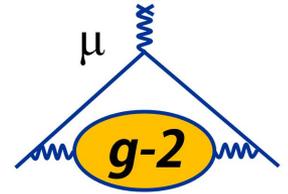


$g-2$ of the muon and electron: anomalous anomalies



Thomas Teubner



- Introduction
- a_μ vs a_e : a new puzzle?
- a_μ^{SM} : overview and update on hadronic contributions
- BSM

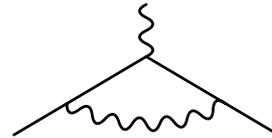
Introduction

- Dirac equation (1928): g is 2 for fundamental fermions $\vec{\mu} = g \frac{Qe}{2m} \vec{s}$
- 1947: small deviations from predictions in hydrogen and deuterium hyperfine structure; Kusch & Foley propose explanation with $g_s = 2.00229 \pm 0.00008$

- 1948: Schwinger calculates the famous radiative correction:

that $g = 2(1+a)$, with

$$a = (g-2)/2 = \alpha/(2\pi) = 0.001161$$



“If you can’t join ‘em, beat ‘em”

This explained the discrepancy and was a crucial step in the development of perturbative QFT and QED

- The anomaly a (Anomalous Magnetic Moment) is from the Pauli term:

$$\delta \mathcal{L}_{\text{eff}}^{\text{AMM}} = -\frac{Qe}{4m} a \bar{\psi}(x) \sigma^{\mu\nu} \psi(x) F_{\mu\nu}(x)$$

This is a dimension 5 operator, non-renormalisable and hence not part of the fundamental (QED) Lagrangian. But it occurs through radiative corrections and is calculable in perturbation theory.

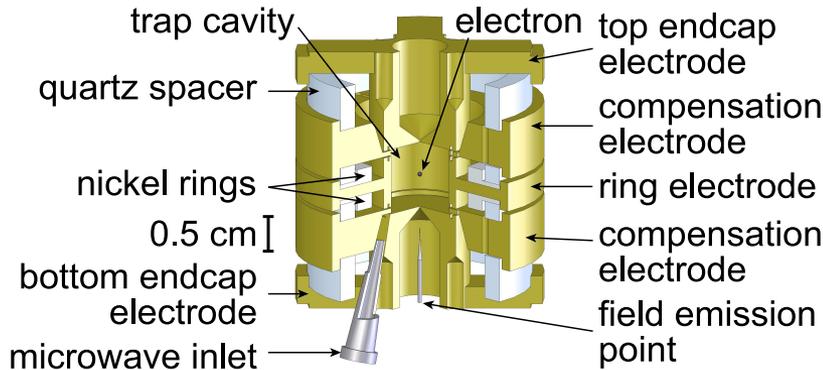
Magnetic Moments: a_e vs. a_μ

$$a_e = 1\,159\,652\,180.73 (0.28) \cdot 10^{-12} \quad [0.24\text{ppb}]$$

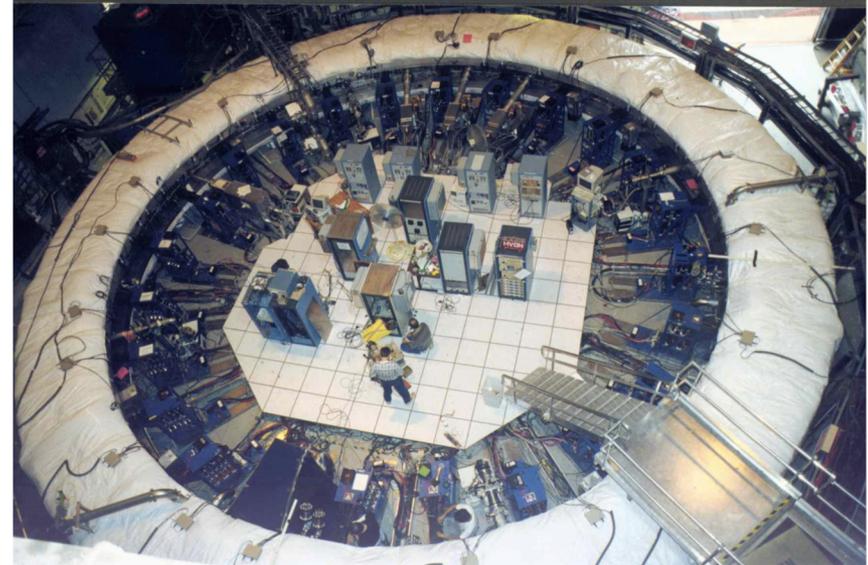
Hanneke, Fogwell, Gabrielse, PRL 100(2008)120801

$$a_\mu = 116\,592\,089(63) \cdot 10^{-11} \quad [0.54\text{ppm}]$$

Bennet et al., PRD 73(2006)072003



one electron quantum cyclotron



- a_e^{EXP} more than 2000 times more precise than a_μ^{EXP} , but for e^- loop contributions come from very small photon virtualities, whereas muon `tests' higher scales
 - dimensional analysis: **sensitivity to NP** (at high scale Λ_{NP}): $a_\ell^{\text{NP}} \sim C m_\ell^2 / \Lambda_{\text{NP}}^2$
- μ wins by $m_\mu^2 / m_e^2 \sim 43000$ for NP, but a_e determines α , tests QED & low scales

Magnetic Moments: a_e^{SM} before very recent shift of α

- General structure: $a_e^{\text{SM}} = a_e^{\text{QED}} + a_e^{\text{hadronic}} + a_e^{\text{weak}}$
- Weak and hadronic contributions suppressed as induced by particles heavy compared to electron, hence a_e^{SM} dominated by QED;
including 5-loop QED and using α measured with Rubidium atoms [α to 0.66 ppb]
[Bouchendir et al., PRL106(2011)080801; Mohr et al., CODATA, Rev Mod Phys 84(2012)1527]

$$a_e^{\text{SM}} = 1\,159\,652\,182.03(72) \times 10^{-12} \quad [\text{Aoyama+Kinoshita+Nio, PRD 97(2018)036001}]$$

small shift from81.78(77) after 2018 update of multi-loop numerics

compared to $a_e^{\text{EXP}} = 1\,159\,652\,180.73(0.28) \times 10^{-12} \rightarrow$ test of QED & low scales physics just o.k.

- Of this only about

$$a_e^{\text{had, LO VP}} = 1.875(18) \times 10^{-12} \quad [\text{or our newer } 1.866(11) \times 10^{-12}]$$

$$a_e^{\text{had, NLO VP}} = -0.225(5) \times 10^{-12} \quad [\text{or our newer } -0.223(1) \times 10^{-12}]$$

$$a_e^{\text{had, L-by-L}} = 0.035(10) \times 10^{-12}$$

$$a_e^{\text{weak}} = 0.0297(5) \times 10^{-12},$$

whose calculations are a byproduct of the μ case which I will discuss in more detail.

- In turn a_e^{EXP} and a_e^{SM} can be used to get a very precise determination of α , to 0.25 ppb, consistent with Rubidium experiments and other determinations.

Magnetic Moments: a_e^{SM} with the recent shift of α

- General structure: $a_e^{\text{SM}} = a_e^{\text{QED}} + a_e^{\text{hadronic}} + a_e^{\text{weak}}$
- $a_e^{\text{SM}} = 1\,159\,652\,182.03(72) \times 10^{-12}$ [Aoyama+Kinoshita+Nio, PRD 97(2018)036001]
small shift from81.78(77) after 2018 update of numerics
using α measured with Rubidium atoms [α to 0.66 ppb]
- is, **due to a new α measurement with Cs-133 atoms** [Parker et al., Science 360 (2018) 191],
now more precise [α to 2×10^{-10} !] and shifted down to

$$a_e^{\text{SM}} = 1\,159\,652\,181.61(23) \times 10^{-12}$$

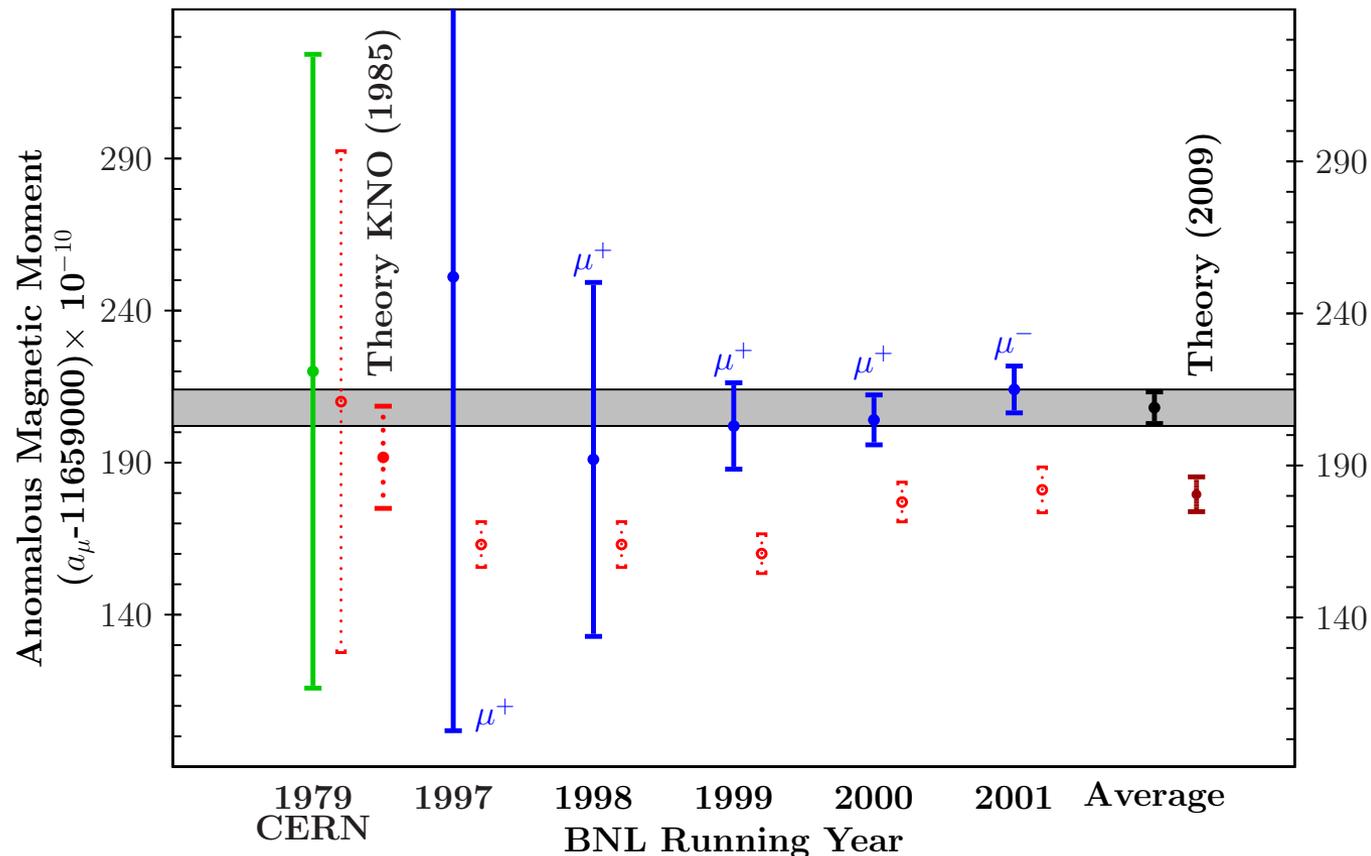
- Comparison with the experimental measurement now gives a
-2.5 σ discrepancy for a_e : $\Delta a_e = a_e^{\text{EXP}} - a_e^{\text{SM}} = -0.88(36) \times 10^{-12}$
- which one may consider together with the muon g-2 discrepancy when discussing possible New Physics contributions

a_μ : back to the future

- CERN started it nearly 40 years ago
- Brookhaven delivered 0.5ppm precision
- E989 at FNAL and J-PARC's g-2/EDM experiments are happening and should give us certainty

g-2 history plot and motto from Fred Jegerlehner's book:

'The closer you look the more there is to see'

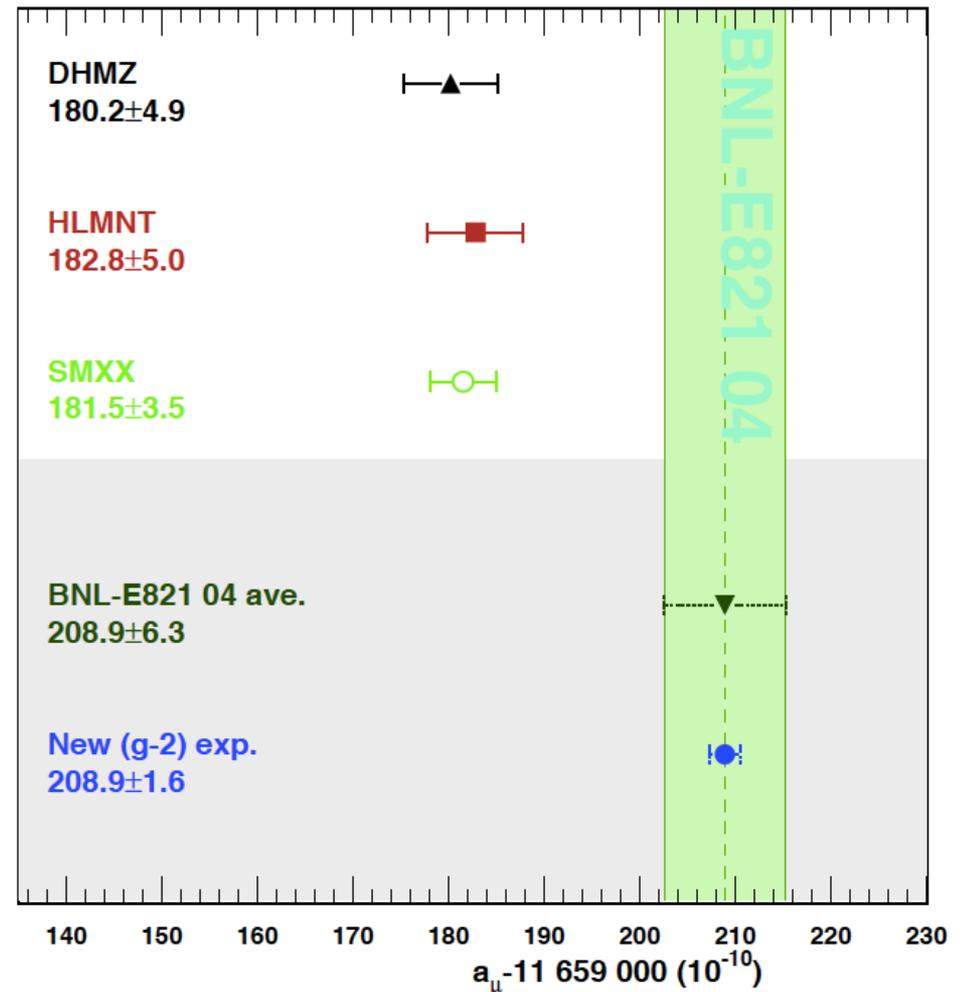


a_μ : Status and future projection → charge for SM TH

$$a_\mu = a_\mu^{\text{QED}} + a_\mu^{\text{EW}} + a_\mu^{\text{hadronic}} + a_\mu^{\text{NP?}}$$

From: arXiv:1311.2198
'The Muon (g-2) Theory Value:
Present and Future'

- if mean values stay and with **no** a_μ^{SM} improvement:
5 σ discrepancy
- if also EXP+TH can improve a_μ^{SM}
'as expected' (consolidation of L-by-L on level of Glasgow consensus, about factor 2 for HVP): NP at 7-8 σ
- or, if mean values get closer, very strong exclusion limits on many NP models (extra dims, new dark sector, xxxSSSM)...



“Muon g-2 theory initiative”, formed in June 2017

for latest June 2018 workshop see: <https://indico.him.uni-mainz.de/event/11/overview>



“map out strategies for obtaining the **best theoretical predictions for these hadronic corrections** in advance of the experimental results”

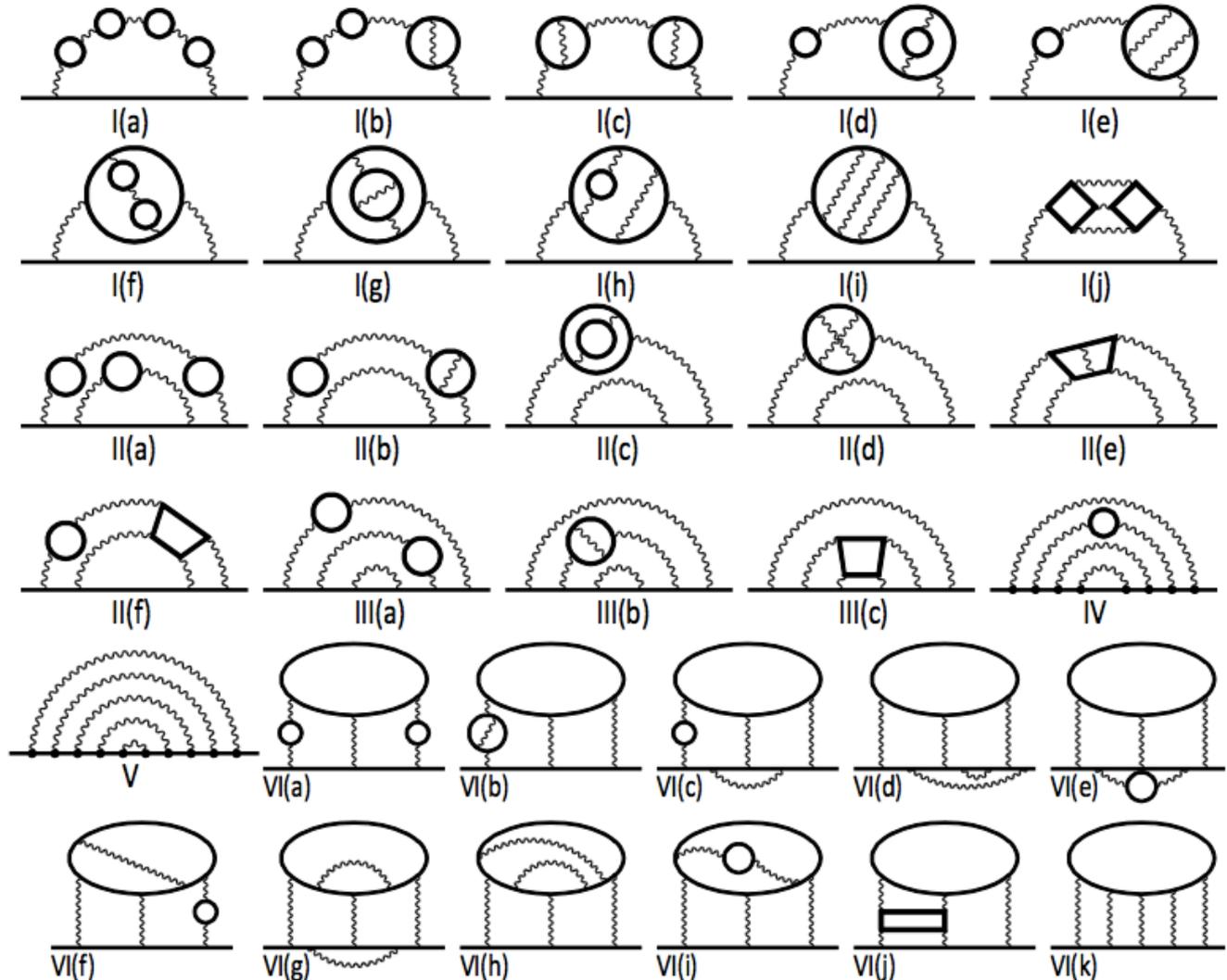
The muon $g - 2$ and $\alpha(M_Z^2)$: a new data-based analysisAlexander Keshavarzi^a, Daisuke Nomura^{b,c} and Thomas Teubner^d^a*Department of Mathematical Sciences, University of Liverpool, Liverpool L69 3BX, U.K.*
Email: a.i.keshavarzi@liverpool.ac.uk^b*KEK Theory Center, Tsukuba, Ibaraki 305-0801, Japan*^c*Yukawa Institute for Theoretical Physics, Kyoto University, Kyoto 606-8502, Japan*
Email: dnomura@post.kek.jp^d*Department of Mathematical Sciences, University of Liverpool, Liverpool L69 3BX, U.K.*
*Email: thomas.teubner@liverpool.ac.uk***Abstract**

This work presents a complete re-evaluation of the hadronic vacuum polarisation contributions to the anomalous magnetic moment of the muon, $a_\mu^{\text{had, VP}}$ and the hadronic contributions to the effective QED coupling at the mass of the Z boson, $\Delta\alpha_{\text{had}}(M_Z^2)$, from the combination of $e^+e^- \rightarrow$ hadrons cross section data. Focus has been placed on the development of a new data combination method, which fully incorporates all correlated statistical and systematic uncertainties in a bias free approach. All available $e^+e^- \rightarrow$ hadrons cross section data have been analysed and included, where the new data compilation has yielded the full hadronic R -ratio and its covariance matrix in the energy range $m_\pi \leq \sqrt{s} \leq 11.2$ GeV. Using these combined data and pQCD above that range results in estimates of the hadronic vacuum polarisation contributions to $g - 2$ of the muon of $a_\mu^{\text{had, LO VP}} = (693.27 \pm 2.46) \times 10^{-10}$ and $a_\mu^{\text{had, NLO VP}} = (-9.82 \pm 0.04) \times 10^{-10}$. The new estimate for the Standard Model prediction is found to be $a_\mu^{\text{SM}} = (11\,659\,182.05 \pm 3.56) \times 10^{-10}$, which is 3.7σ below the current experimental measurement. The prediction for the five-flavour hadronic contribution to the QED coupling at the Z boson mass is $\Delta\alpha_{\text{had}}^{(5)}(M_Z^2) = (276.11 \pm 1.11) \times 10^{-4}$, resulting in $\alpha^{-1}(M_Z^2) = 128.946 \pm 0.015$. Detailed comparisons with results from similar related works are given.

T. Aoyama, M. Hayakawa,
T. Kinoshita, M. Nio (PRLs, 2012)

A triumph for perturbative QFT and computing!

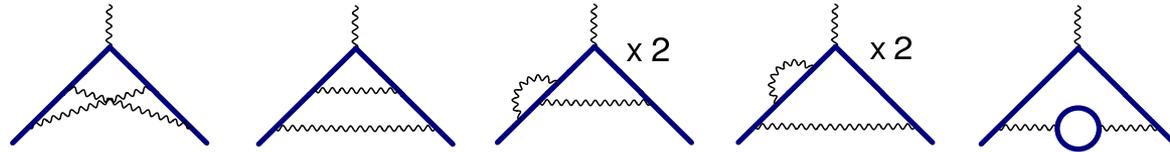
10th
12672
diagrams



- code-generating code, including renormalisation
- multi-dim. numerical integrations

- Schwinger 1948: 1-loop $a = (g-2)/2 = \alpha/(2\pi) = 116\,140\,970 \times 10^{-11}$

- 2-loop graphs:



- 72 3-loop and 891 4-loop diagrams ...

- Kinoshita et al. 2012: 5-loop completed numerically (12672 diagrams):

$$a_\mu^{\text{QED}} = 116\,584\,718.951 (0.009) (0.019) (0.007) (0.077) \times 10^{-11}$$

errors from: lepton masses, 4-loop, 5-loop, α from ^{87}Rb

- QED extremely accurate, and the series is stable: $a_\mu^{\text{QED}} = C_\mu^{2n} \sum_n \left(\frac{\alpha}{\pi}\right)^n$

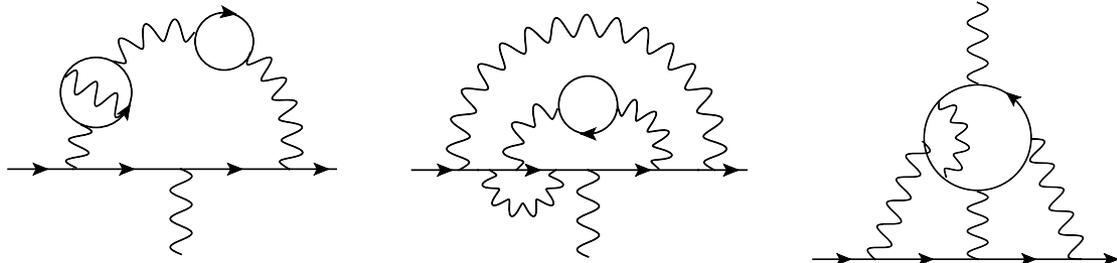
$$C_\mu^{2,4,6,8,10} = 0.5, 0.765857425(17), 24.05050996(32), 130.8796(63), 753.29(1.04)$$

$$\text{contr. to } a_\mu \approx 1 \times 10^{-3}, \quad 4 \times 10^{-6}, \quad 3 \times 10^{-7}, \quad 4 \times 10^{-9}, \quad 5 \times 10^{-11}$$

- Could a_μ^{QED} still be wrong?

Some classes of graphs known analytically ([Laporta](#); [Aguilar, Greynat, deRafael](#)),

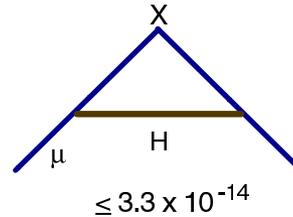
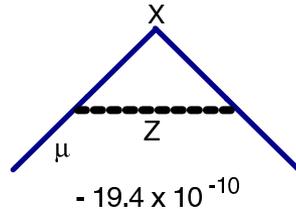
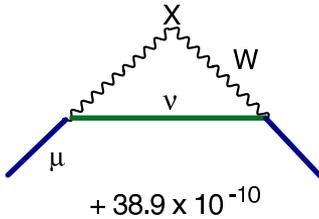
- ... but 4-loop and 5-loop rely heavily on numerical integrations
- Recently several independent checks of 4-loop and 5-loop diagrams:
Baikov, Maier, Marquard [NPB 877 (2013) 647], Kurz, Liu, Marquard, Smirnov AV+VA, Steinhauser [NPB 879 (2014) 1, PRD 92 (2015) 073019, 93 (2016) 053017]:
- all 4-loop graphs with internal lepton loops now calculated independently, e.g.



(from Steinhauser et al., PRD 93 (2016) 053017)

- 4-loop universal (massless) term calculated semi-analytically to 1100 digits (!) by Laporta, PLB772(2017)232, also recent numerical results by Volkov, PRD96(2017)096018
- all agree with Kinoshita et al.'s results, so QED is on safe ground ✓

- Electro-Weak 1-loop diagrams:



$$a_\mu^{\text{EW}(1)} = 195 \times 10^{-11}$$

- known to 2-loop (1650 diagrams, the first full EW 2-loop calculation):
[Czarnecki, Krause, Marciano, Vainshtein](#); [Knecht, Peris, Perrottet, de Rafael](#)
- agreement, a_μ^{EW} relatively small, 2-loop relevant: $a_\mu^{\text{EW}(1+2 \text{ loop})} = (154 \pm 2) \times 10^{-11}$
- with Higgs mass now known, updated by [Gnendiger, Stoeckinger, S-Kim](#),

PRD 88 (2013) 053005

$$a_\mu^{\text{EW}(1+2 \text{ loop})} = (153.6 \pm 1.0) \times 10^{-11} \quad \checkmark$$

- very recent numerical 2-loop EW result, based on GRACE-FORM packages, avoiding the heavy mass expansion used previously:

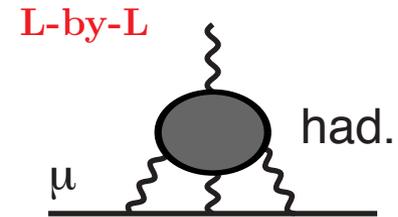
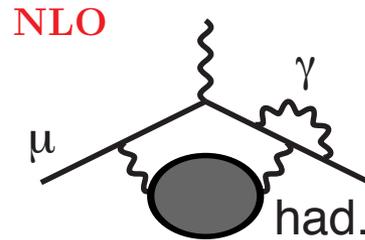
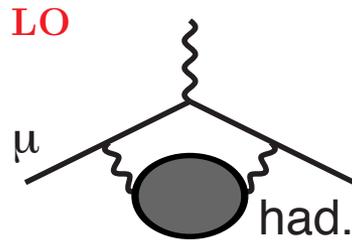
[Ishikawa, Nakazawa, Yasui](#), arXiv:1810.13445

weak 2-loop: $-41.2 (1.0) \rightarrow (-38.6 \pm 1.0) \times 10^{-11}$, i.e. slight shift up by 6%

Compare with $a_\mu^{\text{QED}} = 116\,584\,718.951 (80) \times 10^{-11}$

- Hadronic: **non-perturbative**, the limiting factor of the SM prediction? ~~X~~ → ✓

$$a_\mu^{\text{had}} = a_\mu^{\text{had,VP LO}} + a_\mu^{\text{had,VP NLO}} + a_\mu^{\text{had,Light-by-Light}}$$

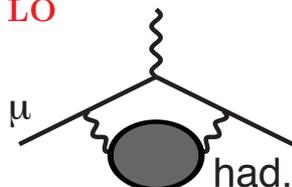


a_μ^{hadronic} : L-by-L one-page summary

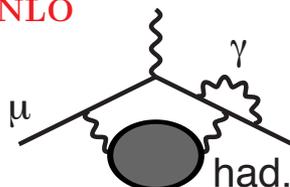
- Hadronic: **non-perturbative**, the limiting factor of the SM prediction $\times \rightarrow \checkmark$

$$a_\mu^{\text{had}} = a_\mu^{\text{had,VP LO}} + a_\mu^{\text{had,VP NLO}} + a_\mu^{\text{had,Light-by-Light}}$$

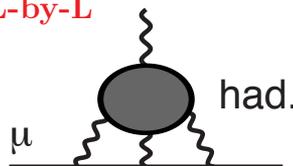
LO



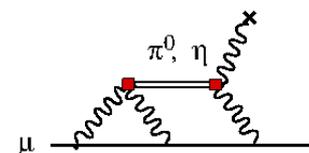
NLO



L-by-L



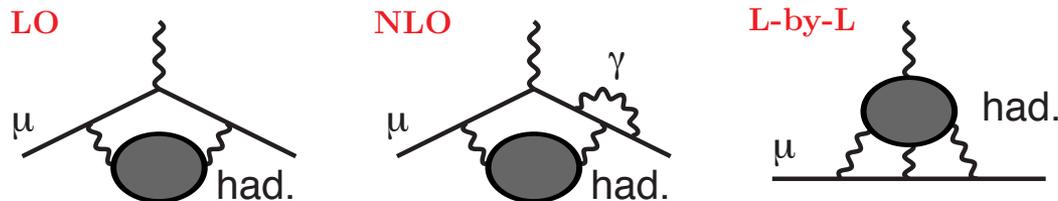
e.g.



- L-by-L**: - so far use of **model calculations** (+ form-factor data and pQCD constraints),
 - but very good news from **lattice QCD**, and
 - from new **dispersive** approaches
- For the moment, still use the '**updated Glasgow consensus**':
 (original by Prades+deRafael+Vainshtein) $a_\mu^{\text{had,L-by-L}} = (98 \pm 26) \times 10^{-11}$
- But first results from new approaches confirm existing model predictions and
- indicate that L-by-L prediction will be improved further soon
- with new results & progress, tell politicians/sceptics: L-by-L can be predicted!**

$a_\mu^{\text{had, VP}}$: Hadronic Vacuum Polarisation

$$a_\mu^{\text{had}} = a_\mu^{\text{had, VP LO}} + a_\mu^{\text{had, VP NLO}} + a_\mu^{\text{had, Light-by-Light}}$$

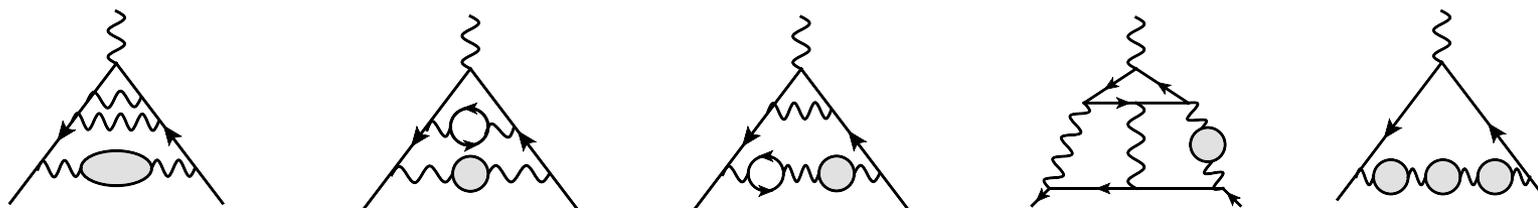


HVP: - most precise prediction by using e^+e^- hadronic cross section (+ tau) data and well known dispersion integrals

- done at LO and NLO (see graphs)

- and recently at NNLO [Steinhauser et al., PLB 734 (2014) 144, also F. Jegerlehner]

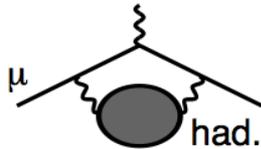
$a_\mu^{\text{HVP, NNLO}} = + 1.24 \times 10^{-10}$ not so small, from e.g.:



- Alternative: lattice QCD, but need QED and iso-spin breaking corrections.
Lots of activity by several groups, errors coming down, QCD+QED started.

Hadronic Vacuum Polarisation, essentials:

Use of data compilation for HVP:



pQCD not useful. Use the **dispersion relation** and the **optical theorem**.

$$\text{had.} = \int \frac{ds}{\pi(s-q^2)} \text{Im had.}$$

$$2 \text{Im had.} = \sum_{\text{had.}} \int d\Phi \left| \text{had.} \right|^2$$

$$a_{\mu}^{\text{had,LO}} = \frac{m_{\mu}^2}{12\pi^3} \int_{s_{\text{th}}}^{\infty} ds \frac{1}{s} \hat{K}(s) \sigma_{\text{had}}(s)$$

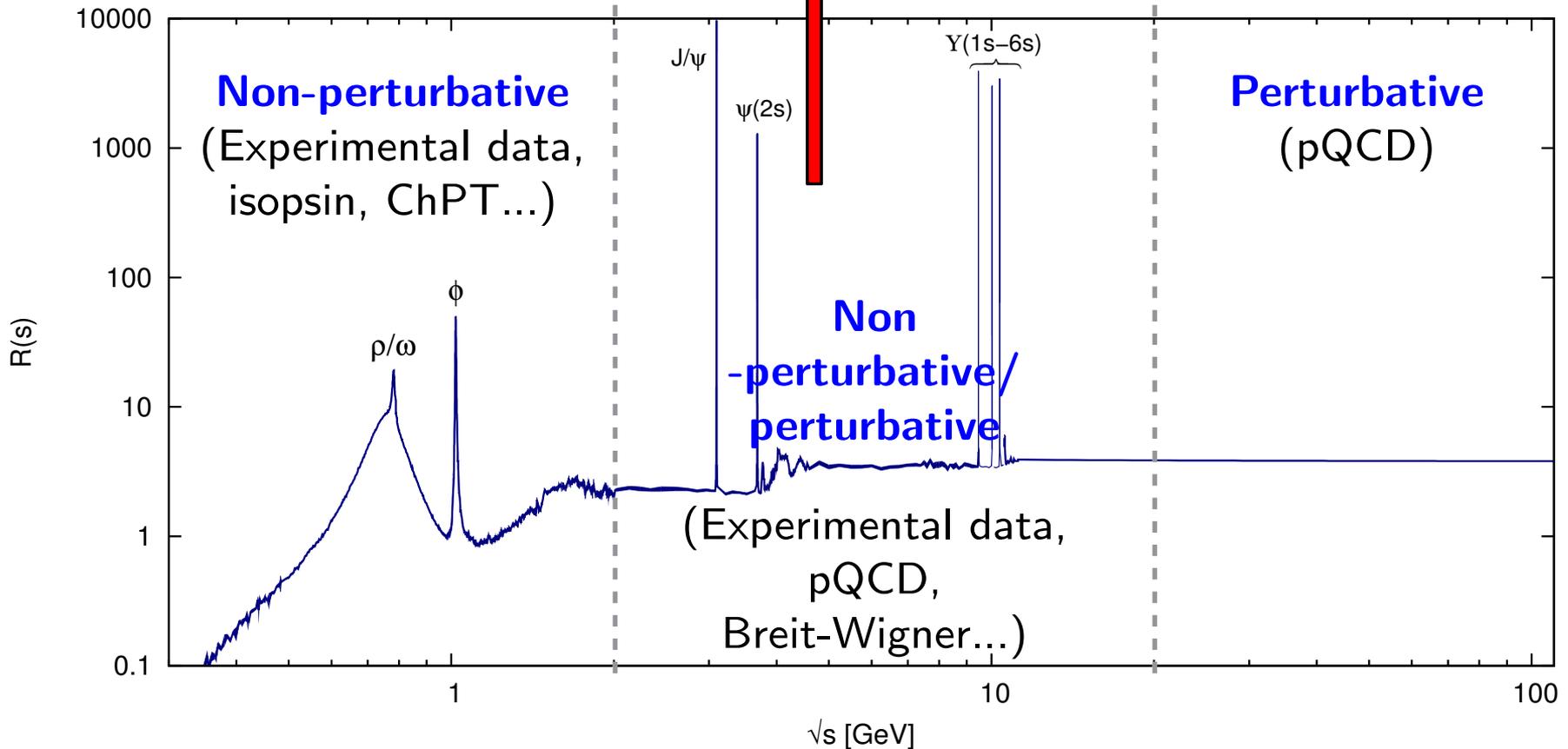
- Weight function $\hat{K}(s)/s = \mathcal{O}(1)/s$
 \Rightarrow **Lower** energies **more important**
 $\Rightarrow \pi^+\pi^-$ channel: 73% of total $a_{\mu}^{\text{had,LO}}$

How to get the most precise σ_{had}^0 ? **e^+e^- data:**

- Low energies: **sum ~35 exclusive channels**, $2\pi, 3\pi, 4\pi, 5\pi, 6\pi, KK, KK\pi, KK\pi\pi, \eta\pi, \dots$, [use iso-spin relations for missing channels]
- Above ~ 1.8 GeV: can start to use **pQCD** (away from flavour thresholds), supplemented by narrow resonances ($J/\psi, \Upsilon$)
- Challenge of **data combination (locally in \sqrt{s})**: many experiments, different energy bins, stat+sys errors from different sources, **correlations**; must avoid **inconsistencies/bias**
- traditional '**direct scan**' (tunable e^+e^- beams) vs. '**Radiative Return**' [+ τ spectral functions]
- σ_{had}^0 means 'bare' σ , but WITH FSR: **RadCorrs** [HLMNT '11: $\delta a_{\mu}^{\text{had, RadCor VP+FSR}} = 2 \times 10^{-10}$!]

HVP cross section input

$$a_{\mu}^{\text{had, LO VP}} = \frac{\alpha^2}{3\pi^2} \int_{s_{th}}^{\infty} \frac{ds}{s} R(s) K(s), \text{ where } R(s) = \frac{\sigma_{\text{had},\gamma}^0(s)}{4\pi\alpha^2/3s}$$

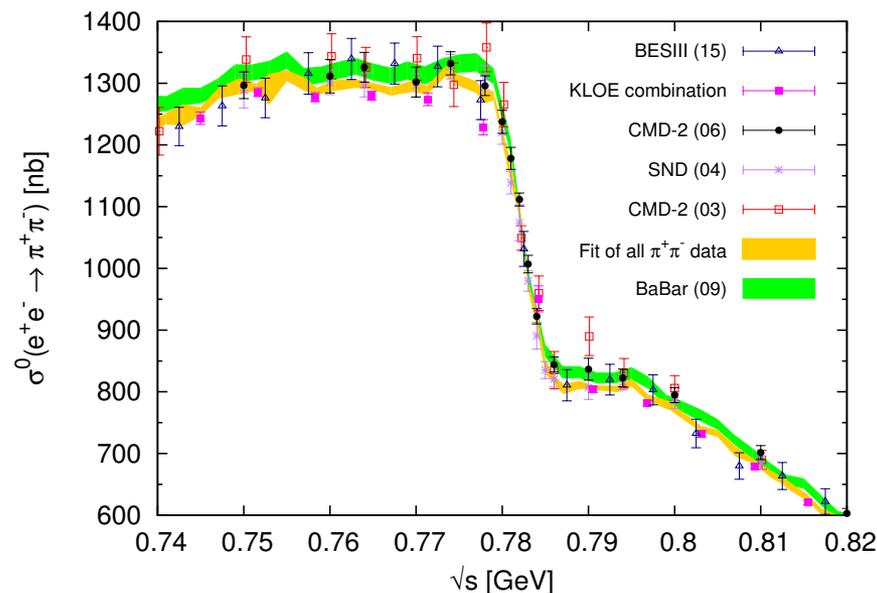
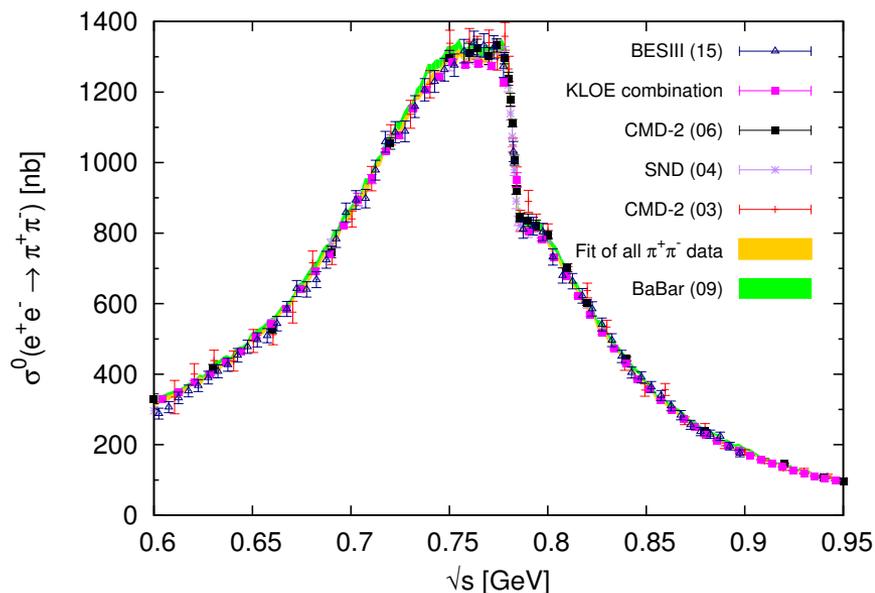


Must build full hadronic cross section/ R -ratio...

HVP: $\pi^+\pi^-$ channel [KNT18, PRD97, 114025]

$\Rightarrow \pi^+\pi^-$ accounts for over 70% of $a_\mu^{\text{had, LO VP}}$

\rightarrow Combines 30 measurements totalling nearly 1000 data points



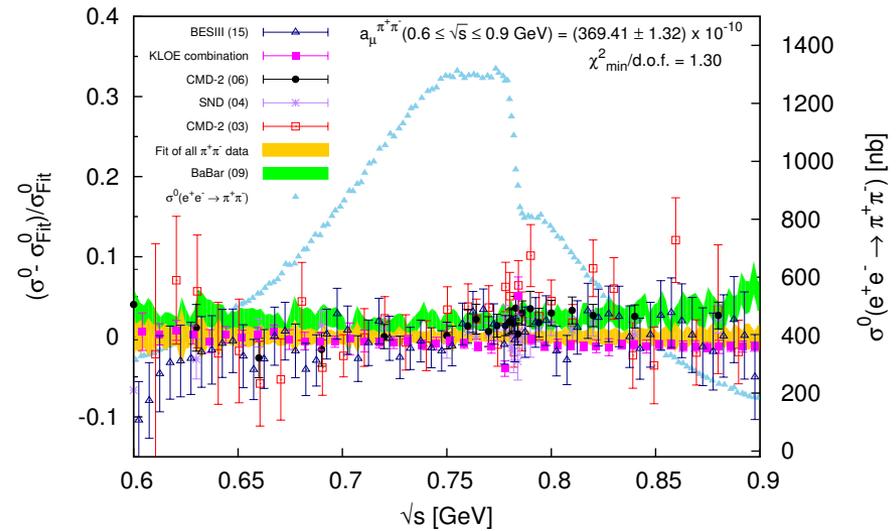
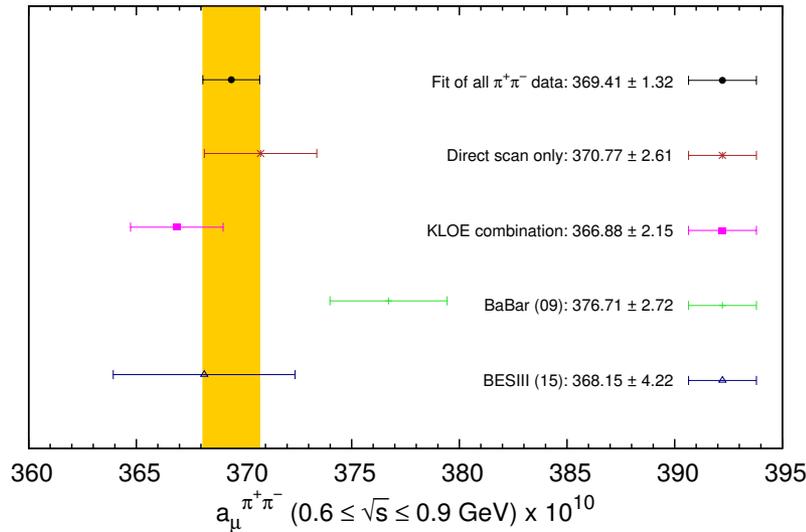
\Rightarrow Correlated & experimentally corrected $\sigma_{\pi\pi(\gamma)}^0$ data now entirely dominant

$$a_\mu^{\pi^+\pi^-} [0.305 \leq \sqrt{s} \leq 1.937 \text{ GeV}] = 502.97 \pm 1.14_{\text{stat}} \pm 1.59_{\text{sys}} \pm 0.06_{\text{vp}} \pm 0.14_{\text{fsr}}$$
$$= 502.97 \pm 1.97_{\text{tot}} \quad \text{HLMNT11: } 505.77 \pm 3.09$$

\Rightarrow 15% local $\chi_{\text{min}}^2/\text{d.o.f.}$ error inflation due to tensions in clustered data

HVP: $\pi^+\pi^-$ channel [KNT18, PRD97, 114025]

- ⇒ Tension exists between BaBar data and all other data in the dominant ρ region.
- Agreement between other radiative return measurements and direct scan data largely compensates this.

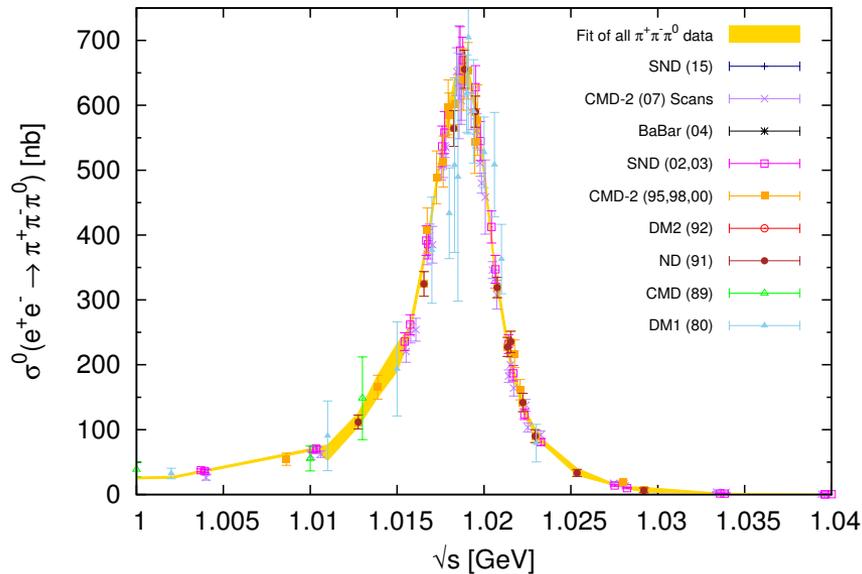
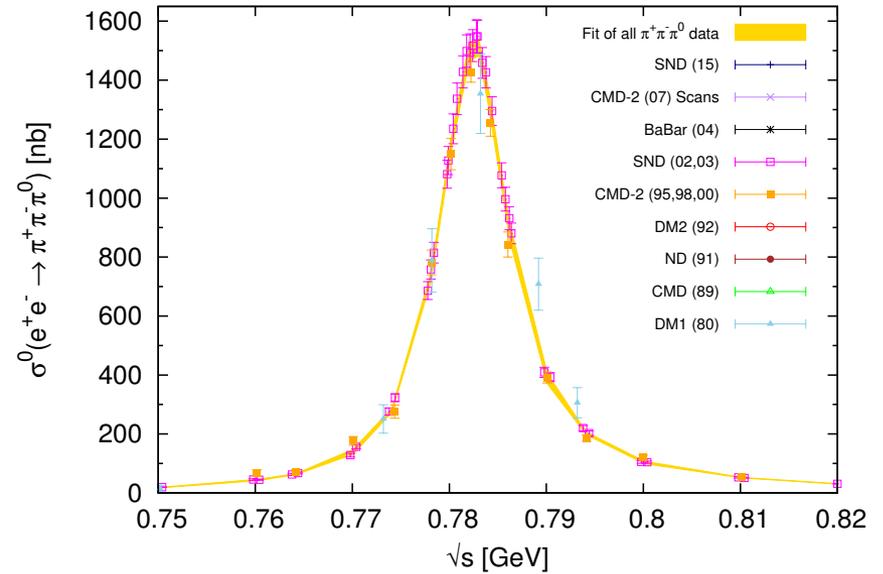
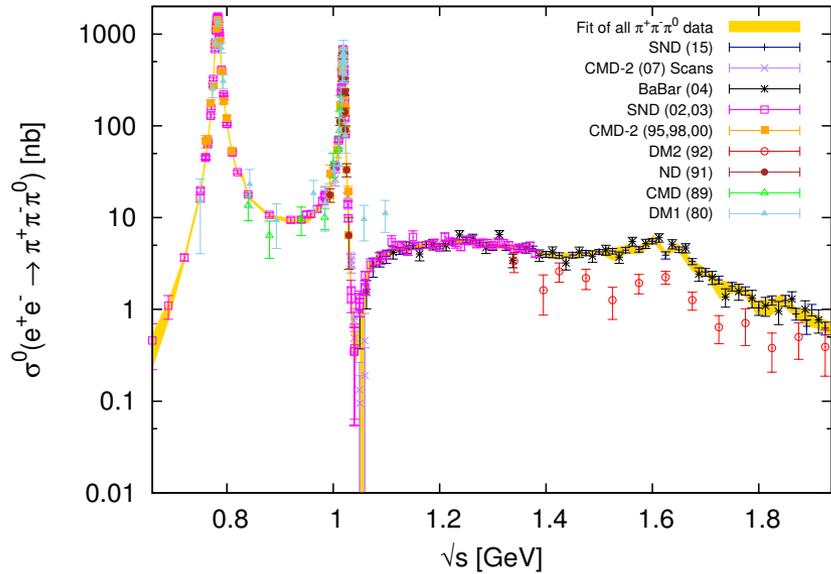


BaBar data alone $\Rightarrow a_{\mu}^{\pi^+\pi^-}$ (BaBar data only) = 513.2 ± 3.8 .

Simple weighted average of all data $\Rightarrow a_{\mu}^{\pi^+\pi^-}$ (Weighted average) = 509.1 ± 2.9 .
(i.e. - no correlations in determination of mean value)

BaBar data dominate when no correlations are taken into account for the mean value
Highlights importance of fully incorporating all available correlated uncertainties

HVP: $\pi^+\pi^-\pi^0$ channel [KNT18, PRD97, 114025]



Improvement for 3π also

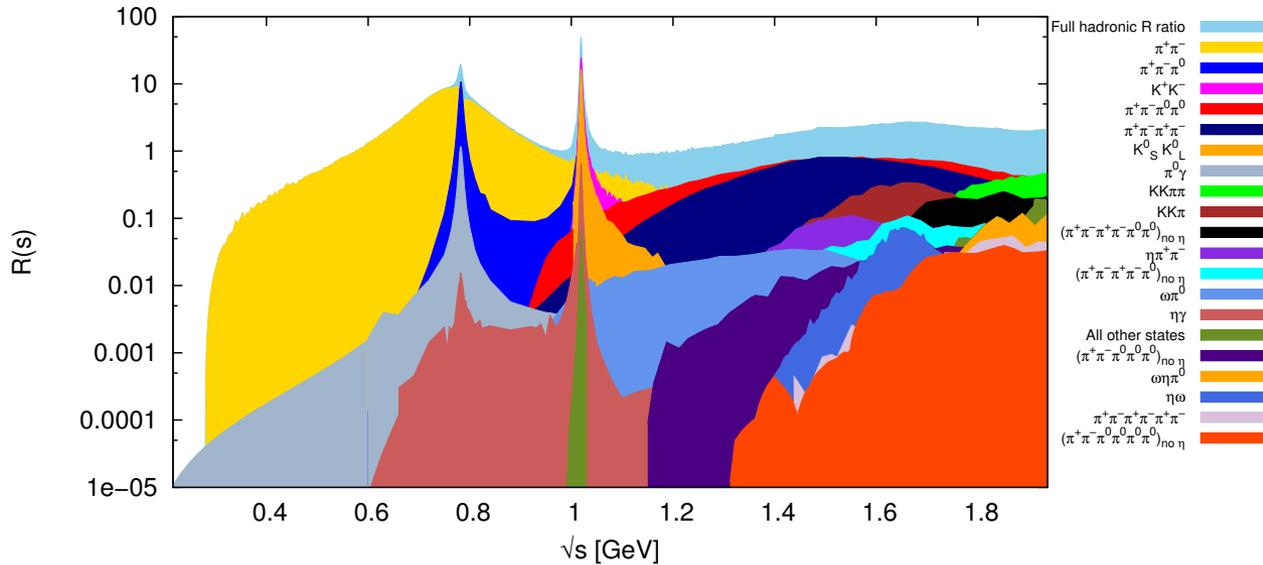
New data:

SND: [J. Exp. Theor. Phys. 121 (2015), 27.]

$$a_\mu^{\pi^+\pi^-\pi^0} = 47.79 \pm 0.22_{\text{stat}} \pm 0.71_{\text{sys}} \pm 0.13_{\text{vp}} \pm 0.48_{\text{fsr}} = 47.79 \pm 0.89_{\text{tot}}$$

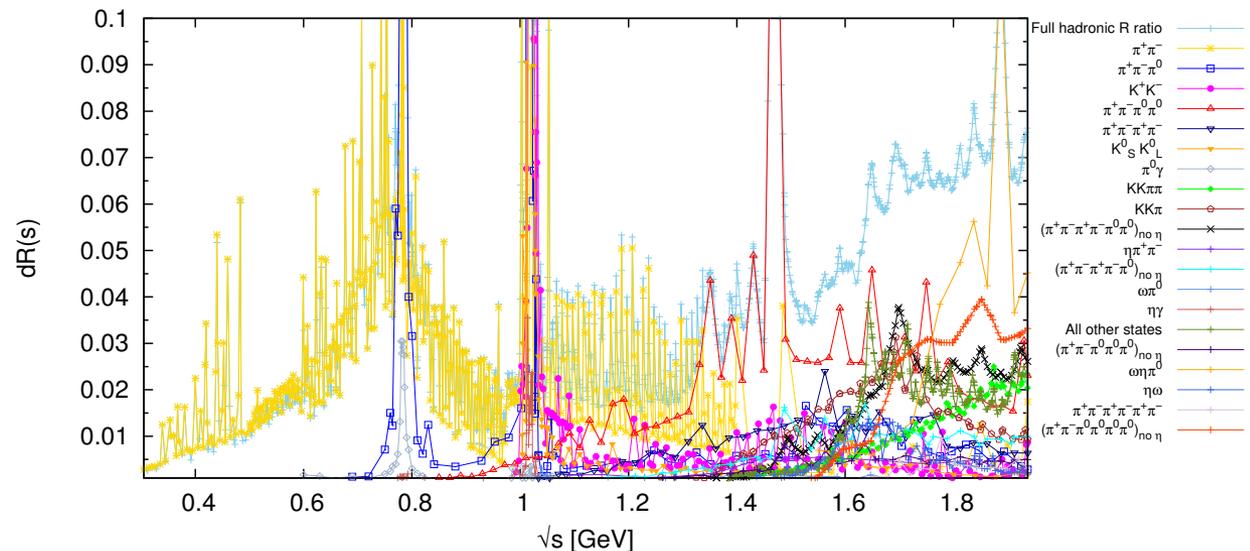
HLMNT11: $47.51 \pm 0.99_{\text{tot}}$

HVP: σ_{had} channels below 2 GeV [KNT18, PRD97, 114025]



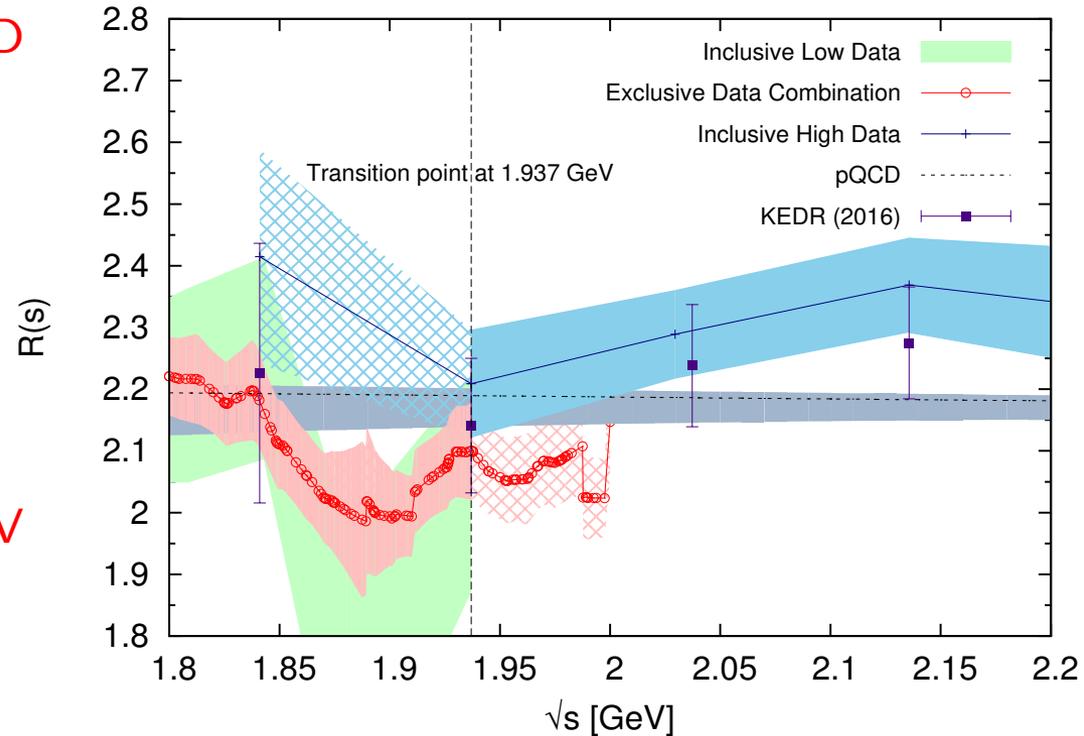
→ Dominance of 2π below 0.9 GeV evident for both cross section and uncertainty

→ Large improvement to cross section and uncertainty from new 4π data



HVP: $\sigma_{\text{had}}^{\text{excl}} \rightarrow \text{inclusive transition [KNT18]}$

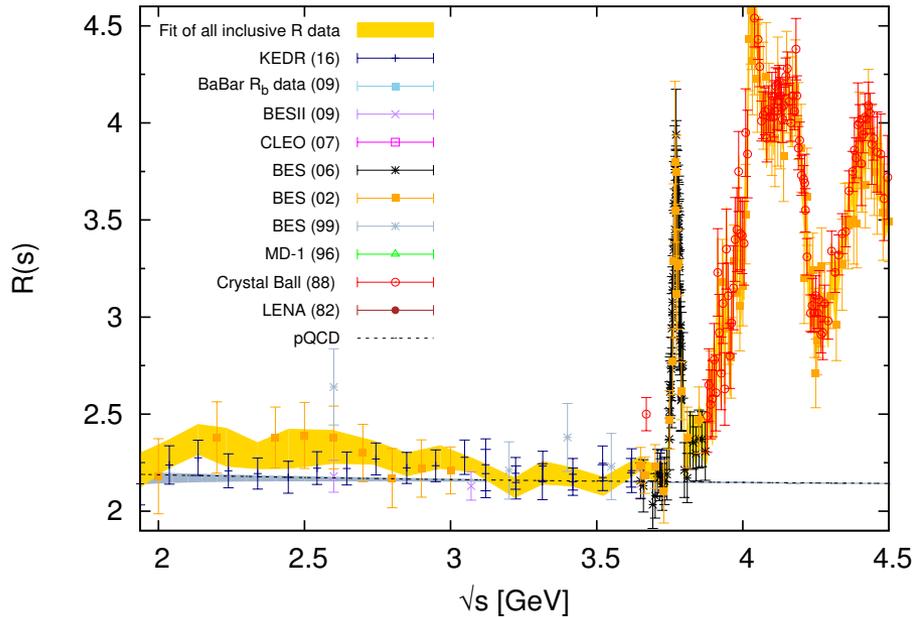
- ⇒ New KEDR data allow **reconsideration of exclusive/inclusive transition point**
- KNT18 aim to **avoid use of pQCD** and **keep a data-driven analysis**
- **Disagreement** between sum of **exclusive states** and **inclusive data/pQCD**
- New $\pi^+\pi^-\pi^0\pi^0$ data result in **reduction of the cross section**
- Previous transition point at **2 GeV** **no longer the preferred choice**
- More natural choice for this **transition point at 1.937 GeV**



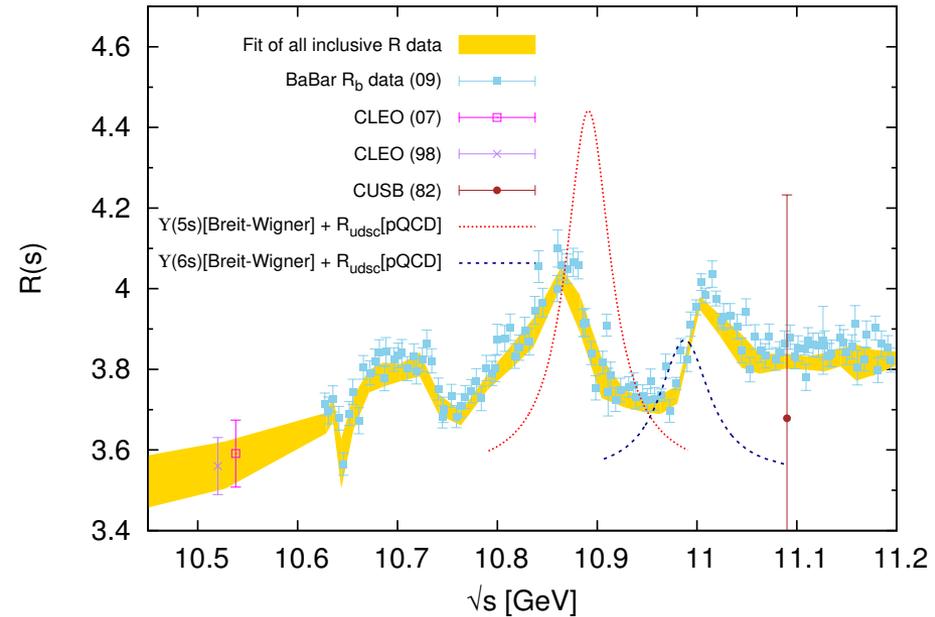
Input	$a_{\mu}^{\text{had, LO VP}} [1.841 \leq \sqrt{s} \leq 2.00 \text{ GeV}] \times 10^{10}$
Exclusive sum	6.06 ± 0.17
Inclusive data	6.67 ± 0.26
pQCD	6.38 ± 0.11
Exclusive ($< 1.937 \text{ GeV}$) + inclusive ($> 1.937 \text{ GeV}$)	6.23 ± 0.13

HVP: σ_{had} inclusive region [KNT18]

⇒ **New KEDR inclusive R data** [Phys.Lett. B770 (2017) 174-181, Phys.Lett. B753 (2016) 533-541] and **BaBar R_b data** [Phys. Rev. Lett. 102 (2009) 012001].



KEDR data improves the inclusive data combination below $c\bar{c}$ threshold



R_b resolves the resonances of the $\Upsilon(5S - 6S)$ states.

⇒ **Choose to adopt entirely data driven estimate from threshold to 11.2 GeV**

$$a_{\mu}^{\text{Inclusive}} = 43.67 \pm 0.17_{\text{stat}} \pm 0.48_{\text{sys}} \pm 0.01_{\text{vp}} \pm 0.44_{\text{fsr}} = 43.67 \pm 0.67_{\text{tot}}$$

HVP: KNT18 total and comparison w. other work

$$\text{HLMNT(11): } 694.91 \pm 4.27$$



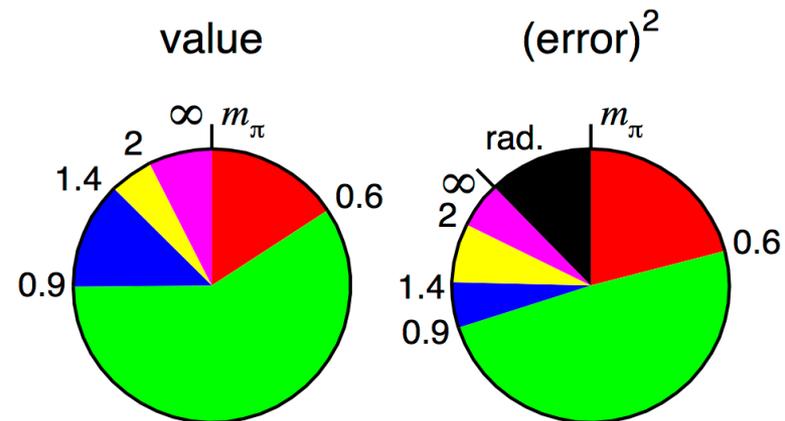
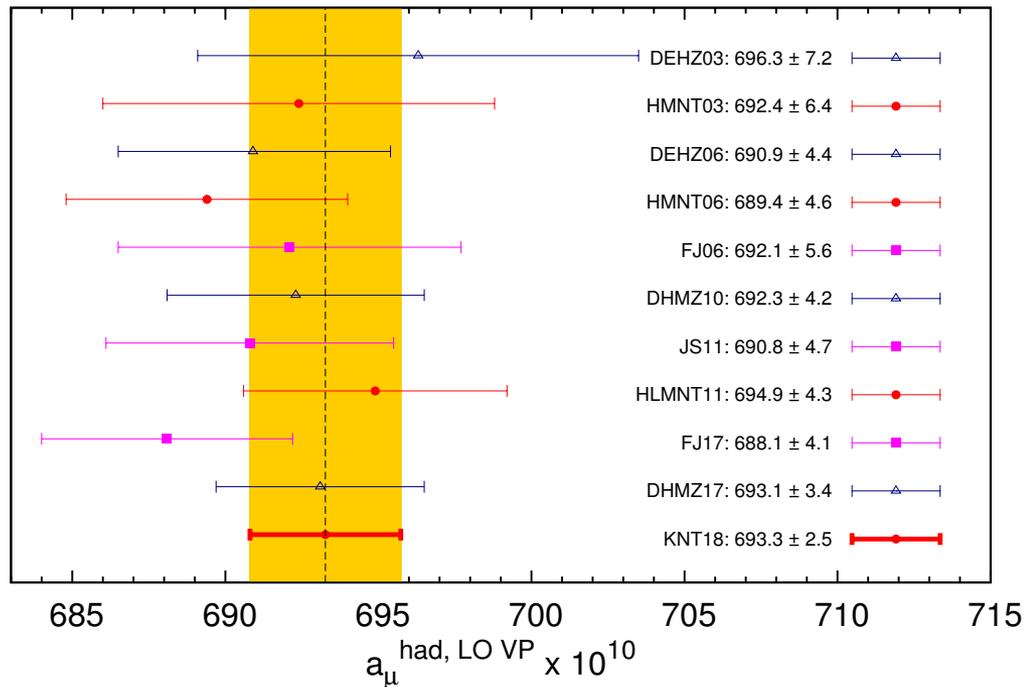
This work: $a_\mu^{\text{had, LO VP}} = 693.27 \pm 1.19_{\text{stat}} \pm 2.01_{\text{sys}} \pm 0.22_{\text{VP}} \pm 0.71_{\text{FSR}}$

$$= 693.27 \pm 2.34_{\text{exp}} \pm 0.74_{\text{rad}}$$

$$= 693.27 \pm 2.46_{\text{tot}}$$

$$a_\mu^{\text{had, NLO VP}} = -9.82 \pm 0.04_{\text{tot}}$$

⇒ Accuracy better than 0.4%
(uncertainties include all available correlations)



⇒ 2π dominance

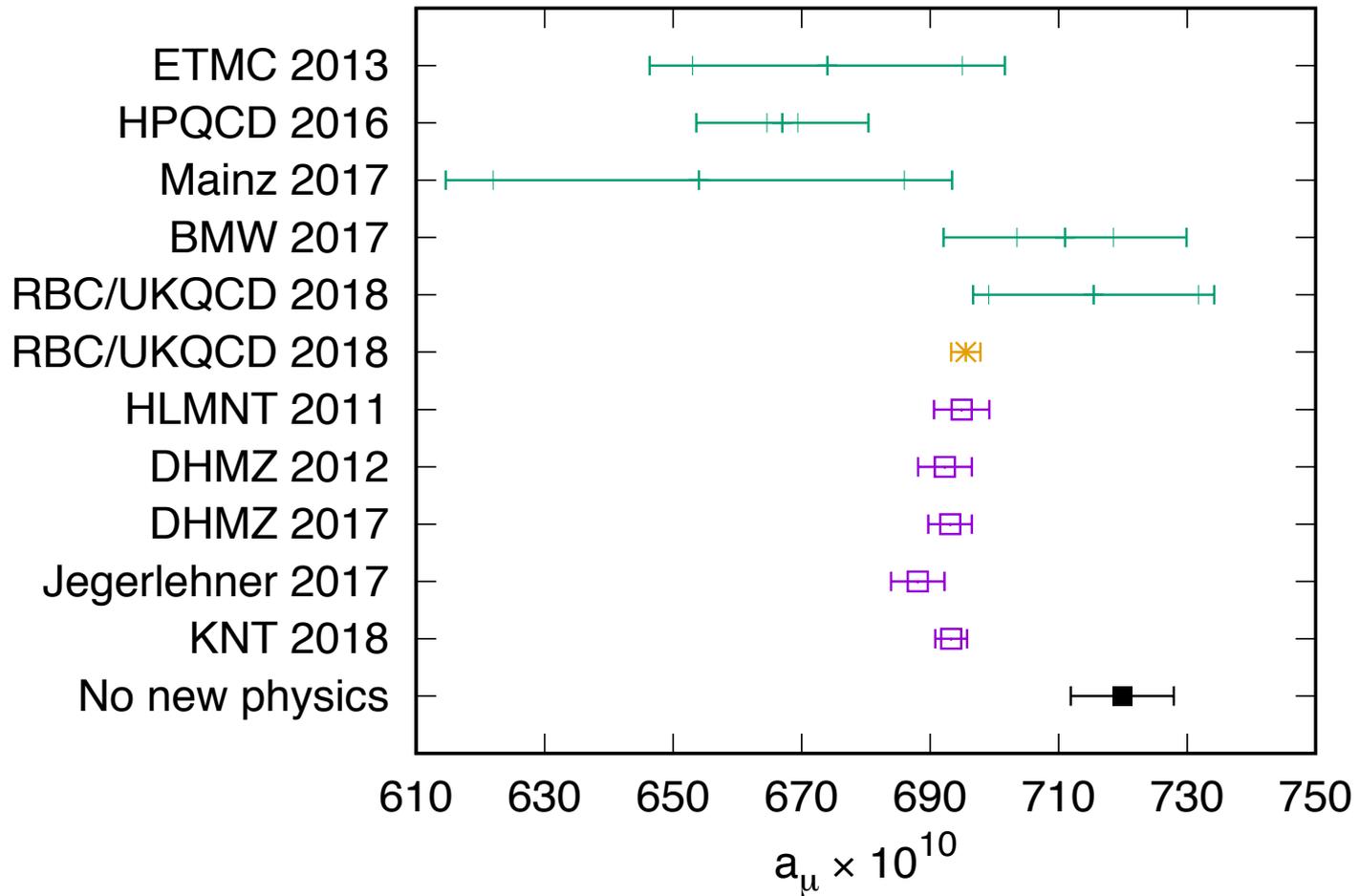
HVP from the lattice

One page summary, for details see the lattice talks at the TGm2 plenary meeting in Mainz, June 2018: <https://indico.him.uni-mainz.de/event/11/>

- Complementary to data-driven ('pheno') DR.
- Need high statistics, and control highly non-trivial systematics:
 - need simulations at physical pion mass,
 - control continuum limit and Finite Volume effects,
 - need to include full QED and Strong Isospin Breaking effects, i.e. full QED+QCD including $m_u \neq m_d$ & disconnected diagrams
- There has been a lot of activity on the lattice, for HVP (& HLbL):
 - Budapest-Marseille-Wuppertal (staggered q 's, also moments)
 - RBC / UKQCD collaboration (Time-Momentum-Representation, DW fermions, window method to comb. 'pheno' with lattice)
 - Mainz (CLS) group ($O(a)$ improved Wilson fermions, TMR)
 - HPQCD & MILC collaborations (HISQ quarks, Pade fits)

HVP from the lattice

Christoph Lehner at the recent meeting of the Theory Initiative for g-2, Mainz, June 2018:

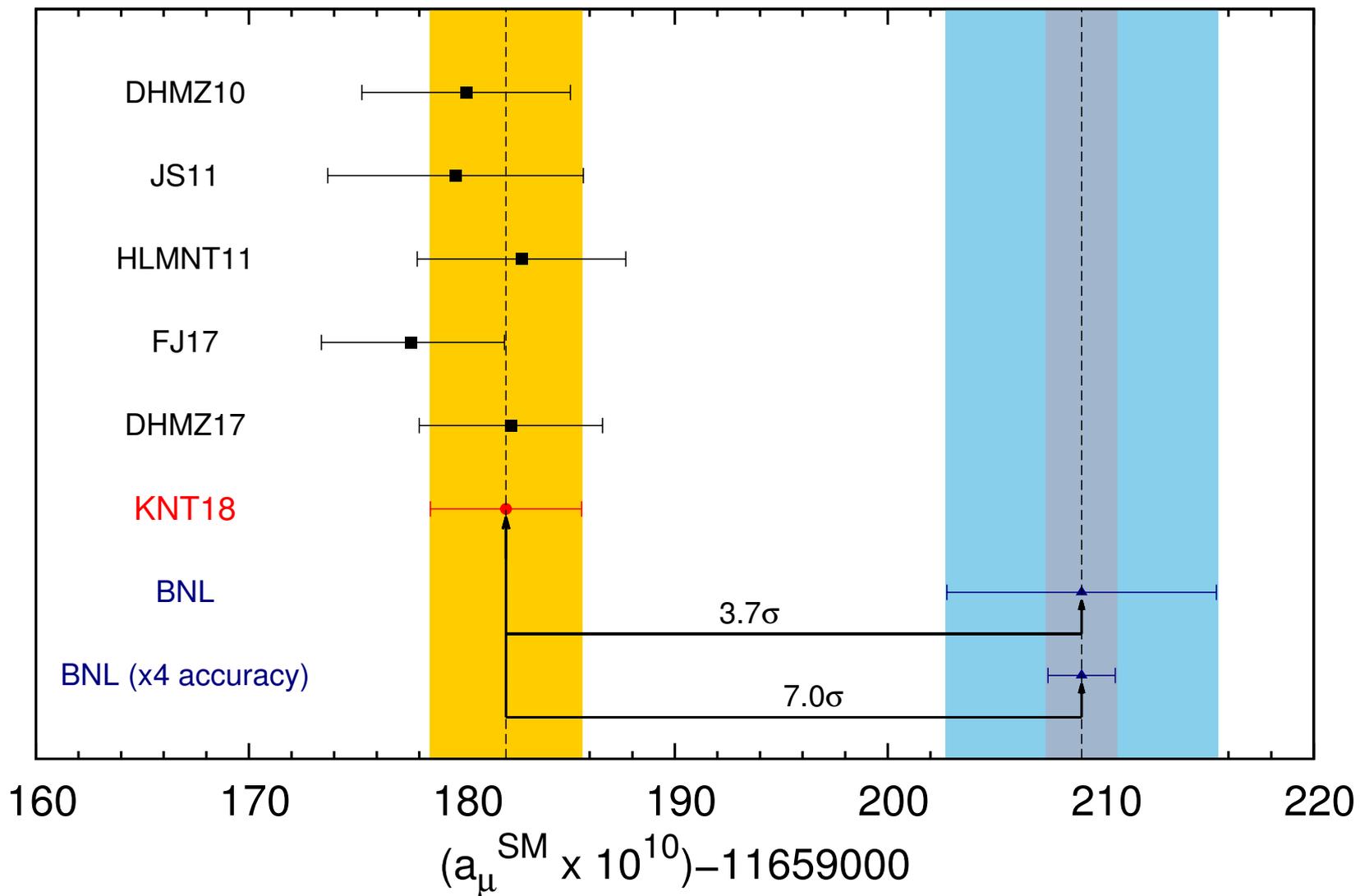


We need to improve the precision of our pure lattice result so that it can distinguish the “no new physics” results from the cluster of precise R-ratio results.

KNT18 a_μ^{SM} update

	<u>2011</u>	→	<u>2017</u>	
QED	11658471.81 (0.02)	→	11658471.90 (0.01)	[arXiv:1712.06060]
EW	15.40 (0.20)	→	15.36 (0.10)	[Phys. Rev. D 88 (2013) 053005]
LO HLbL	10.50 (2.60)	→	9.80 (2.60)	[EPJ Web Conf. 118 (2016) 01016]
NLO HLbL			0.30 (0.20)	[Phys. Lett. B 735 (2014) 90]
<hr/>				
	<u>HLMNT11</u>		<u>KNT18</u>	
LO HVP	694.91 (4.27)	→	693.27 (2.46)	this work
NLO HVP	-9.84 (0.07)	→	-9.82 (0.04)	this work
<hr/>				
NNLO HVP			1.24 (0.01)	[Phys. Lett. B 734 (2014) 144]
<hr/>				
Theory total	11659182.80 (4.94)	→	11659182.05 (3.56)	this work
Experiment			11659209.10 (6.33)	world avg
Exp - Theory	26.1 (8.0)	→	27.1 (7.3)	this work
<hr/>				
Δa_μ	3.3 σ	→	3.7 σ	this work

a_μ^{SM} vs. a_μ^{EXP} discrepancy



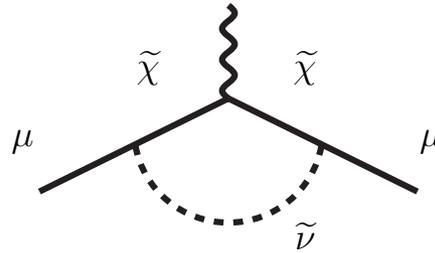
7σ if E989 obtains same mean value with projected improvement in error

a_μ : New Physics?

- Many BSM studies use $g-2$ as constraint or even motivation

- SUSY could easily explain $g-2$

- Main 1-loop contributions:



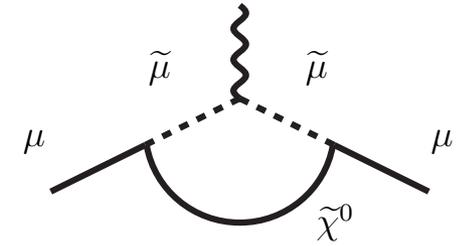
- Simplest case:

$$a_\mu^{\text{SUSY}} \simeq \text{sgn}(\mu) 130 \times 10^{-11} \tan \beta \left(\frac{100 \text{ GeV}}{\Lambda_{\text{SUSY}}} \right)^2$$

- Needs $\mu > 0$, 'light' SUSY-scale Λ and/or large $\tan \beta$ to explain 281×10^{-11}

- This is already excluded by LHC searches in the simplest SUSY scenarios (like CMSSM); causes large χ^2 in simultaneous SUSY-fits with LHC data and $g-2$

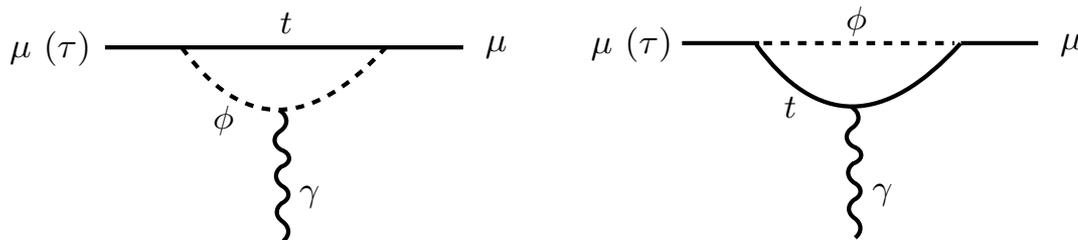
- However:
 - * SUSY does not have to be minimal (w.r.t. Higgs),
 - * could have large mass splittings (with lighter sleptons),
 - * be hadrophobic/leptophilic,
 - * or not be there at all, but don't write it off yet...



New Physics? just a few of many recent studies

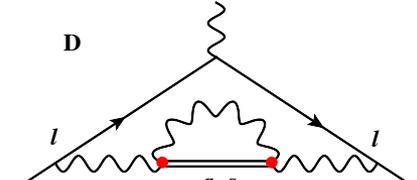
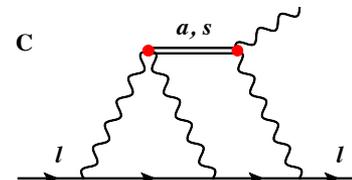
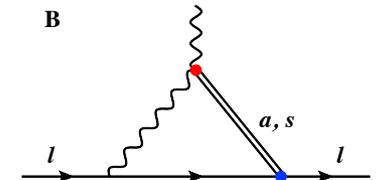
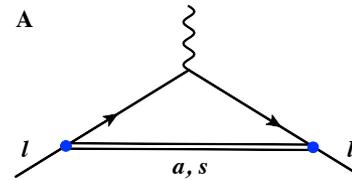
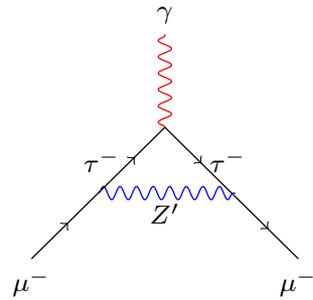
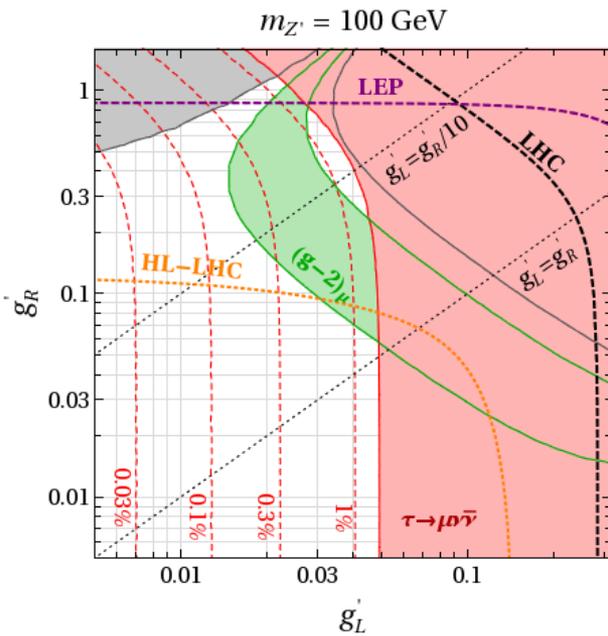
- Don't have to have full MSSM (like coded in GM2Calc [by Athron, ..., Stockinger et al., EPJC 76 (2016) 62], which includes all latest two-loop contributions), and
 - **extended Higgs sector** could do, see, e.g. Stockinger et al., JHEP 1701 (2017) 007, 'The muon magnetic moment in the 2HDM: complete two-loop result'
- lesson: 2-loop contributions can be highly relevant in both cases; one-loop analyses can be misleading
- **1 TeV Leptoquark** Bauer + Neubert, PRL 116 (2016) 141802

one new scalar could explain several anomalies seen by BaBar, Belle and LHC in the flavour sector (e.g. **violation of lepton universality** in $B \rightarrow Kll$, enhanced $B \rightarrow D\tau\nu$) and solve $g-2$, while satisfying all bounds from LEP and LHC



New Physics? just a few of many recent examples

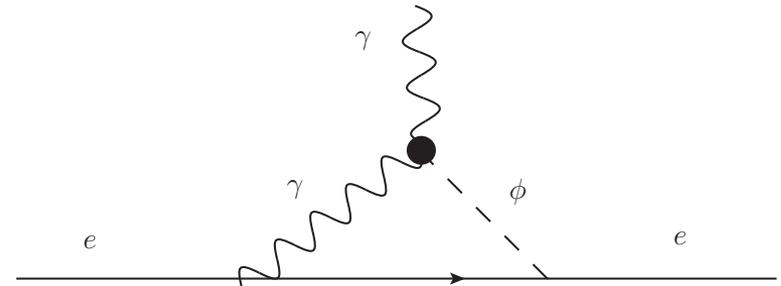
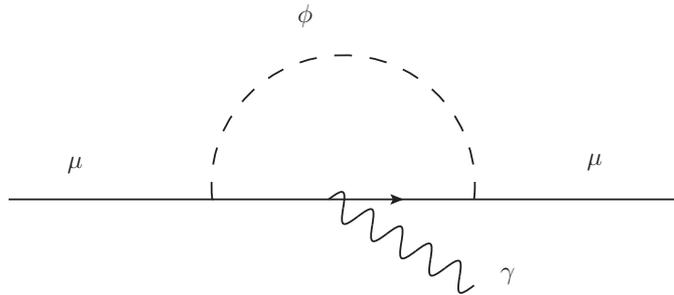
- **light Z'** can evade many searches involving electrons by non-standard couplings preferring heavy leptons (but see BaBar's direct search limits in a wide mass range, PRD 94 (2016) 011102), or invoke flavour off-diagonal Z' to evade constraints [Altmannshofer et al., PLB 762 (2016) 389]



- **axion-like particle (ALP)**, contributing like π^0 in HLbL [Marciano et al., PRD 94 (2016) 115033]
- **'dark photon'** - like fifth force particle [Feng et al., PRL 117 (2016) 071803]

New Physics? Explaining muon and electron g-2

- Davoudiasl + Marciano, 'A Tale of Two Anomalies', PRD96(2018)096018
 use one singlet real scalar Φ with mass $\sim 250\text{-}1000$ MeV and couplings $\sim 10^{-3}$ and $\sim 10^{-4}$ for μ and e , in one- and two-loop diagrams



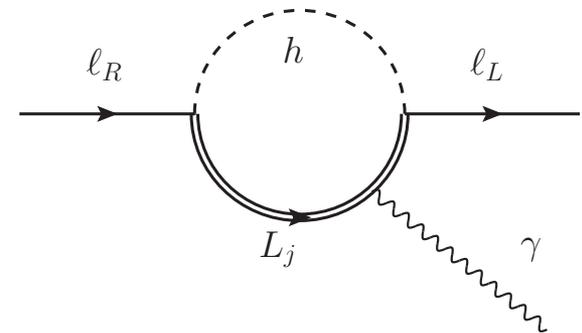
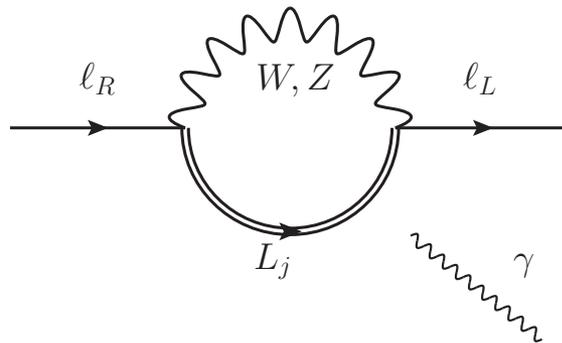
- Crivellin + Hoferichter + Schmidt-Wellenburg, arXiv:1807.11484
 'Combined explanation of $(g-2)_{\mu,e}$ and implications for a large muon EDM'
 discuss UV complete scenarios with vector-like fermions (not minimally flavor violating) which solve both puzzles and at the same time give sizeable muon

EDM contributions,

$$|d_\mu| \sim 10^{-23}\text{-}10^{-21},$$

but escaping constraints from

$$\mu \rightarrow e \gamma.$$



Conclusions/Outlook:

- The still unresolved muon **$g-2$ discrepancy is consolidated at about $3 \rightarrow 4 \sigma$** . It has triggered several new experiments and a lot of theory activities
- The dominant uncertainties from the hadronic contributions will be further squeezed, with L-by-L becoming the bottleneck, but a lot of progress (**lattice + new data driven approaches**) is expected for the next few years
→ TH will be ready for the next round
- Fermilab's $g-2$ experiment has started their data taking, first new analysis result expected for summer next year, J-PARC will take a few years longer, both aiming at bringing the current EXP uncertainty down by a factor of 4
→ **with two completely different EXPs, should get closure/confirmation**
- **We may just see the beginning of a new puzzle with a_e**
- Also expect much improved μ EDM bounds, complementarity w. LFV & MDM
- Many approaches to explain discrepancies with NP, linking $g-2$ with other precision observables, the flavour sector, dark matter and direct searches, but so far NP is only (con)strained.

Thank you for your attention,

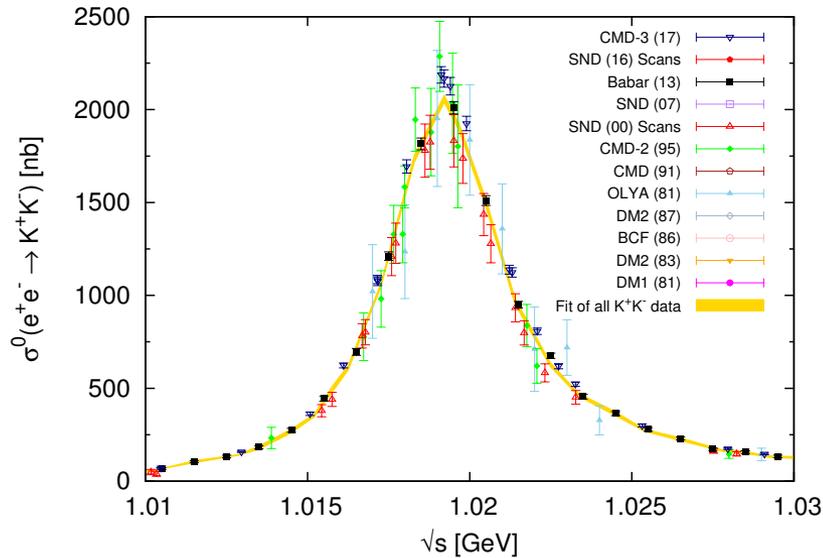
and the organisers and the team from Cosener's

for a great HEP Forum 2018!

Extras

$K\bar{K}$ channels [KNT18: arXiv:1802.02995]

K^+K^-



New data:

BaBar: [Phys. Rev. D 88 (2013), 032013.]

SND: [Phys. Rev. D 94 (2016), 112006.]

CMD-3: [arXiv:1710.02989.]

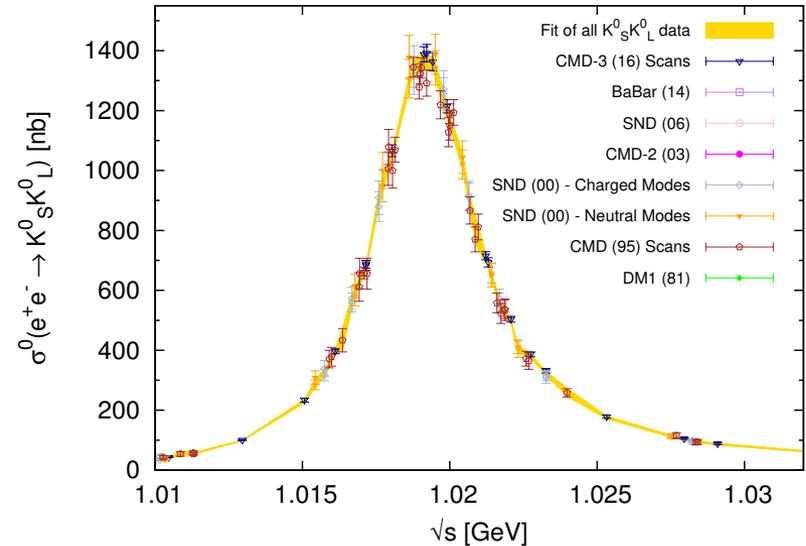
Note: CMD-2 data [Phys. Lett. B 669 (2008) 217.]
omitted as waiting reanalysis.

$$a_{\mu}^{K^+K^-} = 23.03 \pm 0.22_{\text{tot}}$$

HLMNT11: $22.15 \pm 0.46_{\text{tot}}$

Large increase in mean value

$K_S^0K_L^0$



New data:

BaBar: [Phys. Rev. D 89 (2014), 092002.]

CMD-3: [Phys. Lett. B 760 (2016) 314.]

$$a_{\mu}^{K_S^0K_L^0} = 13.04 \pm 0.19_{\text{tot}}$$

HLMNT11: $13.33 \pm 0.16_{\text{tot}}$

Large changes due to new
precise measurements on ϕ

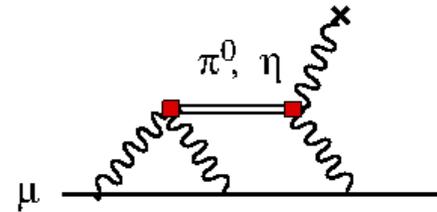
Comparison with other similar works

Channel	This work (KNT18)	DHMZ17	Difference
$\pi^+\pi^-$	503.74 ± 1.96	507.14 ± 2.58	-3.40
$\pi^+\pi^-\pi^0$	47.70 ± 0.89	46.20 ± 1.45	1.50
$\pi^+\pi^-\pi^+\pi^-$	13.99 ± 0.19	13.68 ± 0.31	0.31
$\pi^+\pi^-\pi^0\pi^0$	18.15 ± 0.74	18.03 ± 0.54	0.12
K^+K^-	23.00 ± 0.22	22.81 ± 0.41	0.19
$K_S^0K_L^0$	13.04 ± 0.19	12.82 ± 0.24	0.22
$1.8 \leq \sqrt{s} \leq 3.7$ GeV	34.54 ± 0.56 (data)	33.45 ± 0.65 (pQCD)	1.09
Total	693.3 ± 2.5	693.1 ± 3.4	0.2

- ⇒ Total estimates from two analyses in very good agreement
- ⇒ Masks much larger differences in the estimates from individual channels
- ⇒ Unexpected tension for 2π considering the data input likely to be similar
 - Points to marked differences in way data are combined
 - From 2π discussion: $a_{\mu}^{\pi^+\pi^-}$ (Weighted average) = 509.1 ± 2.9
- ⇒ Compensated by lower estimates in other channels
 - For example, the choice to use pQCD instead of data above 1.8 GeV
- ⇒ FJ17: $a_{\mu, \text{FJ17}}^{\text{had, LO VP}} = 688.07 \pm 41.4$
 - Much lower mean value, but in agreement within errors

$a_\mu^{\text{had, L-by-L}}$: Light-by-Light

- **L-by-L**: $\gamma \rightarrow \text{hadrons} \rightarrow \gamma^* \gamma^* \gamma^*$ non-perturbative, impossible to fully measure ✗
- so far use of **model calculations**, based on **large N_c limit**, **Chiral Perturbation Theory**, plus **short distance constraints** from OPE and pQCD
- **meson exchanges** and **loops** modified by form factor suppression, but with limited experimental information:
 - in principle off-shell form-factors ($\pi^0, \eta, \eta', 2\pi \rightarrow \gamma^* \gamma^*$) needed
 - at most possible, directly experimentally: $\pi^0, \eta, \eta', 2\pi \rightarrow \gamma \gamma^*$
 - additional quark loop, pQCD matching; theory not fully satisfying conceptually ☹️
- several independent evaluations, different in details, but **good agreement for the leading N_c (π^0 exchange) contribution**, differences in sub-leading bits
- mostly used recently:
 - 'Glasgow consensus' (~ 10 years old) by Prades+deRafael+Vainshtein:
$$a_\mu^{\text{had, L-by-L}} = (105 \pm 26) \times 10^{-11}$$
 - compatible with Nyffeler's $a_\mu^{\text{had, L-by-L}} = (116 \pm 39) \times 10^{-11}$



HLbL scattering: Summary of selected results for $a_\mu^{\text{HLbL}} \times 10^{11}$

Contribution	BPP	HKS, HK	KN	MV	BP, MdRR	PdRV	N, JN
π^0, η, η'	85 ± 13	82.7 ± 6.4	83 ± 12	114 ± 10	—	114 ± 13	99 ± 16
axial vectors	2.5 ± 1.0	1.7 ± 1.7	—	22 ± 5	—	15 ± 10	22 ± 5
scalars	-6.8 ± 2.0	—	—	—	—	-7 ± 7	-7 ± 2
π, K loops	-19 ± 13	-4.5 ± 8.1	—	—	—	-19 ± 19	-19 ± 13
π, K loops +subl. N_C	—	—	—	0 ± 10	—	—	—
quark loops	21 ± 3	9.7 ± 11.1	—	—	—	2.3 (c-quark)	21 ± 3
Total	83 ± 32	89.6 ± 15.4	80 ± 40	136 ± 25	110 ± 40	105 ± 26	116 ± 39

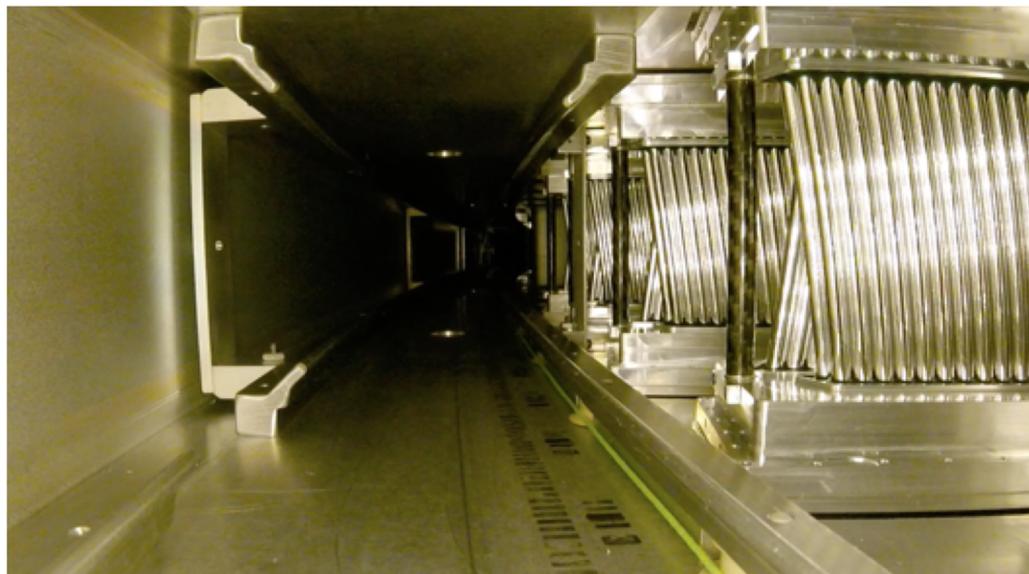
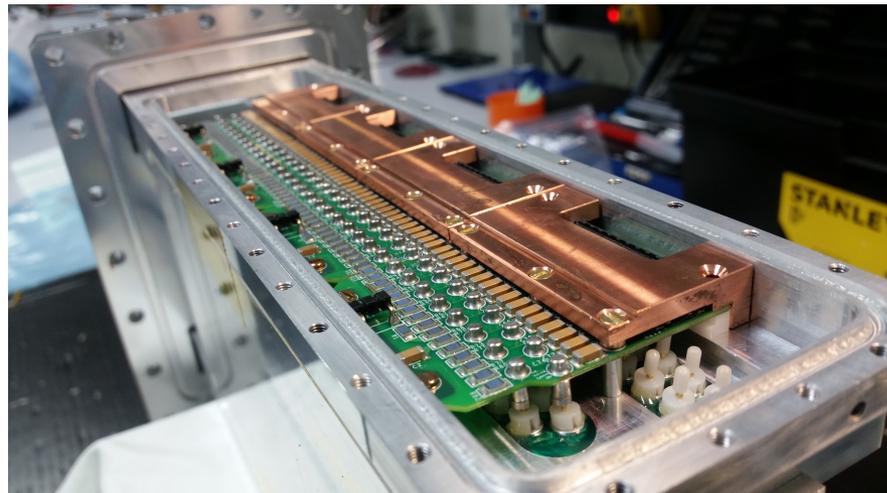
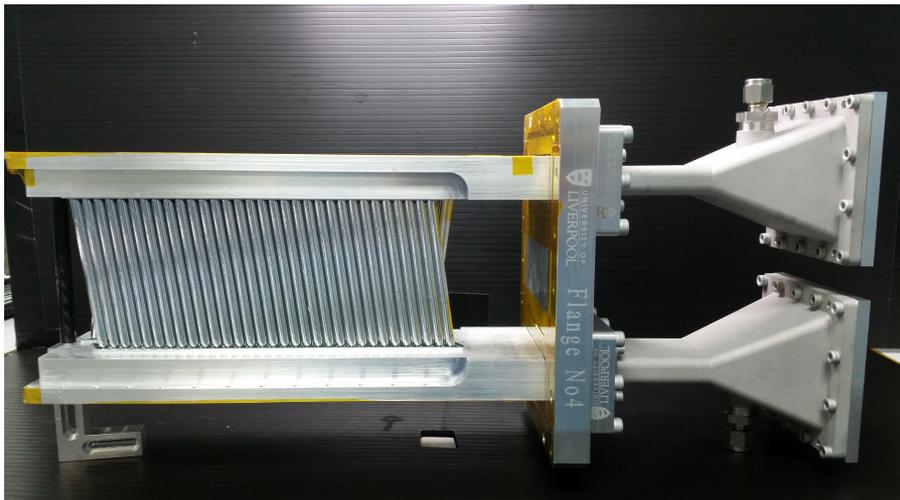
BPP = Bijnsens, Pallante, Prades '95, '96, '02; HKS = Hayakawa, Kinoshita, Sanda '95, '96; HK = Hayakawa, Kinoshita '98, '02; KN = Knecht, AN '02; MV = Melnikov, Vainshtein '04; BP = Bijnsens, Prades '07; MdRR = Miller, de Rafael, Roberts '07; PdRV = Prades, de Rafael, Vainshtein '09; N = AN '09, JN = Jegerlehner, AN '09

- **Pseudoscalar-exchanges dominate numerically.** Other contributions not negligible. **Cancellation** between π, K -loops and quark loops !
 - Note that recent reevaluations of axial vector contribution lead to much smaller estimates than in MV: $a_\mu^{\text{HLbL};\text{axial}} = (8 \pm 3) \times 10^{-11}$ (Pauk, Vanderhaeghen '14; Jegerlehner '14, '15). This would shift central values of compilations downwards: $a_\mu^{\text{HLbL}} = (98 \pm 26) \times 10^{-11}$ (PdRV) and $a_\mu^{\text{HLbL}} = (102 \pm 39) \times 10^{-11}$ (N, JN).
-
- **PdRV:** Analyzed results obtained by different groups with various models and suggested new estimates for some contributions (shifted central values, enlarged errors). **Do not consider dressed light quark loops as separate contribution.** Added all errors in quadrature !
 - **N, JN:** **New evaluation of pseudoscalar exchange contribution imposing new short-distance constraint on off-shell form factors.** Took over most values from BPP, except axial vectors from MV. **Added all errors linearly.**

$a_{\mu}^{\text{had, L-by-L}}$: Light-by-Light Prospects; see recent TGM2 Mainz

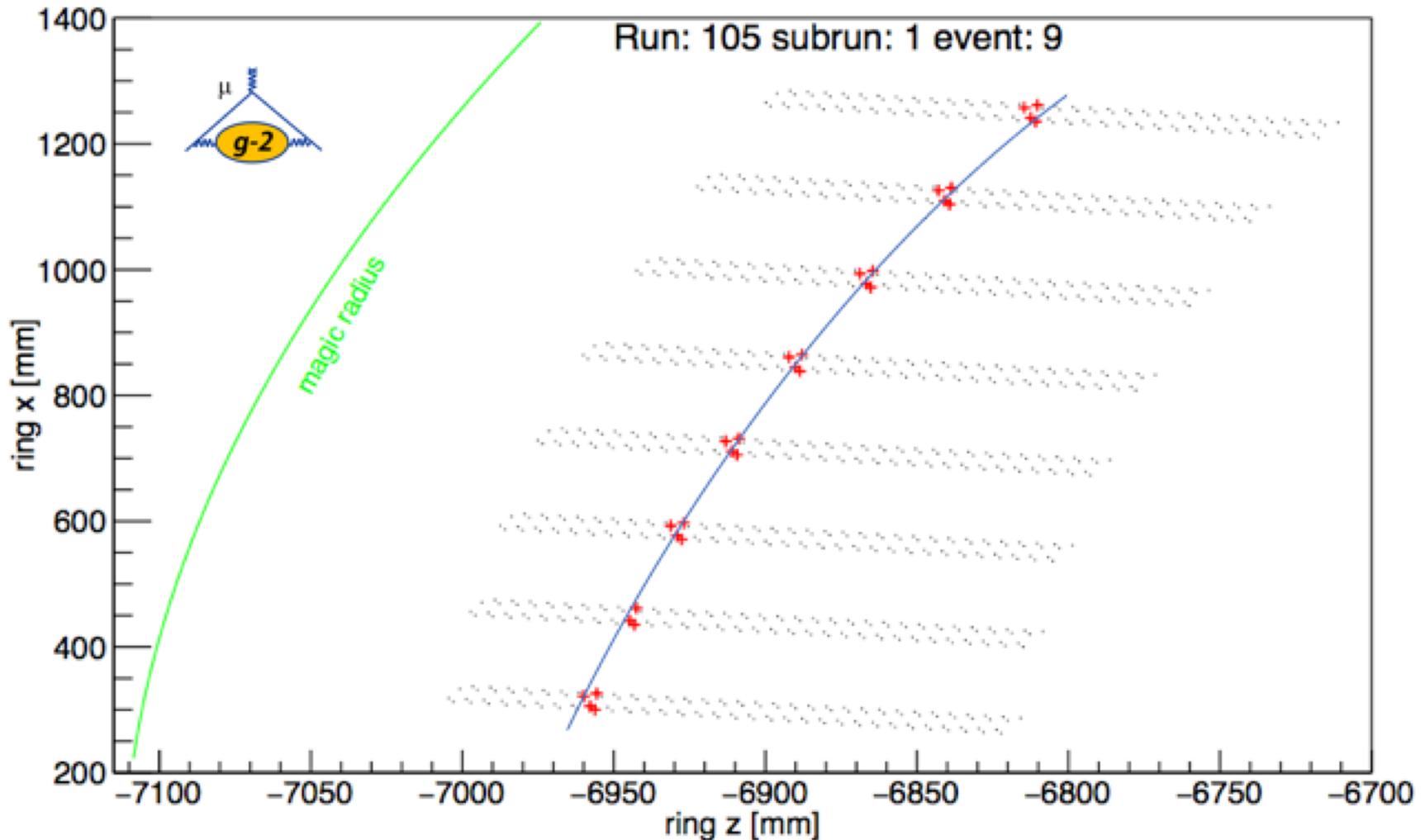
- **Transition FFs** can be measured by **KLOE-2** and **BESIII** using small angle taggers:
 $e^+e^- \rightarrow e^+e^- \gamma \gamma^* \rightarrow \pi^0, \eta, \eta', 2\pi$ expected to constrain leading pole contributions from π, η, η' to $\sim 15\%$ Nyffeler, PRD94, 053006
- or calculate on the lattice: $\pi^0 \rightarrow \gamma^* \gamma^*$ Gerardin, Meyer, Nyffeler, PRD94, 074507
- 1. Breakthrough with **new dispersive approaches**
Pauk, Vanderhaeghen, PRD 90 (2014) 113012
Colangelo, Hoferichter, Procura, Stoffer, JHEP 1704 (2017) 161
 - dispersion relations formulated for the general HLbL tensor or for a_{μ} directly
 - allowing to constrain/calculate the HLbL contributions from data
 - e.g. Colangelo et al. have first results for the π -box contribution from data for $F_V^{\pi}(q^2)$,
and now for the most important pion-pole contribution, JHEP 1810, 141
- 2. Ultimately: 'First principles' full prediction from **lattice QCD+QED**
 - several groups: **Mainz, RBC-UKQCD, ...** much increased effort
 - within few years a 10% estimate may be possible, 30% would already be useful
 - first results very encouraging, **now hunt down errors/systematics**
- **We are already able to defend/confirm the error estimate of the Glasgow consensus, and probably will bring it down significantly** ✓

E989 Liverpool: design and building of trackers



- 3 stations in ring, each 8 modules with straw trackers
- tool for monitoring beam dynamics
- very important for systematics and EDM measurement
- ← photo taken in ring from trolley for NMR probes for B-mapping

E989 Liverpool: design and building of trackers



One of the first tracks recorded by the tracker showing the hits from a single charged particle (likely a proton) through the straw trackers and the (wire)-track fit and the magic radius