Hadronic Matrix Elements (non-lattice)

CP^3 Origins
Cosmology & Particle Physics

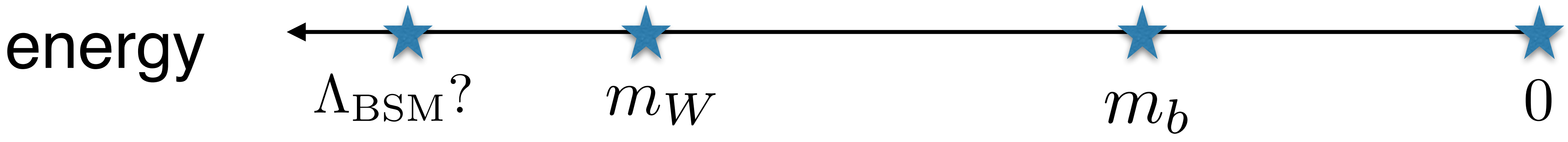
Roman Zwicky
Edinburgh University

Higgs
Centre for Theoretical Physics

The Spice of Flavour 27-28th of Nov 2018

Flavour Physics Basics

- **Flavour physics:** = successful EFT integrating out dof a la Wilson



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amplitude

$$\mathcal{A} = \langle XYZ|H_{\text{eff}}|{\color{red}B}\rangle = \sum_i C_i(m_b)\langle XYZ|\ \underbrace{O_i(m_b)}_{\bar q_1\Gamma_1 q_2 {\color{red}\bar b}\Gamma_2 q_3}\ |{\color{red}B}\rangle$$

perturb. calculable
Wilson coefficient
UV physics (BSM?)

M-element
IR physics (**non-perturbative**)

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- **ΔF=2:** meson mixing $$\langle\bar B|H^{\Delta F=2}|B\rangle \sim \sum_i C_i(\mu)\langle\bar B|O_i(\mu)|B\rangle$$
- **ΔF=1:** next slide……

ΔF=1: classified wrt: 1) final states & 2) tree vs FCNC(rare)

main input

- **leptonic:** no hadron Final State

  not covered

$B^+ \to \ell^+ \nu$

$B_s \to \mu\mu$

FCNC

decay constants f_B

5-10% accuracy

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$B^+ \to \pi_0 \ell^+ \nu$

$B \to K\mu\mu$ FCNC

form factor (FF)

FF & long-distance ME

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- **non-leptonic:** +hadron FS

  not covered

$B \to \pi\pi$ FF & factorisation QCD-fac. BBNS LCSR Khodjamirian,

$D \to \pi\pi$ symmetries, LCSR?

$K \to \pi\pi$ lattice (Sachrajda's talk)

Outline

1) **Form Factor** (local matrix elements)

2) **Long distance** (non-local) matrix elements

3) **Meson mixing** matrix elements

4) **QED-corrections**

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1) **Form Factor** (local matrix elements)

2) **Long distance** (non-local) matrix elements

3) **Meson mixing** matrix elements

4) **QED-corrections**

1-3) can, do and will all **benefit** from **progress in pQCD** technology

r*eduction* to *master integrals*: (Fire,Reduze,Kira,…)
solving master integrals: (differential equation)

1. Form Factors

prototype of hadronic matrix element

Semi-leptonic case study B→π Form Factor

$$\frac{d\Gamma[B \to \pi\ell\nu]}{dq^2} \sim |\,{\color{red}V_{ub}}\, f_+(q^2)\,|^2$$

$$\langle \pi | q\gamma_\mu b | \bar{B} \rangle = (p_B + p_\pi)_\mu f_+(q^2) + \ldots$$

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  - light-cone distribution amplitudes (LCDA)
  - dispersion relations

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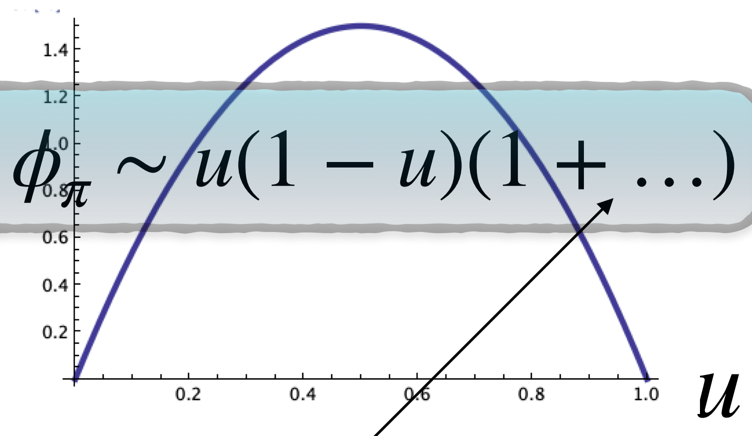
$$\langle \pi | q(x) \gamma_\mu \gamma_5 b(0) | 0 \rangle = f_\pi (p_\pi)_\mu \int_0^1 du e^{iupx} \phi_\pi(u)$$



$$\phi_\pi \sim u(1-u)(1+\ldots)$$

LCDA well understood (ϕ_π ⇔ local matrix elements) -> sum rules & lattice

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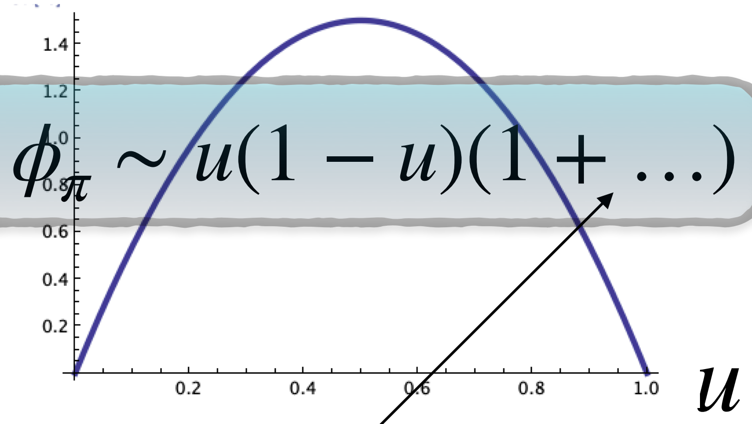

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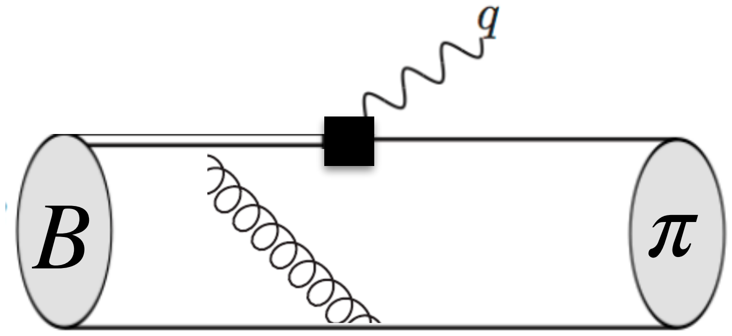
- Both hadrons DAs: Brodsky, Henley Szczepaniak'90

hard mechanism

$$f_+(0)\sim\int_0^1\frac{du\phi_\pi(u)}{(1-u)^2}+\dots\sim$$ **IR-divergent**



LC sum rules: one DA & dispersion relation

- Physics of **IR divergence**: **soft (Feynman) mechanism missing** can be captured by dispersion relation (as in sum rules)

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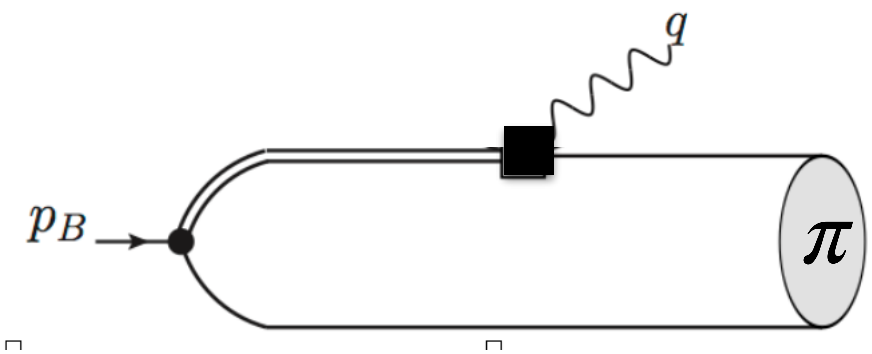
LC-OPE:

$$\langle 0 | T | J_B(x) H^{weak}(0) | \pi \rangle$$



Bagan, Ball, Braun'97	NLO twist 2
Khodjamirian, Ruckl, Weinzierl,..'97	
Ball RZ'01'04	NLO twist 3
Khodjamirian, Offen, Melic… '08	
Bharucha'12	partial NNLO twist 2 q2=0
Rusov'17	LO higher twist 5.6

$$f_+(0) = 0.258(31)$$

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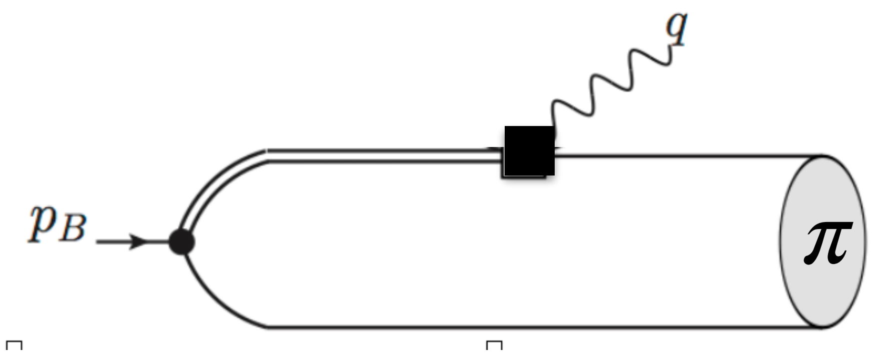

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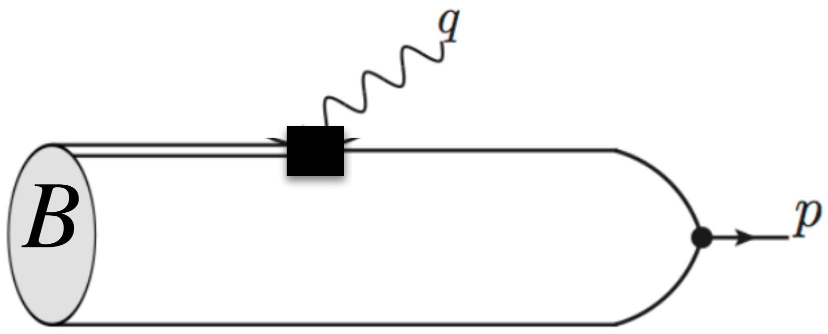

Khodjamirian, Mannel, Offen’05 LO twist 2,3
de Fazio, Hurth, Feldman’05
Shen, Wang’15 NLO twist 2,3

$$f_+(0) \sim \int \frac{d\xi \phi_B(\xi)}{\xi} = \frac{m_B}{\lambda_B} + \ldots$$

λ_B need to discuss …..

Determination of λ_B

- **no** direct **first principle** determination of λ_B

  (not related to local matrix element)

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LHS as input (theory,exp)

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$\lambda_B[MeV]$	mode	order	match	group
460(160)	$B \to \pi\ell\nu$	LO	NLO LCSR Ball,RZ'04	Khodjamirian, Mannel, Offen'05
354(40)	$B \to \pi\ell\nu$	NLO	NLO LCSR Khodj'08	Wang,Shen 15
600	$B \to \gamma\ell\nu$	LO	LO LCSR Ball Kou'03	Descotes-G-Sachrajda'02

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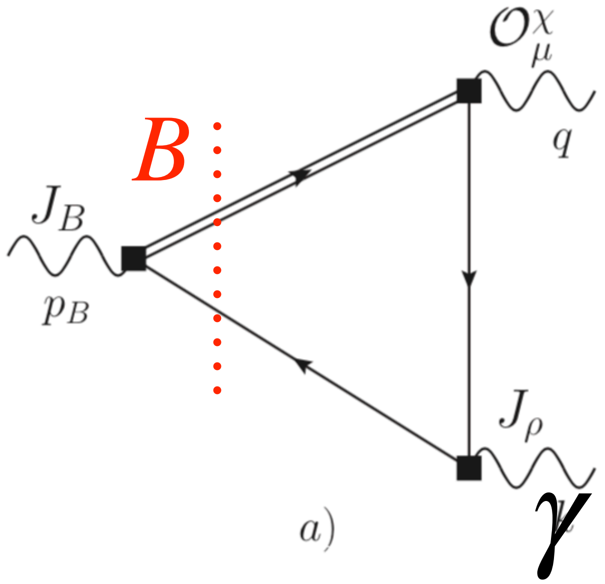
- matching $B \to \gamma \ell \nu$ to BelleII results is seen as "Königsweg" for λB

  Korchemsky,Pirjol,Yan,'99, Descotes-G. Sachrajda'02, Rohrwild,Beneke'11, Braun,Kohodjamirian'12, Wang'16, Braun, Beneke, Ji, Wei'18

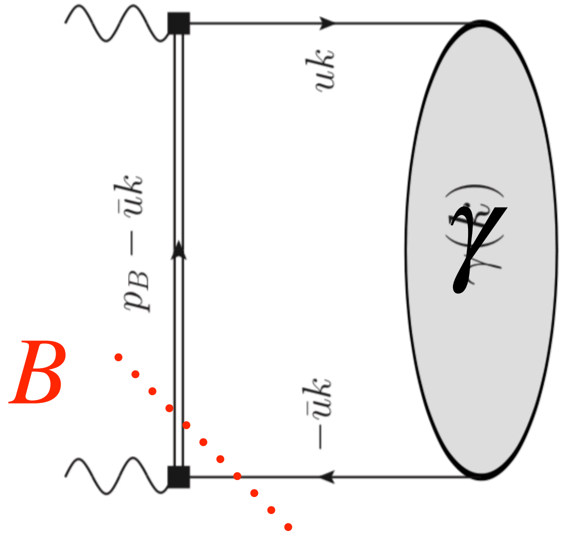
Matching to LCSR might be important as well since …

- Amusing fact: photon is not a point-particle
  mixes with ρ,ω - captured by photon DA (effect is large)



$$\sim \frac{1}{m_b^{1/2}}$$

$+$



$$\sim \frac{1}{m_b^{3/2}}$$

photon-DA correction is of similar size at LO despite 1/m$_b$-suppression!

Investigation at NLO Janowksi, Pullin, RZ in prep (no numbers yet)

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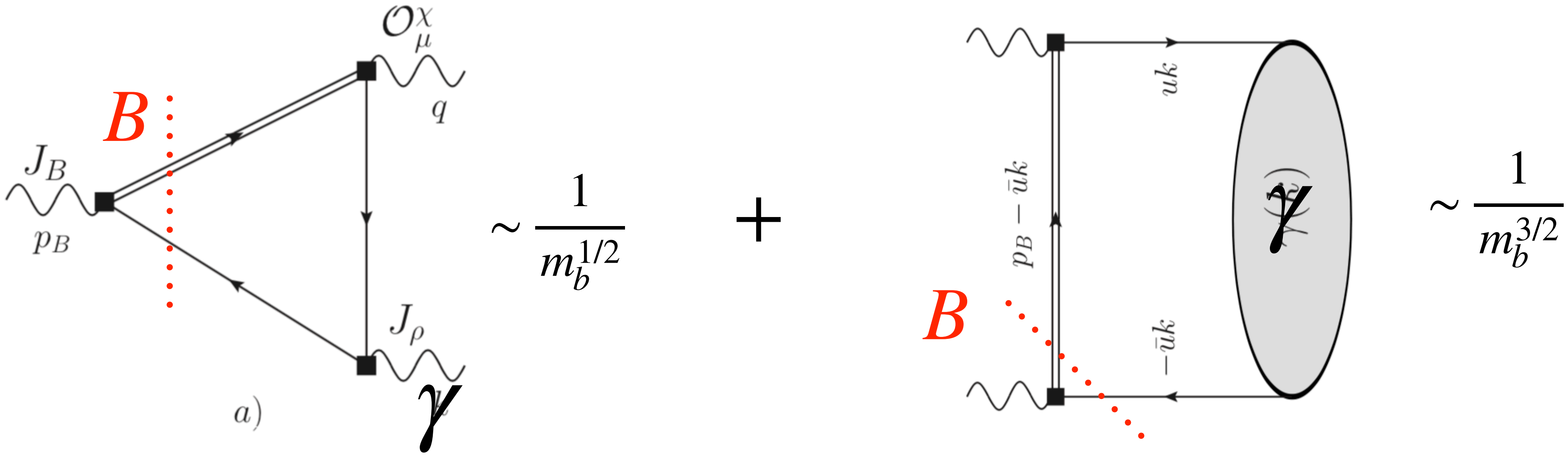

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- Where is this large contribution hidden in B-meson DA approach?
  Wang, Shen'18 add the LCSR contribution (hybrid approach)
  Beneke, Braun, JI, Wei'18 questioned whether there's double counting

- not a closed story - progress ahead theory & experiment (BelleII)

Form Factors summary

- **LCSR light-DA**: relatively mature subject (**$\Delta_{\text{uncert.}}$ ~ 10-15%**)
  Uncertainty grow: D-decays (less light cone, smaller kinematic range)
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  triggered many reevaluations @LO Wang, Shen'18 Lu,Shen, Wang, Wei'18, Gubernari, Kokulu, Dyk'18
  (**$\Delta_{\text{uncert.}} \sim$ 25%**)

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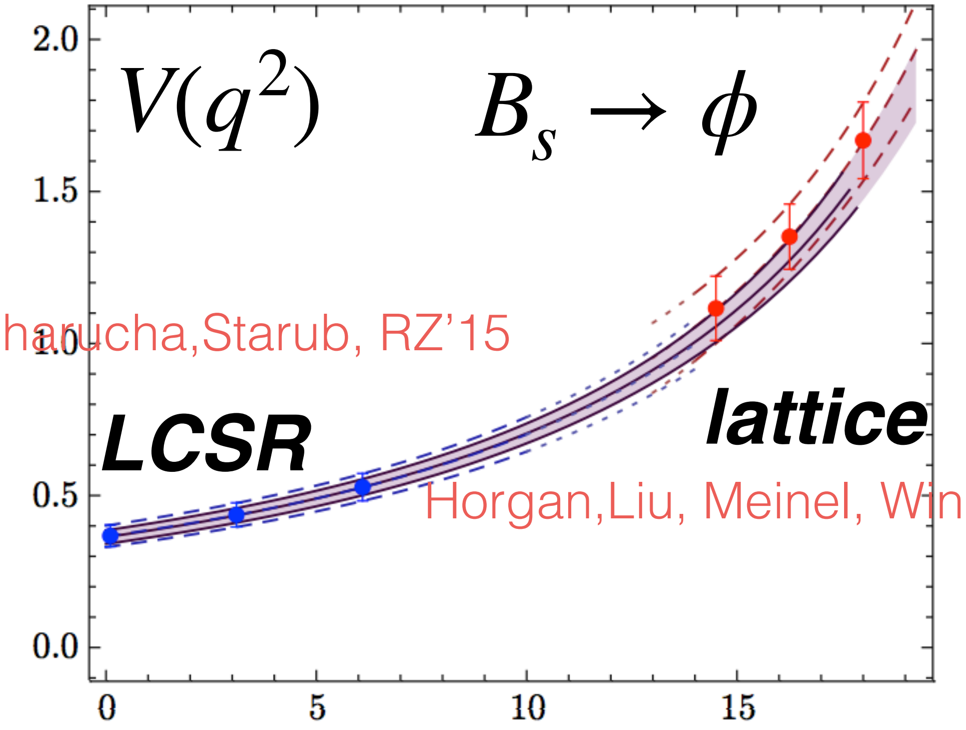
- Computable in kinematic region complementary to lattice QCD
  Generally "good" agreement in interpolation

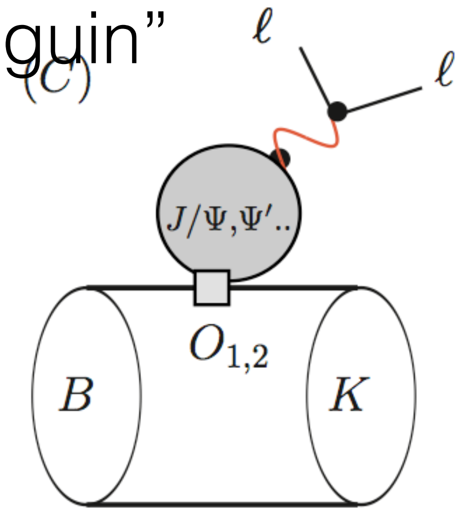


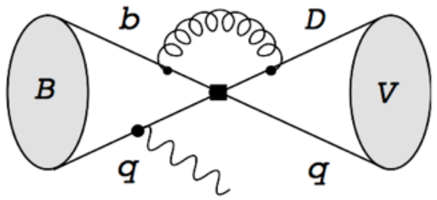
Bharucha,Starub, RZ'15

Horgan,Liu, Meinel, Wingate'13

2. Long distance matrix element

- Typically due to 4-quark operators: aka “charming penguin”
- methods: LCSR & QCD factorisation
- Uncertainties large 50-100%
  can be improved with progress in pQCD
  (even so need crosschecks elsewhere)





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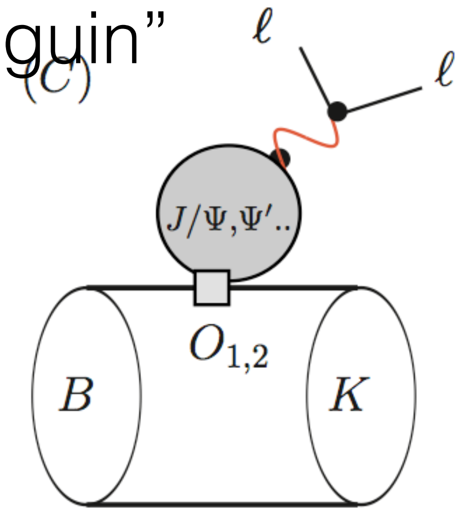

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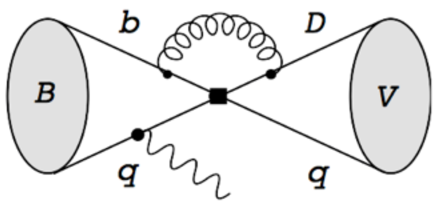

- Present strategy to measure long distance contaminating **R**ight **H**anded **C**urrents [$C'_{7,8,9,10}$]

Gratrex RZ’ 1804.09006 JHEP 1808 (2018) 178
1807.01643 Moriond proceedings

Parity doubling as a tool for RHC-searches

$$H_{eff}^{b\to s\gamma} = C\bar{s}_L \Gamma b O_r + \boxed{C'\bar{s}_R} \Gamma b O_r$$

Right-handed current (RHC)

$$\left.\frac{C'}{C}\right|_{SM} = \frac{m_s}{m_b}, tiny \qquad \Rightarrow \qquad \delta C' = \text{BSM-RHC visible?}$$

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- Long distance matrix-element can perturb this structure ….

The trouble with RHC - hadronic m-elements

Form Factor (SD)*

$$A_L^{B_s \to \phi \gamma_L} = \mathcal{N}(1 +$$

$$A_R^{B_s \to \phi \gamma_R} = \mathcal{N}(\frac{m_s}{m_b} + \delta \hat{C}' +$$

---

* $\gamma_5 \sigma_{\mu\nu} = \frac{i}{2} \epsilon_{\mu\nu\mu'\mu'} \sigma^{\mu'\nu'}$ $\Rightarrow T_1(0) = T_2(0)$ [exact relation]

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Long distance (LD)
(4-quark operators)

$$A_L^{B_s\to\phi\gamma_L} = \mathcal{N}(1 + \epsilon_L(C, C')\,)$$

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Problem: distinguishing RHCs from LD-terms induced by large C$_{\text{Wilson}}$

(assuming we can measure A$_R$)

⇒ non-perturbative **QCD** (LD) can **blur RHC** and **LHC** in amplitudes

....

---

* $\gamma_5\sigma_{\mu\nu} = \frac{i}{2}\epsilon_{\mu\nu\mu'\mu'}\sigma^{\mu'\nu'}$ ⇒ $T_1(0) = T_2(0)$ [exact relation]

Parity-doubling as proposed solution

- **Chiral** symmetry **restoration** limit:

$$m_q, \langle \bar{q} q \rangle, .. \rightarrow 0$$

restored flavour-symmetry

$$SU(N_f)_V \times \mathbf{SU(N_f)_A} \times "\mathbf{U(1)_A}"$$

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* Afonin'07 (history) - tested on the lattice hight T Glozman, Lang, Cossu, KEK-collab. '14

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We **propose degeneracies**
in full **amplitudes :**

$$A^{B_s \to \phi\gamma}(C, C') = A^{B_s \to f_1(1420)\gamma}(-C, C')$$

---

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Proof of amplitude relation (symmetry limit)

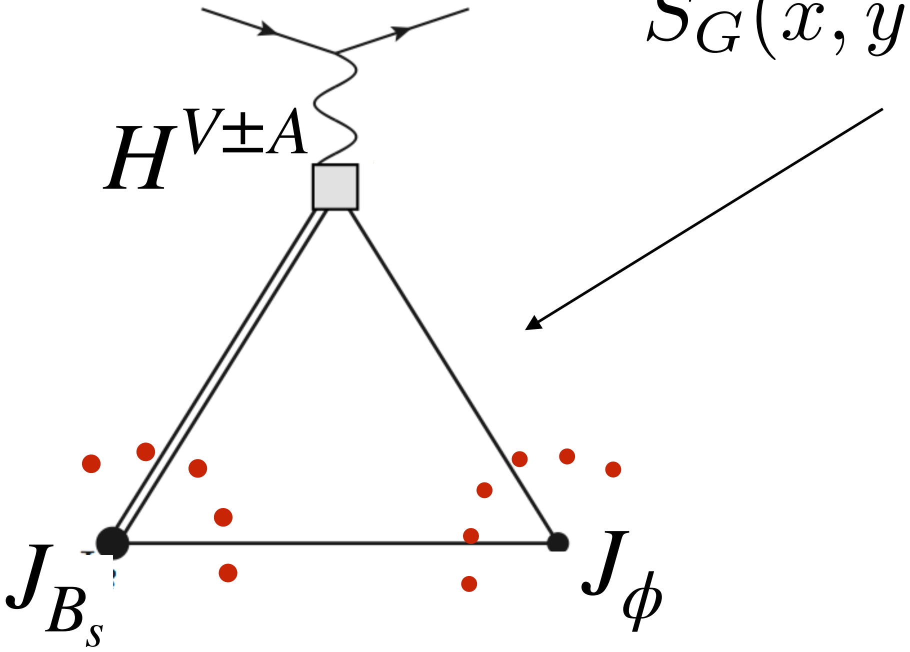
briefly show
(no time to discuss

- Any $B_s \to \phi\gamma$ matrix elements $\subset$ 3-pt function:

$$\langle T J_{B_s}(x) J_\phi(y) H^{V\pm A}(0)\rangle =$$

$$\int DG_\mu \det(D+im) e^{iS(G)} \times$$

"as in lattice QCD (here Mink. space)"



$$S_G(x,y) = \langle x|\frac{1}{\not{D}}|y\rangle$$

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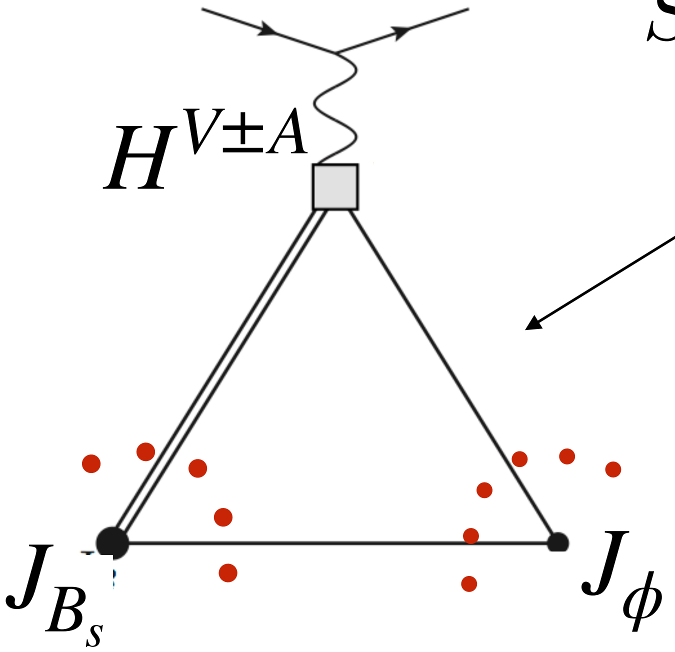

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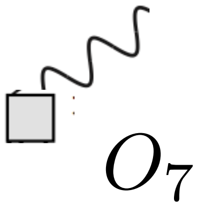
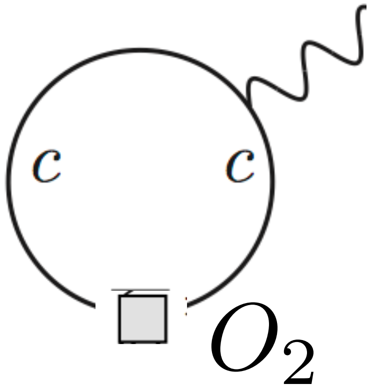
$H^{V\pm A}$ either be

local operator (=SD=FF)

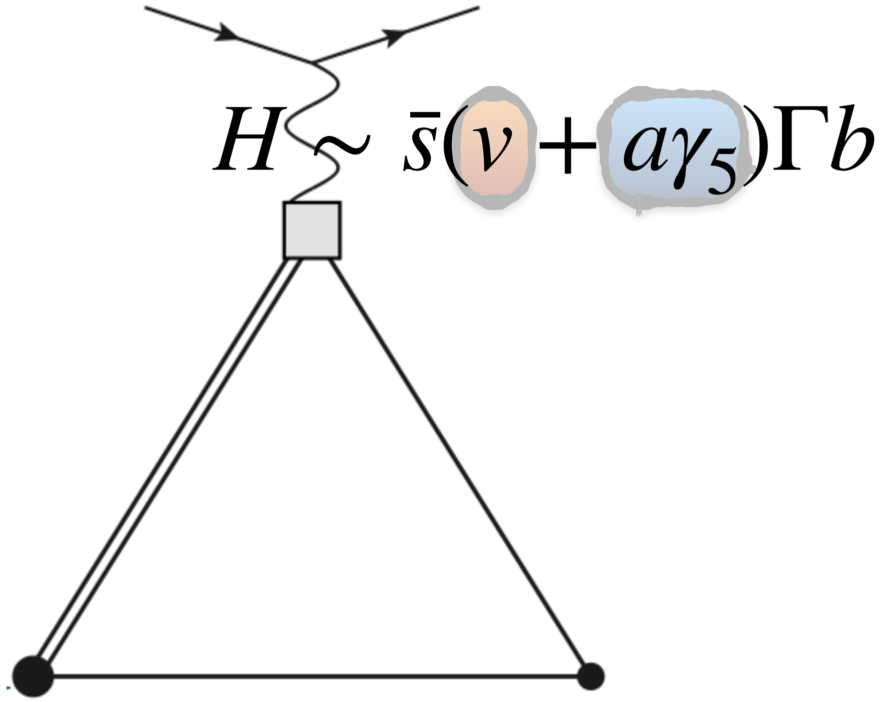
charm loop





An exact equality in the symmetry limit

$$B_s \to \phi\gamma$$



$$H \sim \bar{s}(v + a\gamma_5)\Gamma b$$

$$J_{B_s}$$

$$\phi_\mu = \bar{s}\gamma_\mu[\gamma_5^2]s$$

$$\gamma_5 S_G^{(q)} = -\, S_G^{(q)}\gamma_5$$

$$\downarrow$$

$$=$$

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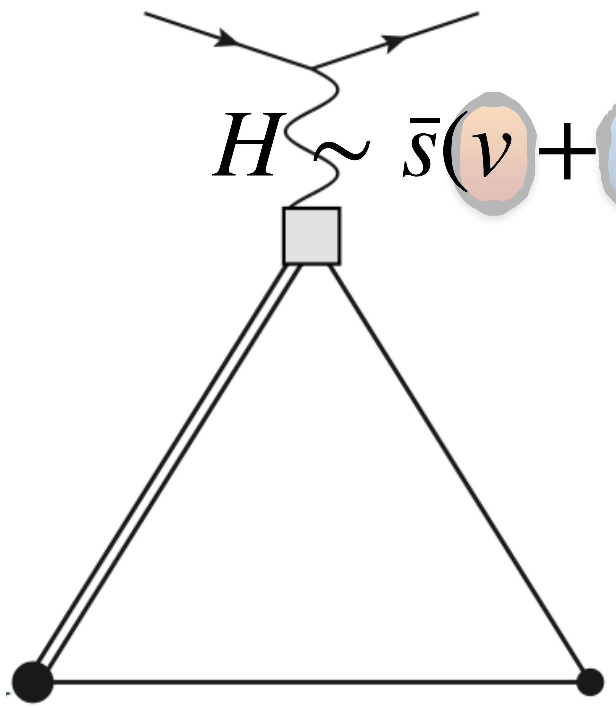

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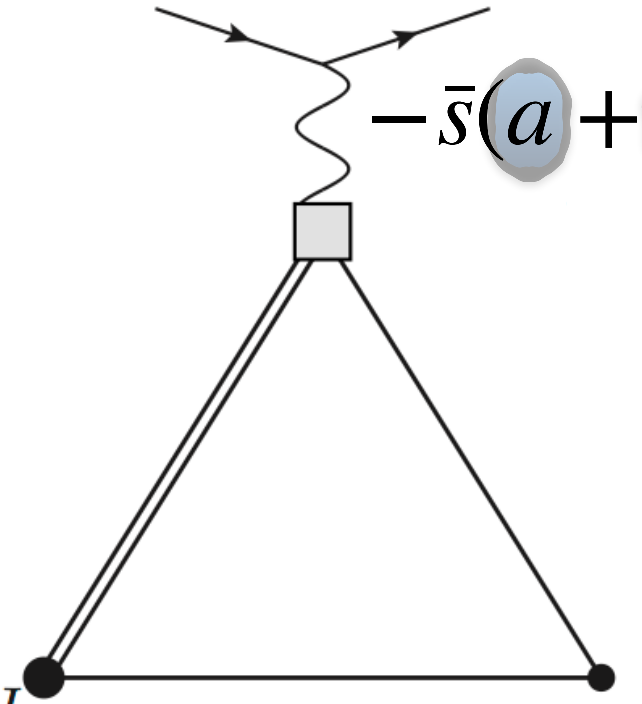
$\downarrow$

$$=$$

$B_s \to f_1(1420)\gamma$



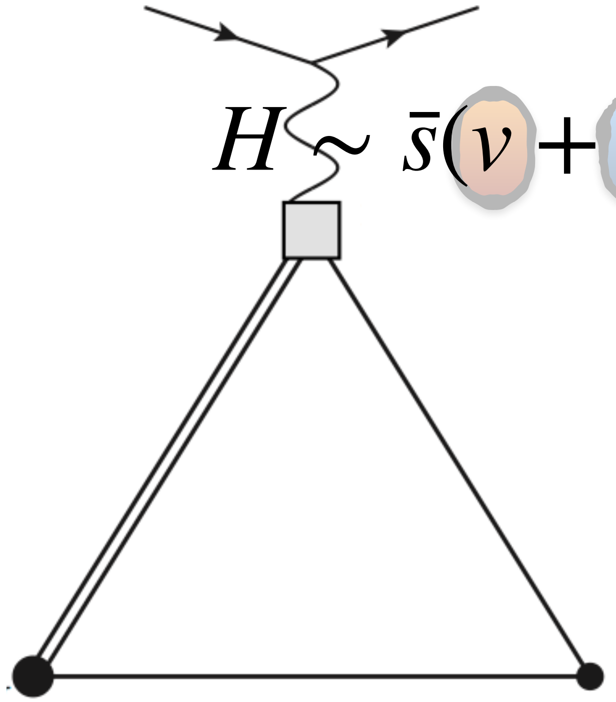
$-\bar{s}(a + v\gamma_5)\Gamma b$

$J_{B_s}^{J_B}$

$(f_1)_\mu = \bar{s}\gamma_\mu\gamma_5 s$

An exact equality in the symmetry limit

$B_s \to \phi\gamma$



$H \sim \bar{s}(v + a\gamma_5)\Gamma b$

J_{B_s}

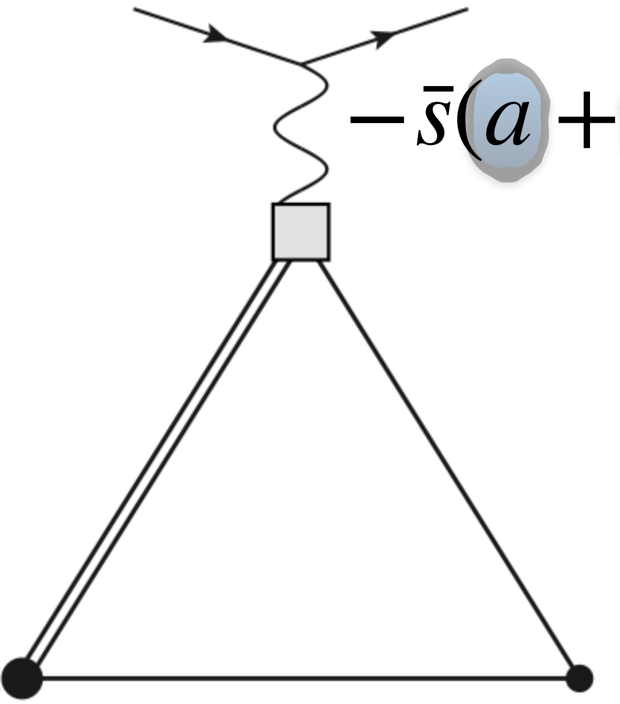
$\phi_\mu \doteq \bar{s}\gamma_\mu[\gamma_5^2]s$

$$\gamma_5 S_G^{(q)} = - S_G^{(q)}\gamma_5$$

$$\downarrow$$

$$=$$

$B_s \to f_1(1420)\gamma$



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$J_{B_s}^{J_B}$

$(f_1)_\mu = \bar{s}\gamma_\mu\gamma_5 s$

Since: $C(C') \leftrightarrow (v, a) = (1, \mp 1)$

$$\Rightarrow \quad \boxed{A^{B_s \to \phi\gamma}(C, C') = A^{B_s \to f_1(1420)\gamma}(-C, C')}$$

(4) In practice beyond the symmetry limit

- Right-handed amplitude (crucial sign)

$$A_R^{B_s \to \phi[f_1]\gamma_R} = -\mathcal{N}_{\phi[f_1]}\left(\frac{m_s}{m_b} + \delta\hat{C}' \pm \epsilon_R^{\phi[f_1]}(C)\right)$$

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What to do with it?

… there are observables linear in A_R!

[1] Normalisation $\mathcal{N}$ drops in asymmetries e.g. time-dependent rate*

$$H_{B_s\to\phi\gamma} + H_{B_s\to f_1\gamma} = -2\mathrm{Re}[\epsilon_R^{\phi} + \epsilon_R^{f_1}] = -2\mathrm{Re}[\epsilon_R^{\phi}](1+\mathbb{R}_{\phi f_1})$$

Experiment

⇒ sum of LD contribution RH-amplitude measurable

---

* $H_{B_s\to\phi\gamma} = -0.98(50)(20)$ @LCHb'16 - $H_{B_s\to\phi\gamma} = 0.047(25)$ Muheim,Xie,RZ PLB08

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[2] Compute ratio (improved situation) then “know everything”

Theory

$$\mathbb{R}_{\phi f_1} = \frac{\mathrm{Re}[\epsilon_R^{f_1}]}{\mathrm{Re}[\epsilon_R^{\phi}]} \simeq 1.3(1) = 1 + O(m_q, \langle \bar{q}q\rangle, \ldots)$$

tentative

---

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tentative

⇒ crucial error (0.1) and not deviation from unity (0.3)

work
in progress

---

* $H_{B_s\to\phi\gamma} = -0.98(50)(20)$ @LCHb'16 - $H_{B_s\to\phi\gamma} = 0.047(25)$ Muheim,Xie,RZ PLB08

Long distance summary

- **Data driven program:** assess “1/2” LD contributions
  Experiment: measure opposite parity channel
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$$\begin{array}{cccc|cc} C_7' & C_8' & - & - & B \to V(A)\gamma & \Lambda_b \to \Lambda(\tilde{\Lambda})\gamma \\ C_7' & C_8' & C_9' & C_{10}' & B \to V(A)\ell\ell & \Lambda_b \to \Lambda(\tilde{\Lambda})\ell\ell \end{array}$$

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- Assessing RH-Long-distance contribution is important:

a) 1/2 LD-input into P_5' prediction [possibility to crosscheck]

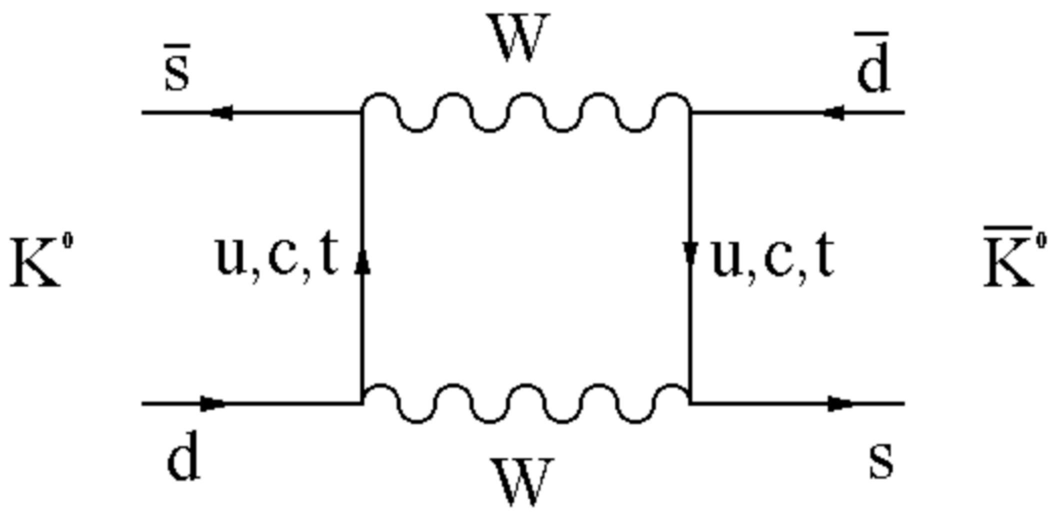
b) argued to be large in other context [we can test]

3. Neutral meson mixing

progress due to new master integrals
and further efforts …

Meson mixing matrix elements

- Physics: K_0,$D_{0,}$$B_q$,$B_s$ mix antiparticles



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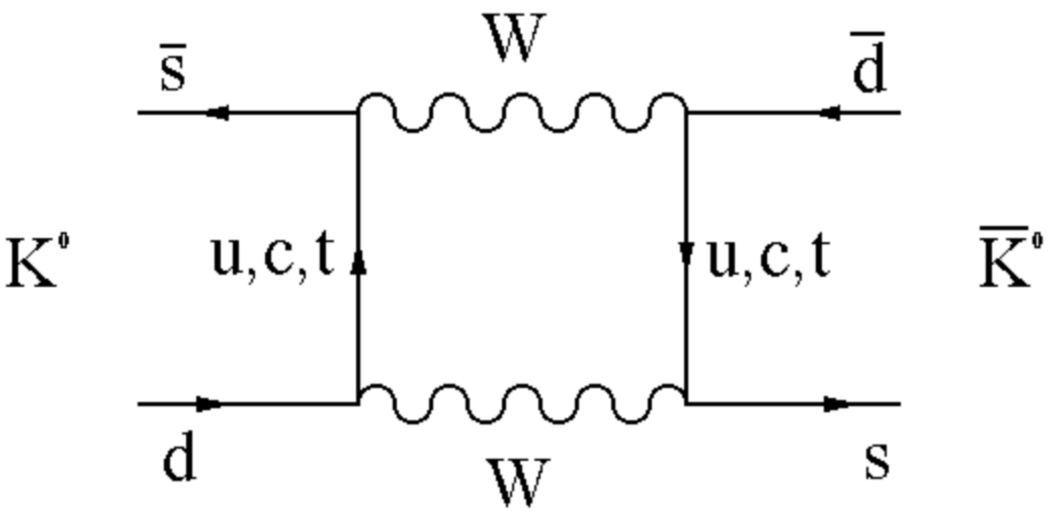
- General task is to compute…



$$\langle \bar{B} | Q_i | B \rangle = f_B^2 m_B^2 f(N_c) B_{Q_i}$$

….for a set of QCD (or BSM) operators

"bag"-parameter

B_Q~1+…

Hatree-Fock app. (VFH)

$$Q_1 = \bar{b}_i \gamma_\mu (1-\gamma^5) q_i \; \bar{b}_j \gamma^\mu (1-\gamma^5) q_j,$$

$$Q_2 = \bar{b}_i (1-\gamma^5) q_i \; \bar{b}_j (1-\gamma^5) q_j, \qquad Q_3 = \bar{b}_i (1-\gamma^5) q_j \; \bar{b}_j (1-\gamma^5) q_i,$$

$$Q_4 = \bar{b}_i (1-\gamma^5) q_i \; \bar{b}_j (1+\gamma^5) q_j, \qquad Q_5 = \bar{b}_i (1-\gamma^5) q_j \; \bar{b}_j (1+\gamma^5) q_i.$$

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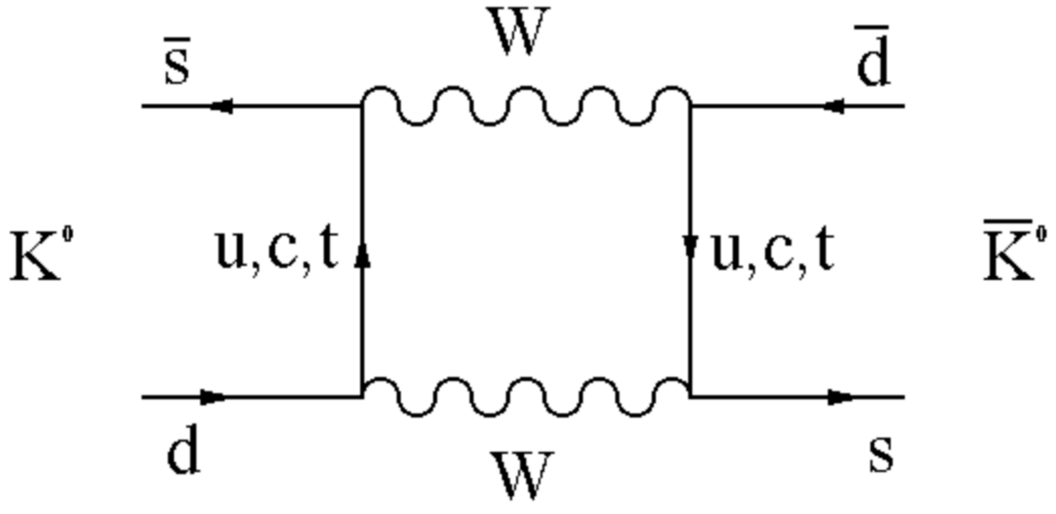

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\end{aligned}$$

- Usually thought to be a lattice affair by now but thanks to **progress** in **pQCD technology** sum rules can contribute too!

Mixing matrix elements from QCD sum rules

new developments

Grozin, Lee'08 Master integrals
Grozin,Klein, Mannel, Pivovarov'16
Kirk, Lenz, Rauh'17

- 1) matching computation to HQET
- 2) matrix elements evaluated
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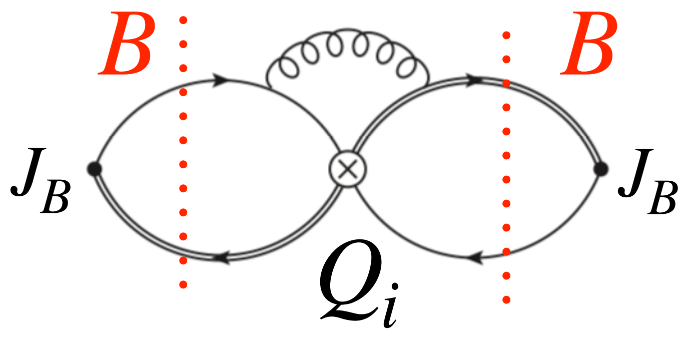

- condensates small
- perturbation theory dominant
- dominant error from matching (improvable)

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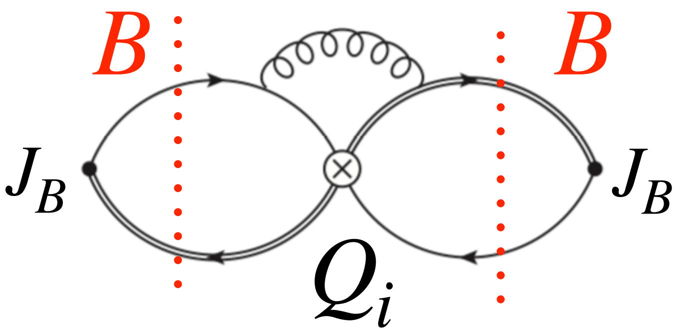

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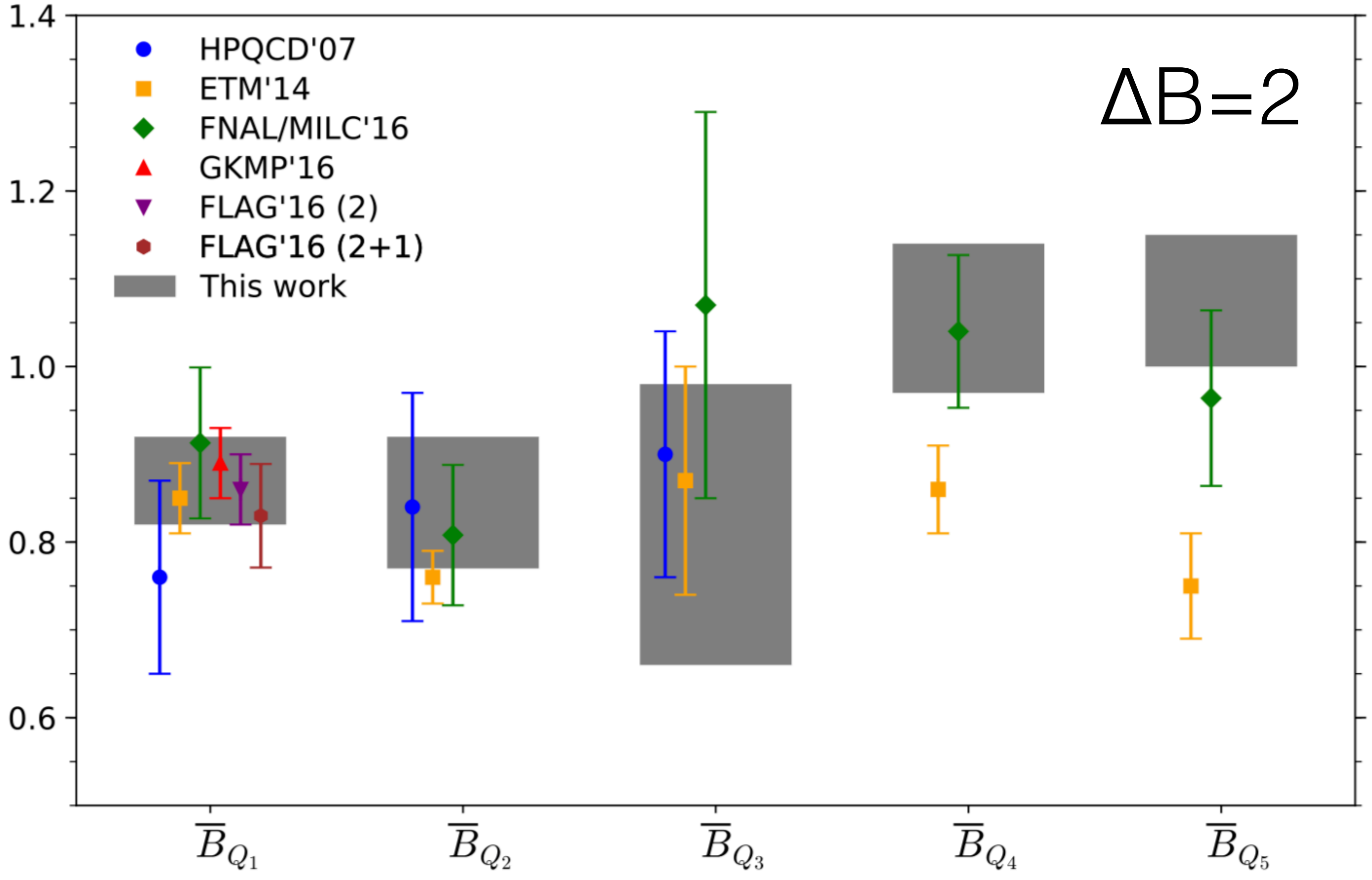
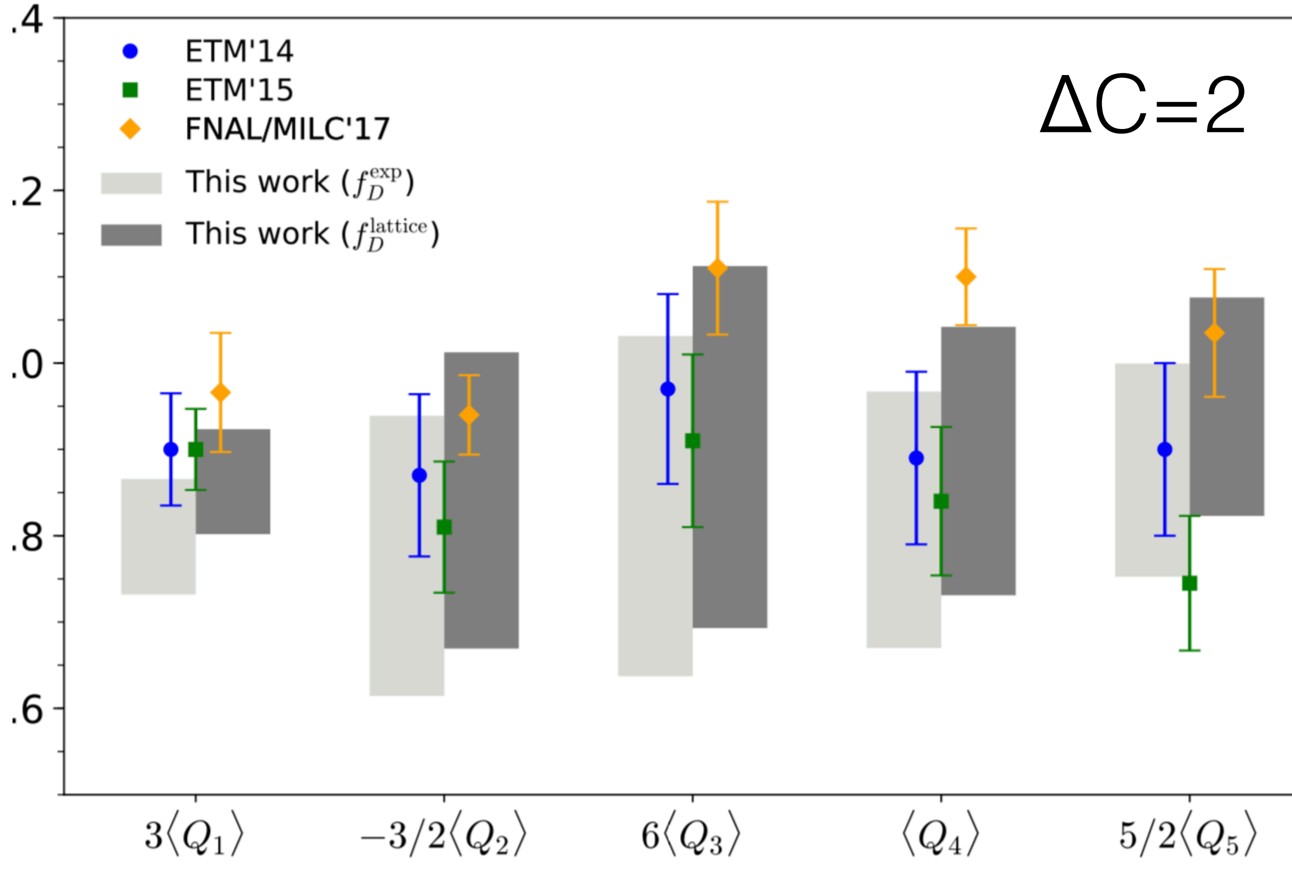


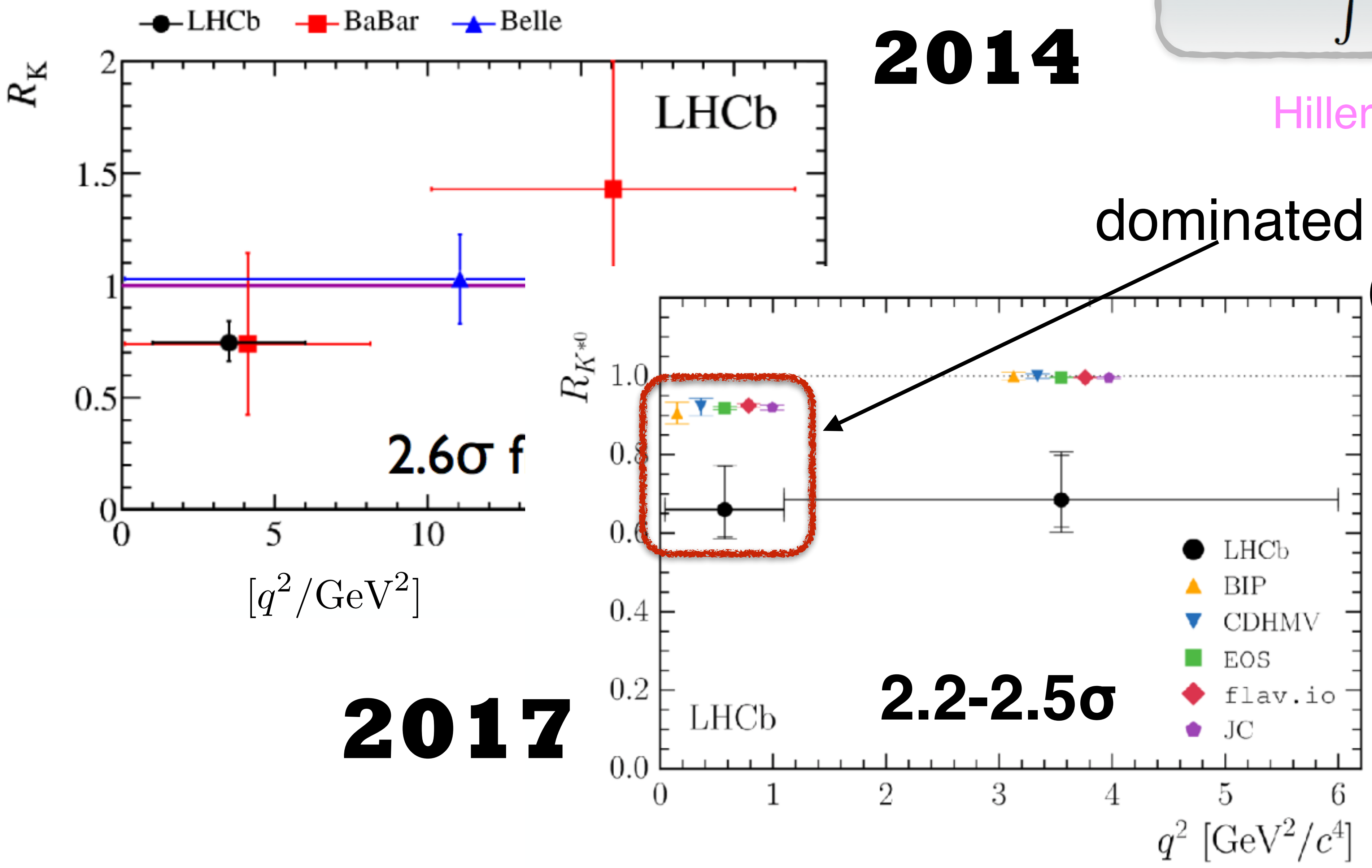


4. QED corrections

- lattice precision calls for it
- violation lepton flavour universality as well..

$$R_H = \frac{\int \frac{d\Gamma(B\to H\mu^+\mu^-)}{dq^2}\, dq^2}{\int \frac{d\Gamma(B\to He^+e^-)}{dq^2}\, dq^2}$$

Hiller Kruger'03



2014

2.6σ f

2017

2.2-2.5σ

dominated by photon pole!?
(non-universality
unlikely unless
light-resonance)

enhanced collinear & soft logs? $\sim \alpha \ln^2\left(\frac{m_e}{m_\mu}\right)$

QED in semileptonic/rad. B-decays

- scalar QED: mesons = point-particle

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1) Effects up to 15% for electrons (depending m_B-cuts)
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3) Estimate structure dependent part to O(**1%)** for R_K, R_{K^*}

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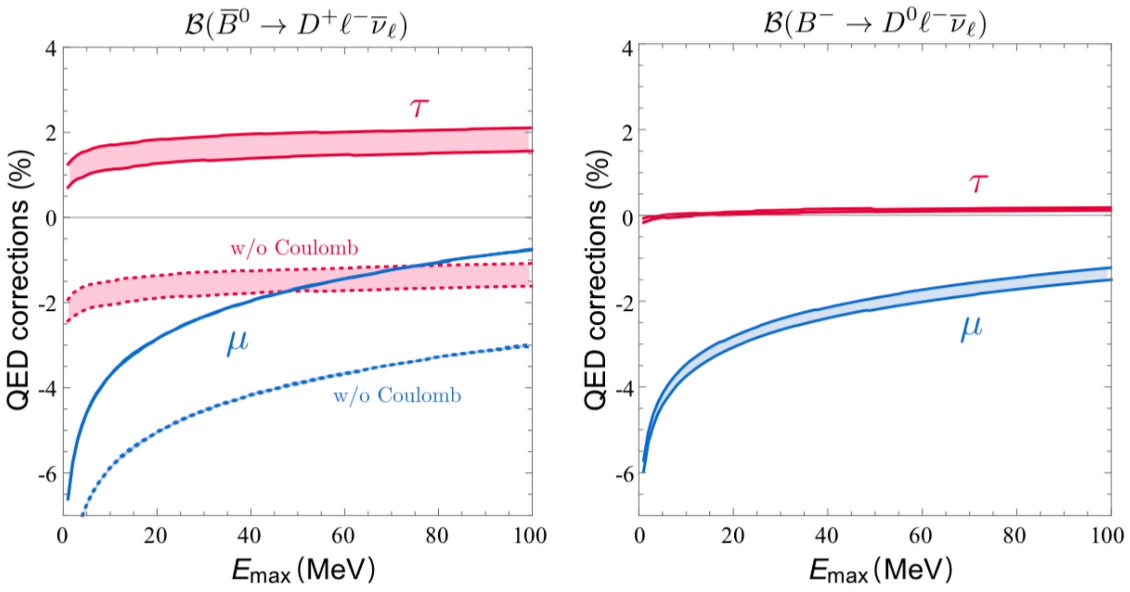
$B \to D\ell\nu$ computed real radiation and virtual radiation

deBoer, Kitahara ,NIsandcisc'18



3-4% effects on R_D
no full comparison with Photos…

Calls for further investigation of QED effects

Hopfer, Gratrex, RZ'15

- Experimental assessment possible:
  QED is measurable in higher moments partial waves in B->K$^{(*)}$ll etc

$$H_{d=6}^{b\to s\ell\ell} \sim \bar{s}_L\gamma_\mu b \boxed{\ell\gamma^\mu(\gamma_5)\ell} \longrightarrow$$

S- and **P-wave (ℓ=0,1)**

QED: no restriction in partial waves

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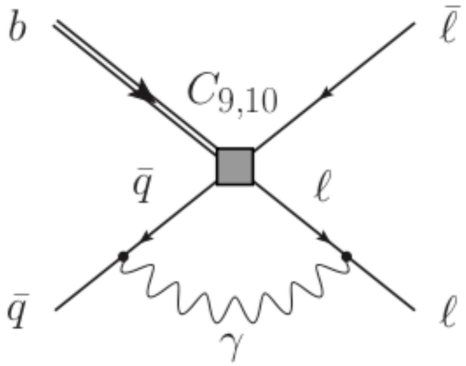
QED: no restriction in partial waves

- $B_s \to \mu\mu$, QED-correction taking into account the **structure**



Beneke, Bobeth, Szafron,'17

$$C_{10} \to C_{10} + \frac{\alpha_{\rm em}}{4\pi} Q_\ell Q_q \Delta_{\rm QED}\,.$$

$$\Delta_{\rm QED} = (33-119) + i\,(9-23) \qquad (\ell=\mu)$$

- error mainly due to B-meson DA!

- net effect only 1% (non-Photos)

Very brief Summary

There is room for improvement in predicting hadronic matrix elements

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Thanks for you attention

Very brief Summary

There is room for improvement in predicting hadronic matrix elements

- Thanks to progress in pQCD = N^{n}LO-technology [FFs,LD,mixing]
- Experimental input (data driven) [parity doubling]
- More input from lattice QCD [e.g. 3-parton matrix elements]
- More thorough assessment of QED needed

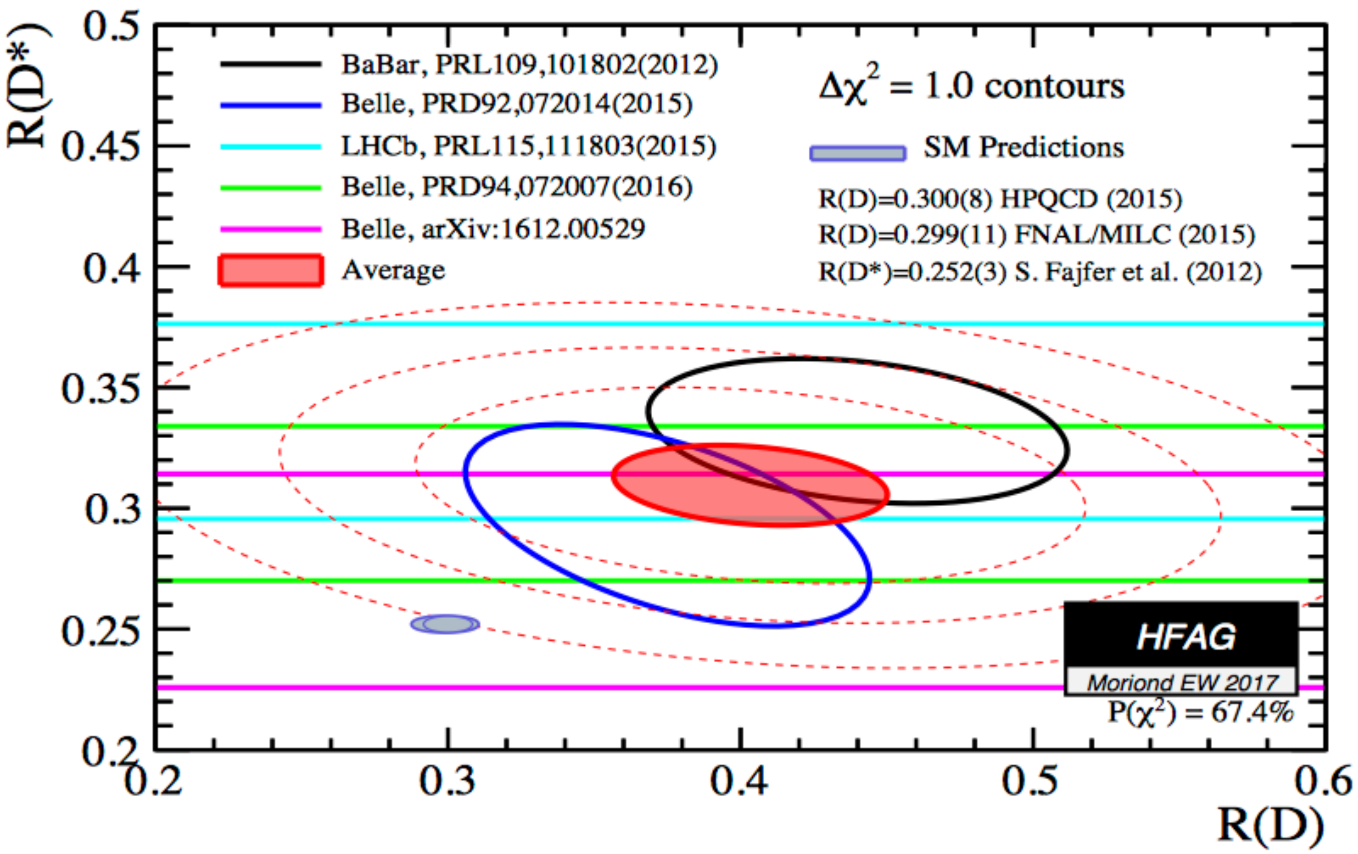
Thanks for you attention

BACKUP

(b) R_{D^*} Lepton Flavour Universality I

$$R_{D^{(*)}} = \frac{\mathcal{B}(B \to D^{(*)}\tau\nu)}{\mathcal{B}(B \to D^{(*)}(e,\mu)\nu)}$$

3.9σ



New results:

LHCb@FPCP’17

$R_{D^*} = 0.285(19)(25)(14)$

- **However,**

1) Using **BelleII angular-data** Schacht et al (cf. Robinson et al 17xx.)

R_{D^*} = 0.262(10) [as average of diff. methods/imputs]

compare R_{D^*} = 0.252(3), Fajfer et al'13

R_{D^*} = 0.304(13)(7) HFAG

2) **τ** difficult particle: 2 exclusive modes saturate incl. rate?

$$BF(B \to X_c \tau \nu) = \begin{cases} 2.42(06) \cdot 10^{-2} & \text{Ligeti, Tackman(theory)} \\ 2.41(23) \cdot 10^{-2} & \text{LEP(experiment)} \end{cases}$$

$$BF(B \to D\tau\nu) + BF(B \to D^*\tau\nu) = \begin{cases} Kamenik, Fajfer'12 & BaBar'12, LHCb'15 & Belle'15 \\ 2.01(7) \cdot 10^{-2} & 2.78(25) \cdot 10^{-2} & 2.39(32) \cdot 10^{-2} \end{cases}$$

D(2400) states contribute ca 10% [PDG]

Perspectives (reducing errors)

- **Theory**:

1) CLN-expansion can be partly improved $O(\alpha_s^2, \alpha_s/m_c, 1/m_c^2)$

2) lattice computation on the way …

3) $B \to D^* \tau \nu$ angular distributions (LHCb?) =

info on unconstrained scalar form factor (contributes 10% to R_{D^*})

- **Experiment**:

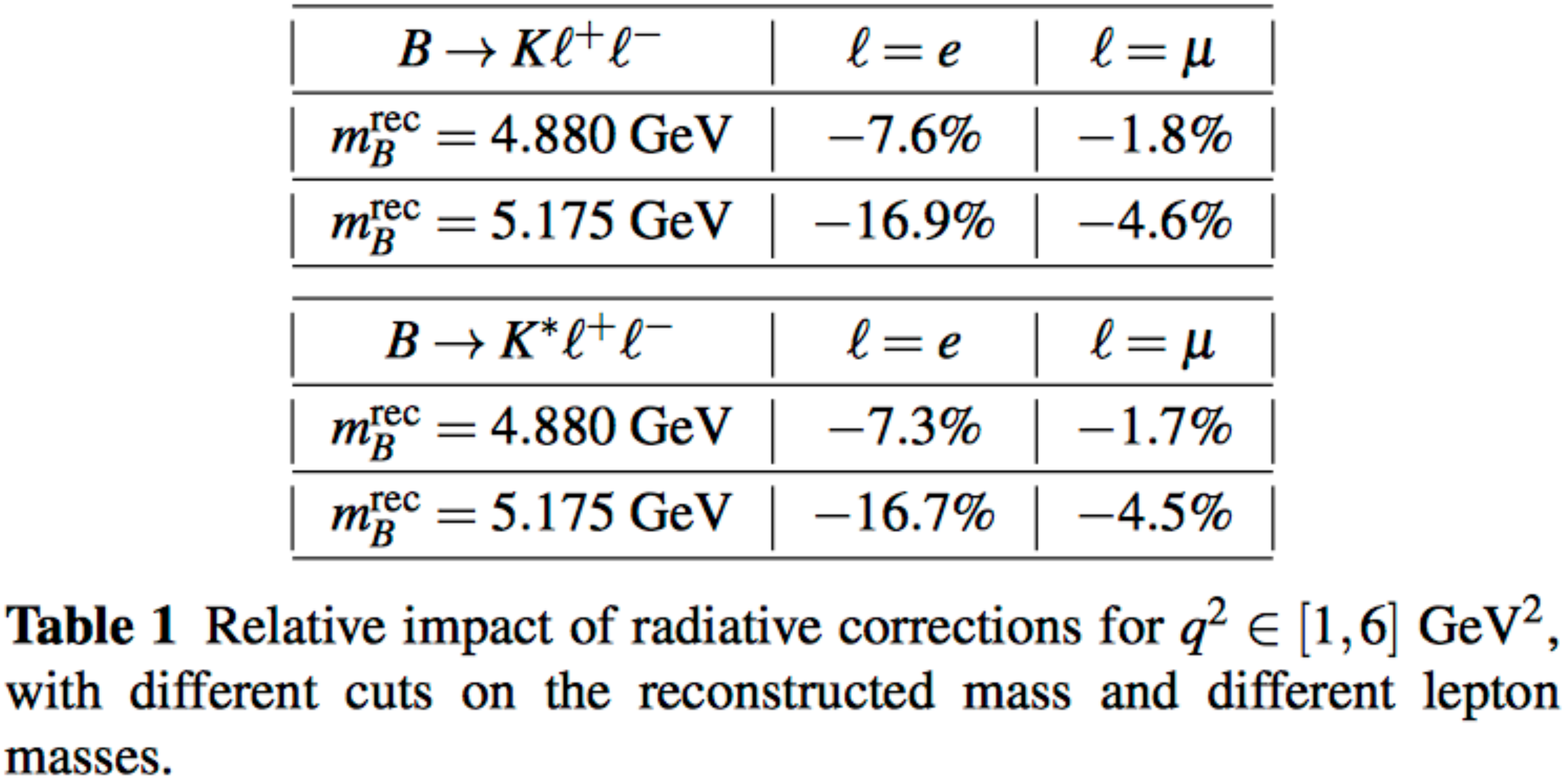
1) BelleII@50/ab competitive with theory error

2) BelleII redo LEP's $B \to X_c \tau \nu$

3) LHCb Run2 4% on R_{D^*}

More details QED-corrections

Bordone, Isidori, Pattori'16



$B \to K\ell^+\ell^-$	$\ell = e$	$\ell = \mu$
$m_B^{\text{rec}} = 4.880$ GeV	-7.6%	-1.8%
$m_B^{\text{rec}} = 5.175$ GeV	-16.9%	-4.6%

$B \to K^*\ell^+\ell^-$	$\ell = e$	$\ell = \mu$
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$m_B^{\text{rec}} = 5.175$ GeV	-16.7%	-4.5%

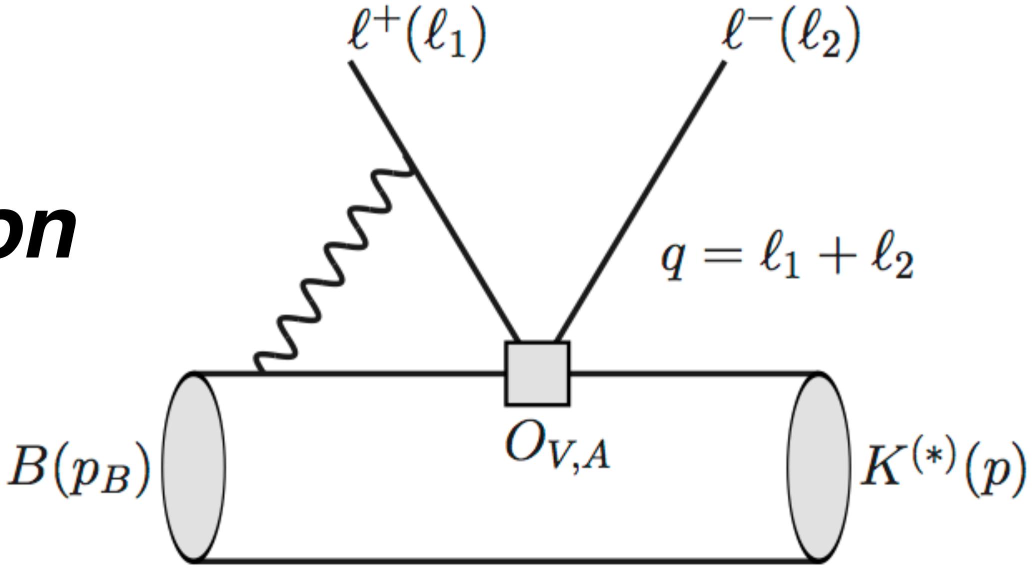
Table 1 Relative impact of radiative corrections for $q^2 \in [1,6]$ GeV^2, with different cuts on the reconstructed mass and different lepton masses.

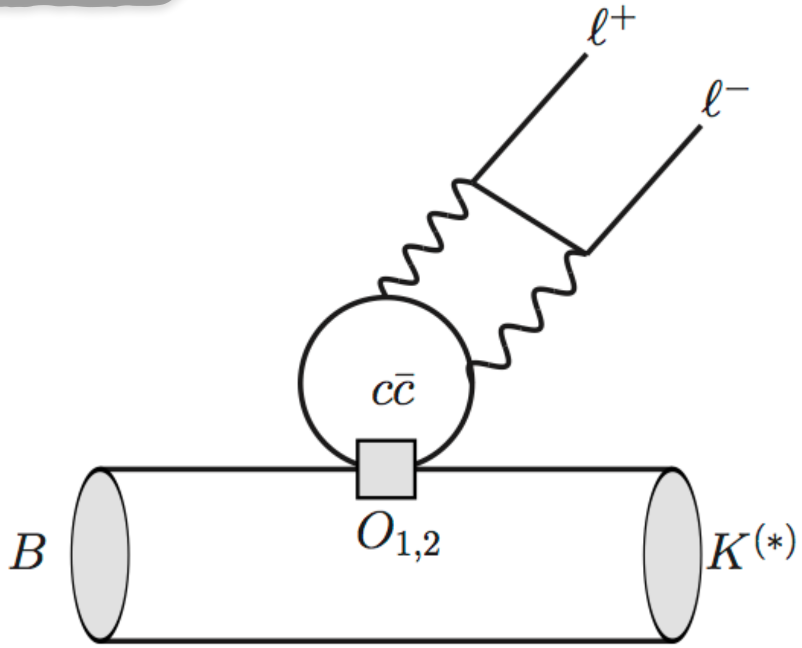
m_B^{rec}=m_B-Detector-Resolution

non-factorisable QED corrections

effects:
A_{FB} without axial interaction

photon





- Becomes a proper 1→3 process and by crossing a 2→2 with Mandelstam variables

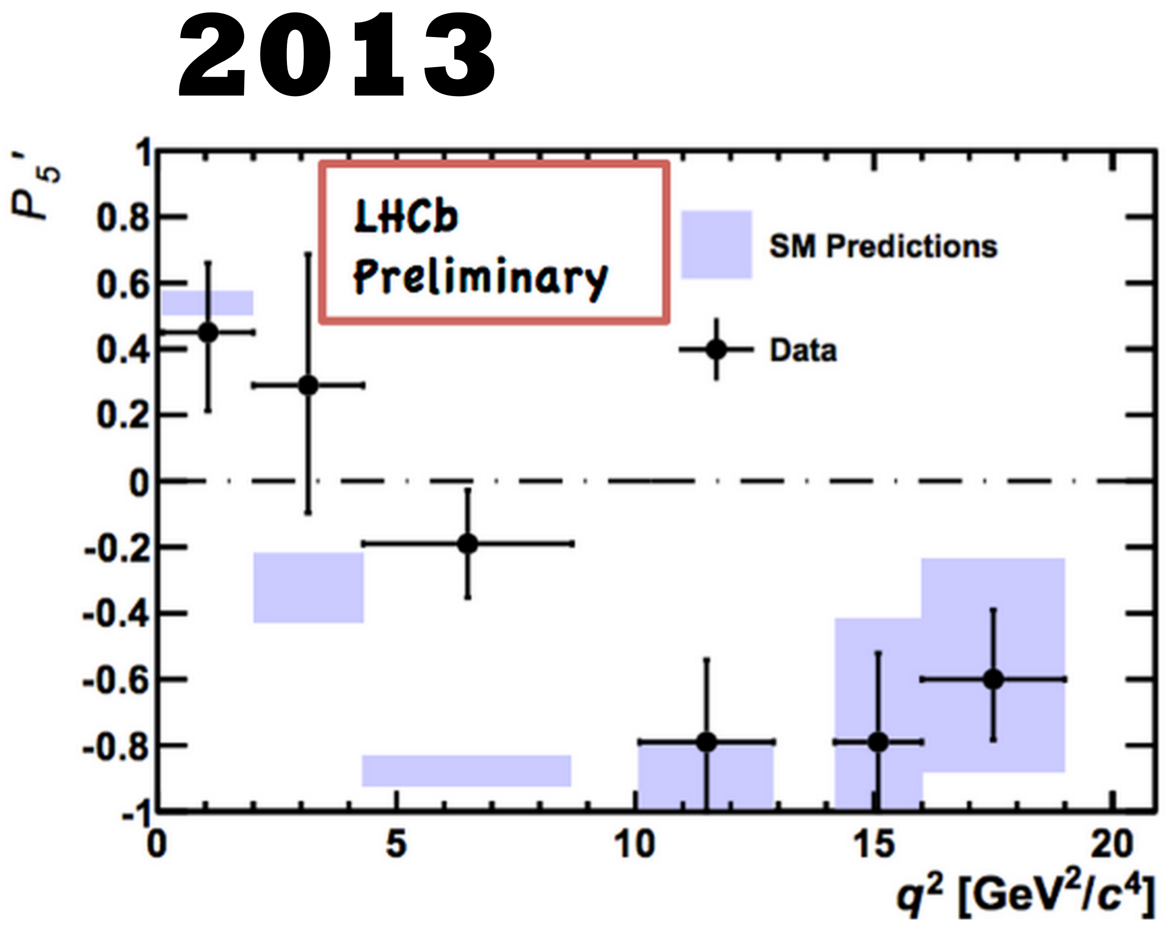
$$B(p_B) + \ell^-(-\ell_1) \to K(p) + \ell^-(\ell_2) \; ,$$

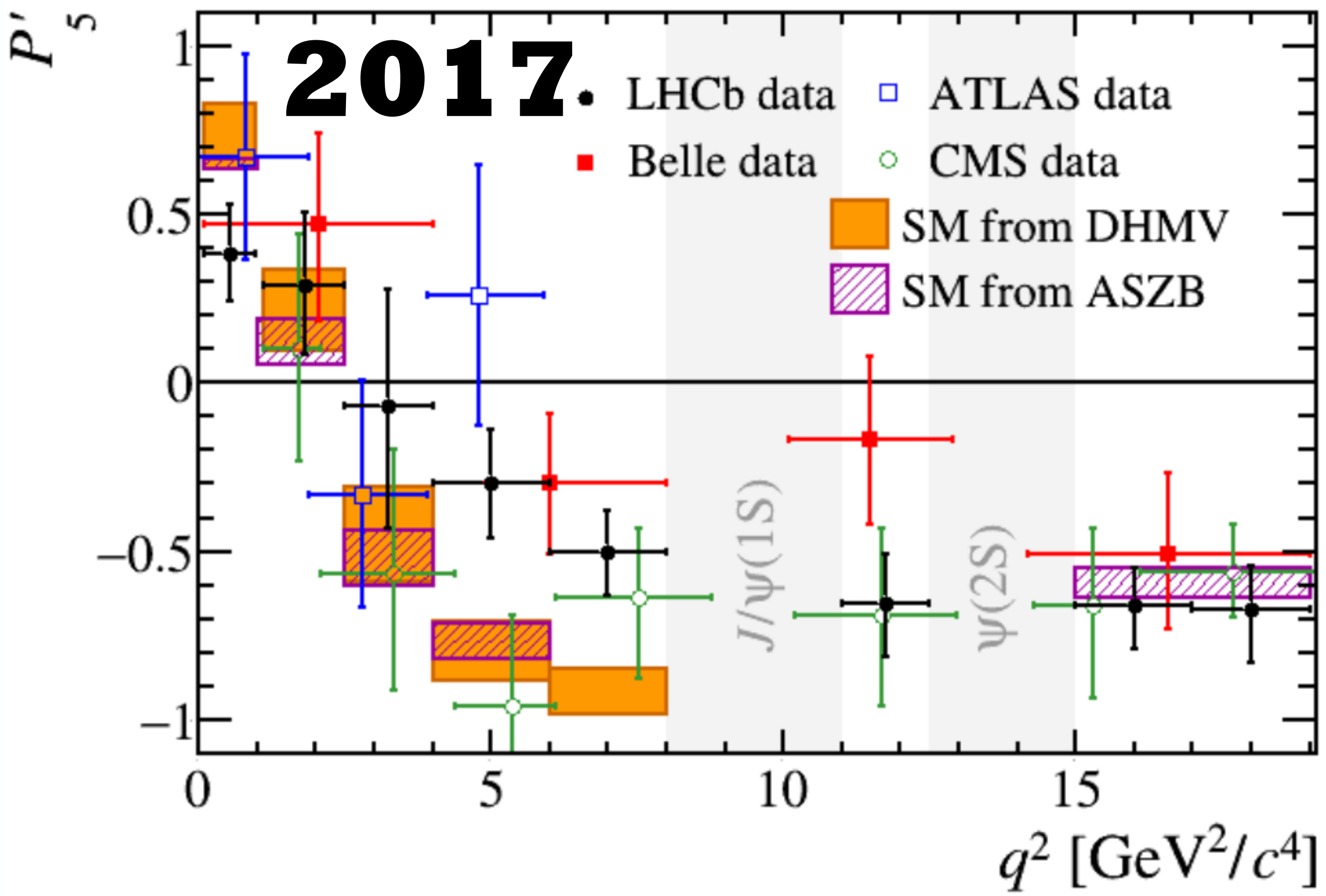
$$s[u] = (p \pm \ell_2[\ell_1])^2 = \frac{1}{2}\left[(m_B^2 + m_K^2 + 2m_\ell^2 - q^2) \pm \beta_\ell \sqrt{\lambda} \cos\theta_\ell\right]$$

- $\Longrightarrow$ s[u] enter logs $\Longrightarrow$ **no restriction sin(θ_l),cos(θ_l)-powers;** Legendre polynomial [or $\Omega_m^{|k,|l|}$] serves as a complete basis (non-vanishing higher moments)

$$\frac{d^2\Gamma(B \to K\ell^+\ell^-)}{dq^2 \; d\cos\theta_\ell} = \sum_{l_\ell \geq 0} G^{(l_\ell)} P_{l_\ell}(\cos\theta_\ell)$$

(a) Tension angular observables B→K*μμ





- **e.g. P_5'** odd lepton partial wave A_{FB}-like

$$\langle P_5'\rangle_{\rm bin}\Big|_{\rm LHCb} = \frac{\left\langle \mathrm{Re}\left[G_1^{2,1}\right]\right\rangle_{\rm bin}}{2\sqrt{3}\,\mathcal{N}'_{\rm bin}}$$

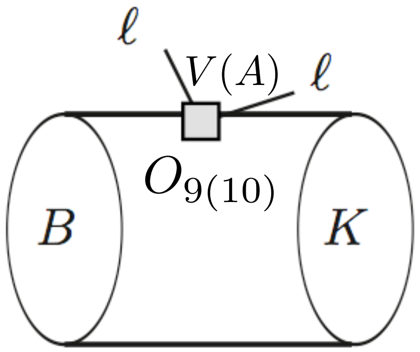
very sensitive to
polarisation

➪ need to understand what is behind polarisation (dynamics)

B→K$^{(*)}$ll under microscope

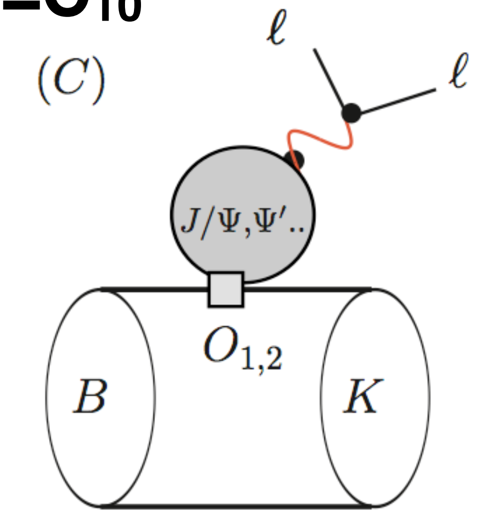
- SM Wilson-coeff: **$C_S=C_P=C_T\simeq 0$**, **$C_V=C_9+\delta C_9^{\text{eff}}(q^2)$**, **$C_A=C_{10}$**

short distance ⇔ form factor



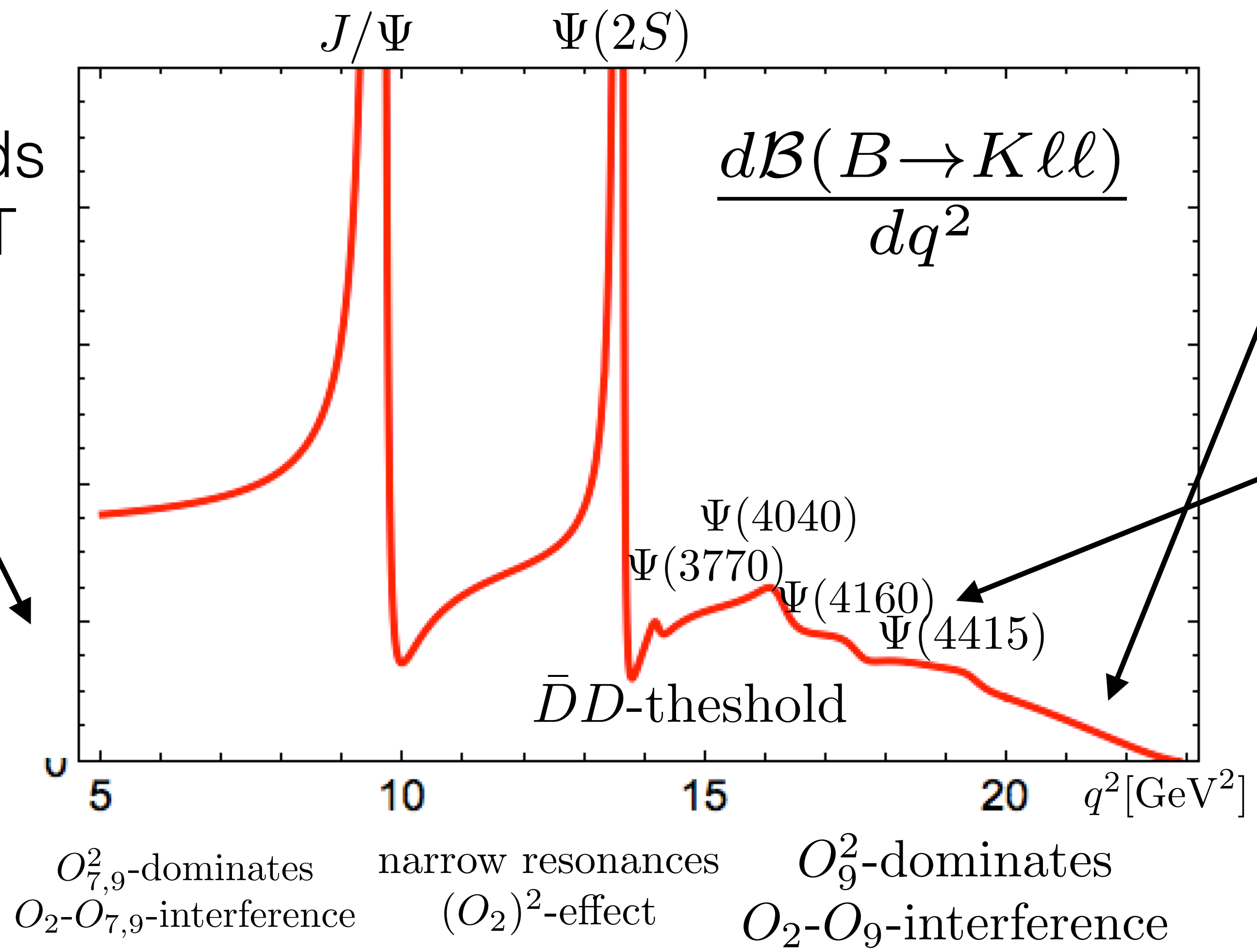
bs$\ell\ell$-operators ($O_{7,9,10}$)

long distance



4-quark ($O_{1\text{-}6}$)

K **fast**:
- **light-cone** methods
LCSR, QCDF/SCET



K **slow**:
- high-q² **"OPE"**
-**endpoint relations**

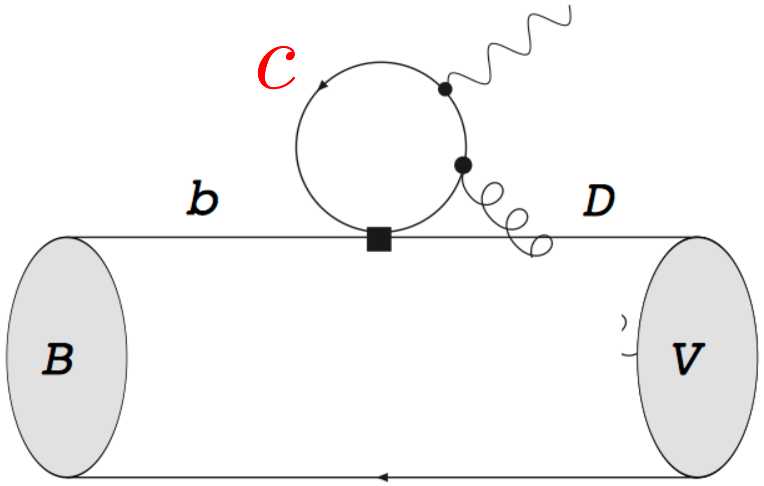
diagnostic shape for charm

Theory outlook

- **Form factors:** believe to known reasonable well
- **Charm:** divides into partonic and hadronic methods and ideally we relate them via dispersion relation

e.g. cross-checks with semileptonics

partonic **(below charm threshold)**



known only in **factorisation-limit:**
LD(q^2)xFormFactor(q^2)

Comment: **problematic** as polarisation-sensitive

Cure: **compute** or argue polarisation dependence to be small

hadronic **(above threshold)**

- **Fact: no duality** in **exclusive** processes for branching fraction

Since not related to n-point function (duality at level of amplitude).
Note: Br(e^+e^- → hadrons) is inclusive and a misleading example

- ➪ if we want to enter resonance region have to deal with hadrons

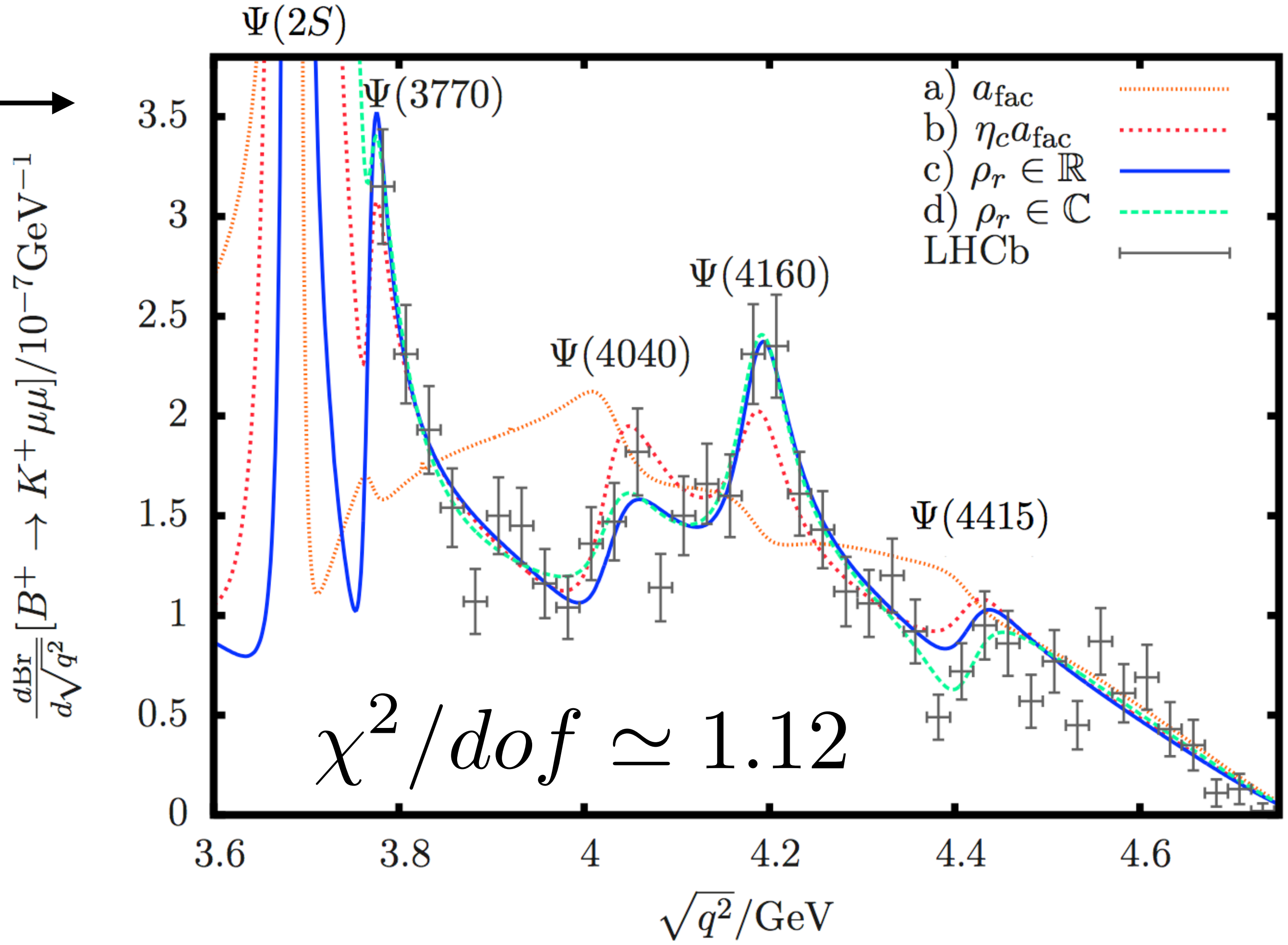
  ➪ charmonium-SD interference phases $\delta_{\psi K^{(*)}}$ have to be fitted!

- B→Kμμ done for **broad resonances** Lyon, RZ ’14

  Results:

  1) large effects $\delta_{\psi \mathrm{broadK}} \simeq \pi$

  2) severe violation of
  naive factorisation
  (using e^+e^-data)

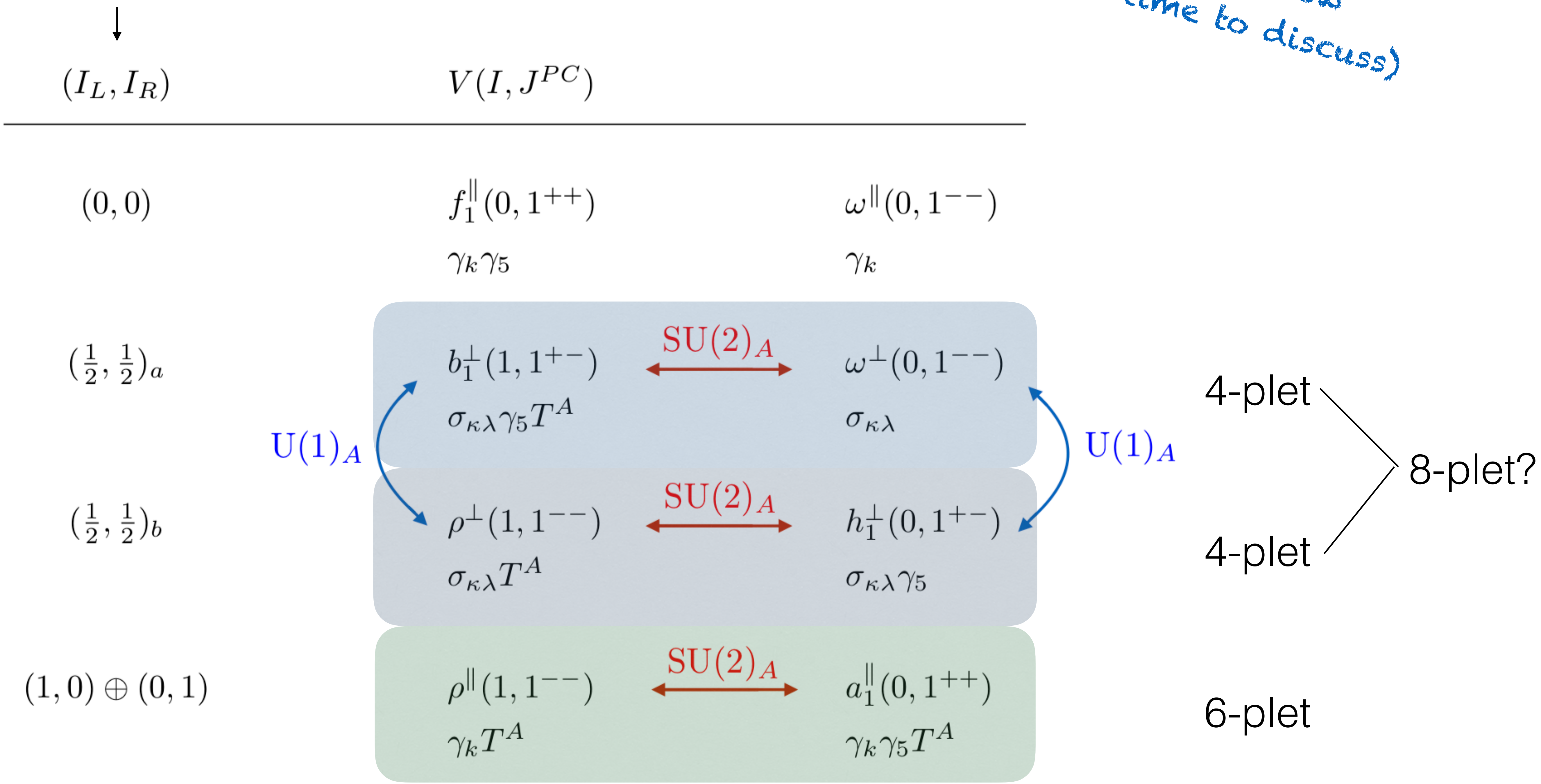


- B→Kμμ redone LHCb’16 **& narrow resonances**
  4-fold degeneracy — $\delta_{J/\psi K}=\pm\pi = \delta_{\psi(2S)K} = \pm\pi$

- B→K*μμ ongoing LHCb better perspectives as more observables

How the symmetries work for N_f=2

briefly show
(no time to discuss)



(I_L, I_R)	$V(I, J^{PC})$		
$(0,0)$	$f_1^{\parallel}(0, 1^{++})$ $\gamma_k\gamma_5$		$\omega^{\parallel}(0, 1^{--})$ γ_k
$(\frac{1}{2},\frac{1}{2})_a$	$b_1^{\perp}(1, 1^{+-})$ $\sigma_{\kappa\lambda}\gamma_5 T^A$	$\mathrm{SU}(2)_A$	$\omega^{\perp}(0, 1^{--})$ $\sigma_{\kappa\lambda}$
$(\frac{1}{2},\frac{1}{2})_b$	$\rho^{\perp}(1, 1^{--})$ $\sigma_{\kappa\lambda} T^A$	$\mathrm{SU}(2)_A$	$h_1^{\perp}(0, 1^{+-})$ $\sigma_{\kappa\lambda}\gamma_5$
$(1,0)\oplus(0,1)$	$\rho^{\parallel}(1, 1^{--})$ $\gamma_k T^A$	$\mathrm{SU}(2)_A$	$a_1^{\parallel}(0, 1^{++})$ $\gamma_k\gamma_5 T^A$

A few references

- Paper arguing charmloops could be large

  Grinstein, Grossman, Ligeti, Pirjol'04 [based on inclusive decay]
  Fedele, Franco, Ciuchini, Mishima, Paul, Silvestrini,.Vialli. JHEP'16

- Papers extracting information on LD from experiment

  Lyon, RZ'14, LHCb'16

- Papers with concrete computation on charmloops

  Ball, RZ PLB'06, Ball, Jones & RZ'PRD'07 Khodjamirian, Mannel, Pivovarov, Wang JHEP'10

- Papers aiming to eliminate the hadronic contribution

  Atwood, Gershon, Hazumi, Soni PRD'05

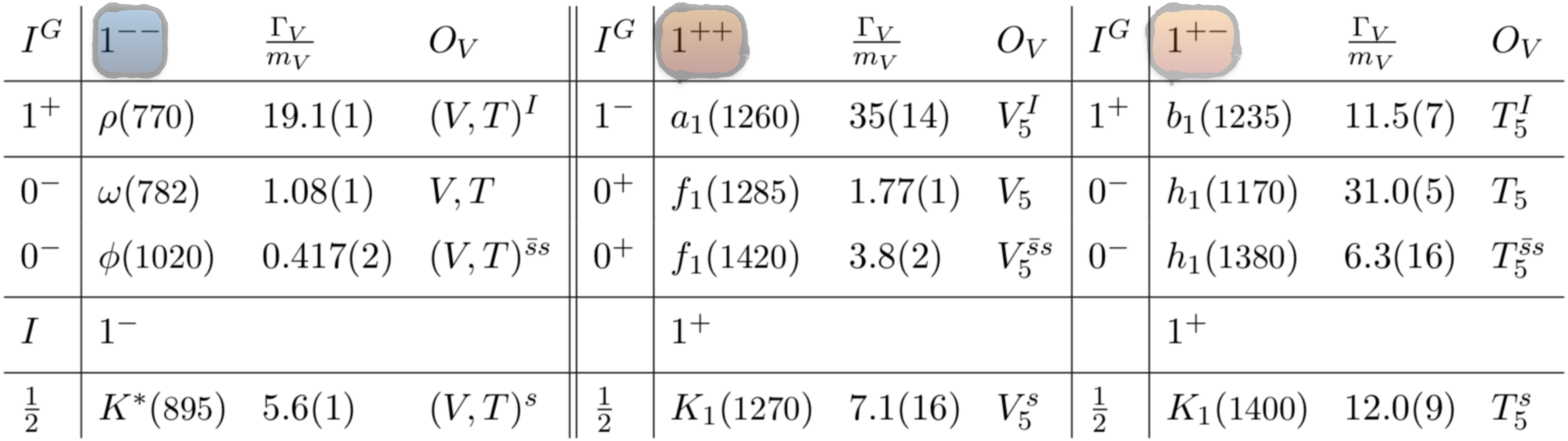
- Authors investigating RHC in bs-transitions (incomplete list)

  Kou, Becirevic, Hiller, Matias, Lunghi, Schneider, Mannel …..

- Authors parameterising LD-contributions (input-depdendent)

  Bobeth, Chrasz, vDyk, Virto

Table of “parity doublers”



I^G	1^{--}	$\frac{\Gamma_V}{m_V}$	O_V	I^G	1^{++}	$\frac{\Gamma_V}{m_V}$	O_V	I^G	1^{+-}	$\frac{\Gamma_V}{m_V}$	O_V
1^+	$\rho(770)$	$19.1(1)$	$(V,T)^I$	1^-	$a_1(1260)$	$35(14)$	V_5^I	1^+	$b_1(1235)$	$11.5(7)$	T_5^I
0^-	$\omega(782)$	$1.08(1)$	V,T	0^+	$f_1(1285)$	$1.77(1)$	V_5	0^-	$h_1(1170)$	$31.0(5)$	T_5
0^-	$\phi(1020)$	$0.417(2)$	$(V,T)^{\bar{s}s}$	0^+	$f_1(1420)$	$3.8(2)$	$V_5^{\bar{s}s}$	0^-	$h_1(1380)$	$6.3(16)$	$T_5^{\bar{s}s}$
I	1^-				1^+				1^+		
$\frac{1}{2}$	$K^*(895)$	$5.6(1)$	$(V,T)^s$	$\frac{1}{2}$	$K_1(1270)$	$7.1(16)$	V_5^s	$\frac{1}{2}$	$K_1(1400)$	$12.0(9)$	T_5^s

Going back to example of $B_s \to \phi\gamma$
& beyond symmetry limit (QCD)

Quick summary: experiment & theory numbers

- **Experiment:**
- $S_{K^*\gamma}$ and $S_{\rho\gamma}$ good @ B-factories

$S_{B\to K^*\gamma}$ = -0.16(22)
$S_{B\to\rho\gamma}$ = -0.83(65)(18)

Belle, Babar
(HFAG-values)

- $H_{\phi\gamma}$ feasible @ LHCb

Muheim, Xie, RZ'08

$H_{Bs\to\phi\gamma}$ = -0.98(50)(20)

LHCb'16

- **Theory:**

$$S_{K^*\gamma} = -\frac{m_s}{m_b}\sin(2\beta) + \mathrm{LD} = -2.3(16)\%$$

$$S_{\rho\gamma} = \frac{m_d}{m_b} + \mathrm{LD} = 0.2(16)\%$$

$$H_{\phi\gamma} = \frac{m_s}{m_b} + \mathrm{LD} = 4.7(25)\%$$

Ball, Jones, RZ'06

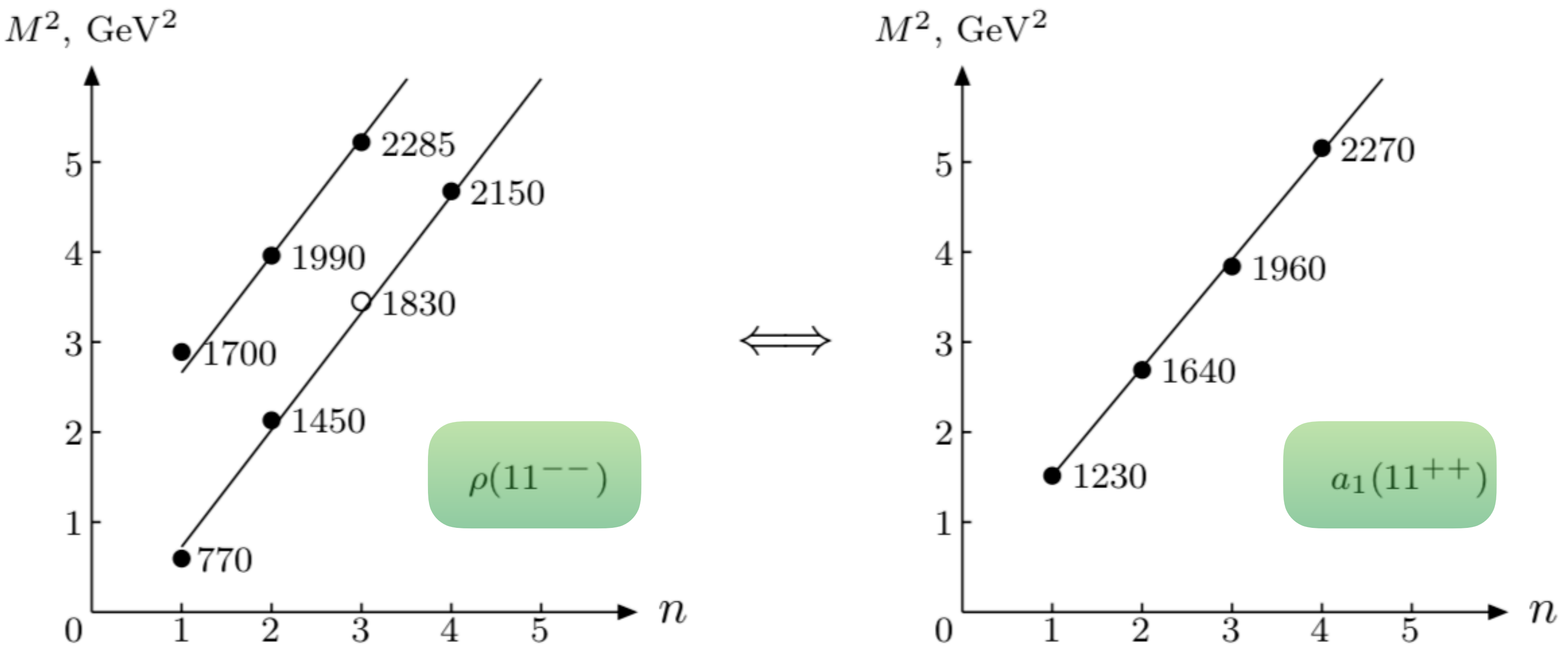
show for completeness

$$\mathrm{BelleII@50}ab^{-1} : \ \Delta S_{K^*\gamma} = 3\% \, , \Delta S_{\rho^0\gamma} = 6\%$$

So what's the trouble (besides statistics)?

2. Parity Doubling* - Global Symmetries

- QCD is parity symmetric - (parity not spontaneously broken Vafa, Witten'84)
- Parity discrete symmetry: Z_2 with irreps **1** and **1'**
  particles parity-eigenstates - either **singlet** or **doublet** of parity
- Reality-check: Anisovich'04



Doubling pattern but not exact. Need a little help from ……

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* **Parity Doubling:** 50 years history Afonin'07 motivated by Regge theory, bootstrap models,..

Intermezzo: test of Symmetry on the Lattice

- Tested on lattice: $T > T_\chi$ Rohrhofer, Aoki, Cossu, Fukaya, Glozman, Hashimoto, Lang. Prelovsek'17

  truncate low Dirac eigenmodes Denissenya, Glozman,Lang '14'15

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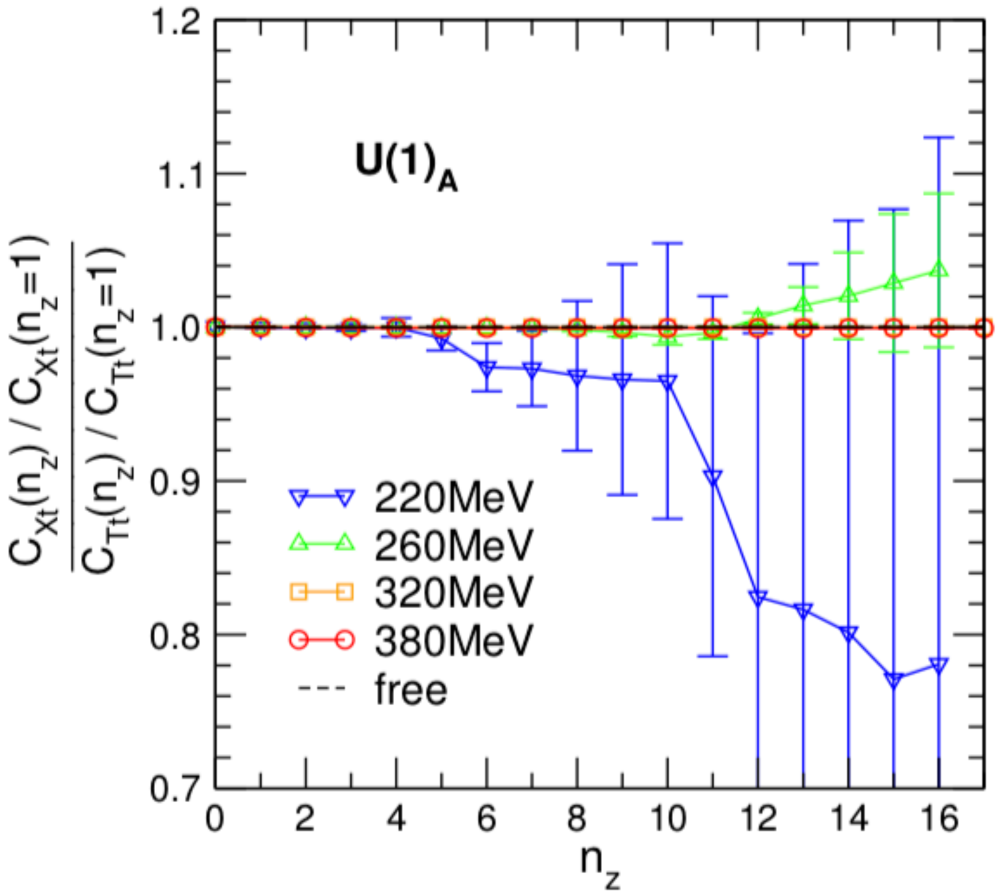
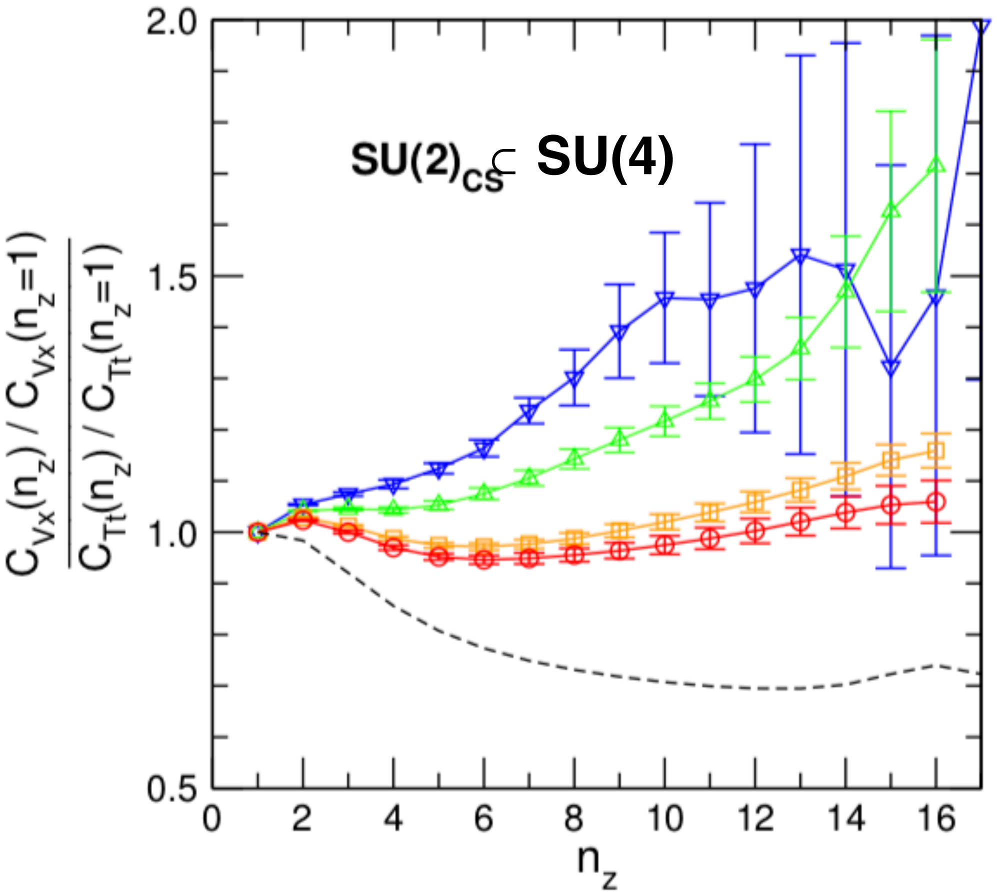


$U(1)_A$ restoration



SU(2)Chiral spin emergence!

Weinberg Sum Rules - parity splitting controlled by condensates

- combining **dispersion relations** and **group theory**

Weinberg'67

$$\Pi_{LR}^{ab} \sim \begin{cases} \int_0^\infty ds \frac{\rho_V(s)-\rho_A(s)}{s-q^2-i0} \\ \frac{\langle \bar{q}\gamma_\mu T^a \lambda^i q_L \bar{q}\gamma^\mu T^b \lambda^i q_R \rangle}{q^6} + \ldots \end{cases}$$

$$\rho_A(s) = F_\pi^2 \delta(s-m_\pi^2) + F_{a_1}^2 \delta(s-m_{a_1}^2) + \ldots \qquad \rho_V(s) = F_\rho^2 \delta(s-m_\rho^2) + \ldots$$

massless quarks

$$(\Pi_{LR}^{a,b})_{\mu\nu}(q^2) = i \int d^4x e^{iq\cdot x} \langle T J_\mu^{a,L}(x) J_\nu^{b,R}(x) \rangle = (q_\mu q_\nu - q^2 g_{\mu\nu}) \Pi_{LR}^{a,b}(q^2) \;;$$

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- Assuming perturbation theory to dominate above a₁-meson:

2-Weinberg sum rules:

$$F_\rho^2 - F_\pi^2 - F_{a_1}^2 = 0 \,,$$

$$m_\rho^2 F_\rho^2 - m_{a_1}^2 F_{a_1}^2 = 0 \,,$$

3rd sum rule:

$$m_\rho^4 F_\rho^2 - m_{a_1}^4 F_{a_1}^2 = (c\alpha_s + \dots) \underbrace{\langle \bar q .. q_L q .. q_R \rangle}_{\simeq \langle \bar q q \rangle^2} \,.$$