High Precision QCD in NNLOJET

hep-ph/1812.06104

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Part of the NNLOJET Collaboration

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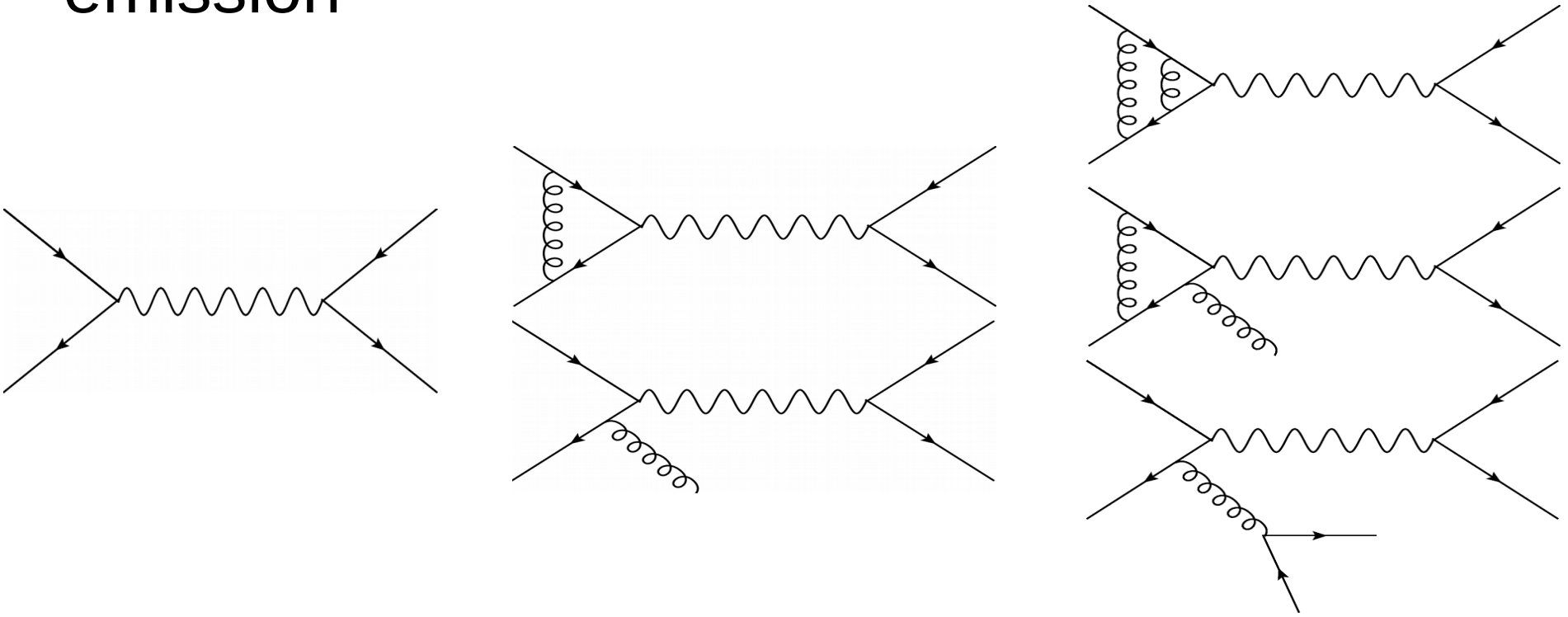
Fixed Order Perturbative QCD

- Expand in strong coupling constant – High energy regime so coupling ~ 0.118
- Each order adds 1 loop (virtual)/radiative (real) emission



Fixed Order Perturbative QCD

- Expand in strong coupling constant – High energy regime so coupling ~ 0.118
- Each order adds 1 loop (virtual)/radiative (real) emission
- Calculate order by order to improve prediction
- Reduces scale dependence, improved description of kinematics across phase space
- More orders, better science!

Goal: Improve predictions faster than experimental results

Some Jargon

- LO ↔ Leading Order
- NLO ↔ Next-to-Leading Order
- NNLO ↔ Next-to-next-to-Leading Order
- N3LO ↔ Next-to-next-to-next-to-Leading Order

  …
- Inclusive calculation

  - Often in closed analytic form

  - Final state information integrated out over **full** phase space
- Exclusive calculation

  - Generally require numerical integration

  - Full control of final state kinematics over **arbitrary** phase space region

Perturbative QCD – State of the Art

- NLO exclusive automated in multiple programs
- Exclusive NNLO benchmark for $2 \rightarrow 1$ and $2 \rightarrow 2$ scattering from many groups
- N3LO available inclusively for DIS, ggH and VFH(H)
- e+e- → 2j known inclusively to N4LO

- Exclusive is the gold standard for usable predictions! Gives full control over all kinematics, allowing:
  - Arbitrary cuts/jet algorithms
  - Direct map to the phase space coverage of experiment rather than extrapolating

NNLOJET includes many state of the art NNLO calculations:
pp → V,VJ, ggH, ggHJ, VFH JJ, G, GJ, JJ, GG
ep → j, jj
ee → 3j

Now interfacing with resummation, EW corrections for even more N

IR Singularities

- Beyond LO, real and virtual contributions contain divergences
- Real singularities implicit (appear after PS integration)
  - come from soft, collinear QCD emissions
- V divergences explicit (Laurent expansion in regulator coming from loop calculations)
- Cancel order by order for meaningful predictions! But...

  Live in different phase spaces. Can’t do this directly or numerically. Need to be clever!

Antenna Subtraction

Shown at YTF many times in the last decade, so I won't go into too much detail!

- Idea → replicate divergent structure of real contribution using known functions that can be integrated analytically + reduced matrix element (requires factorisation theorem for phase space)

  - Easiest functions to use? Matrix elements from well known processes.

- Antenna subtraction → Use ratios of matrix elements from simplest processes to construct divergent limits (ggH, inclusive Z)

  - All necessary subtraction terms known for NNLO massless QCD with 0,1,2 incoming partons

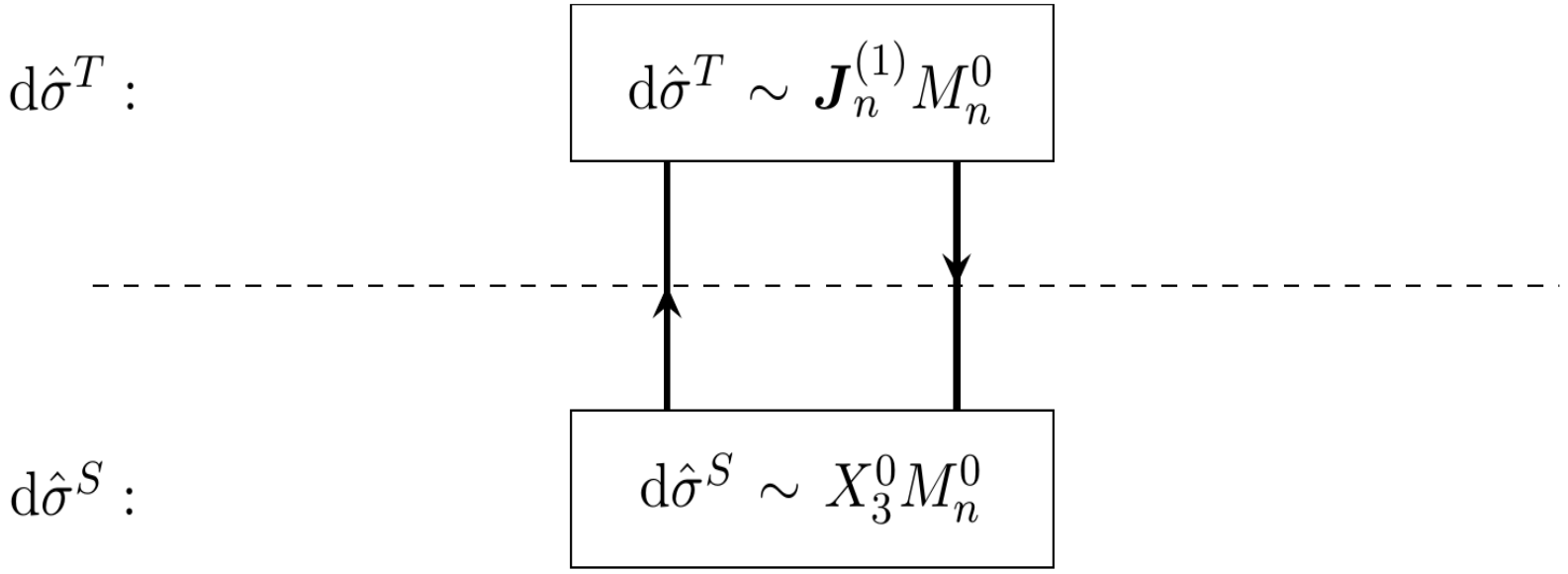
- Add these (analytically) integrated values back in at virtual level to achieve two finite integrals. Job done!

$$d\sigma_{NLO} = \int_{N+1} [d\sigma_R - d\sigma_S] + \int_N [d\sigma_V - d\sigma_T] \qquad \int_{N+1} d\sigma_S + \int_N d\sigma_T = 0$$

NLO to NNLO

- Idea the same, but **much** more complicated!
- Now have RR, RV and VV contributions
- Becomes a real technical challenge to implement, but now mostly solved for $2 \to 2$

  - For all cases where there are 2 loop matrix elements available at least.



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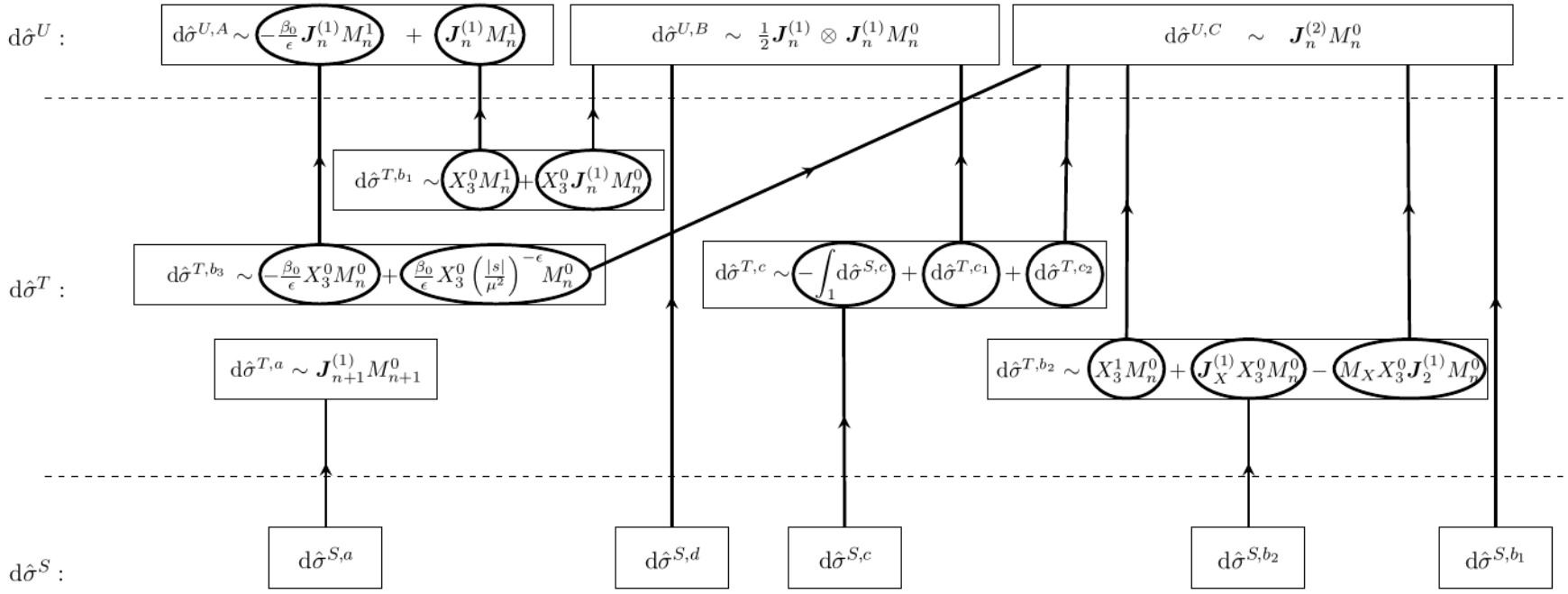
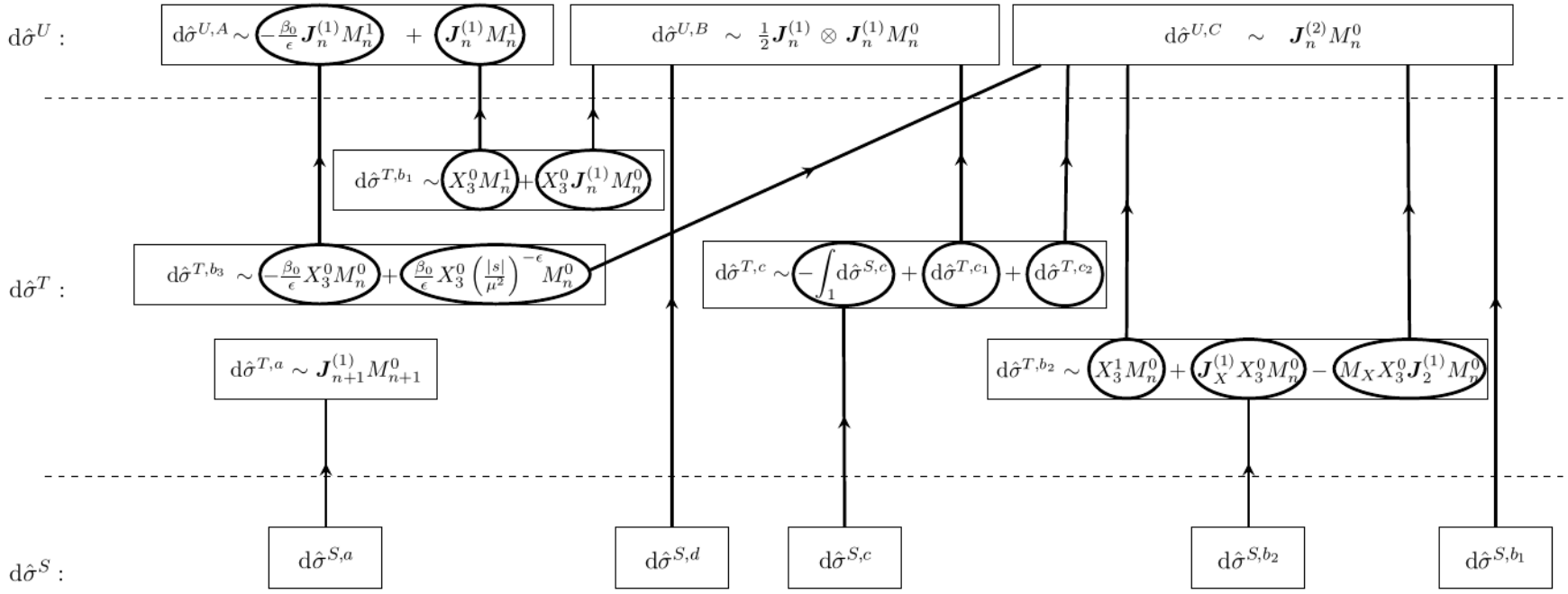


NLO to NNLO

Example: H+J subtraction terms
~ 75 pages of LaTeX.
~ 400k lines of FORTRAN
Not trivial to implement without typos/bugs!
Lots of **automation** and **testing** required to get it all right (but we've managed).



To N3LO?

- New loop and real radiation contributions
- Subtraction counterterms required for finiteness
- Numerical integration expected to be much more challenging

→ Projection to Born!

Projection To Born

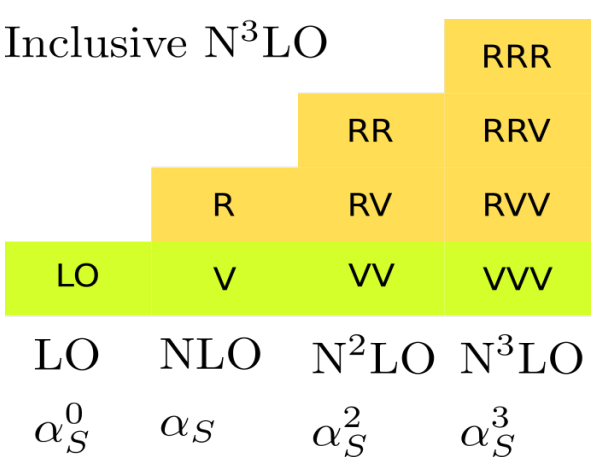
- Combine exclusive X+J @ $\mathrm{N}^{k-1}\mathrm{LO}$ with inclusive $\mathrm{N}^{3}\mathrm{LO}$ to get fully exclusive $\mathrm{N}^{3}\mathrm{LO}$
- Requires a kinematic mapping from X+J phase space to inclusive phase space

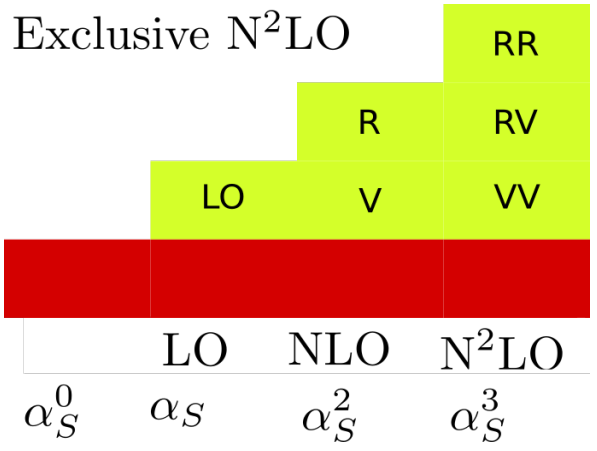
$$\mathrm{d}\mathcal{O} \xrightarrow{\mathrm{P2B}} \mathrm{d}\mathcal{O}_B$$

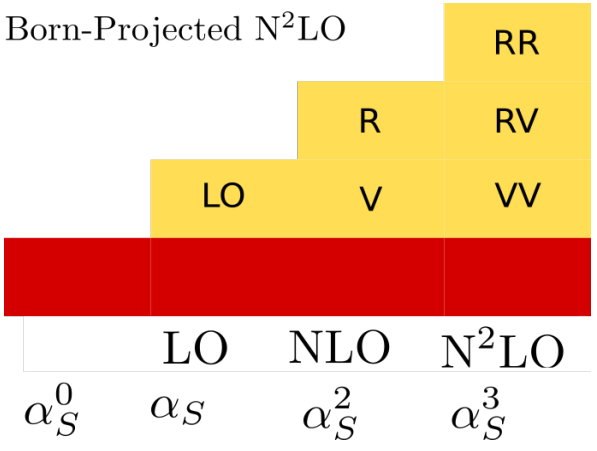
- This mapping takes X+J into the limit where J is fully unresolved (soft/collinear)
- Mapping is process dependent

Projection To Born

- Idea → Use Born (LO) kinematics to define subtraction term, weight ME (mapped to Born) for X+J calculation
- Perfectly cancels divergences we can't handle, where we become Born like (radiation in X+J becomes soft/collinear)
- Then inclusive calculation becomes the Born level subtraction term+ME!







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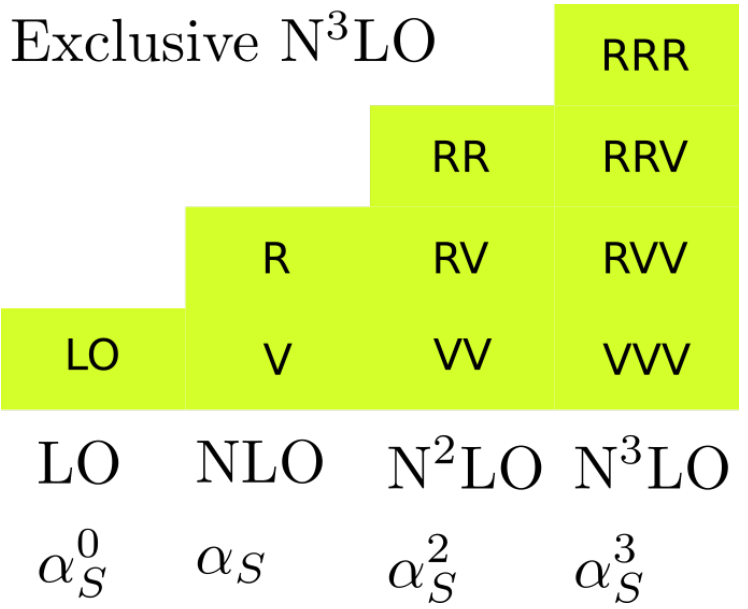
$$\frac{\mathrm{d}\sigma_X^{\mathrm{N}^k\mathrm{LO}}}{\mathrm{d}\mathcal{O}} = \frac{\sigma_{X+j}^{\mathrm{N}^{k-1}\mathrm{LO}}}{\mathrm{d}\mathcal{O}} - \frac{\sigma_{X+j}^{\mathrm{N}^{k-1}\mathrm{LO}}}{\mathrm{d}\mathcal{O}_B} + \frac{\sigma_X^{\mathrm{N}^k\mathrm{LO,incl}}}{\mathrm{d}\mathcal{O}_B}$$



Projection To Born

- Works independent of order! $\mathrm{N}^k\mathrm{LO}$ subtraction here we come...
- However, only works for processes with unambiguous mapping and inclusive calculation available

  (pp → V, pp → H, pp-VHF, DIS, e+e- → 2j)
- Easy/fast to implement

  - Just need to map kinematics so no need for large numbers of boutique subtraction terms for $\mathrm{N}^k\mathrm{LO}$
- Numerically challenging – integrating a function that = 0 much of the time, so hard to adapt to integrand

Charged Current DIS

- Deep Inelastic Scattering

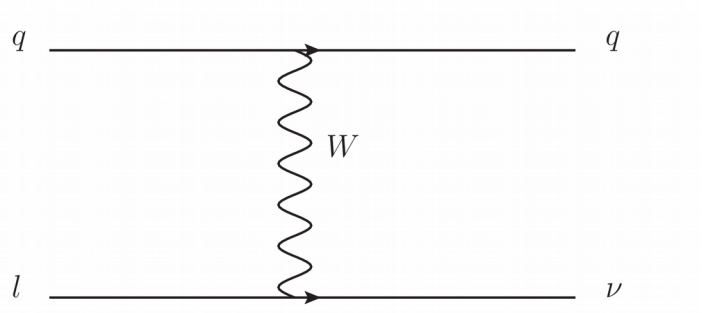
  → lepton + proton collisions

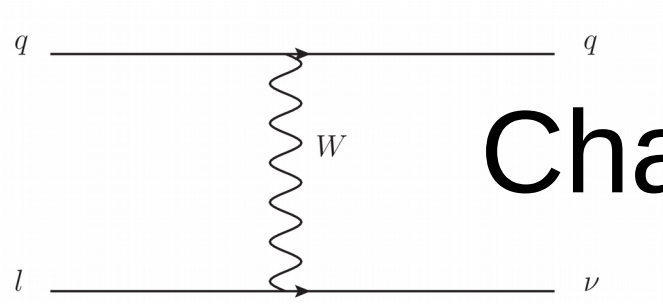
  → Lower energy than proton-proton collisions

  → More precise QCD (only one proton)

- Charged Current – W boson exchange

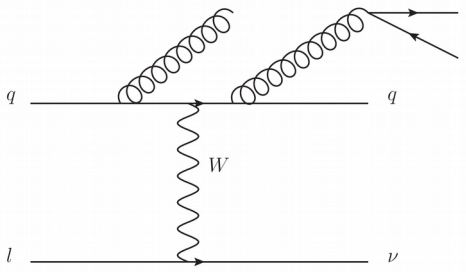
  → Cross section decomposes into 3 structure functions F_2, F_3, F_L





Charged Current DIS

Why do we care?



- PDFs – great for quark content
- Chiral SM test through polarised results
- LheC [if it happens...]
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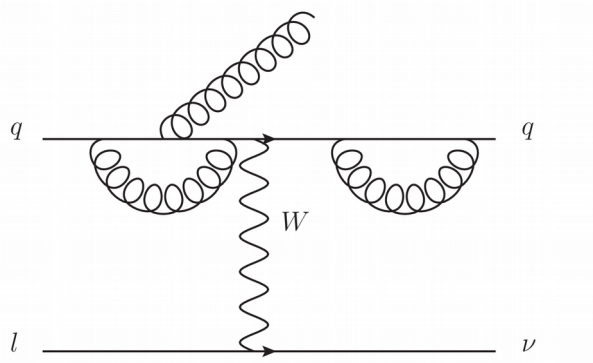
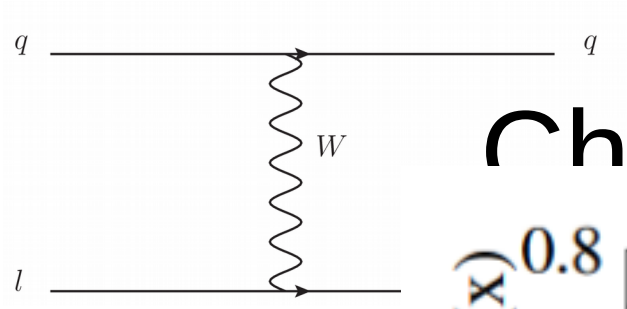
Ideal for P2B!





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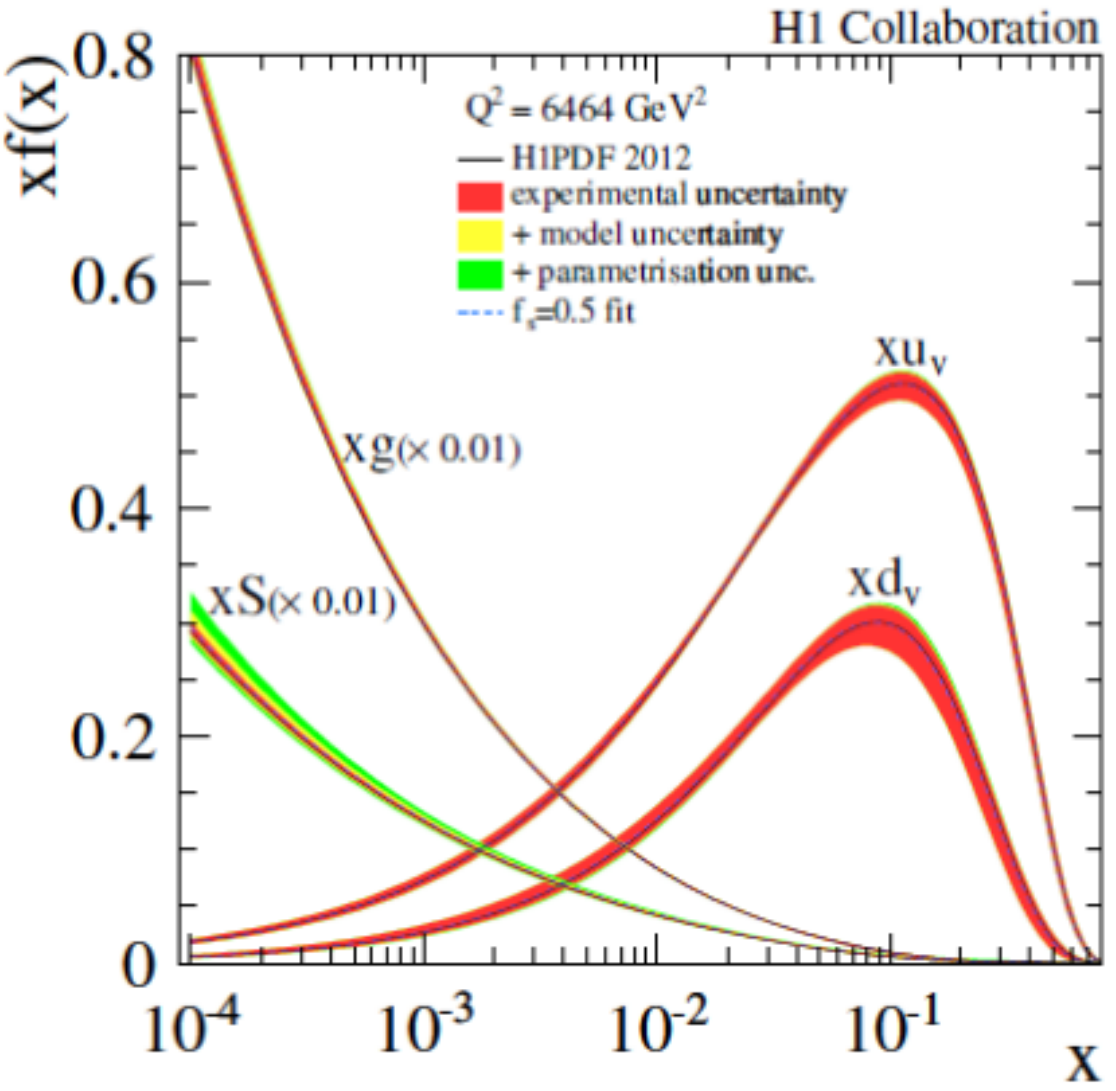

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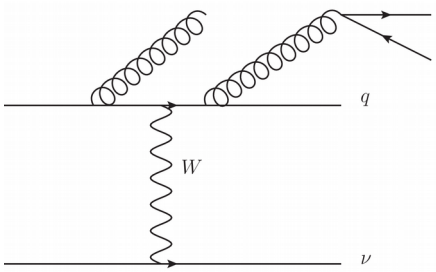

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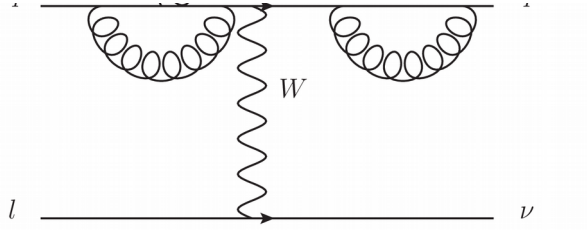

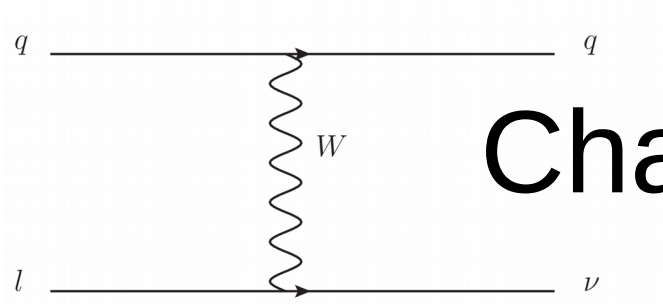

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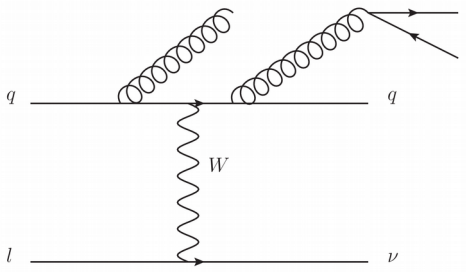

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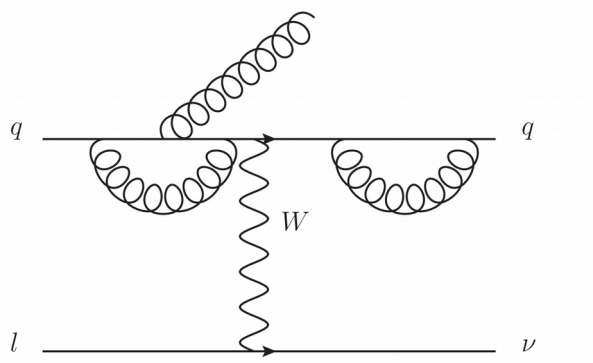

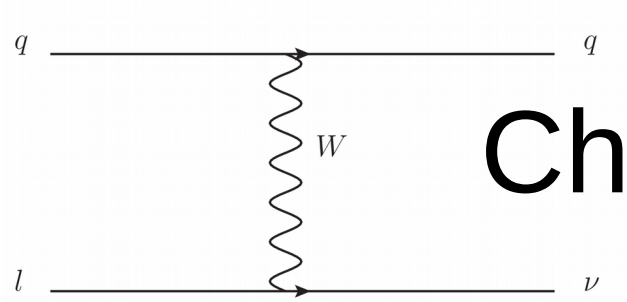

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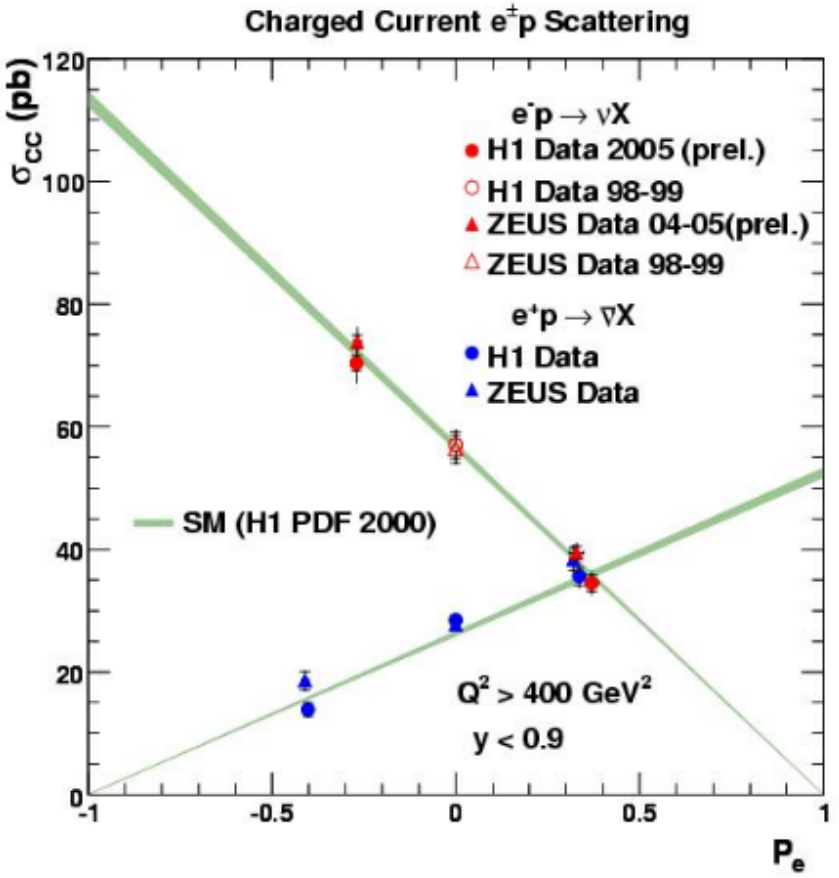

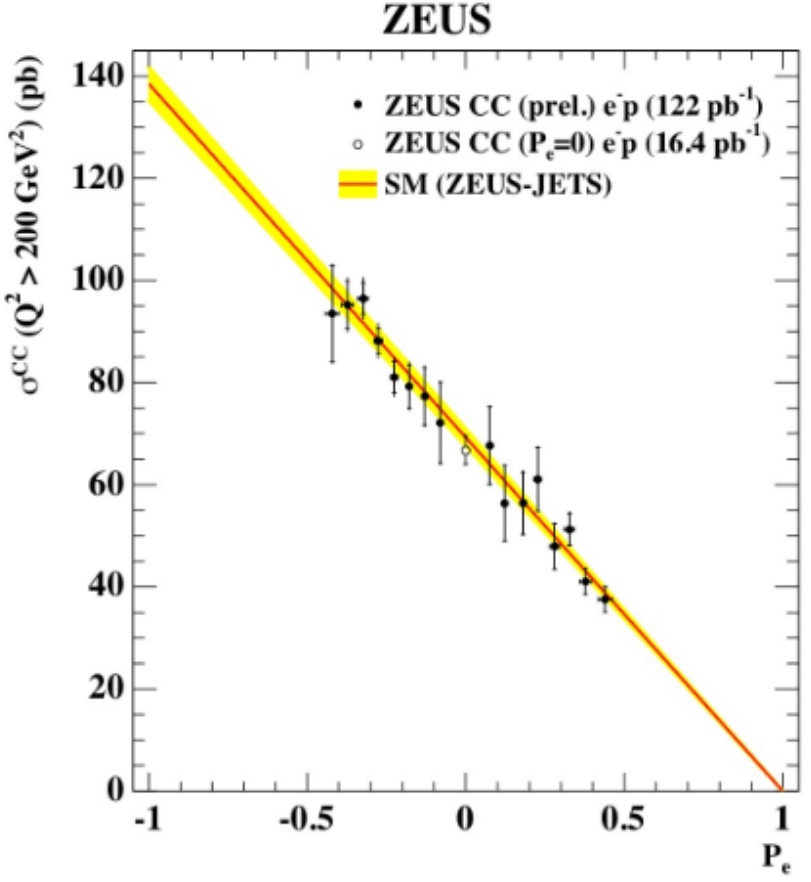

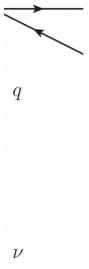

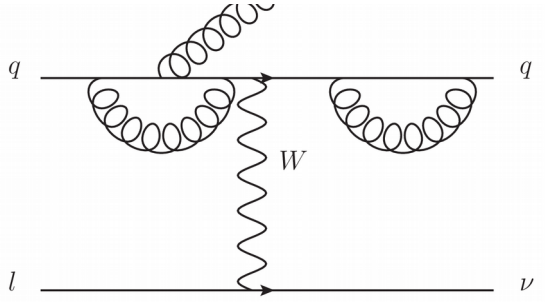

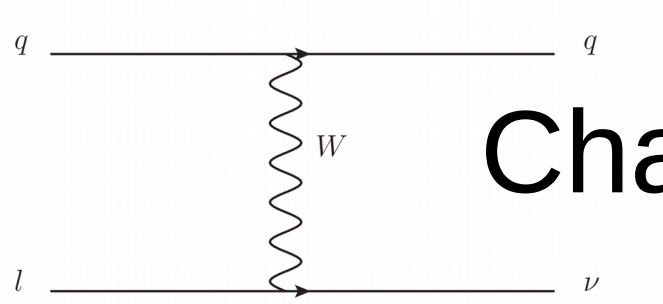

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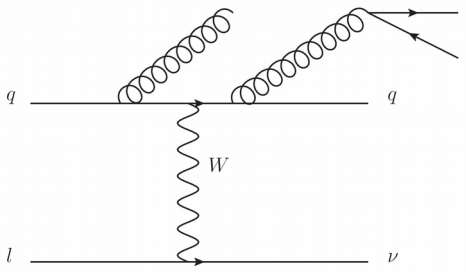

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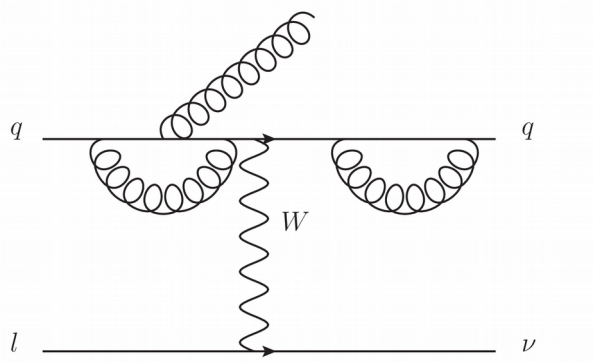

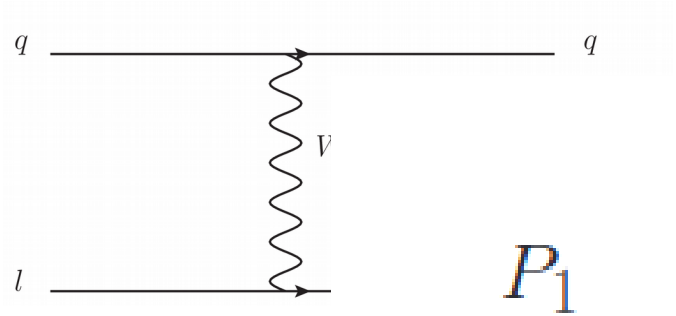

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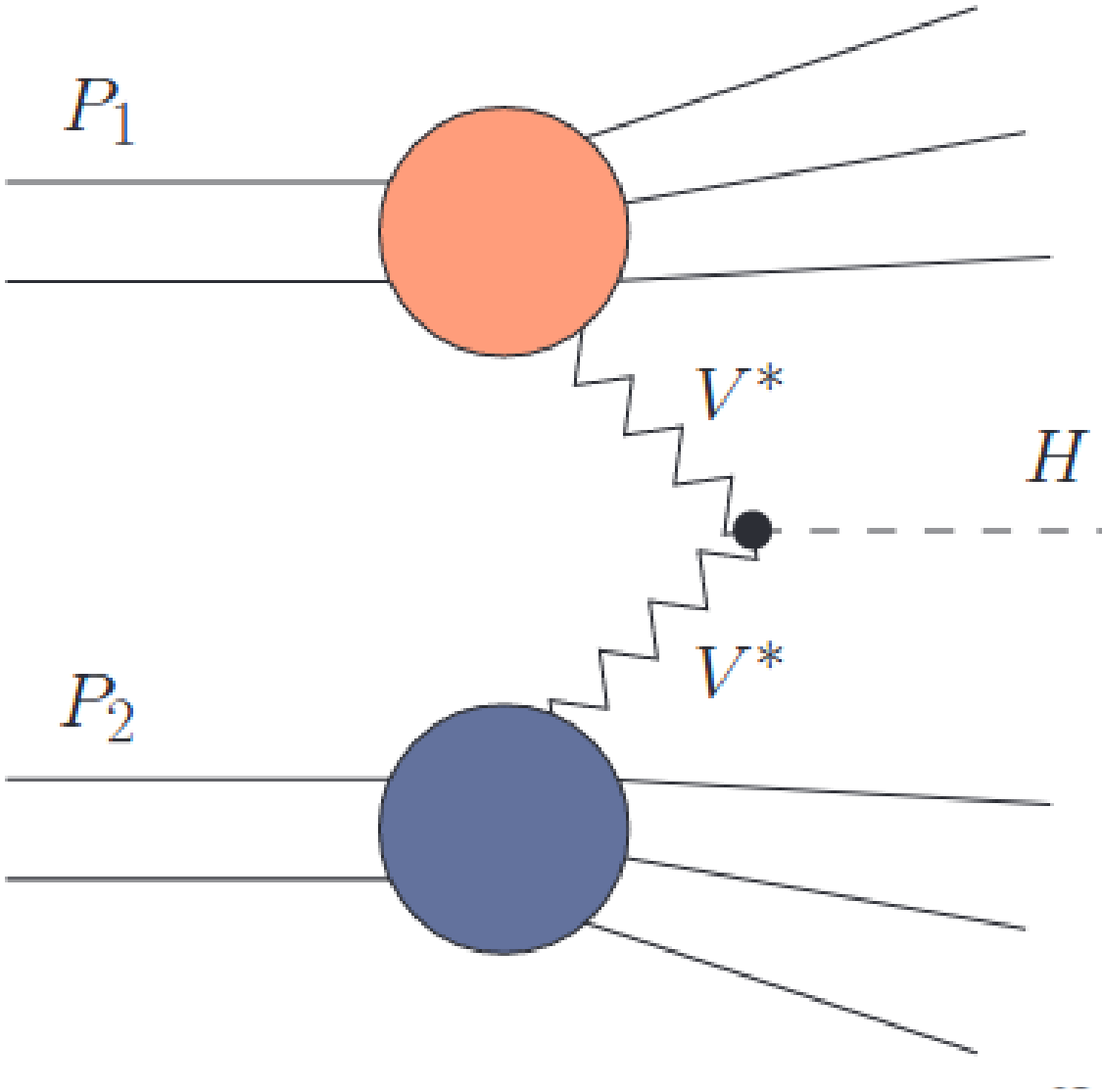

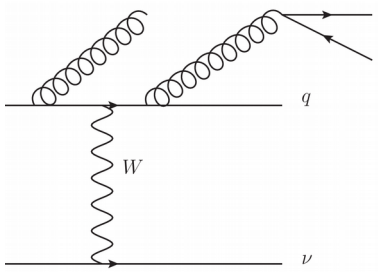

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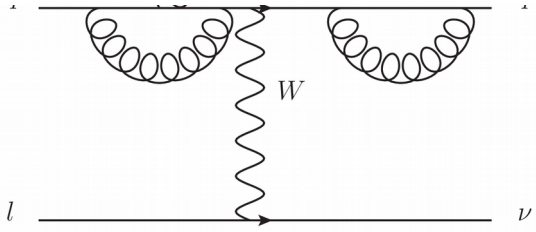

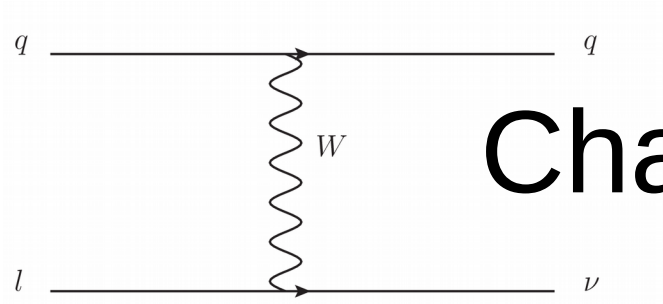

Charged Current DIS

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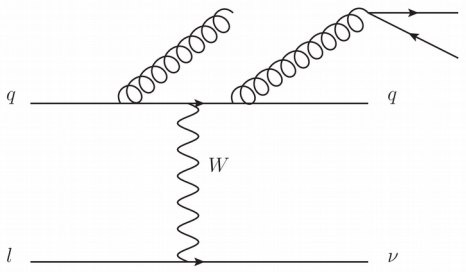

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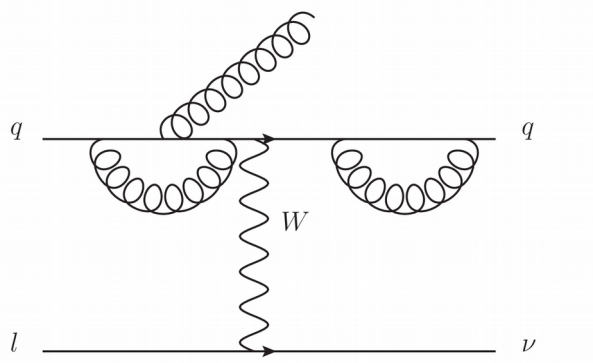

Charged Current DIS Mapping

Standard DIS variables: $p(P) + l(k) \rightarrow \nu(k') + X(p_X)$

$$x = \frac{Q^2}{2P \cdot q} \qquad q = k - k'$$

$$Q^2 = -q^2 > 0$$

DIS Mapping to Born PS: $p_{in,B} = xP$

$$p_{out,B} = xP + q$$

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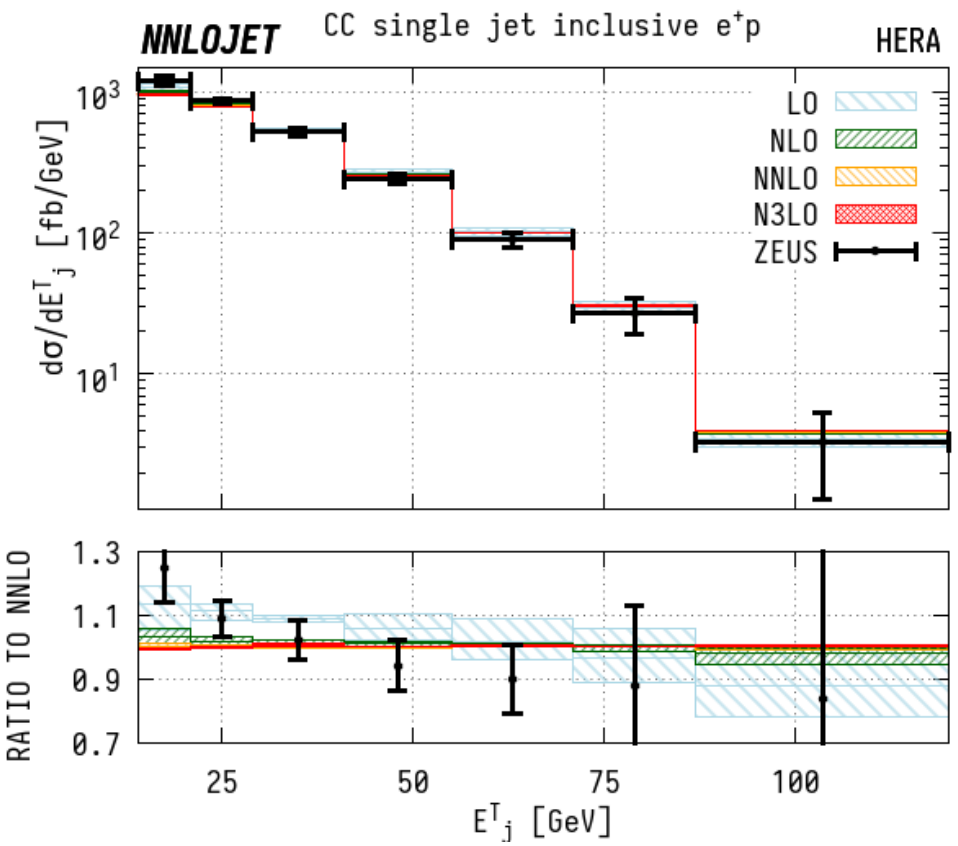
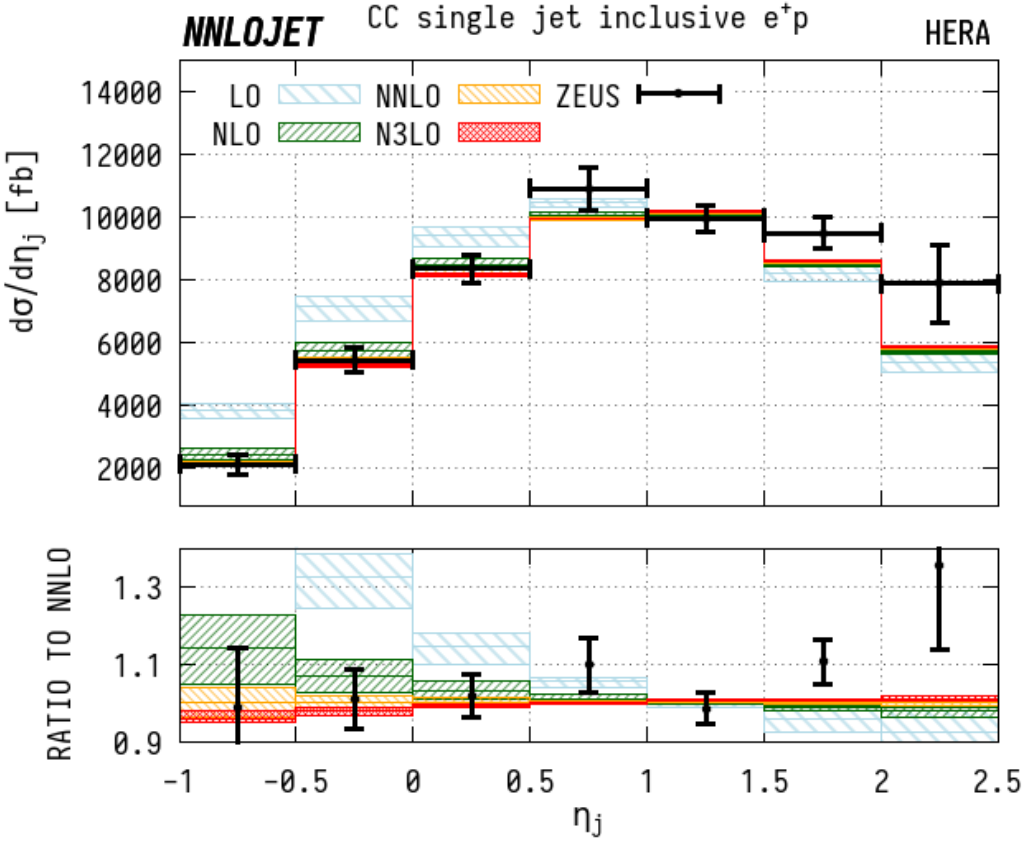
Results!

500,000 CPU hours later...





Scale variation band is 0.2% on total x-section

Conclusions / Outlook

- Projection to Born is a useful all orders subtraction scheme for IR divergences, and very simple to implement
- Using N3LO inclusive results plus CC DIS dijet calculation, both implemented in NNLOJET, we now have exclusive N3LO CC DIS
- Next stop: exclusive VBF @ N3LO!
- More orders, better science!