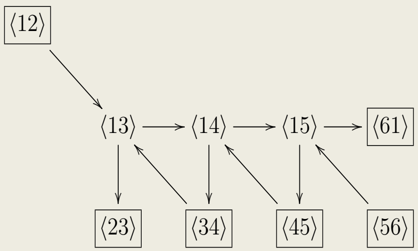
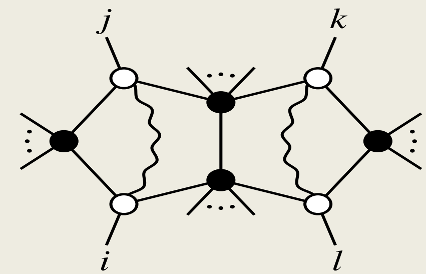
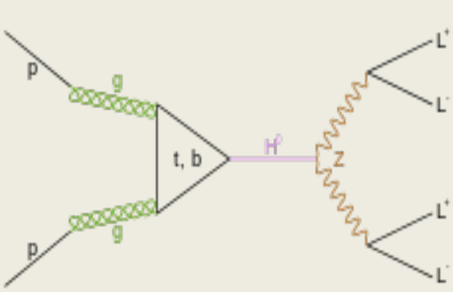


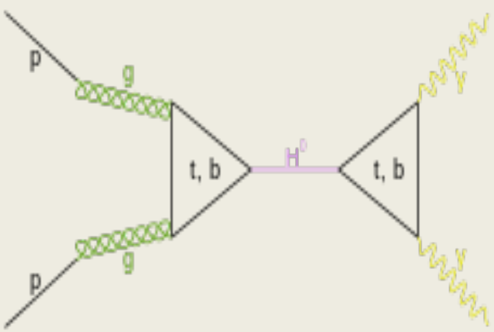
Cluster Algebras, Landau Singularities and Scattering Amplitudes





Anastasia Volovich

Brown University



Igor Prlina, Marcus Spradlin,

James Stankowicz, Stefan Stanojevic

Planar N=4 Yang-Mills

- N=4 Yang-Mills is a quantum field theory which describes "real world" particle interactions. Many techniques we develop for YM have been applied to QCD and Standard Model.
- N=4 Yang-Mills is a string theory (via AdS/CFT), so learning about N=4 Yang-Mills teaches us about quantum gravity.
- N=4 Yang-Mills is solvable (integrable) which is a non-trivial fact for four-dimensional quantum field theory.

N=4 Yang-Mills Amplitudes

The goal is to explore the hidden structures of amplitudes and to exploit these structures as much as possible to make previously impossible computations possible.

- Twistor string description
- Dual conformal and Yangian symmetries
- Wilson loops/amplitudes/correlation funcs triality
- Color/kinematic duality
- Amplituhedron, positivity, on-shell diagrams
- Simplifying limits: fishnet, multi-Regge

Bern, Dixon, Kosower; Witten; Drummond, Henn, Korchemsky, Sokachev, Brandhuber, Heslop, Spence, Travaglini, Arkani-Hamed, Cheung, Cachazo, Kaplan, Trnka, Bourjaily, Bern, Carrasco, Johansson, Caron-Huot, He, Beisert, Eden, Staudacher, Mason, Skinner, Alday, Madalcena, Basso, Sever, Viera, Roiban, Spradlin, many others

Plan

- Introduction
- N=4 Yang-Mills Amplitudes Bootstrap Program:
  Input = Symbol Alphabet

  How Symbol Alphabet be determined?

- Symbol Alphabet and Landau Singularities
- Symbol Alphabet and Cluster Algebra
- Conclusions

N=4 Planar Yang-Mills Amplitudes

L-loop n-points N^kMHV

- 0- and 1-loops: all n, k
- 2-loop: all n-point MHV, 6, 7 NMHV
- 3, 4-loops: 6, 7-point MHV & NMHV
- 5-loops: 6-point MHV & NMHV
- 6-loops: 6-point MHV
- L-loops: 4, 5-point MHV

[Bern, Dixon, Kosower][Caron-Huot, Dixon, Drummond, Duhr, Foster, Gurdogan, Harrington, Henn, McLeod, Papathanasiou, Spradlin, von Hippel 2015-present]

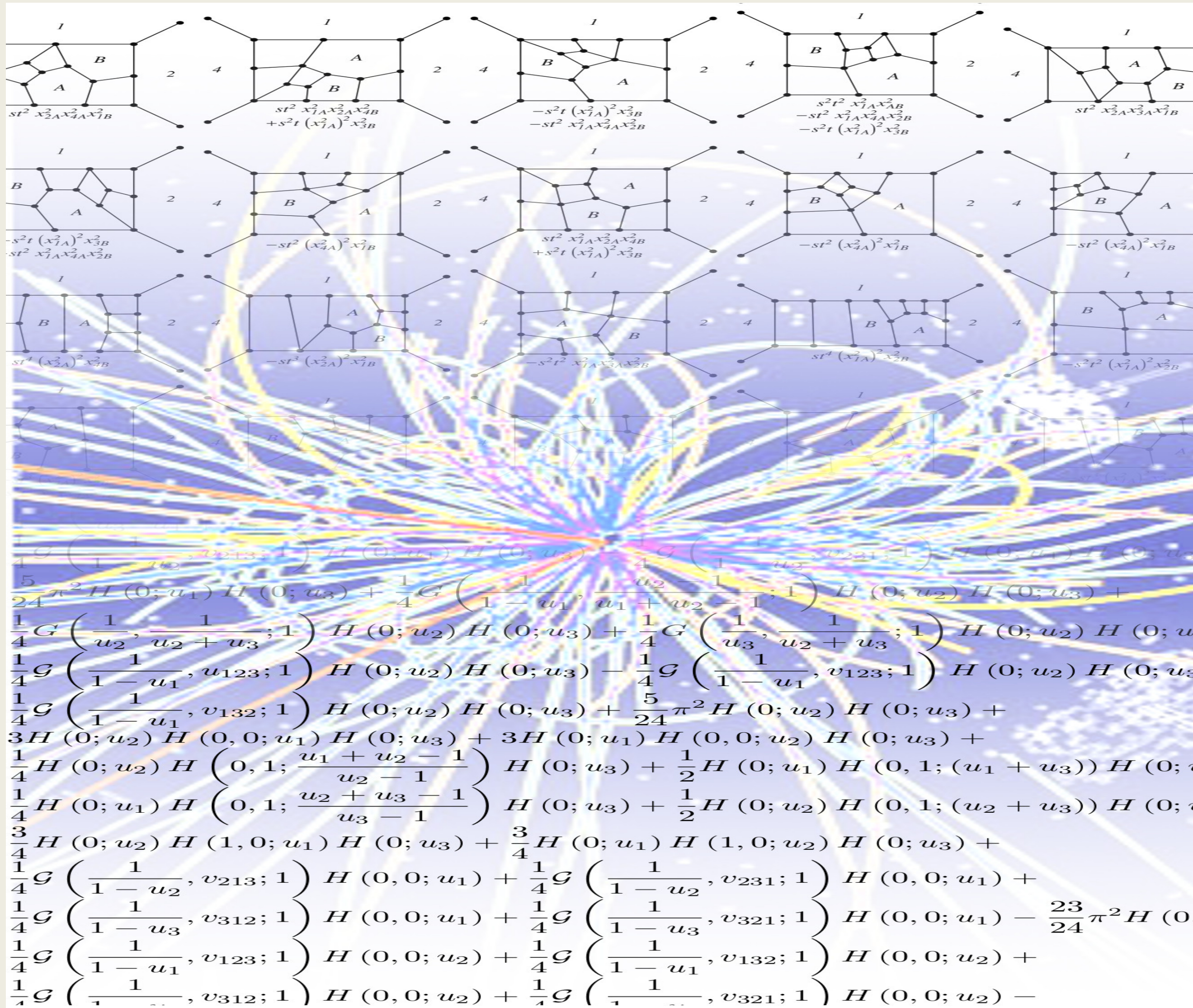
Quantum Field Theory Textbooks:

$$Amplitude = \int d(loop\ momenta)\ (Integrand)$$

Integrand

hard

Amplitude



N=4 Yang-Mills Amplitudes

Integrands are known for all L, k and n, but as we saw on the previous slide the resulting integral is only known for a few cases….

- We need new tools to make these calculations possible.
- Having more data is crucial for identifying hidden structures of these amplitudes.
- Modern approaches to computing amplitudes avoid knowledge of integrands completely….

Amplitudes Bootstrap

Amplitude Bootstrap : Old vs New

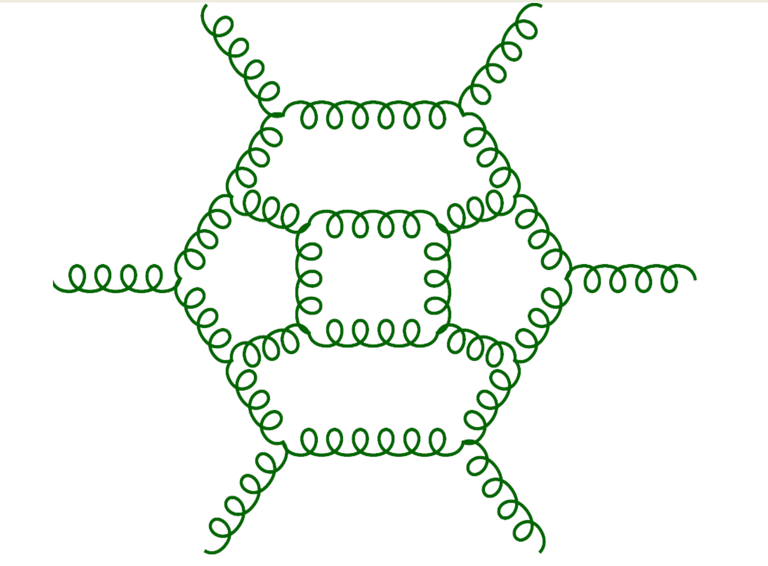
It has long been a goal of the S-matrix program to be able to construct scattering amplitudes based on a few physical principles and a thorough understanding of their analytic structure.

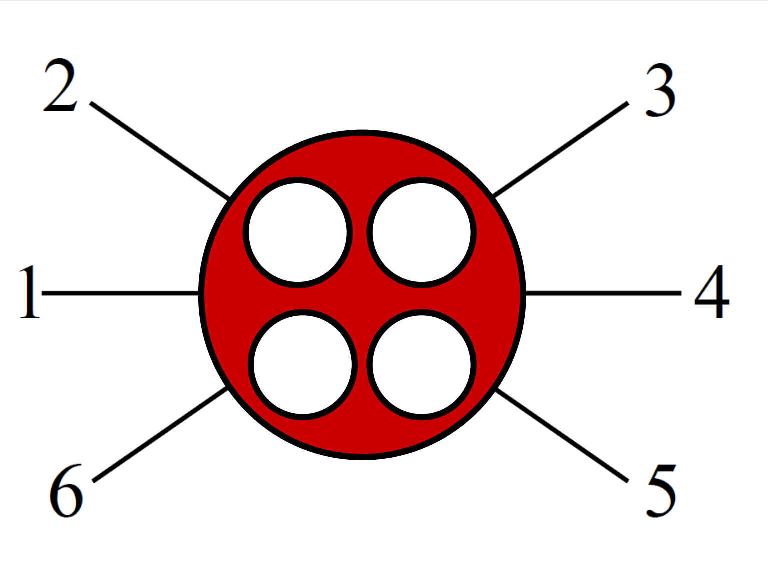
Important New Ingredients

1. Momentum twistors [Penrose, Hodges]
2. Symbol of Polylogarithms [GSVV]
3. Amplituhedron [Arkani-Hamed, Trnka]

Amplitude Bootstrap

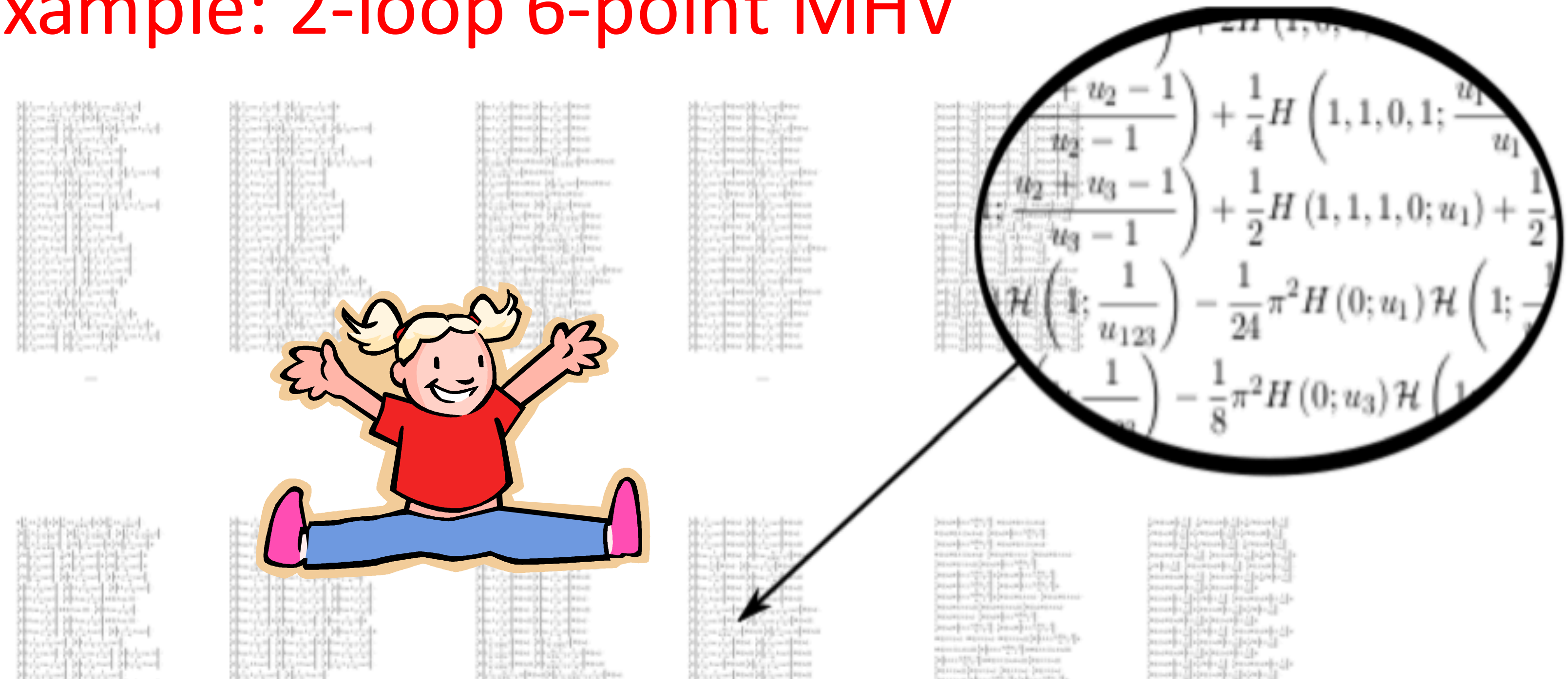
Scattering Amplitudes:
functions of the geometry
of scattering configurations





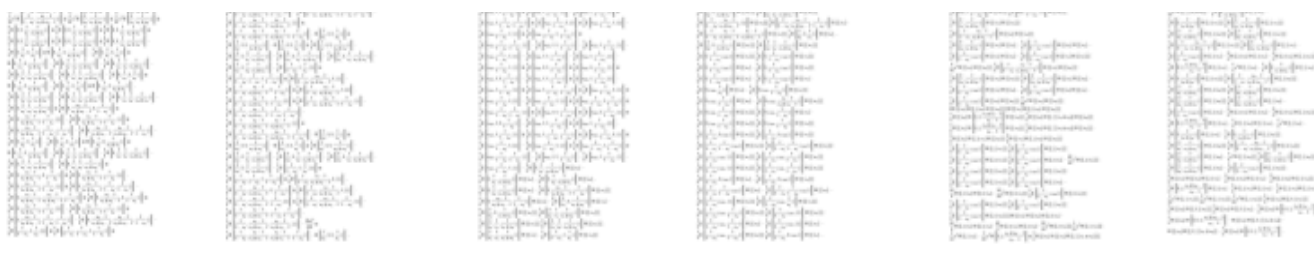
Use mathematical and physical properties
to determine these functions directly.

Example: 2-loop 6-point MHV



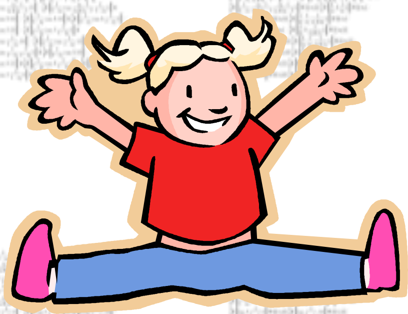
$$R_6^{(2)} = \sum_{\text{cyclic}} \mathrm{Li}_4\left(-\frac{\langle 1234\rangle\langle 2356\rangle}{\langle 1236\rangle\langle 2345\rangle}\right) - \frac{1}{4}\,\mathrm{Li}_4\left(-\frac{\langle 1246\rangle\langle 1345\rangle}{\langle 1234\rangle\langle 1456\rangle}\right)$$

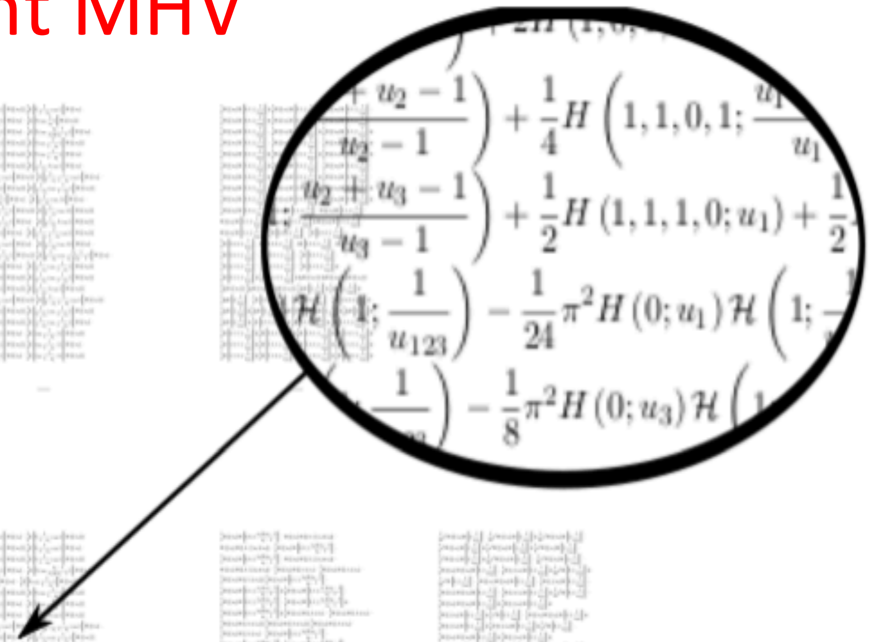
$+$ products of $\mathrm{Li}_k(-x)$ functions of lower weight



GSVV

Example: 2-loop 6-point MHV





$$R_6^{(2)} = \sum_{\text{cyclic}} \mathrm{Li}_4\left(-\frac{\langle 1234\rangle\langle 2356\rangle}{\langle 1236\rangle\langle 2345\rangle}\right) - \frac{1}{4}\,\mathrm{Li}_4\left(-\frac{\langle 1246\rangle\langle 1345\rangle}{\langle 1234\rangle\langle 1456\rangle}\right)$$

$+$ products of $\mathrm{Li}_k(-x)$ functions of lower weight

Function and its Argument

GSVV

Polylogarithm Functions

- Uniform Transcendentality 2L @ L-loops
- Logarithm Functions $\log^{2L} x$
- Polylogarithm Functions

$$Li_{2L}(x) = \int_0^x \frac{dt}{t} Li_{2L-1}(t) \quad Li_1(x) = -\log(1-x)$$

- Generalized Polylogarithm Functions

$$G(a_1, \ldots, a_{2L}; x) = \int_0^x \frac{dt}{t-a_1} G(a_2, \ldots, a_{2L}; t)$$

- Elliptic Polylogarithms (n=10 L=2 k=3)

$$\widetilde{\Gamma}\left(\begin{smallmatrix} n_1 & \cdots & n_k \\ z_1 & \cdots & z_k \end{smallmatrix} ; z, \tau \right) = \int_0^z dz' \, g^{(n_1)}(z' - z_1, \tau) \, \widetilde{\Gamma}\left(\begin{smallmatrix} n_2 & \cdots & n_k \\ z_2 & \cdots & z_k \end{smallmatrix} ; z', \tau \right)$$

Arguments of the Polylogarithms

Null vectors in 4d Minkowski space subject to momentum conservation can be represented by momentum twistors

$$\begin{pmatrix} | & | & \cdots & | \\ Z_1 & Z_2 & \cdots & Z_n \\ | & | & \cdots & | \end{pmatrix}, \qquad Z_i \in \mathbb{P}^3$$

Hodges

$$\langle ijkl \rangle = \det(Z_i Z_j Z_k Z_l)$$

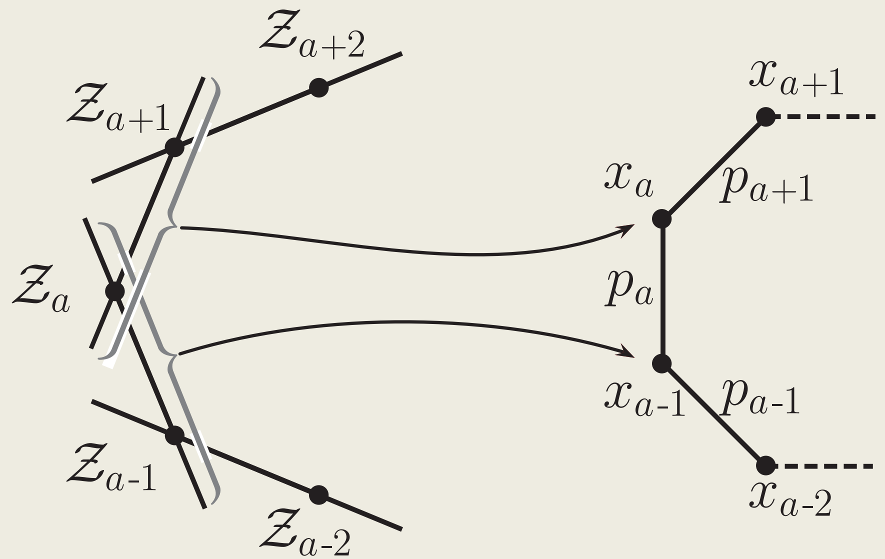
n=6

$$v_1 = \frac{\langle 1246\rangle\langle 1345\rangle}{\langle 1234\rangle\langle 1456\rangle}, \qquad v_2 = \frac{\langle 1235\rangle\langle 2456\rangle}{\langle 1256\rangle\langle 2345\rangle}, \qquad v_3 = \frac{\langle 1356\rangle\langle 2346\rangle}{\langle 1236\rangle\langle 3456\rangle},$$

$$x_1^+ = \frac{\langle 1456\rangle\langle 2356\rangle}{\langle 1256\rangle\langle 3456\rangle}, \qquad x_2^+ = \frac{\langle 1346\rangle\langle 2345\rangle}{\langle 1234\rangle\langle 3456\rangle}, \qquad x_3^+ = \frac{\langle 1236\rangle\langle 1245\rangle}{\langle 1234\rangle\langle 1256\rangle},$$

$$x_1^- = \frac{\langle 1234\rangle\langle 2356\rangle}{\langle 1236\rangle\langle 2345\rangle}, \qquad x_2^- = \frac{\langle 1256\rangle\langle 1346\rangle}{\langle 1236\rangle\langle 1456\rangle}, \qquad x_3^- = \frac{\langle 1245\rangle\langle 3456\rangle}{\langle 1456\rangle\langle 2345\rangle},$$

Relation to momentum



$$\langle 1\,4\,5\,6\rangle \sim \frac{(p_1+p_2)^2(p_3+p_4)^2}{(p_2+p_3+p_4)^2}\,\frac{u_1+u_2+u_3-1+\sqrt{\Delta}}{2u_1u_2u_3},$$

$$\Delta = (1-u_1-u_2-u_3)^2 - 4u_1u_2u_3$$

$$u_1 = \frac{(p_1+p_2)^2(p_4+p_5)^2}{(p_1+p_2+p_3)^2(p_4+p_5+p_6)^2}, \qquad u_2, u_3 = \text{cyclic}$$

Symbol of Polylogarithm

17-pages can be simplified using a very useful tool from modern mathematics

Symbol

$$log(R) \rightarrow R$$

$$Li_2(R) \rightarrow -(1-R) \otimes R$$

$$Li_2(x) + Li_2(-x) = \frac{1}{2} Li_2(x^2)$$

$$-(1-x) \otimes x - (1+x) \otimes (-x) = -(1-x^2) \otimes x = -\frac{1}{2}(1-x^2) \otimes x^2$$

QCD computations

Symbol Alphabet: n=6

- Amplitude = Function of Cross-Ratios

$$R_6^{(2)} = \sum_{\text{cyclic}} \text{Li}_4\left(-\frac{\langle 1234\rangle\langle 2356\rangle}{\langle 1236\rangle\langle 2345\rangle}\right) - \frac{1}{4}\text{Li}_4\left(-\frac{\langle 1246\rangle\langle 1345\rangle}{\langle 1234\rangle\langle 1456\rangle}\right)$$

$+$ products of $\text{Li}_k(-x)$ functions of lower weight

- Full Symbol

$$\langle 1256\rangle \otimes \langle 1346\rangle \otimes \langle 1246\rangle \otimes \langle 1456\rangle + \ldots 7272 \text{ terms}$$

- Symbol Alphabet: 9 letters

$$\langle 1235\rangle, \langle 2345\rangle, \langle 1345\rangle, \langle 2456\rangle, \langle 1356\rangle, \langle 1246\rangle, \langle 1245\rangle, \langle 2356\rangle, \langle 1346\rangle$$

Amplitudes Bootstrap

start with symbol alphabet, impose constraints, find unique solution

Constraint	$L=1$	$L=2$	$L=3$	$L=4$	$L=5$
0. Functions	(10,10)	(82,88)	(639,761)	(5153,6916)	(???,???)
1. Steinmann	(7,7)	(37,39)	(174,190)	(758,839)	(3105,3434)
2. Symmetry	(3,5)	(11,24)	(44,106)	(174,451)	(???,???)
3. Final-entry	(2,2)	(5,5)	(19,12)	(72,32)	(272,83)
4. Collinear	(0,0)	(0,0)	(1,1)	(3,5)	(9,15)
5. Regge	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)

[Caron-Huot, Dixon, Drummond, McLeod, Papathanasiou, Spradlin, Von Hippel]

Bootstrap in QCD amplitudes: Almelid, Duhr, Gardi, McLeod, White; Chicherin, Henn, Mitev

How Do We Determine Symbol Alphabet?

1. Symbol Alphabet and Cluster Algebra

2. Symbol Alphabet and Landau Singularities

Symbol Alphabet and Cluster Algebra

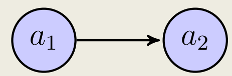
Symbol Alphabet is given by
a subset of cluster coordinates of
Grassmannian Cluster Algebra

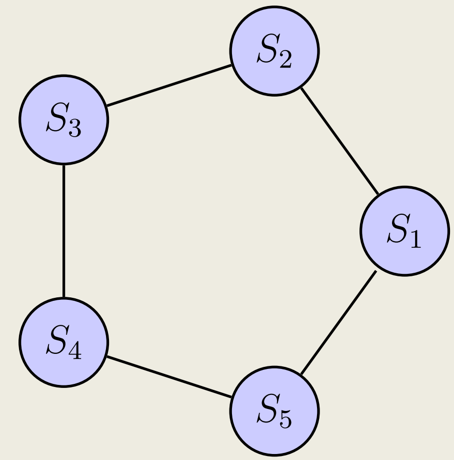
$$Gr(4, n)$$

Golden, Goncharov, Spradlin, Vergu, AV

A_2 Cluster Algebra

Initial Quiver





Mutation Rule

$$a_{n+1} = \frac{1 + a_n}{a_{n-1}}$$

Cluster Coordinates

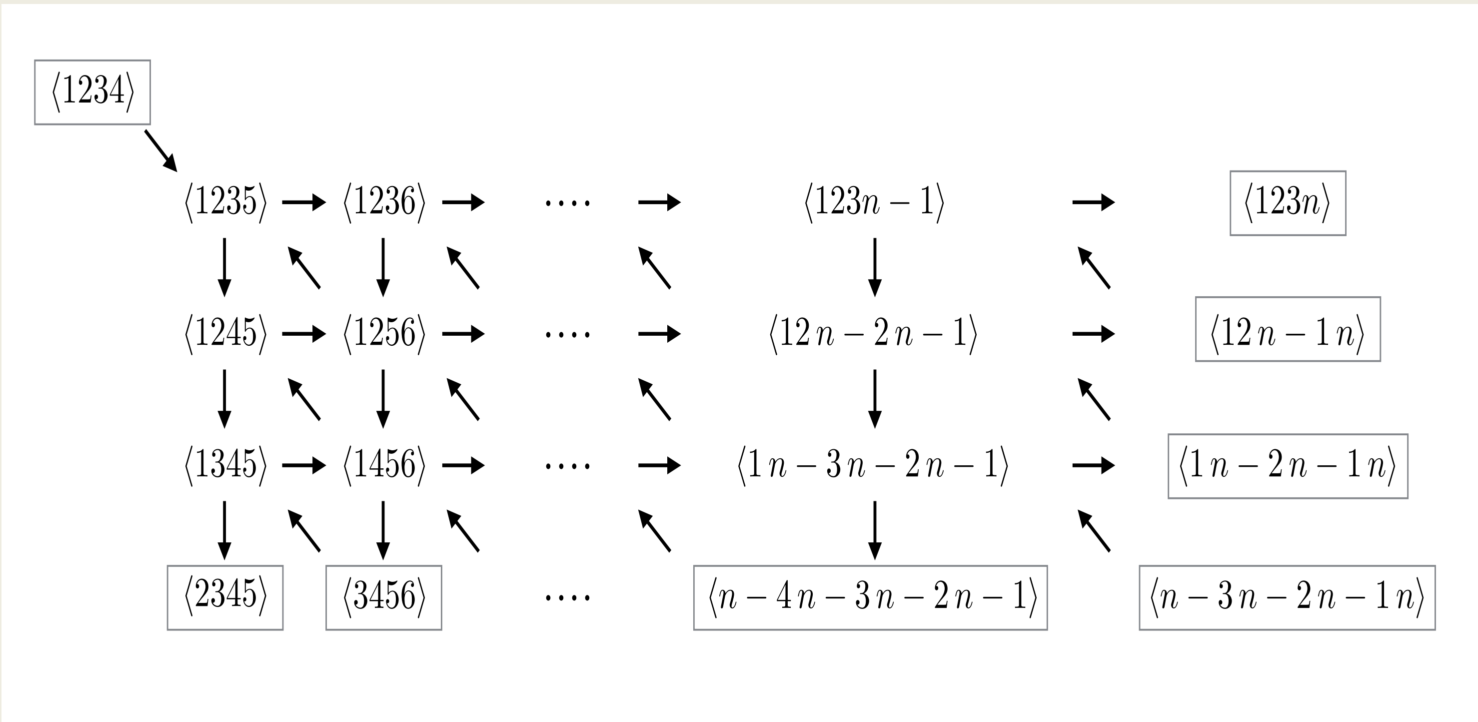
$$a_1,\ a_2,\ a_3 = \frac{1 + a_2}{a_1},\ a_4 = \frac{1 + a_1 + a_2}{a_1 a_2},\ a_5 = \frac{1 + a_1}{a_2}$$

Fomin, Zelevinsky

Grassmannian Cluster Algebra

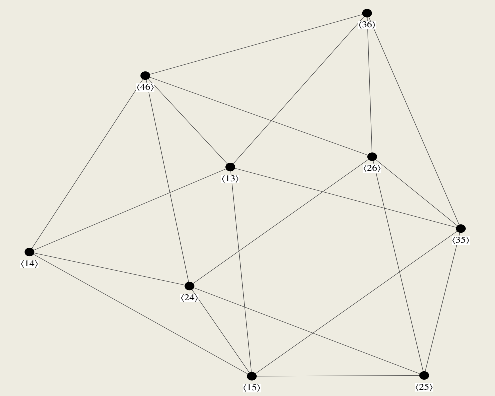
$$Gr(4,n)$$

$3(n-5)$ Quiver + Mutation Rule



Fomin, Zelevinsky; Scott; Gekhtman, Shapiro, Vainshtein

3-loop 6-point Symbol

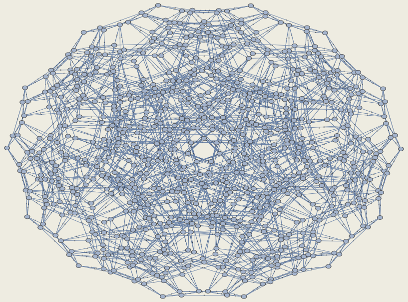


Symbol Alphabet: 9 letters

$$\langle 1235\rangle, \langle 2345\rangle, \langle 1345\rangle, \langle 2456\rangle, \langle 1356\rangle, \langle 1246\rangle, \langle 1245\rangle, \langle 2356\rangle, \langle 1346\rangle$$

```
-12 T[x2, x1, x1, x1, x1, x1] - 12 T[x2, x1, x1, x1, x1, x6] +
 T[x2, x1, x1, x1, x2, x1] + T[x2, x1, x1, x1, x2, x3] + T[x2, x1, x1, x1, x2, x4] +
 T[x2, x1, x1, x1, x2, x6] - 12 T[x2, x1, x1, x1, x6, x1] -
 12 T[x2, x1, x1, x1, x6, x6] + T[x2, x1, x1, x1, x8, x1] +
 T[x2, x1, x1, x1, x8, x5] + T[x2, x1, x1, x1, x8, x6] + T[x2, x1, x1, x1, x8, x9] +
 T[x2, x1, x1, x2, x1, x1] + T[x2, x1, x1, x2, x1, x3] + ··· 5376 ··· +
 T[x8, x9, x9, x8, x9, x6] + T[x8, x9, x9, x8, x9, x9] - 12 T[x8, x9, x9, x9, x5, x5] -
 12 T[x8, x9, x9, x9, x5, x9] + T[x8, x9, x9, x9, x7, x3] + T[x8, x9, x9, x9, x7, x4] +
 T[x8, x9, x9, x9, x7, x5] + T[x8, x9, x9, x9, x7, x9] + T[x8, x9, x9, x9, x8, x1] +
 T[x8, x9, x9, x9, x8, x5] + T[x8, x9, x9, x9, x8, x6] + T[x8, x9, x9, x9, x8, x9] -
 12 T[x8, x9, x9, x9, x9, x5] - 12 T[x8, x9, x9, x9, x9, x9]
```

3-loop 7-point Symbol



Symbol Alphabet: 42 letters

```
-48 T[a11, a11, a11, a11, a11, a21] - 48 T[a11, a11, a11, a11, a11, a22] +
 48 T[a11, a11, a11, a11, a11, a25] - 48 T[a11, a11, a11, a11, a11, a31] +
 48 T[a11, a11, a11, a11, a11, a34] - 48 T[a11, a11, a11, a11, a11, a37] +
 12 T[a11, a11, a11, a11, a14, a21] - 12 T[a11, a11, a11, a11, a14, a24] -
 12 T[a11, a11, a11, a11, a14, a25] - 12 T[a11, a11, a11, a11, a14, a33] -
 12 T[a11, a11, a11, a11, a14, a34] + ... 703 674 ... + 12 T[a17, a66, a66, a66, a37, a37] -
 12 T[a17, a66, a66, a66, a45, a23] - 12 T[a17, a66, a66, a66, a45, a37] -
 12 T[a17, a66, a66, a66, a47, a25] - 12 T[a17, a66, a66, a66, a47, a32] -
 12 T[a17, a66, a66, a66, a55, a21] - 12 T[a17, a66, a66, a66, a55, a32] -
 12 T[a17, a66, a66, a66, a57, a23] - 12 T[a17, a66, a66, a66, a57, a34]
```

Drummond, Papathanasiou, Spradlin

Cluster Adjacency

Two distinct $\mathcal{A}$-coordinates can appear consecutively in a symbol only if there exists a cluster where they both appear.

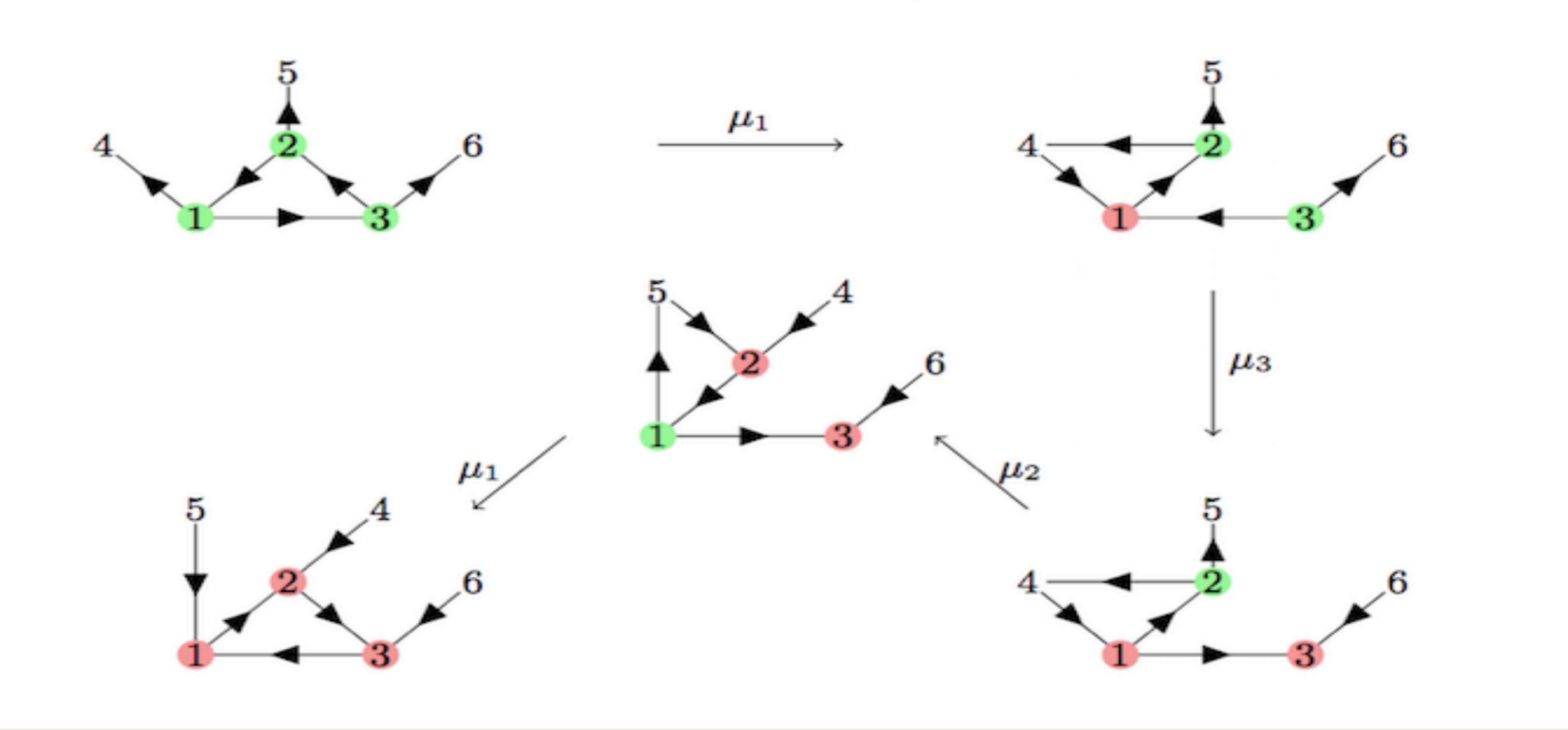
Drummond, Foster, Gurdogan

- Works for all known examples

- It is not known whether there is a physical or mathematical principle which requires this to be true, nor how general it is…

In Progress

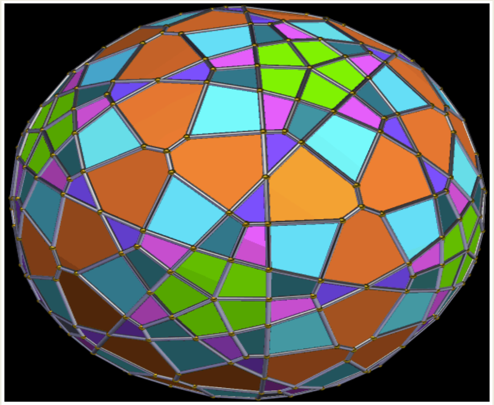
- Write symbol in terms of preferred clusters
- Look at triples, quads, etc



Beyond Cluster Algebra

Arkani-Hamed, Lam, Spradlin

- For n>7: cluster algebra is infinite dimensional and amplitudes involve square roots.
- Natural object: $Gr(4,8)/T^7$
- It is a finite polytope with 360 faces.
- There is is a natural role for square roots to play in the variables associated to this polytope.



Other Interesting Applications of Cluster Algebras to Amplitudes

- Coproduct: only cluster X-coordinates appear which constraints the class of functions to cluster polylogarithm functions [GPSV]
- Special kinematics: [Duhr et al]
- Applications to other theories: [Henn et al]
- Cluster algebras also show up in the integrand: [Arkani-Hamed et al]

Symbol Alphabet and Landau Singularities

Symbol and Landau Singularities

- Symbol captures the information about the singularity structure of the polylogarithm functions.
- Given the integrand, the location of its singularities is described by Landau equations.
- There should be a close connection between solutions of Landau Equations (integrand) and Symbol Alphabet (integral).

Maldacena, Simons-Duffin, Zhiboedov

Abreu, Britto, Duhr, Gardi

Landau Singularities

Landau 1959, ELOP

Landau Equations for a given Feynman integral are a set of kinematic constraints that are necessary for the appearance of a pole or a branch point in the integrated function:

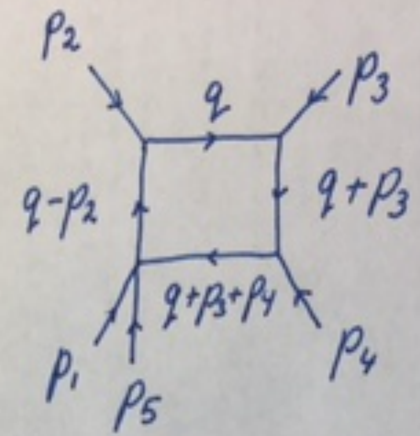
$$\sum_{i \in \text{loop}} \alpha_i q_i^{\mu} = 0 \quad \forall \text{ loops},$$

$$\alpha_i (q_i^2 - m_i^2) = 0 \quad \forall i.$$

Landau Singularities occur for external momenta such that the Landau equations have solutions.

Example

$$\int \frac{d^4 q}{q^2 (q-p_2)^2 (q+p_3)^2 (q+p_3+p_4)^2}$$



$$\begin{cases} (q-p_2)^2 = q^2 = (q+p_3)^2 = (q+p_3+p_4)^2 = 0 \\ \alpha_1 (q-p_2) + \alpha_2 q + \alpha_3 (q+p_3) + \alpha_4 (q+p_3+p_4) = 0 \end{cases}$$

$\Longleftrightarrow \quad (p_2+p_3)^2 (p_3+p_4)^2 = 0$

$$\langle 1234 \rangle \langle 2345 \rangle = 0$$

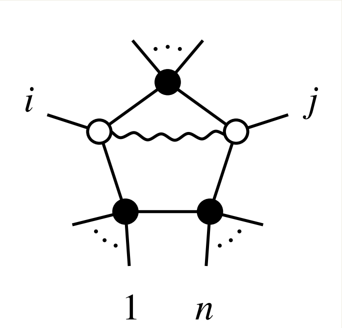
One-loop n-point MHV

Bern, Dixon, Dunbar, Kosower

Arkani-Hamed, Bourjaily, Cachazo, Trnka

Chiral pentagon integrals

$$\frac{\mathcal{A}^{1-\text{loop}}_{\text{MHV}}}{\mathcal{A}^{\text{tree}}_{\text{MHV}}} = \int_{AB} \sum_{1<i<j<n}$$



Landau Singularities

$$\langle \bar{i} \bar{j} \rangle \quad \langle i-1\; i\; j-1 j\rangle$$

Dennen, Spradlin, AV

$$\langle j(j-1\; j+1)(i\; i+1)(k\; k+1)\rangle$$

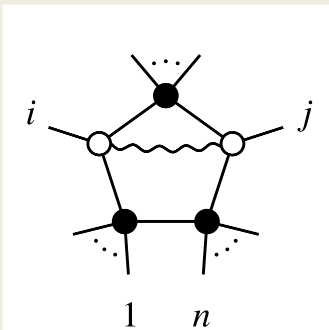
$$\langle (a\,b\,c) \cap (d\,e\,f)\, g\, h\rangle = \langle a\,b\,c\,g\rangle\langle d\,e\,f\,h\rangle - \langle a\,b\,c\,h\rangle\langle d\,e\,f\,g\rangle$$

$$\langle (a\,b\,c) \cap (d\,e\,c)\, g\, h\rangle \equiv \langle c\,(a\,b)(d\,e)(g\,h)\rangle \qquad \bar{a} = (a-1, a, a+1)$$

Symbol: one-loop n-point MHV

Bern, Dixon, Dunbar, Kosower

Arkani-Hamed, Bourjaily, Cachazo, Trnka

$\int_{AB}$ 

$$= \mathrm{Li}_2\left(1-u_{n,i-1,i,j}\right) - \mathrm{Li}_2\left(1-u_{j,n,i,j-1}\right) - \mathrm{Li}_2\left(1-u_{i,j-1,n,i-1}\right) - \mathrm{Li}_2\left(1-u_{i,j-1,n,i-1}\right) + \mathrm{Li}_2\left(1-u_{i,j-1,j,i-1}\right) + \log\left(u_{j,n,i-1,j-1}\right)\log\left(u_{n,i-1,i,j}\right)$$

- One-loop n-point MHV Symbol Alphabet

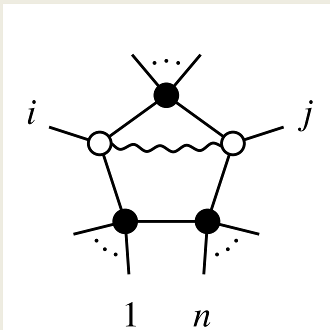
$$\langle i{-}1\, i\, j{-}1\, j\rangle \qquad \langle i\, \bar{j}\rangle$$

Symbol vs LS: one-loop n-point MHV

Bern, Dixon, Dunbar, Kosower

Arkani-Hamed, Bourjaily, Cachazo, Trnka

$\int_{AB}$



$$= \mathrm{Li}_2\left(1-u_{n,i-1,i,j}\right) - \mathrm{Li}_2\left(1-u_{j,n,i,j-1}\right) - \mathrm{Li}_2\left(1-u_{i,j-1,n,i-1}\right) - \mathrm{Li}_2\left(1-u_{i,j-1,n,i-1}\right) + \mathrm{Li}_2\left(1-u_{i,j-1,j,i-1}\right) + \log\left(u_{j,n,i-1,j-1}\right)\log\left(u_{n,i-1,i,j}\right)$$

- One-loop n-point MHV Symbol Alphabet

$$\langle i{-}1\, i\, j{-}1\, j\rangle \qquad \langle i\, \bar{j}\rangle$$

- Landau Singularities

$$\langle i{-}1\, i\, j{-}1\, j\rangle \qquad \langle i\, \bar{j}\rangle \qquad \langle j\,(j{-}1\, j{+}1)(i\, i{+}1)(k\, k{+}1)\rangle$$

Spurious Landau Singularity

Landau Singularities vs Symbol Alphabet

All symbol entries are Landau Singularities.
There are spurious Landau Singularities
which are not in Symbol Alphabet.
[not surprising: numerators, sum of integrals, etc]

Q: How to eliminate spurious Landau Singularities?
A: Use Amplituhedron.

≡ TIME

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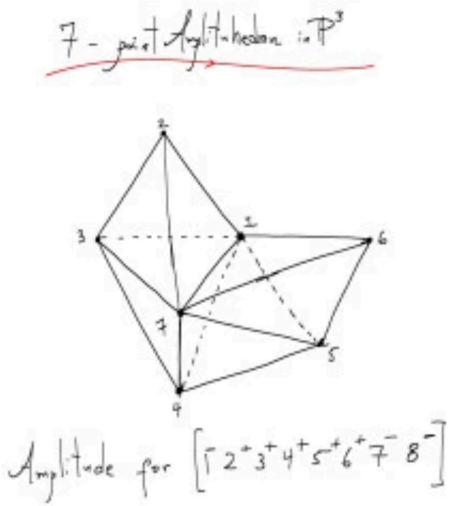
The Amplituhedron

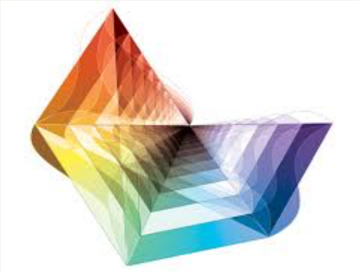
By TIME Staff | Nov. 13, 2013

Physicists at the Institute for Advanced Study in Princeton, N.J., recently found a major shortcut for predicting subatomic-particle collisions. The new method **represents probabilities as pyramid-like structures,** then combines the pyramids into one elegant gemstone-like structure called an amplituhedron, thereby massively simplifying the task of calculating particle interactions. Ultimately the amplituhedron could lead to the long-sought quantum theory of gravity.



NIMA

The integrand of MHV amplitude is a canonical form defined by having logarithmic singularities only on the boundary of the Amplituhedron.



Amplituhedron

Arkani-Hamed, Trnka

The integrand of MHV amplitude is a canonical form defined by having logarithmic singularities only on the boundary of the Amplituhedron.

$$\mathcal{L}_{\alpha}^{(\ell)I} = \sum_{i=1}^{n} D_{\alpha i}^{(\ell)} Z_i^I$$

Loop Momenta

$G_+(2,n)$

D-matrices

$G_+(4,n)$

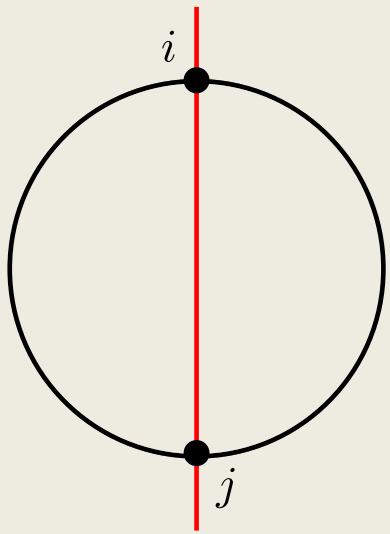
External Momenta

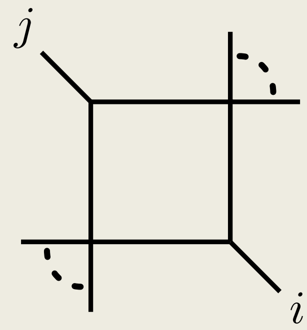
Inside Amplituhedron

$$\langle \mathcal{L}^{(\ell)}\, i\, j \rangle > 0 \text{ for } i < j \text{ and all } \ell$$

$$\langle \mathcal{L}^{(\ell_1)}\, \mathcal{L}^{(\ell_2)} \rangle > 0 \text{ for all } \ell_1, \ell_2.$$

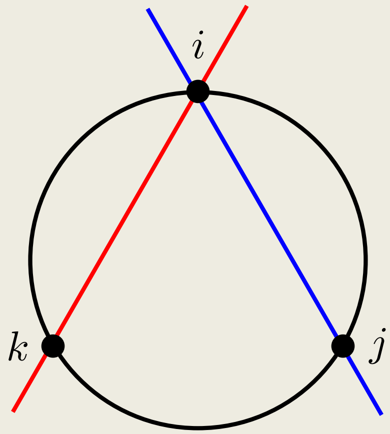
One-loop MHV Amplitudes

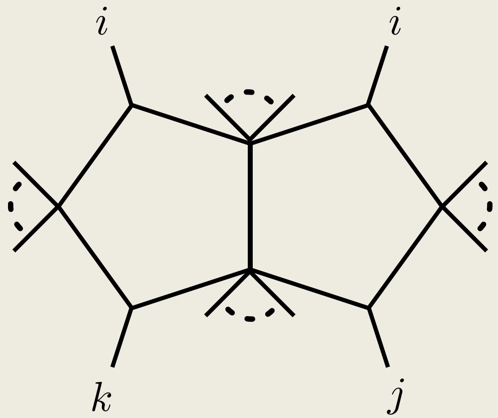




$$\langle i\!-\!1\; i\; j\!-\!1\; j \rangle \qquad \langle i\; \bar{j} \rangle$$

Two-Loop MHV Amplitudes





$$\langle a\,\bar{b}\rangle$$

$$\langle a\,b\,c\,c{+}1\rangle$$

$$\langle a\,a{+}1\,\bar{b}\cap\bar{c}\rangle$$

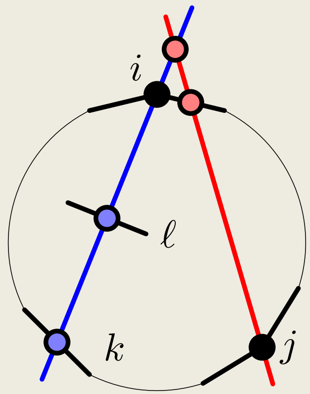
$$\langle a\,(a{-}1\,a{+}1)(b\,b{+}1)(c\,c{+}1)\rangle$$

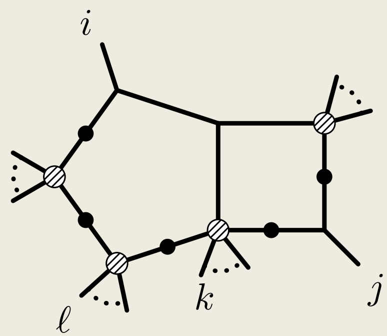
Two-loop NMHV and Three-loop MHV

Prlina, Spradlin, Stankowicz, Stanojevic

- Two-loop NMHV: allow for sign flips

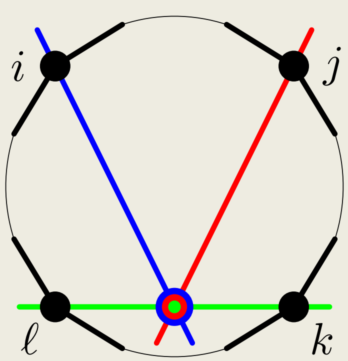
Arkani-Hamed, Thomas, Trnka

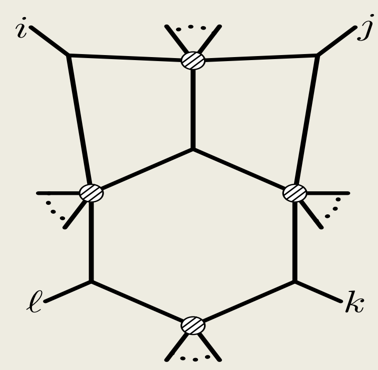




$$\langle a\, a+1\, b\, b+1\rangle$$
$$\langle a\, b\, c\, c+1\rangle$$
$$\langle a\,(b,b+1)\,(c,c+1)\,(d,d+1)\rangle$$
$$\langle a\,(a-1,a+1)\,(b,b+1)\,(c,c+1)\rangle$$
$$\langle a\,(a-1,d)\,(b,b+1)\,(c,c+1)\rangle$$
$$\langle \bar{b}\cap(d,c,c+1)\,(a,a+1)\rangle$$
$$\langle \bar{b}\cap(a\,a+1\,b)\,,\ \bar{d}\cap(d,c,c+1)\rangle$$
$$\langle b\,(b-1,b+1)\,(c,c+1)\,(a,d)\rangle,$$

- Three-loop MHV

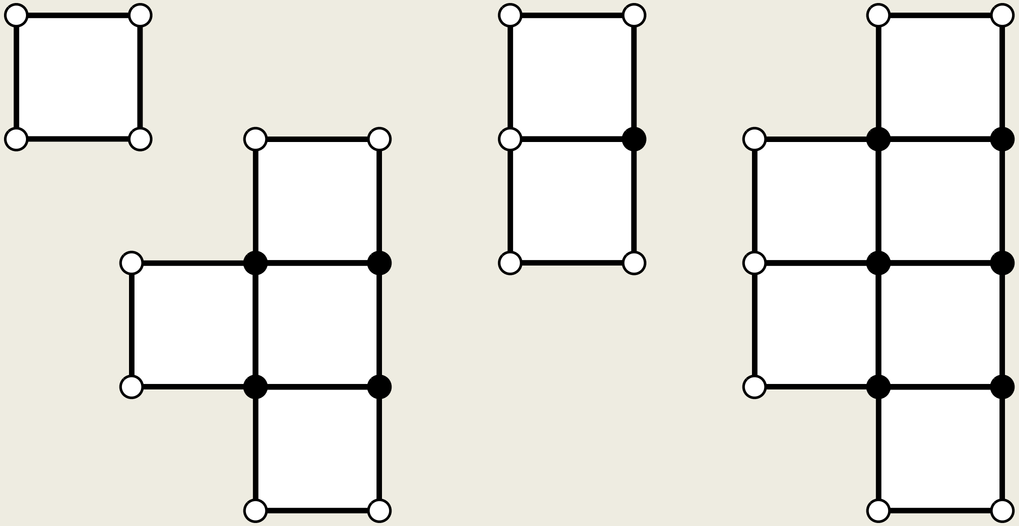




Ziggurat Graphs

Prlina, Spradlin, Stanojevic

- Landau singularities of any n-particle amplitude in any massless, planar theory are a subset of those of a special type of ``ziggurat graph.''
- For n=6 consistent with symbol alphabet.



Conclusion

- N=4 Yang-Mills Amplitudes Bootstrap Program
- Symbol Alphabet and Cluster Algebra
- Symbol Alphabet and Landau Singularities
- Many Open Questions Remain

Thank you!