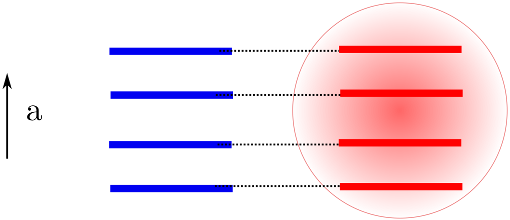
Hilbert space of Quantum Gravity does not factorize as

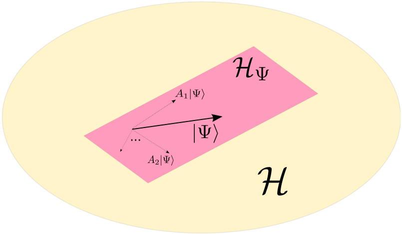
$$\mathcal{H} \neq \mathcal{H}_{\text{inside}} \otimes \mathcal{H}_{\text{outside}}$$

Solves problem of Monogamy of Entanglement

Concrete realization of "Black Hole Complementarity". We showed it is consistent with approximate locality in effective field theory

State-dependence of operators



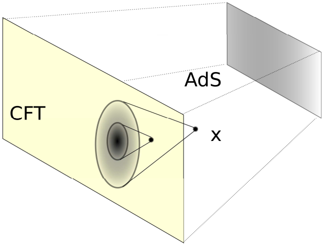


- Interior operators defined by

$$\widetilde{\mathcal{O}}|\Psi\rangle = e^{-\frac{\beta H}{2}}\mathcal{O}e^{\frac{\beta H}{2}}|\Psi\rangle \qquad \widetilde{\mathcal{O}}\mathcal{O}...\mathcal{O}|\Psi\rangle = \mathcal{O}...\mathcal{O}\widetilde{\mathcal{O}}|\Psi\rangle$$

- We notice the specific black hole microstate $|\Psi\rangle$ entering the equation
- Operators depend on the state, they are defined in "patches" on the Hilbert space
- Unusual in Quantum Mechanics, needs further study

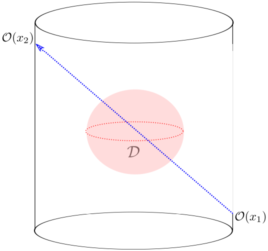
Local physics vs "S-matrix"



Direct local reconstruction

Hamilton-Kabat-Lifschytz-Lowe (HKLL) construction

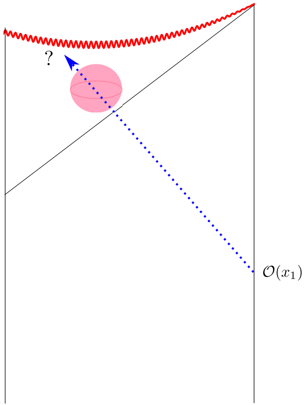
$$\phi(x) = \int dY \, K(Y, x) \mathcal{O}(Y)$$



Indirect experiments

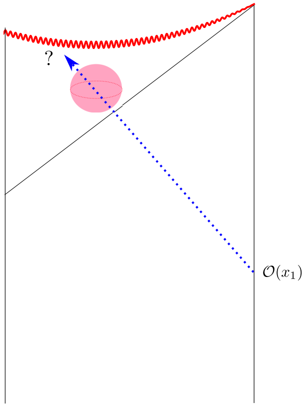
"S-matrix" $\Leftrightarrow$ CFT correlators

Local physics vs “S-matrix”



Can we probe BH interior by an S-matrix-like experiment?

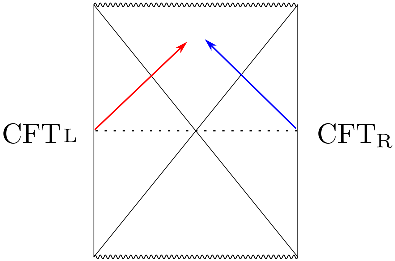
Local physics vs "S-matrix"

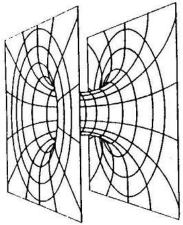


Can we probe BH interior by an S-matrix-like experiment?
Maybe! Traversable-wormhole protocol by Gao-Jafferis-Wall (2016)

Eternal AdS black hole

[Maldacena]





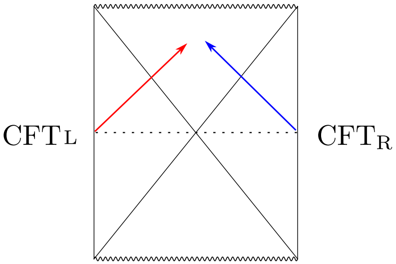
Two identical non-interacting CFTs

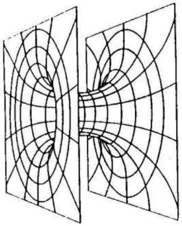
$$H = H_L + H_R$$

in an entangled state

$$|\mathrm{TFD}\rangle = \frac{1}{\sqrt{Z}} \sum_E e^{-\frac{\beta E}{2}} |E\rangle_L \otimes |E\rangle_R$$

Eternal AdS black hole

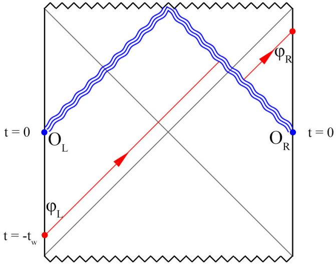




In the bulk they are connected by a wormhole (Einstein-Rosen bridge).

It is not traversable, consistent with the fact that CFTs are non-interacting

Gao-Jafferis-Wall protocol



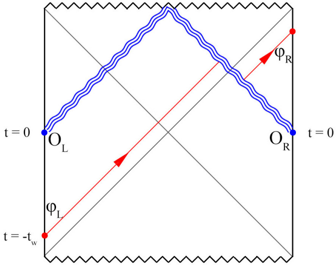
at $t = 0$ we briefly couple the CTFs by a double-trace interaction

$$H = H_L + H_R + g f(t) \mathcal{O}_L \mathcal{O}_R$$

For given sign of g this creates negative energy shockwaves in the bulk. Probe undergoes time advance when crossing shockwaves

Wormhole becomes traversable!

Gao-Jafferis-Wall protocol



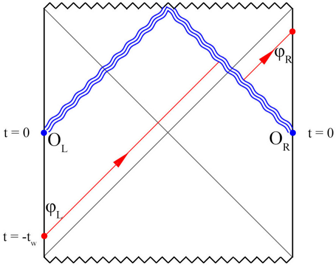
Change of CFT energy

$$\delta\langle H_R\rangle \propto g\langle \mathcal{O}_L\mathcal{O}_R\rangle + O(g^2)$$

Black hole horizon shrinks somewhat, probe can cross the wormhole
CFTs briefly interacted via $O_L O_R$ at $t = 0$, so information can be exchanged
Notice ϕ vs O

Gao-Jafferis-Wall protocol

analysis by [Maldacena-Stanford-Yang]



We create the probe on the left by

$$e^{i\epsilon\phi_L(-t)}|\mathrm{TFD}\rangle$$

At $t = 0$ we apply double-trace perturbation coupling the two CFTs

$$e^{igO_L O_R(0)} e^{i\epsilon\phi_L(-t)}|\mathrm{TFD}\rangle$$

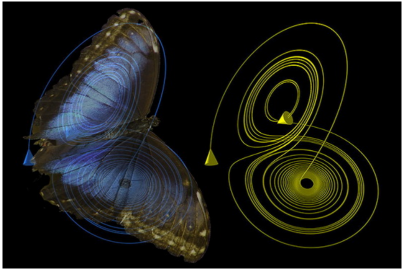
We measure the operator $\phi_R(t)$ on this state. To leading order in ϵ we need

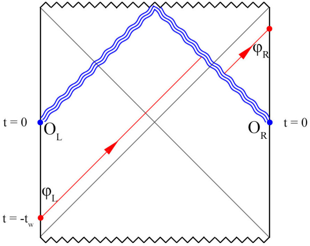
$$\langle \mathrm{TFD}|[\phi_L(-t), e^{-igO_L O_R(0)}\phi_R(t)e^{igO_L O_R(0)}]|\mathrm{TFD}\rangle$$

Expanding in g

$$\langle \mathrm{TFD}|[\phi_L(-t), O_L(0)][\phi_R(t), O_R(0)]|\mathrm{TFD}\rangle$$

Traversable wormholes and quantum chaos



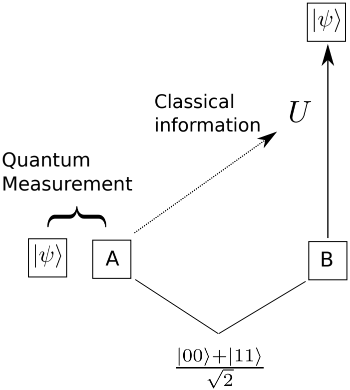


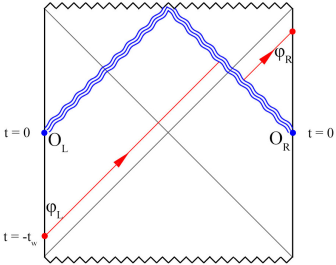
Growth of out-of-time-order-correlators (OTOC) due to **quantum chaos**

$$\langle \mathrm{TFD}|[\phi_L(-t), O_L(0)][\phi_R(t), O_R(0)]|\mathrm{TFD}\rangle \sim e^{\frac{2\pi}{\beta}t}$$

Including higher orders in g, we find that the commutator is zero up to scrambling time $t \approx \beta \log S$, when it becomes nonzero and we get a nontrivial signal, corresponding to the probe appearing in the right CFT.

Quantum Teleportation Interpretation





Measure O_L on CFT$_L$, then apply

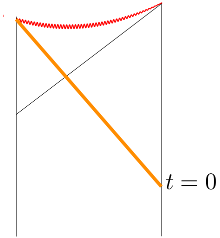
$$e^{igo_L\mathcal{O}_R}$$

on CFT$_R$. The probe ϕ is teleported.

- Identified an S-matrix-like experiment which probes the interior of eternal black hole
- CFT correlators contain information about geometry inside horizon
- Computations provide evidence for smoothness of horizon of eternal black hole, dual to the TFD state, and ER/EPR proposal
- However, the real difficulty in reconciling unitarity with the smoothness of the black hole horizon is not for the TFD (which is a very special, atypical state), but rather for *typical black hole microstates*.
- Can we find a way of applying a similar protocol to (1-sided) typical black hole microstates, which will allow us to probe their interior?

Collapsing vs Typical black holes

Black holes formed by gravitational collapse are *a-typical*



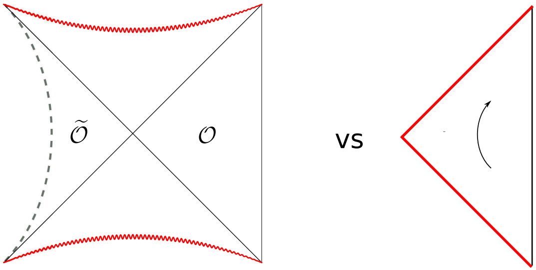
Instead, a *typical black hole microstate* is defined as

$$|\Psi\rangle = \sum_i c_i |E_i\rangle$$

where random coefficients c_i selected by the Haar measure.
Notice that typical states are almost time-independent

$$\langle\Psi|\frac{dA}{dt}|\Psi\rangle = O(e^{-S})$$

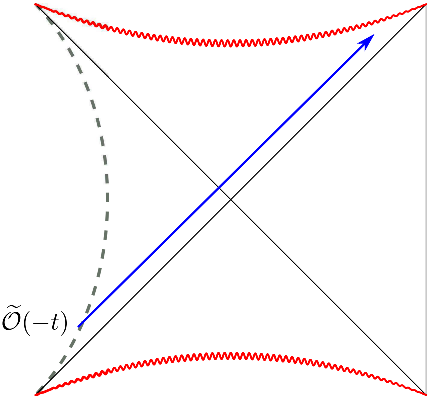
Geometry of a typical state



If horizon is smooth, we expect interior region to be consistent with (approximate) Killing isometry.

Notice "cutoff" in left region related to $\omega < \omega_*$

Exciting the left region

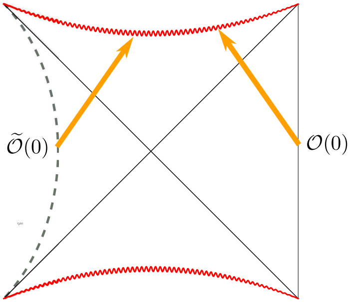


Mirror quench: we perturb the CFT Hamiltonian by $\widetilde{\mathcal{O}}$ at $-t$

Excitation is invisible by simple CFT operators

Creating negative energy shockwaves for 1-sided black hole

[J. de Boer, R. van Breukelen, S. Lokhande, KP, E. Verlinde, arXiv: 1804.10580]+to appear

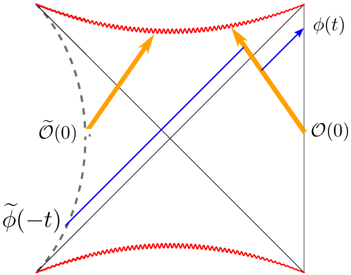


At $t = 0$ we perturb CFT Hamiltonian by

$$g f(t) \mathcal{O} \widetilde{\mathcal{O}}(0)$$

Compute effect on bulk correlators $\Rightarrow$ generates negative energy shockwaves for appropriate choice of g

The experiment



We create a probe in the left region of the black hole by acting with $\widetilde{\phi}(-t)$.

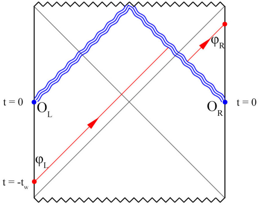
Then at $t = 0$ we perturb the CFT by $g f(t)\mathcal{O}(0)\widetilde{\mathcal{O}}(0)$. Finally we detect the probe by measuring $\phi(t)$.

The postulated Penrose diagram makes a prediction about CFT correlators (singal around $t = \beta \log S$)

$$\langle\Psi_0|[\widetilde{\phi}(-t), e^{-ig\widetilde{\mathcal{O}}\mathcal{O}(0)}\phi(t)e^{ig\widetilde{\mathcal{O}}\mathcal{O}(0)}]|\Psi_0\rangle$$

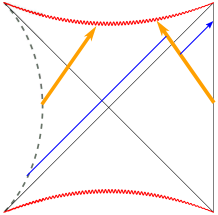
Comparison

Eternal BH



$$C = \frac{1}{Z}\mathrm{Tr}[e^{-\beta H}\mathcal{X}(\phi, O)]$$

1-sided BH



$$C' = \langle\Psi_0|\mathcal{X}(\phi, O)|\Psi_0\rangle$$

Using properties of the TFD state and the mirror operators we find that both experiments are governed by the expectation value of **exactly the same** string of ordinary CFT operators $\chi(\phi, \mathcal{O})$. Moreover, in stat-mech we have

$$C' = \mathrm{Tr}[\rho_m \mathcal{X}(\phi, \mathcal{O})] + O(e^{-S})$$

Condition for CFT correlators

$$C = \frac{1}{Z}\mathrm{Tr}[e^{-\beta H}\mathcal{X}(\phi, O)] \qquad C'' = \mathrm{Tr}[\rho_m \mathcal{X}(\phi, O)]$$

A necessary condition for horizon of typical BH mircostate to be smooth is

$$\boxed{\lim_{N\to\infty} C = \lim_{N\to\infty} C''}$$

- Not obvious, trace-distance $||\rho_\beta - \rho_m||$ between ensembles is almost maximal.
- $\mathcal{X}(\phi, O)$ is a complicated observable, product of operators at time separation $\Delta t \sim \beta \log S$
- Condition is related to whether $\mathcal{X}(\phi, O)$ obeys Eigenstate Thermalization Hypothesis (ETH)

$$\langle E_i|\mathcal{X}|E_j\rangle = f(E_i)\delta_{ij} + R_{ij}. \tag{1}$$

with $\frac{df}{dE} \sim O(1/S)$

Condition for CFT correlators

- Interesting effect comes from subleading corrections of the form

$$\frac{1}{N^2} e^{\frac{2\pi t}{\beta}}$$

  At scrambling time they become O(1).

  Are these "chaos-enhanced" $1/N^2$ corrections the same in typical pure states and thermal ensemble?

- Our condition requires that correlators agree *even after analytic continuation* by $t \to t - i\frac{\beta}{2}$ (keeping frequencies up to ω_*)

Evidence

1. ETH holds for products of operators at small time separation. We can show that it also holds for very large time separations (when chaos saturates). It is natural to expect that it holds for intermediate times of order $\beta \log S$

2. In 2d CFTs with large c and sparse spectrum correlators are dominated by Virasoro identity block. In this case the conjecture is true.

3. Numerical evidence in SYK model

The SYK model

N-Majorana fermions in $0+1$d

$$\{\psi^i, \psi^j\} = \delta^{ij}$$

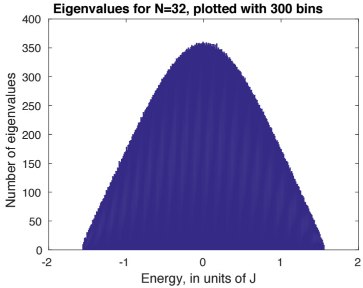
$$H = \sum_{ijkl} J_{ijkl} \psi^i \psi^j \psi^k \psi^l$$

where J_{ijkl} random couplings

$$dim\mathcal{H} = 2^{\frac{N}{2}}$$

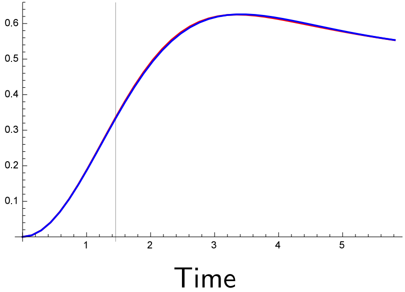
Flows to strongly coupled CFT in IR

Model of black hole in AdS_2



[figure from Maldacena, Stanford]

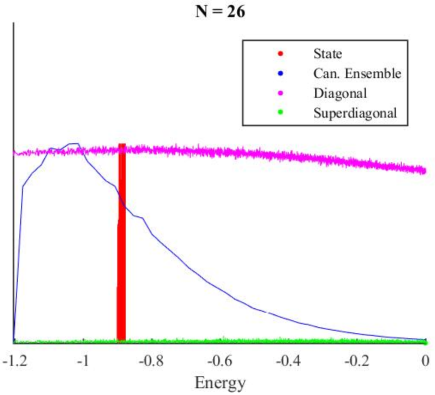
Pure vs thermal state OTOC in SYK



$$\langle \{\psi^i(t), \psi^i(0)\}^2 \rangle$$

on thermal state (red) vs typical pure state (blue).

ETH for chaotic observables in SYK

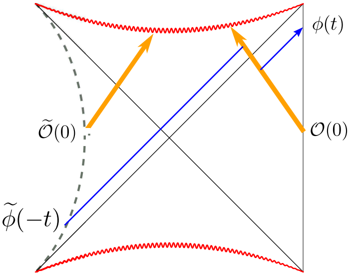


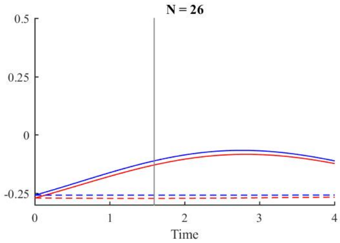
Matrix elements in SYK of

$$\{\psi^i(t), \psi^i(0)\}^2$$

for $t \approx \beta \log S$

Extracting particle from behind the horizon





Recovering information from a black hole

We throw a qubit into black hole. How long do we need to wait to recover the information from Hawking radiation?

$$t_{evap} \sim G^2 M^3$$

Hayden Preskill (2007): if we have access to more than half of Hawking radiation we only need to wait scrambling time

$$t_S \sim GM \log S$$

to recover information. For the protocol to work we need to know the initial state of the black hole.

Hayden-Preskill protocol

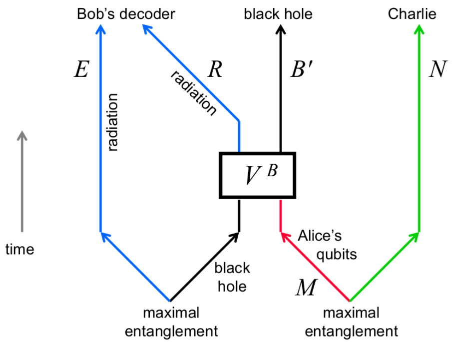
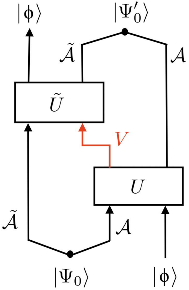
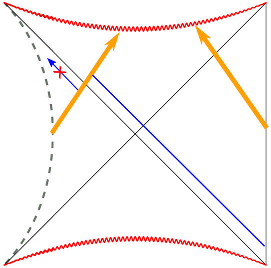


Figure from Hayden and Preskill (arXiv: 0708.4025)

Reformulated by Maldacena-Stanford-Yang in terms of traversable wormholes

A realization of Hayden-Preskill





We throw qubit $\phi(-t_s)$ into black hole

At $t = 0$ we act with $\mathcal{O}\widetilde{\mathcal{O}}$

After scrambling time we can *extract* the quantum information of the qubit my measuring operator $\widetilde{\phi}(t_s)$.

This provides an explicit decoding Hayden-Preskill protocol

Knowledge of the quantum state related to state-dependent $\widetilde{\mathcal{O}}$.

Summary

- The nature of space-time behind the horizon remains mysterious
- Presented a proposal for describing the black hole interior in AdS/CFT. Important aspects: non-locality and state-dependence
- Developments related to traversable wormholes: new calculational tools to probe BH interior
- Interesting connections with quantum teleportation, thermalization and quantum chaos in pure states.