Precision kaon physics and lattice QCD

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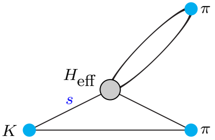
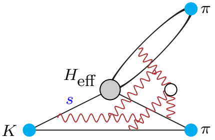
Motivation for Precision Flavour Physics

- Precision Flavour Physics is a key approach, complementary to the large E_T searches at the LHC, in exploring the limits of the Standard Model of Particle Physics and in searches for new physics.
  - If the LHC experiments discover new elementary particles BSM, then precision flavour physics will be necessary to understand the underlying framework.
  - The discovery potential of precision flavour physics should also not be underestimated. (In principle, the reach is about two-orders of magnitude deeper than the LHC!)
- Precision flavour physics requires control of hadronic effects for which lattice QCD simulations are essential.
- For example, a schematic cartoon of $K \to \pi\pi$ decays:

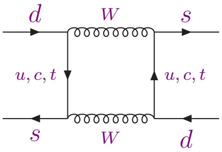


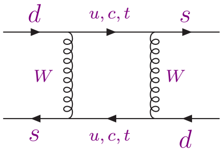
means



“Standard" Quantities in Lattice (Kaon) Physics

- Let me illustrate the discussion with a study of K^0 - $\bar{K}^0$ mixing and the evaluation of ε_K:





- This is an example of the use of the OPE to separate the short distance (perturbative) and long distance (non-perturbative) contributions:

$$\text{Physics} = \sum_i V_{\text{CKM},i}\, C_i(Q^2/\mu^2) \times \langle f\,|O_i(0)|\,i\rangle_{\frac{\mu^2}{\Lambda^2}}$$

  - In the 1990/2000s, the Wilson coefficients C_i were calculated at reasonably high orders in the $\overline{\text{MS}}$ scheme. The $\overline{\text{MS}}$ scheme is purely perturbative.
  - In many cases the matrix elements have now been computed to very good precision, in schemes which can be simulated.
  - This still requires the perturbative matching between the schemes used in lattice simulations and the perturbative $\overline{\text{MS}}$ scheme. This is becoming a significant component of the final systematic uncertainty.

- I define "standard" quantities in lattice flavour physics as being ones relying on the evaluation of matrix elements of the form:
  1. $\langle 0|O_i(0)|h(p)\rangle$, (e.g. leptonic decay constants)
  2. $\langle h_2(p_2)|O_i(0)|h_1(p_1)\rangle$ (e.g. semileptonic decays or neutral-meson mixing)

  where h, $h_{1,2}$ are single hadrons the O_i are local composite operators and the matrix elements are evaluated in iso-symmetric QCD.
- In this talk I will describe our attempts to go beyond these standard calculations.

Outline of talk:

1. General introduction
2. Adding isospin breaking effects (including radiative corrections)
3. $K \to \pi\pi$ decays
4. Long-distance contributions in kaon physics
5. Summary, prospects and conclusions

Quantity	■	$N_f=2+1+1$	■	$N_f=2+1$	■	$N_f=2$
m_s(MeV)	2	$93.9(1.1)$	5	$92.0(2.1)$	2	$101(3)$
m_{ud}(MeV)	1	$3.70(17)$	5	$3.373(80)$	1	$3.6(2)$
m_s/m_{ud}	2	$27.30(34)$	4	$27.43(31)$	1	$27.3(9)$
m_d(MeV)	1	$5.03(26)$	Flag(4)	$4.68(14)(7)$	1	$4.8(23)$
m_u(MeV)	1	$2.36(24)$	Flag(4)	$2.16(9)(7)$	1	$2.40(23)$
m_u/m_d	1	$0.470(56)$	Flag(4)	$0.46(2)(2)$	1	$0.50(4)$
m_c/m_s	3	$11.70(6)$	2	11.82	1	11.74
$f_+^{K\pi}(0)$	1	$0.9704(24)(22)$	2	$0.9667(27)$	1	$0.9560(57)(62)$
f_{K^+}/f_{π^+}	3	$1.193(3)$	4	$1.192(5)$	1	$1.205(6)(17)$
f_K(MeV)	3	$155.6(4)$	3	$155.9(9)$	1	$157.5(2.4)$
f_π(MeV)			3	$130.2(1.4)$		
$\Sigma^{\frac{1}{3}}$(MeV)	1	$280(8)(15)$	4	$274(3)$	4	$266(10)$
F_π/F	1	$1.076(2)(2)$	5	$1.064(7)$	4	$1.073(15)$
$\bar{\ell}_3$	1	$3.70(7)(26)$	5	$2.81(64)$	3	$3.41(82)$
$\bar{\ell}_4$	1	$4.67(3)(10)$	5	$4.10(45)$	2	$4.51(26)$
$\hat{B}_K$	1	$0.717(18)(16)$	4	$0.7625(97)$	1	$0.727(22)(12)$

- The precision of “standard" isosymmetric QCD calculations is now such that in order to improve the precision still further isospin breaking (IB) effects (including electromagnetism) need to be included.
- These are

$$O\left(\frac{m_u - m_d}{\Lambda_{\rm QCD}}\right) \quad \text{and} \quad O(\alpha)\,,$$

  i.e. $O(1\%)$ or so.
- {The separation of IB corrections into those due to $m_u \neq m_d$ and those due to electromagnetism requires a convention. It is only the sum which is physical.}
- Such calculations for the spectrum have been performed for a few years now, with perhaps the most noteworthy result being BMW Collaboration, arXiv:1406.4088

$$m_n - m_p = 1.51(16)(23)\,\text{MeV}$$

  to be compared to the experimental value of 1.2933322(4) MeV.
- I stress that including electromagnetic effects, where the photon is massless of course, requires considerable theoretical progress, e.g.

$$\int \frac{d^4k}{(2\pi)^4}\,\frac{1}{k^2}\cdots \Rightarrow \frac{1}{L^3T}\sum_k \frac{1}{k^2}\cdots$$

  and we have to control the contribution of the zero mode in the sum.

This section is based on ongoing work and the following papers:

1. *QED Corrections to Hadronic Processes in Lattice QCD,*
   N.Carrasco, V.Lubicz, G.Martinelli, C.T.Sachrajda, N.Tantalo, C.Tarantino and M.Testa,
   Phys. Rev. D **91** (2015) no.7, 07450 [arXiv:1502.00257 [hep-lat]].

2. *Finite-Volume QED Corrections to Decay Amplitudes in Lattice QCD*,
   V.Lubicz, G.Martinelli, C.T.Sachrajda, F.Sanfilippo, S.Simula and N.Tantalo,
   Phys. Rev. D **95** (2017) no.3, 034504 [arXiv:1611.08497 [hep-lat]].

3. *First Lattice Calculation of the QED Corrections to Leptonic Decay Rates*,
   D.Giusti, V.Lubicz, G.Martinelli, C.T.Sachrajda, F.Sanfilippo, S.Simula, N.Tantalo and C.Tarantino,
   Phys. Rev. Lett. **120** (2018) 072001 [arXiv:1711:06537]

4. *Radiative corrections to decay amplitudes in lattice QCD,*
   D.Giusti, V.Lubicz, G.Martinelli, C.T.Sachrajda, F.Sanfilippo, S.Simula and N.Tantalo,
   arXiv:1811.06364 [hep-lat].

Procedure for handling infrared divergences

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- Calculating electromagnetic corrections to decay amplitudes has an added major complication, not present in computations of the spectrum,

  the presence of infrared divergences

- This implies that when studying weak decays, such as e.g. $K^+ \to \ell^+\nu$ the physical observable must include soft photons in the final state

$$\Gamma(K^+ \to \ell^+\nu_\ell(\gamma)) = \Gamma(K^+ \to \ell^+\nu_\ell) + \Gamma(K^+ \to \ell^+\nu_\ell\gamma)\,.$$

F.Bloch and A.Nordsieck, PR 52 (1937) 54

- The question for the lattice community is how best to combine this understanding with lattice calculations of non-perturbative hadronic effects.
- This is a generic problem if em corrections are to be included in the evaluation of a decay process.
- In 2015 we proposed a method for including electromagnetic corrections in decay amplitudes.

  N.Carrasco, V.Lubicz, G.Martinelli, CTS, N.Tantalo, C.Tarantino & M.Testa, arXiv:1502.00257

- I stress that in order to implement this method successfully, it will be be necessary to work with the experimental community to ensure that we are calculating quantities which correspond to the experimental measurements.

$$\begin{aligned}\Gamma(K^+\to\ell^+\nu_\ell(\gamma)) &= \Gamma(K^+\to\ell^+\nu_\ell)+\Gamma(K^+\to\ell^+\nu_\ell\gamma)\\ &\equiv \Gamma_0+\Gamma_1\end{aligned}$$

- In principle, it is possible to compute Γ_1 nonperturbatively over a larger range of photon energies.
- At present we do not propose to compute Γ_1 nonperturbatively. Rather we consider only photons which are sufficiently soft for the point-like (pt) approximation to be valid.
  - The calculation of Γ_1 non-perturbatively is however, likely to happen in the medium term.
  - For pions and kaons at least, a cut-off ΔE of $O(10\text{-}20\,\mathrm{MeV})$ appears to be appropriate both experimentally and theoretically.

    F.Ambrosino et al., KLOE collaboration, hep-ex/0509045. arXiv:0907.3594, NA62
  - Question: What is the best way to translate the photon energy and angular resolutions at LHC, Belle II etc. into the rest frame of the decaying mesons?

Lattice computations of $\Gamma(K^- \to \ell^- \bar{\nu}_\ell(\gamma))$ at $O(\alpha)$

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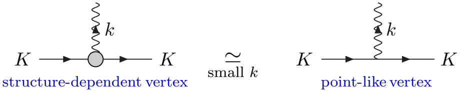
- The observable we calculate is $\Gamma_0(K \to \ell\bar{\nu}_\ell) + \Gamma_1(K \to \ell\bar{\nu}_\ell\gamma)$ where (in the kaon rest-frame) $E_\gamma < \Delta E$ and ΔE is sufficiently small for the structure dependence of K to be neglected ($\Delta E \lesssim 20\,\mathrm{MeV}$).

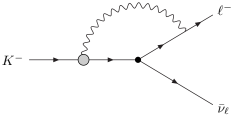


- We now write

$$\boxed{\Gamma_0 + \Gamma_1(\Delta E) = \lim_{V\to\infty}(\Gamma_0 - \Gamma_0^{\mathrm{pt}}) + \lim_{V\to\infty}(\Gamma_0^{\mathrm{pt}} + \Gamma_1(\Delta E))\,.}$$

  where pt stands for *point-like*.

  - The second term on the rhs can be calculated in perturbation theory. It is infrared convergent, but does contain a term proportional to $\log \Delta E$.
  - The first term is also free of infrared divergences.
  - Γ_0 is calculated non-perturbatively and Γ_0^{pt} in perturbation theory.



- Writing

$$\frac{\Gamma(K_{\mu2})}{\Gamma(\pi_{\mu2})} = \left|\frac{V_{us}}{V_{ud}}\frac{f_K^{(0)}}{f_\pi^{(0)}}\right|^2 \frac{m_\pi^3}{m_K^3}\left(\frac{m_K^2-m_\mu^2}{m_\pi^2-m_\mu^2}\right)^2 (1+\delta R_{K\pi})$$

where $m_{K,\pi}$ are the physical masses, we find

$$\delta R_{K\pi} = -0.0122(16)\,.$$

D.Giusti et al., arXiv:1711.06537

This first calculation can certainly be improved.

- $f_P^{(0)}$ are the decay constants obtained in iso-symmetric QCD with the renormalized $\overline{\mathrm{MS}}$ masses and coupling equal to those in the full QCD+QED theory extrapolated to infinite volume and to the continuum limit.
- This result can be compared to the PDG value, based on ChPT, is $\delta R_{K\pi} = -0.0112(21)$.
- Our result, together with $V_{ud} = 0.97417(21)$ from super-allowed nuclear β-decays gives $V_{us} = 0.22544(58)$ and

$$V_{ud}^2 + V_{us}^2 + V_{ub}^2 = 0.99985(49)\,.$$

- Our emphasis now is on improving the renormalisation and on extending the formalism to semileptonic decays.

- $K \to \pi\pi$ decays are a very important class of processes for standard model phenomenology with a long and noble history.
  - It is in these decays that both indirect and direct CP-violation was discovered.
- Bose Symmetry $\Rightarrow$ the two-pion state has isospin 0 or 2.
- Among the very interesting issues are the origin of the $\Delta I = 1/2$ rule (Re A_0/Re $A_2 \simeq 22.5$) and an understanding of the experimental value of ε'/ε, the parameter which was the first experimental evidence of direct CP-violation.
- The evaluation of $K \to \pi\pi$ matrix elements requires an extension of the standard computations of $\langle 0 \,|\, O(0) \,|\, h \rangle$ and $\langle h_2 \,|\, O(0) \,|\, h_1 \rangle$ matrix elements with a single hadron in the initial and/or final state.

The RBC & UKQCD collaborations

BNL and BNL/RBRC

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Mattia Bruno
Taku Izubuchi
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Chulwoo Jung
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Renwick Hudspith

- In 2015 RBC-UKQCD published our first result for ε'/ε computed at physical quark masses and kinematics, albeit still with large relative errors:

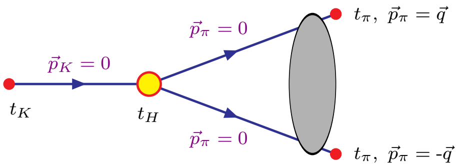
$$\left.\frac{\varepsilon'}{\varepsilon}\right|_{\text{RBC-UKQCD}} = (1.38 \pm 5.15 \pm 4.59) \times 10^{-4}$$

to be compared with

$$\left.\frac{\varepsilon'}{\varepsilon}\right|_{\text{Exp}} = (16.6 \pm 2.3) \times 10^{-4}\,.$$

RBC-UKQCD, arXiv:1505.07863

- This is by far the most complicated project that I have ever been involved with.
- This single result hides much important (and much more precise) information which we have determined along the way.
- We are updating the results with about ≥ 6 times the statistics and much improved techniques for reducing the systematic uncertainties. Updated results will be published early in the new year.



- $K \to \pi\pi$ correlation function is dominated by lightest state, i.e. the state with two-pions at rest. Maiani and Testa, PL B245 (1990) 585

$$C(t_\pi) = A + B_1 e^{-2m_\pi t_\pi} + B_2 e^{-2E_\pi t_\pi} + \cdots$$

  - Solution 1: Study an excited state. Lellouch and Lüscher, hep-lat/0003023
  - Solution 2: Introduce suitable boundary conditions such that the $\pi\pi$ ground state is $|\pi(\vec{q})\pi(-\vec{q})\rangle$. RBC-UKQCD, C.h.Kim hep-lat/0311003

For B-decays, with so many intermediate states below threshold, this is the main obstacle to producing reliable calculations.

Results for A_2

- The amplitude A_2 is considerably simpler to evaluate that A_0.
- Our first results for A_2 at physical kinematics were obtained at a single, rather coarse, value of the lattice spacing ($a \simeq 0.14$ fm). Estimated discretization errors at 15%.
  arXiv:1111.1699, arXiv:1206.5142
- Our latest results were obtained on two new ensembles, 48^3 with $a \simeq 0.11$ fm and 64^3 with $a \simeq 0.084$ fm so that we can make a continuum extrapolation:

$$\begin{aligned} \mathrm{Re}(A_2) &= 1.50(4)_{\mathrm{stat}}(14)_{\mathrm{syst}} \times 10^{-8}\ \mathrm{GeV}. \\ \mathrm{Im}(A_2) &= -6.99(20)_{\mathrm{stat}}(84)_{\mathrm{syst}} \times 10^{-13}\ \mathrm{GeV}. \end{aligned}$$

arXiv:1502.00263

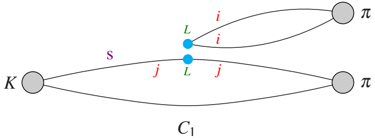
- The experimentally measured value is $\mathrm{Re}(A_2) = 1.479(4) \times 10^{-8}$ GeV.
- Although the precision can still be significantly improved (partly by perturbative calculations), the calculation of A_2 at physical kinematics can now be considered as standard.

RBC-UKQCD Collaboration, arXiv:1212.1474

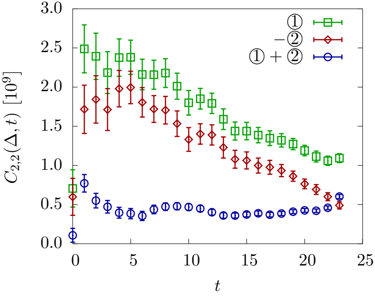
- Re A_2 is dominated by a simple operator:

$$O^{3/2}_{(27,1)} = (\bar{s}^i d^i)_L \left\{ (\bar{u}^j u^j)_L - (\bar{d}^j d^j)_L \right\} + (\bar{s}^i u^i)_L \, (\bar{u}^j d^j)_L$$

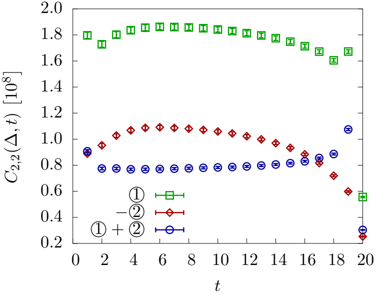
and two diagrams:



- Re A_2 is proportional to $C_1 + C_2$.
- The contribution to Re A_0 from Q_2 is proportional to $2C_1 - C_2$ and that from Q_1 is proportional to $C_1 - 2C_2$ with the same overall sign.
- Colour counting might suggest that $C_2 \simeq \frac{1}{3} C_1$.
- We find instead that $C_2 \approx -C_1$ so that A_2 is significantly suppressed!
- **We believe that the strong suppression of Re A_2 and the (less-strong) enhancement of Re A_0 is a major factor in the $\Delta I = 1/2$ rule.**



Physical Kinematics



$m_\pi \simeq 330$ MeV at threshold.

- Notation ⓘ $\equiv C_i,\ i = 1, 2$.
- Of course before claiming a quantitative understanding of the $\Delta I = 1/2$ rule we needed to compute Re A_0 at physical kinematics and found a results of $\simeq 31 \pm 12$ to be compared to the experimental value of 22.5.
- Much early phenomenology was based on the vacuum insertion approach. although the qualitative picture we find had been suggested by Bardeen, Buras and Gerard in 1987.

- RBC-UKQCD Collaborations are developing and exploiting techniques to evaluate long-distance contributions to kaon physics.
  - Long-distance here means scales $\gtrsim \frac{1}{m_c}$.
  - As well as computing the non-perturbative long-distance contributions from scales of $O(\Lambda_{\rm QCD})$, we aim to avoid the necessity of performing perturbation theory at the scale of m_c. For Δm_K this has proved particularly slowly convergent. J.Brod & M.Gorbahn, arXiv:1108.2036

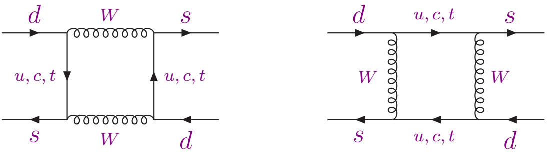
- The techniques are being applied to
  1. $\Delta m_K = m_{K_L} - m_{K_S}$ and ε_K;
  2. Rare kaon decays $K^+ \to \pi^+ \ell^+ \ell^-$ and $K_S \to \ell^+ \ell^-$;
  3. The rare kaon decay $K^+ \to \pi^+ \nu \bar{\nu}$.

  - I will illustrate the generic issues in evaluating long-distance contributions with Δm_K.

The K_L-K_S mass difference

- Consider the neutral-kaon system:
  - Strong interaction eigenstates: $|K_0\rangle = |\bar{s}d\rangle$ and $|\bar{K}_0\rangle = |s\bar{d}\rangle$.
  - CP-eigenstates: $|K_{1,2}\rangle = \frac{1}{\sqrt{2}}(|K_0\rangle \pm |\bar{K}_0\rangle)$.
  - Mass eigenstates: $|K_S\rangle \propto (|K_1\rangle + \varepsilon|K_2\rangle)$ and $|K_L\rangle \propto (|K_2\rangle + \varepsilon|K_1\rangle)$.

$$\Delta m_K \equiv m_{K_L} - m_{K_S} = 3.483(6)\times 10^{-12}\ \text{MeV} \ll \Lambda_{\text{QCD}}.$$



- It is frequently said that Flavour Physics can probe scales which are unreachable in colliders.
  - Here, if we could reproduce the experimental value of Δm_K in the SM to 10% accuracy and if we imagine an effective new-physics $\Delta S = 2$ contribution $\frac{1}{\Lambda^2}(\bar{s}\cdots d)(\bar{s}\cdots d)$ then $\Lambda \gtrsim (10^3 - 10^4)$ TeV.
- The RBC-UKQCD collaborations are well on the way to an *ab initio* calculation of Δm_K. Preliminary result $\Delta m_K = 5.5(1.7)\times 10^{-12}\,\text{MeV}$.

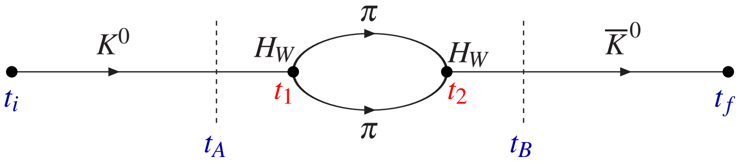
Z.Bai, N.H.Christ, CTS; EPJ Web Conf 175 (2018) 13017.

(a) The fiducial volume

(b) Unphysical exponentially growing contributions

(c) Finite-volume corrections

(d) Renormalization

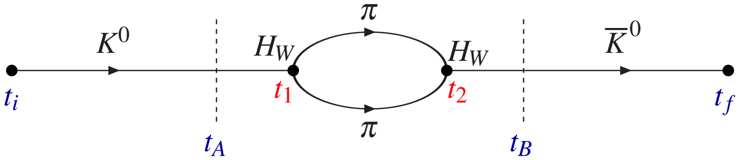


- How do you prepare the states $h_{1,2}$ in the generic integrated correlation function:

$$\int d^4x \int d^4y \, \langle h_2 \,|\, T\{O_1(x)\, O_2(y)\} \,|\, h_1 \rangle \,,$$

  when the time of the operators is integrated?
- The practical solution is to integrate over a large subinterval in time $t_A \leq t_{x,y} \leq t_B$, but to create h_1 and to annihilate h_2 well outside of this region.
- This is the natural modification of standard field theory for which the asymptotic states are prepared at $t \to \pm\infty$ and then the operators are integrated over all time.
- This approach has been successfully implemented in all our projects.

(b) Exponentially growing exponentials



- Δm_K is given by

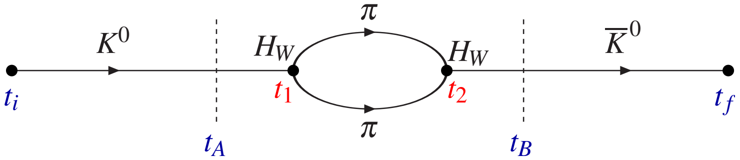
$$\Delta m_K \equiv m_{K_L} - m_{K_S} = 2\mathscr{P}\sum_\alpha \frac{\langle \bar{K}^0\,|\,\mathscr{H}_W\,|\,\alpha\rangle\,\langle\alpha\,|\,\mathscr{H}_W\,|\,K^0\rangle}{m_K - E_\alpha} = 3.483(6)\times 10^{-12}\,\mathrm{MeV}.$$

- The above correlation function gives ($T = t_B - t_A + 1$)

$$\begin{aligned} C_4(t_A, t_B; t_i, t_f) \quad &= \quad |Z_K|^2 e^{-m_K(t_f - t_i)} \sum_n \frac{\langle \bar{K}^0\,|\,\mathscr{H}_W\,|\,n\rangle\,\langle n\,|\,\mathscr{H}_W\,|\,K^0\rangle}{(m_K - E_n)^2} \times \\ &\qquad \left\{ e^{(m_K - E_n)T} - (m_K - E_n)T - 1 \right\}. \end{aligned}$$

- From the coefficient of T we can therefore obtain

$$\Delta m_K^{\mathrm{FV}} \equiv 2\sum_n \frac{\langle \bar{K}^0\,|\,\mathscr{H}_W\,|\,n\rangle\,\langle n\,|\,\mathscr{H}_W\,|\,K^0\rangle}{(m_K - E_n)}\,.$$



$$C_4(t_A,t_B;t_i,t_f) = |Z_K|^2 e^{-m_K(t_f-t_i)} \sum_n \frac{\langle \bar{K}^0\,|\,\mathscr{H}_W\,|\,n\rangle\,\langle n\,|\,\mathscr{H}_W\,|\,K^0\rangle}{(m_K-E_n)^2} \times \left\{ e^{(m_K-E_n)T} - (m_K-E_n)T - 1 \right\}.$$

- The presence of terms which (potentially) grow exponentially in T is a generic feature of calculations of matrix elements of bilocal operators.

- The message here is that the finite-volume corrections in terms of the $\pi\pi$ phase-shifts have been calculated.
  - This is an extension of the Luscher formalism for the spectrum and that of Lellouch and Luscher (and others) for matrix elements.
- The FV correction for the two-pion intermediate state is given by

N.H. Christ, X. Feng, G. Martinelli and C.T. Sachrajda, arXiv:1504.01170

$$\Delta m_K - \Delta m_K^{FV} = 2\mathscr{P}\int \mathrm{dE}\,\rho_V(E)\,\frac{f(E)}{m_K-E} - 2\sum_n \frac{f(E_n)}{m_K-E_n} = -2\left(f(m_K)\cot(h)\,\frac{\mathrm{dh}}{\mathrm{dE}}\right)_{E=m_K},$$

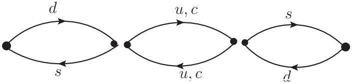
where

$$f(m_K) = {}_V\langle \bar{K}^0|H_W|(\pi\pi)_{E=m_K}\rangle_V\ {}_V\langle(\pi\pi)_{E=m_K}|H_W|K_0\rangle_V \quad \text{and} \quad h(k) = \delta(k)+\phi(k)\,.$$

$$\int d^4x \, \langle h_2 \,|\, T\{O_1(x)\, O_2(0)\} \,|\, h_1\rangle \,,$$

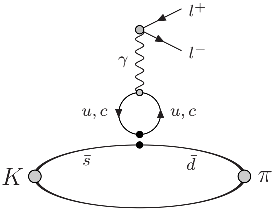
- The local operators $O_{1,2}$ are renormalised in a standard way, e.g. non-perturbatively into a RI-SMOM scheme & then perturbatively into the $\overline{\mathrm{MS}}$ scheme if appropriate.
- However, additional ultraviolet divergences may arise as $x \to 0$.
- This does not happen in two of our cases in the four-flavour theory:

1. Δm_K



• Taking the u-quark component of the operators $\Rightarrow$ a quadratic divergence.
• GIM mechanism & $V-A$ nature of the currents $\Rightarrow$ elimination of both quadratic and logarithmic divergences.

2 $K \to \pi \ell^+ \ell^-$ decays:

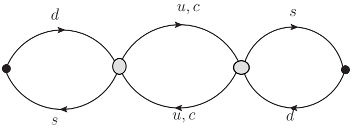


- Quadratic divergence is absent by gauge invariance $\Rightarrow$ Logarithmic divergence.
  - Checked explicitly for Wilson and Clover at one-loop order.
    G.Isidori, G.Martinelli and P.Turchetti, hep-lat/0506026
  - Absence of power divergences does not require GIM.
  - Logarithmic divergence cancelled by GIM.

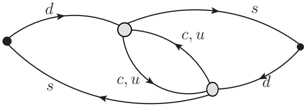
- Such an absence of additional divergences as $x \to 0$ is not generic and for example, is not the case for ε_K or for $K \to \pi \nu \bar{\nu}$ rare kaons decays.
- The formalism for performing the renormalisation in such cases was presented in
  N.H.Christ, Xu Feng, A.Portelli, CTS, arXiv:1605.04442.

RBC-UKQCD, **Z.Bai** et al.

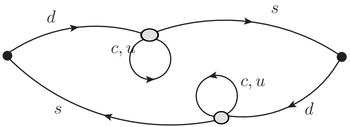
- There are four types of diagram to be evaluated:



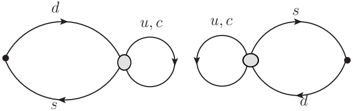
Type 1



Type 2

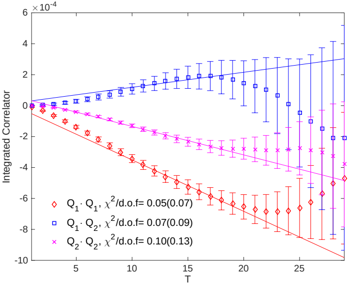


Type 3

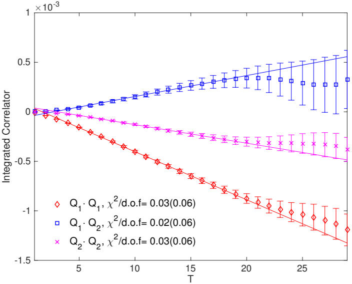


Type 4

- Following the development of the theoretical background and exploratory numerical studies, we presented the first numerical results at physical masses at Lattice 2017. Z.Bai, N.H.Christ, CTS; EPJ Web Conf 175 (2018) 13017.
- The calculation was performed on a $64^3 \times 128 \times 12$ lattice with Möbius DWF and the Iwasaki gauge action. a^-1=2.359(7) GeV, $m_\pi = 135.9(3)$ MeV and $m_K = 496.9(7)$ MeV. T.Blum et al., RBC-UKQCD Collabs., arXiv:1411.7017
  Charm-physics studies with this action $\Rightarrow am_c \simeq 0.32 - 0.33$. We have used $am_c \simeq 0.31$ and studied the dependence on m_c.



all diagrams



Type 1 & 2 diagrams only

- Lines here correspond to uncorrelated fits in the range $10 < T < 20$.

- We have performed the first non-perturbative calculations of the $K_L - K_S$ mass difference, now with physical quark masses.
- Our preliminary result based on an analysis of 59 configurations is

$$\Delta m_K = (5.5 \pm 1.7) \times 10^{-12}\,\mathrm{MeV}\,,$$

to be compared to the physical value

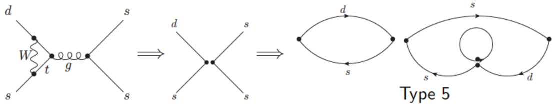
$$(\Delta m_K)^{\mathrm{phys}} = 3.483(6) \times 10^{-12}\,\mathrm{MeV}\,.$$

- We are finishing the present calculation by performing measurements on 160 configurations, aiming to reduce the uncertainty to about 1.0×10^{-12} MeV.
- Longer term, we are developing a strategy which will include an improved determination of Δm_K together with other elements of our kaon physics programme.

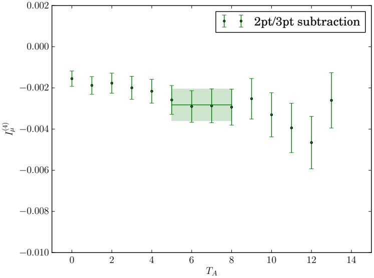
- In all cases the theoretical framework has been developed.
- For ε_K we have preliminary results obtained from 200 configurations on a $N_f = 2+1$ flavour ensemble using DWF and Iwasaki gauge action on a $24^3 \times 64 \times 16$ lattice with $a^{-1} = 1.78\,$GeV. Z.Bai, arXiv:1611.06601
  - The quark masses are unphysical, $m_\pi = 339\,$MeV, $m_K \simeq 592\,$MeV and $m_c^{\overline{\rm MS}}(2\,{\rm GeV}) = 968\,$MeV.
- Our preliminary result for the LD contribution at these unphysical masses is

$$\varepsilon_K^{\rm LD} = 0.11(0.08) \times 10^{-3}\,,$$

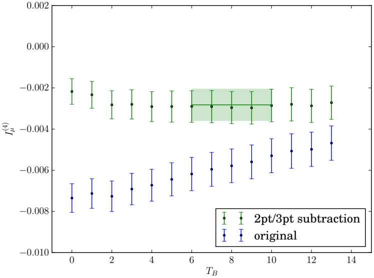
  whose central value is consistent with the expected 5% ($\varepsilon_K^{\rm Exp} = 2.228(11) \times 10^{-3}$).
- We need Im $M_{\bar{0}0} \Rightarrow t$-quark contributions not suppressed $\Rightarrow$ QCD penguin operators contribute and we have a Type 5 topology.



- We have published the results from a numerical study performed on a $24^3 \times 64$ DWF+Iwasaki RBC-UKQCD ensembles with $m_\pi \simeq 420$ MeV, $m_K \simeq 625$ MeV), $a^{-1} \simeq 1.78$ fm. N.Christ, X.Feng, A.Jüttner, A.Lawson, A.Portelli and CTS, arXiv:1608.07585
  - 128 configurations were used with $\vec{k} = \vec{0}$ and $\vec{p} =$(1,0,0), (1,1,0) and (1,1,1) in units of $2\pi/L$. With this kinematics we are in the unphysical region, $q^2 < 0$ and the charm quark is also lighter than physical $m_c^{\overline{\text{MS}}}(2\,\text{GeV}) \simeq 520$ MeV.



$$\int_{t_J - T_A}^{t_J+8} \tilde{\Gamma}_0^{(4)}\, dt_H$$



$$\int_{t_J - 6}^{t_J+T_B} \tilde{\Gamma}_0^{(4)}\, dt_H$$

$$A_0(q^2) = -0.0028(6).$$

- NA62 ($K^+ \to \pi^+ \nu \bar{\nu}$) and KOTO ($K_L \to \pi^0 \nu \bar{\nu}$) are beginning their experimental programme to study these decays. These FCNC processes provide ideal probes for the observation of new physics effects.
- The dominant contributions from the top quark $\Rightarrow$ they are also very sensitive to V_{ts} and V_{td}.
- Experimental results and bounds:

$$\mathrm{Br}(K^+ \to \pi^+ \nu \bar{\nu})_{\mathrm{exp}} = 1.73^{+1.15}_{-1.05} \times 10^{-10}$$

A.Artamonov et al. (E949), arXiv:0808.2459

$$\mathrm{Br}(K_L \to \pi^0 \nu \bar{\nu}) \leq 2.6 \times 10^{-8} \text{ at 90\% confidence level},$$

J.Ahn et al. (E291a), arXiv:0911.4789

- Sample recent theoretical predictions:

$$\begin{aligned} \mathrm{Br}(K^+ \to \pi^+ \nu \bar{\nu})_{\mathrm{SM}} &= (9.11 \pm 0.72) \times 10^{-11} \\ \mathrm{Br}(K_L \to \pi^0 \nu \bar{\nu})_{\mathrm{SM}} &= (3.00 \pm 0.30) \times 10^{-11}, \end{aligned}$$

A.Buras, D.Buttazzo, J.Girrbach-Noe, R.Knejgens, arXiv:1503.02693

- To what extent can lattice calculations reduce the theoretical uncertainty?

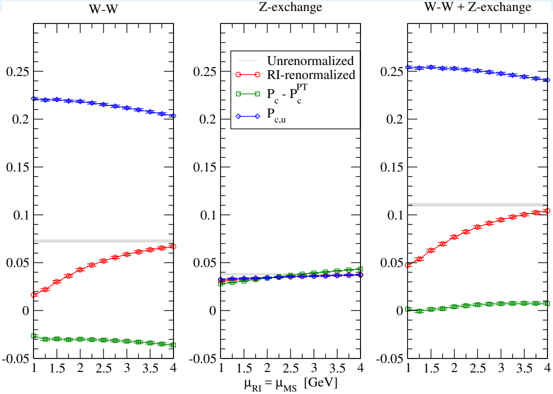
- To what extent can lattice calculations reduce the theoretical uncertainty?
- $K\to\pi\nu\bar\nu$ decays are SD dominated and the hadronic effects can be determined from CC semileptonic decays such as $K^+\to\pi^0e^+\nu$.
  - Lattice calculations of the $K_{\ell3}$ form factors are well advanced,

FLAG review, S.Aoki et al, arXiv:1607.00299

- LD contributions, i.e. contributions from distances greater than $1/m_c$ are negligible for K_L decays and are expected to be $O(5\%)$ for K^+ decays.
  - K_L decays are therefore one of the cleanest places to search for the effects of new physics.
  - The aim of our study is to compute the LD effects in K^+ decays. These provide a significant, if probably still subdominant, contribution to the theoretical uncertainty (which is dominated by the uncertainties in CKM matrix elements).
  - A phenomenological estimate of the long distance effects, estimated these to enhance the branching fraction by 6% with an uncertainty of 3%.

G.Isidori, F.Mescia and C.Smith, hep-ph/0503107

- Lattice QCD can provide a first-principles determination of the LD contribution with controlled errors.
  - Given the NA62 experiment, it is timely to perform a lattice QCD calculation of these effects.



- Details of simulation: 800 configs on a $16^3 \times 32$ lattice with $N_f = 2+1$ DWF, $a^{-1} \simeq 1.73\,\text{GeV}$, $m_\pi \simeq 420\,\text{MeV}$, $m_K \simeq 563\,\text{MeV}$ and $m_c^{\overline{\text{MS}}}(2\,\text{GeV}) \simeq 863\,\text{MeV}$.
- For this unphysical kinematics, we find

$$P_c = 0.2529(\pm 13)(\pm 32)(-45) \quad \text{and} \quad \Delta P_c = 0.0040(\pm 13)(\pm 32)(-45)\,.$$

- Large cancellation between WW and Z-exchange contributions.

- In this talk I have presented some of the exciting physics which is beginning to be done using lattice simulations.
- This builds on the enormous improvement in precision in the evaluation of standard quantities, which has been made in the last 10 years or so.
- This precision is such that isospin breaking effects (including electromagnetism) must be included if further progress in determining the CKM matrix elements is to be made. This is underway for leptonic decays and is being developed for semileptonic decays.
- The RBC-UKQCD collaboration has also demonstrated that $K \to \pi\pi$ decays are amenable to lattice computations and have calculated both the real and imaginary parts of A_2 and A_0 (and hence ε'/ε).
  - The priority now is to reduce the errors on A_0 and to consolidate the result for ε'/ε.
- The theoretical framework for evaluating long-distance contributions has been developed by RBC-UKQCD collaboration and is being applied to the evaluation of Δm_K, ε_K and the rare kaon decays $K \to \pi\ell^+\ell^-$ and $K^+ \to \pi^+\nu\bar{\nu}$.

- So much more to be done!

1. *QED Corrections to Hadronic Processes in Lattice QCD,*
   N.Carrasco, V.Lubicz, G.Martinelli, C.T.Sachrajda, N.Tantalo, C.Tarantino and M.Testa,
   Phys. Rev. D **91** (2015) no.7, 07450 [arXiv:1502.00257 [hep-lat]].
2. *Finite-Volume QED Corrections to Decay Amplitudes in Lattice QCD,*
   V.Lubicz, G.Martinelli, C.T.Sachrajda, F.Sanfilippo, S.Simula and N.Tantalo,
   Phys. Rev. D **95** (2017) no.3, 034504 [arXiv:1611.08497 [hep-lat]].
3. *First Lattice Calculation of the QED Corrections to Leptonic Decay Rates,*
   D.Giusti, V.Lubicz, G.Martinelli, C.T.Sachrajda, F.Sanfilippo, S.Simula, N.Tantalo and C.Tarantino,
   Phys. Rev. Lett. **120** (2018) 072001 [arXiv:1711:06537]
4. *Radiative corrections to decay amplitudes in lattice QCD,*
   D.Giusti, V.Lubicz, G.Martinelli, C.T.Sachrajda, F.Sanfilippo, S.Simula and N.Tantalo,
   arXiv:1811.06364 [hep-lat].

1. A_0 and A_2 amplitudes with unphysical quark masses and with the pions at rest.

   "K to $\pi\pi$ decay amplitudes from lattice QCD,"
   T.Blum, P.A.Boyle, N.H.Christ, N.Garron, E.Goode, T.Izubuchi, C.Lehner, Q.Liu, R.D. Mawhinney, C.T.S, A.Soni, C.Sturm, H.Yin and R. Zhou, Phys. Rev. D **84** (2011) 114503 [arXiv:1106.2714 [hep-lat]].

   "Kaon to two pions decay from lattice QCD, $\Delta I = 1/2$ rule and CP violation"
   Q.Liu, Ph.D. thesis, Columbia University (2010)

2. A_2 at physical kinematics and a single coarse lattice spacing.

   "The $K \to (\pi\pi)_{I=2}$ Decay Amplitude from Lattice QCD,"
   T.Blum, P.A.Boyle, N.H.Christ, N.Garron, E.Goode, T.Izubuchi, C.Jung, C.Kelly, C.Lehner, M.Lightman, Q.Liu, A.T.Lytle, R.D.Mawhinney, C.T.S., A.Soni, and C.Sturm
   Phys. Rev. Lett. **108** (2012) 141601 [arXiv:1111.1699 [hep-lat]],

   "Lattice determination of the $K \to (\pi\pi)_{I=2}$ Decay Amplitude A_2"
   Phys. Rev. D **86** (2012) 074513 [arXiv:1206.5142 [hep-lat]]

   "Emerging understanding of the $\Delta I = 1/2$ Rule from Lattice QCD,"
   P.A. Boyle, N.H. Christ, N. Garron, E.J. Goode, T. Janowski, C. Lehner, Q. Liu, A.T. Lytle, C.T. Sachrajda, A. Soni, and D.Zhang, Phys. Rev. Lett. **110** (2013) 15, 152001 [arXiv:1212.1474 [hep-lat]].

3. A_2 at physical kinematics on two finer lattices $\Rightarrow$ continuum limit taken.
   "$K \to \pi\pi\ \Delta I = 3/2$ decay amplitude in the continuum limit,"
   T.Blum, P.A.Boyle, N.H.Christ, J.Frison, N.Garron, T.Janowski, C.Jung, C.Kelly, C.Lehner, A.Lytle, R.D.Mawhinney, C.T.S., A.Soni, H.Yin, and D.Zhang
   Phys. Rev. D **91** (2015) 7, 074502 [arXiv:1502.00263 [hep-lat]].
4. A_0 at physical kinematics and a single coarse lattice spacing.
   "Standard-model prediction for direct CP violation in $K \to \pi\pi$ decay,"
   Z.Bai, T.Blum, P.A.Boyle, N.H.Christ, J.Frison, N.Garron, T.Izubuchi, C.Jung, C.Kelly, C.Lehner, R.D.Mawhinney, C.T.S, A. Soni, and D. Zhang,
   Phys. Rev. Lett. **115** (2015) 21, 212001 [arXiv:1505.07863 [hep-lat]].

See also: "Calculation of ε'/ε on the lattice" C.Kelly et al. PoS FPCP2016 (2017) 017

1. "Long-distance contribution to the $K_L - K_S$ mass difference,"
   N.H. Christ, T. Izubuchi, CTS, A. Soni and J. Yu,
   Phys.Rev. D88 (2013) 014508 (arXiv:1212.5931)
2. "$K_L - K_S$ mass difference from Lattice QCD,"
   Z. Bai, N.H. Christ, T.Izubuchi, CTS, A.Soni and J. Yu,
   Phys.Rev.Lett. 113 (2014) 112003 (arXiv:1406.0916)
3. "Neutral Kaon Mixing from Lattice QCD"
   Z.Bai, Columbia University Thesis (2017)
4. "The K_L-K_S mass difference"
   Z.Bai, N.H.Christ and CTS, Proceedings of the 2017 International Symposium on Lattice Field Theory.
   EPJ Web Conf 175 (2018) 13017

1. "Prospects for a lattice computation of rare kaon decay amplitudes: I, $K \to \pi\ell^+\ell^-$ decays"
   N.H.Christ, X.Feng, A.Portelli and CTS Phys.Rev. D. **92** (2015) 094512 [arXiv:1507.03094]
2. "Prospects for a lattice computation of rare kaon decay amplitudes: II, $K \to \pi\nu\bar{\nu}$ decays"
   N.H.Christ, X.Feng, A.Portelli and CTS Phys.Rev. D **93** (2016) 114517 [arXiv:1605:04442]
3. "First exploratory calculation of the long distance contributions to the rare kaon decay $K \to \pi\ell^+\ell^-$"
   N.H.Christ, X.Feng, A.Jüttner, A.Lawson, A.Portelli and CTS
   Phys.Rev. D **94** (2016) 114516 [arXiv:1608.07585]
4. "Exploratory lattice QCD study of the rare kaon decay $K^+ \to \pi^+\nu\bar{\nu}$"
   Z.Bai, N.H.Christ, X.Feng, A.Jüttner, A.Lawson, A.Portelli and CTS
   Phys.Rev.Lett. **118** (2017) 252001 [arXiv:1701.02858]
5. "$K^+ \to \pi^+\nu\bar{\nu}$ decay amplitude from lattice QCD"
   Z.Bai, N.H.Christ, X.Feng, A.Jüttner, A.Lawson, A.Portelli and CTS
   Phys.Rev. D **98** (2018) 074509 [arXiv:1806.11520]