An Amplitude level parton shower

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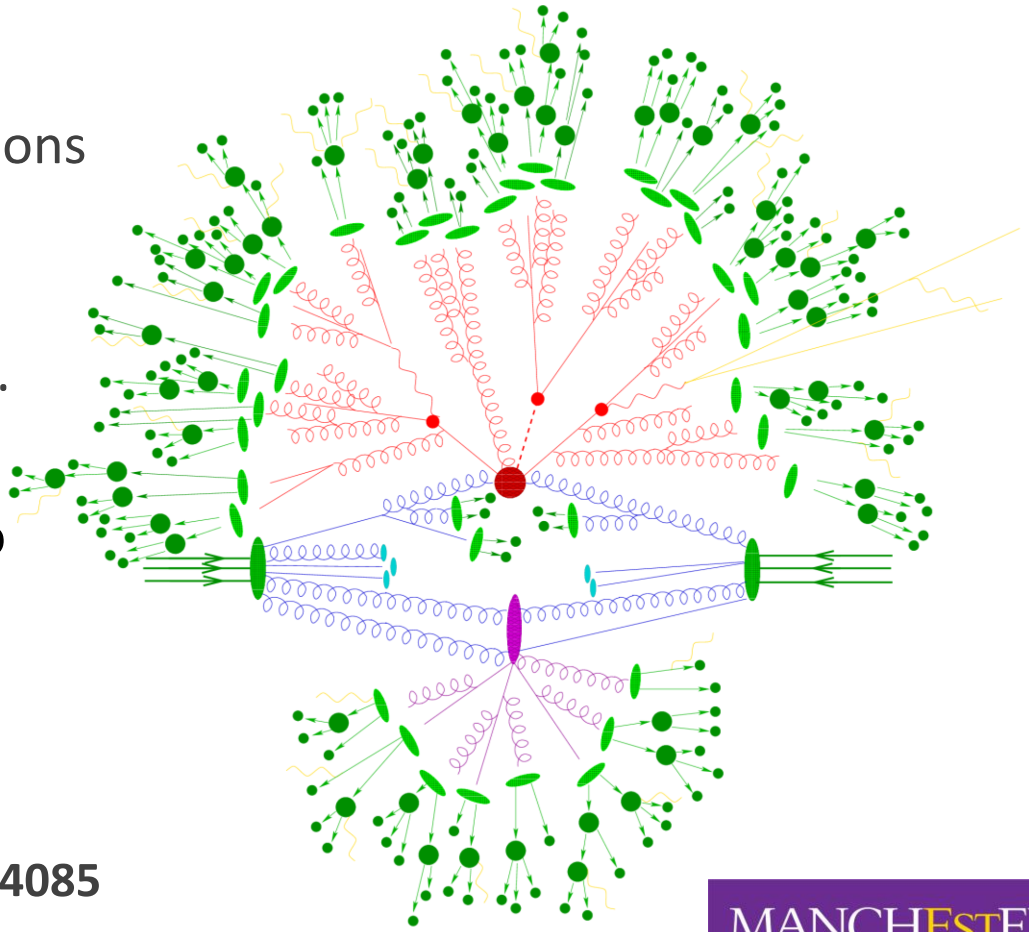
Why parton showers?

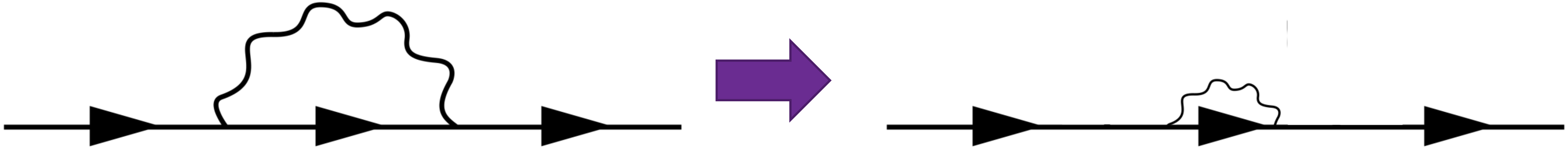
- Parton showers approximate the cascade of emissions we see at colliders. These cascades produce jets. This is the nice intuitive picture at least.
- Really parton showers approximate resummations.
- Resummations are crucial for making accurate predictions with QCD. However they are difficult to perform.



Höche, **arXiv:1411.4085**

Why resummations?

Let's look at the types of divergences.

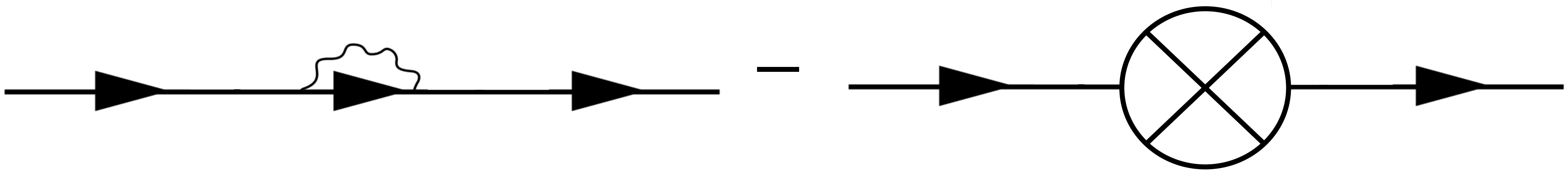


- Position space: probing smaller length scale with higher momentum (UV divergence)

- Momentum space: either the energy goes to 0 (soft IR divergence) or $(p-k)^2$ goes to 0 (collinear IR divergence).

Why resummations?

How do we sort this? Let's look at an UV divergence.



$$\sim \ln \frac{Q}{\mu}$$

- We renormalize the theory. This is a clever trick that introduces terms that exactly cancel divergences by definition. To do this we add a new parameter, the scale the theory is defined at, μ.
- We can do a second clever trick. We can make μ be a variable so that we can use the theory at any scale.

Why resummations?

- This gives the running couplings. This can be viewed as a resummation of the logarithms involved.

$$\bar{\alpha}(q) = \frac{\alpha}{1 - (\alpha/3\pi)\log(-q^2/Am^2)}$$

$$= \alpha\left(1 + \frac{\alpha}{3\pi}\log\frac{-q^2}{Am^2} - \frac{\alpha^2}{9\pi^2}\left(\log\frac{-q^2}{Am^2}\right)^2 + \mathcal{O}(\alpha^3)\right)$$

- We could actually derive this result simply by looking at the diagrams involved and trying to find some careful statements that are true to all orders. We can also use RG flow.

Why resummations?

What about the IR divergences?

- These look a little different. The Bloch-Nordsieck theorem guarantees exact cancelations of divergences from emissions against those from loops. (The real part at least)

- However the functional form of terms containing the real emissions depends on your observable. Therefore every observable has unique cancellations!

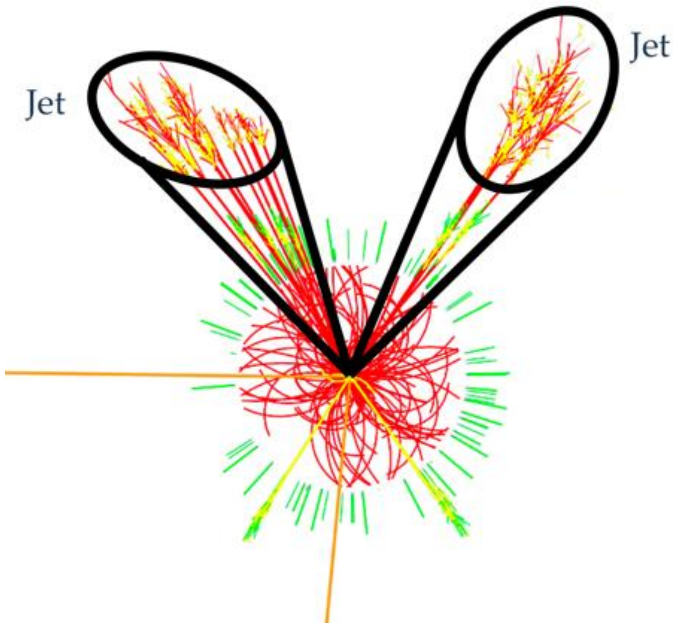
- The cancelations generate logarithms dependant on your observable.

- This is the hard part of perturbative QCD!

Why resummations?

Consider a simple example.

- You want to accept jets into your analysis and veto everything else.
- However your detector has a minimum energy it is sensitive to, Q_0.
- Perform this calculation and logarithms emerge $\ln\frac{Q}{Q_0} \gg 1$.
- In fact they emerge at each order in α_s as ${\alpha_s}^n \left(\ln\frac{Q}{Q_0}\right)^n$. This can completely spoil convergence.



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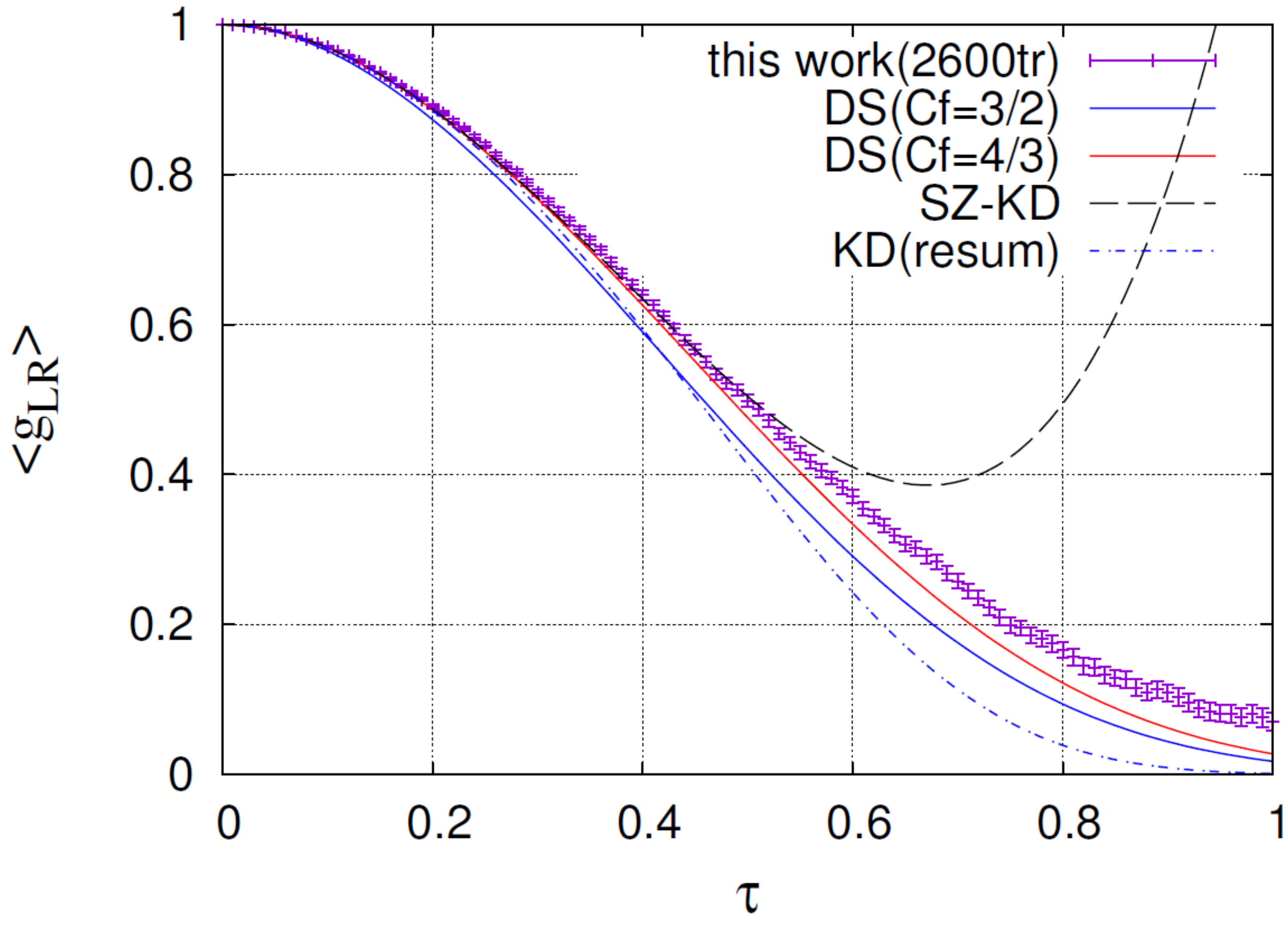
Why parton showers?

- In order to restore convergence we want to resum them to all orders just as we did with the running coupling. However the is time consuming to do analytically. We need an algorithm for calculating arbitrary resummations.
- This is been achieved exactly at NLL at leading colour for global observables. CAESAR by Banfi et al, **arXiv:0304148** (2003)
- Parton showers are approximations to resummations. Not analytically derived! Pieced together by intuition and tuned to match resummations/data. Recent studies question their accuracy. Dasgupta et al, **arXiv:1805.09327** (2018)
- We want an analytically correct algorithm that exactly reproduces resummations at LL and with full colour but can also be used as a parton shower!

Why our parton shower?



Jet hemisphere mass resummation
arXiv:1507.07641

The tool box

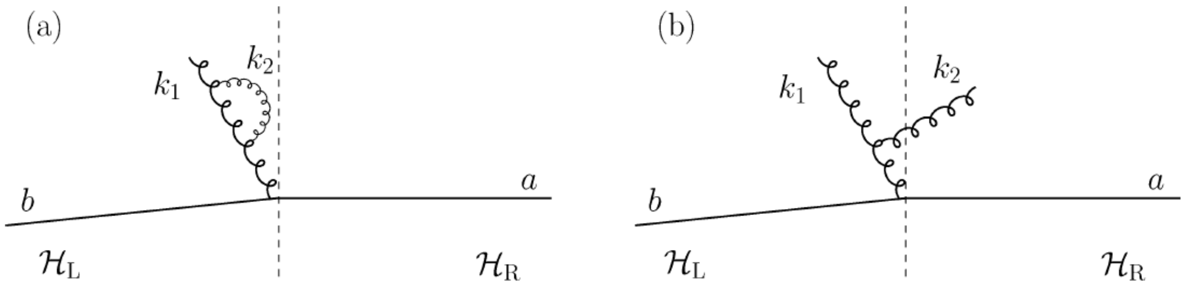
Known structures and good ansatzes

- Double logarithms -> Pole of order $\epsilon^{-2n}\alpha^n$. From the soft-collinear region. $L^{2n-2m}\alpha^n, m \in \mathbb{Z}^+$.
- Single -> Poles of order $\epsilon^{-n}\alpha^n$. From hard collinear or wide angle soft regions. $L^{n-m}\alpha^n, m \in \mathbb{Z}^+$.
- Combined we get $L^{2n-m}\alpha^n, m \in \mathbb{Z}^+$ the full spectrum.
- Global logarithms.

  Catani et al, **Nuclear Physics B407 (1993) 3-42**
- Non-global logarithms

  Dasgupta et al, **arXiv:hep-ph/0104277** (2001)
- Superleading logarithms

  Forshaw et al, **arXiv:0808.1269** (2008)



- Well known fixed order results back these up: cusp anomalous dimensions, calculations to α^5, etc
- All order results and evolution equations: SCET

$$\Sigma(v) = \left(1 + \sum_n C_n \bar{\alpha}_s^n\right) e^{L g_1(\alpha_s L) + g_2(\alpha_s L) + \cdots}$$

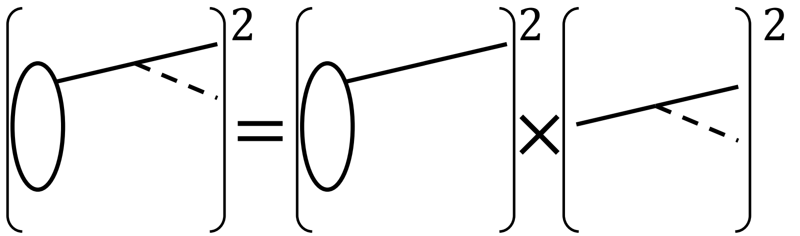
The tool box

Factorisation:

$$\mathcal{M}^{n+1}_{s_1\ldots s_i\lambda_j}(\ldots,p_i,p_j) = igT^a_f\epsilon^{*a}_{\lambda_j\,\mu}(p_j)\bar{u}_{s_i}(p_i)\gamma^\mu\frac{i\not{p}_{ij}}{p_{ij}^2+i\epsilon}\hat{\mathcal{M}}^n_{s_1\ldots}(\ldots,p_{ij})$$

- Soft limit $p_j\sim = \lambda p_j$ where $\lambda \to 0$. We find an Eikonal current

$$\mathcal{M}^{n+1}\quad(\ldots,p_i,p_j) = g\frac{\epsilon^a_j\cdot p_i}{p_i\cdot p_j}T^a_f\,\mathcal{M}^n\quad(\ldots,p_{ij})$$



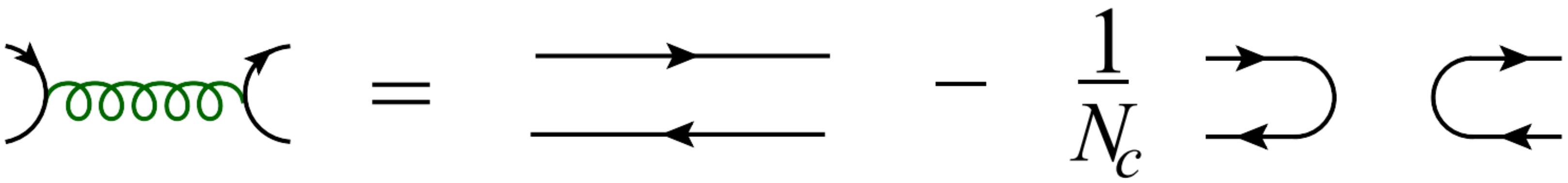
- Collinear limit $p_i \cong z(p_i+p_j)$ where $p_{j\,\mathrm{T}} \to 0$. We find splitting functions

$$\mathcal{M}^{n+1}_{s_1\ldots s_i\lambda_j}(\ldots,p_i,p_j) = gT_f\sqrt{\frac{\mathcal{P}_{qq}}{\mathcal{C}_\mathrm{F}(1+z^2)}}\frac{1}{\langle p_jp_i\rangle}\mathcal{M}^n_{s_1\ldots\frac{1}{2}}(\ldots,p_{ij})\delta_{s_i\frac{1}{2}}\delta_{\lambda_j1} + gT_f\sqrt{\frac{z^2\mathcal{P}_{qq}}{\mathcal{C}_\mathrm{F}(1+z^2)}}\frac{1}{[p_ip_j]}\mathcal{M}^n_{s_1\ldots\frac{1}{2}}(\ldots,p_{ij})\delta_{s_i\frac{1}{2}}\delta_{\lambda_j-1}$$
$$+ gT_f\sqrt{\frac{z^2\mathcal{P}_{qq}}{\mathcal{C}_\mathrm{F}(1+z^2)}}\frac{1}{\langle p_jp_i\rangle}\mathcal{M}^n_{s_1\ldots-\frac{1}{2}}(\ldots,p_{ij})\delta_{s_i-\frac{1}{2}}\delta_{\lambda_j1} + gT_f\sqrt{\frac{\mathcal{P}_{qq}}{\mathcal{C}_\mathrm{F}(1+z^2)}}\frac{1}{[p_ip_j]}\mathcal{M}^n_{s_1\ldots-\frac{1}{2}}(\ldots,p_{ij})\delta_{s_i-\frac{1}{2}}\delta_{\lambda_j-1}.$$

The tool box

Playing with colour:

- Fierz Identity



$$(T^a)_{i_1}^{\;\bar{j}_1}\,(T^a)_{i_2}^{\;\bar{j}_2} = \delta_{i_1}^{\;\bar{j}_2}\,\delta_{i_2}^{\;\bar{j}_1} - \frac{1}{N_c}\,\delta_{i_1}^{\;\bar{j}_1}\,\delta_{i_2}^{\;\bar{j}_2}$$

Dixon et al, **arXiv:1310.5353** (2013)

The tool box

Playing with colour:

- Basis independent notation

$$(T_i^a)_{dc} = \begin{cases} t^a_{dc} & i = \bar{u}, \\ t^a_{dc} & i = \bar{v}, \\ -t^a_{cd} & i = u, \\ -t^a_{dc} & i = v, \\ i f^{dac} & i = \epsilon, \\ -i f^{cad} & i = \epsilon^*. \end{cases}$$

$$\mathcal{M}^{c_1 \ldots c_n}(p_1, \ldots, p_n) = \langle c_1 \ldots c_n | \, n \rangle$$

$$\langle d_i | \, \mathbb{T}_i^a \, | c_i \rangle = (T_i^a)_{dc},$$

$$\langle d_1, \ldots, d_i, \ldots, d_n | \, \mathbb{T}_i^a \, | c_1, \ldots, c_i, \ldots, c_n \rangle = \delta_{d_1 c_1} \ldots (T_i^a)_{d_i c_i} \ldots \delta_{d_n c_n},$$

$$[\mathbb{T}_i^a, \mathbb{T}_j^b] = \begin{cases} i f^{abc} \mathbb{T}_i^c & i = j, \\ 0 & i \neq j. \end{cases}$$

$$\sum_{i \neq j} \mathbb{T}_i \cdot \mathbb{T}_j = -\mathbb{T}_i \cdot \mathbb{T}_i.$$

$$\mathbb{T}_i^\dagger \cdot \mathbb{T}_i = \mathcal{C}_i \mathbb{1}.$$

The tool box

Playing with colour:

- Colour flow basis

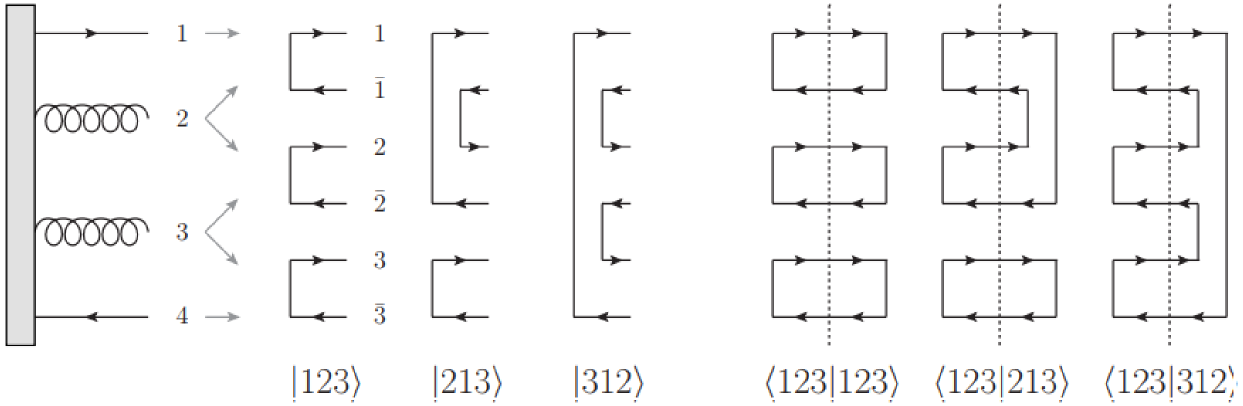
$$|\sigma\rangle = \left| \begin{array}{ccc} 1 & \cdots & n \\ \sigma(1) & \cdots & \sigma(n) \end{array} \right\rangle = \delta^{\alpha_1}_{\bar{\alpha}_{\sigma(1)}} \cdots \delta^{\alpha_n}_{\bar{\alpha}_{\sigma(n)}}$$

$$\mathbf{T}_i = \lambda_i \, \mathbf{t}_{c_i} - \bar{\lambda}_i \, \bar{\mathbf{t}}_{\bar{c}_i} - \frac{1}{N}(\lambda_i - \bar{\lambda}_i) \, \mathbf{s}$$

$$\mathbf{t}_\alpha|\sigma\rangle = \mathbf{t}_\alpha \left| \begin{array}{ccccc} 1 & \cdots & \alpha & \cdots & n \\ \sigma(1) & \cdots & \sigma(\alpha) & \cdots & \sigma(n) \end{array} \right\rangle = \left| \begin{array}{cccccc} 1 & \cdots & \alpha & \cdots & n & n+1 \\ \sigma(1) & \cdots & n+1 & \cdots & \sigma(n) & \sigma(\alpha) \end{array} \right\rangle$$

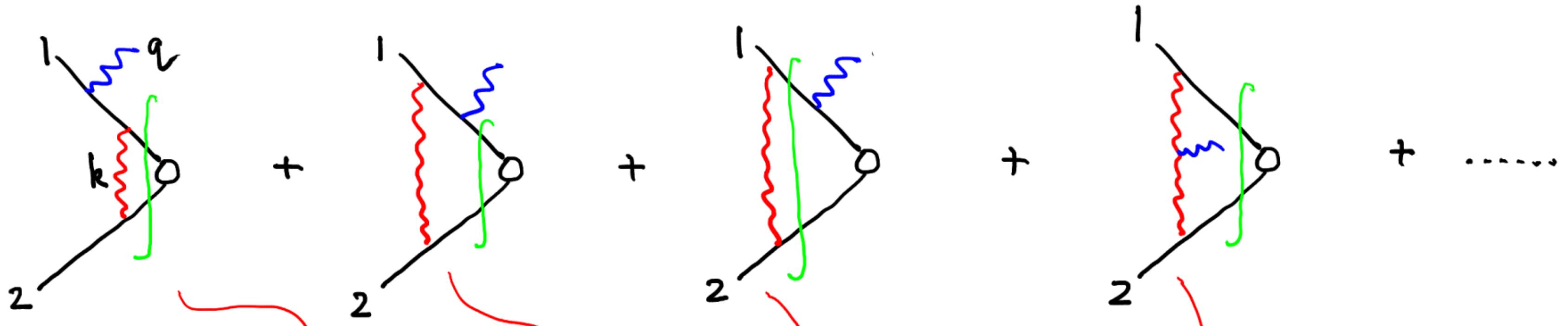
$$\bar{\mathbf{t}}_{\bar{\alpha}}|\sigma\rangle = \mathbf{t}_{\sigma^{-1}(\bar{\alpha})}|\sigma\rangle \; ,$$

$$\mathbf{s}|\sigma\rangle = \mathbf{s} \left| \begin{array}{cccc} 1 & \cdots & \cdots & n \\ \sigma(1) & \cdots & \cdots & \sigma(n) \end{array} \right\rangle = \left| \begin{array}{ccccc} 1 & \cdots & \cdots & n & n+1 \\ \sigma(1) & \cdots & \cdots & \sigma(n) & n+1 \end{array} \right\rangle$$



Martínez et al, **arXiv:1802.08531**

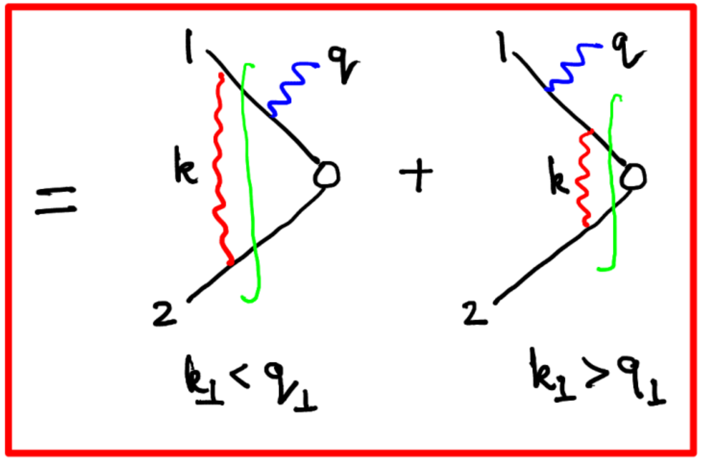
Start with one emission case

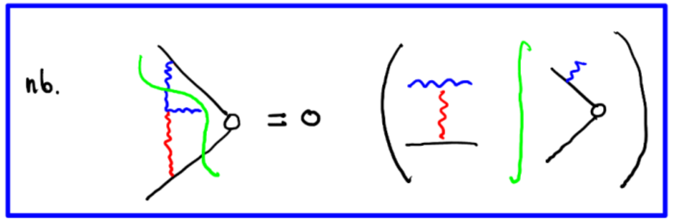


$$i\pi \int \frac{d^2k_\perp}{k_\perp^2}\,\frac{p_1\cdot\epsilon^*}{p_1\cdot q}\left\{ t_1\,(t_1\cdot t_2) - T_1\cdot T_2\, t_1 + T_1\cdot T_2\, t_1 \right\}$$

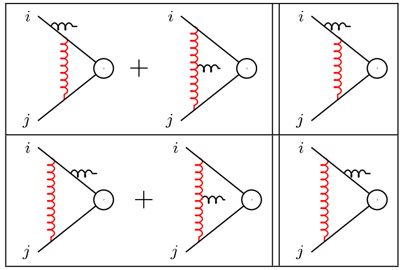
$$+\, i\pi \int \frac{d^2k_\perp}{k_\perp^2}\,\frac{q_\perp^2}{k_\perp^2+q_\perp^2}\,\frac{p_1\cdot\epsilon^*}{p_1\cdot q}\left\{ T_1\cdot T_2\, t_1 - t_1\, t_1\cdot t_2 \right\}$$

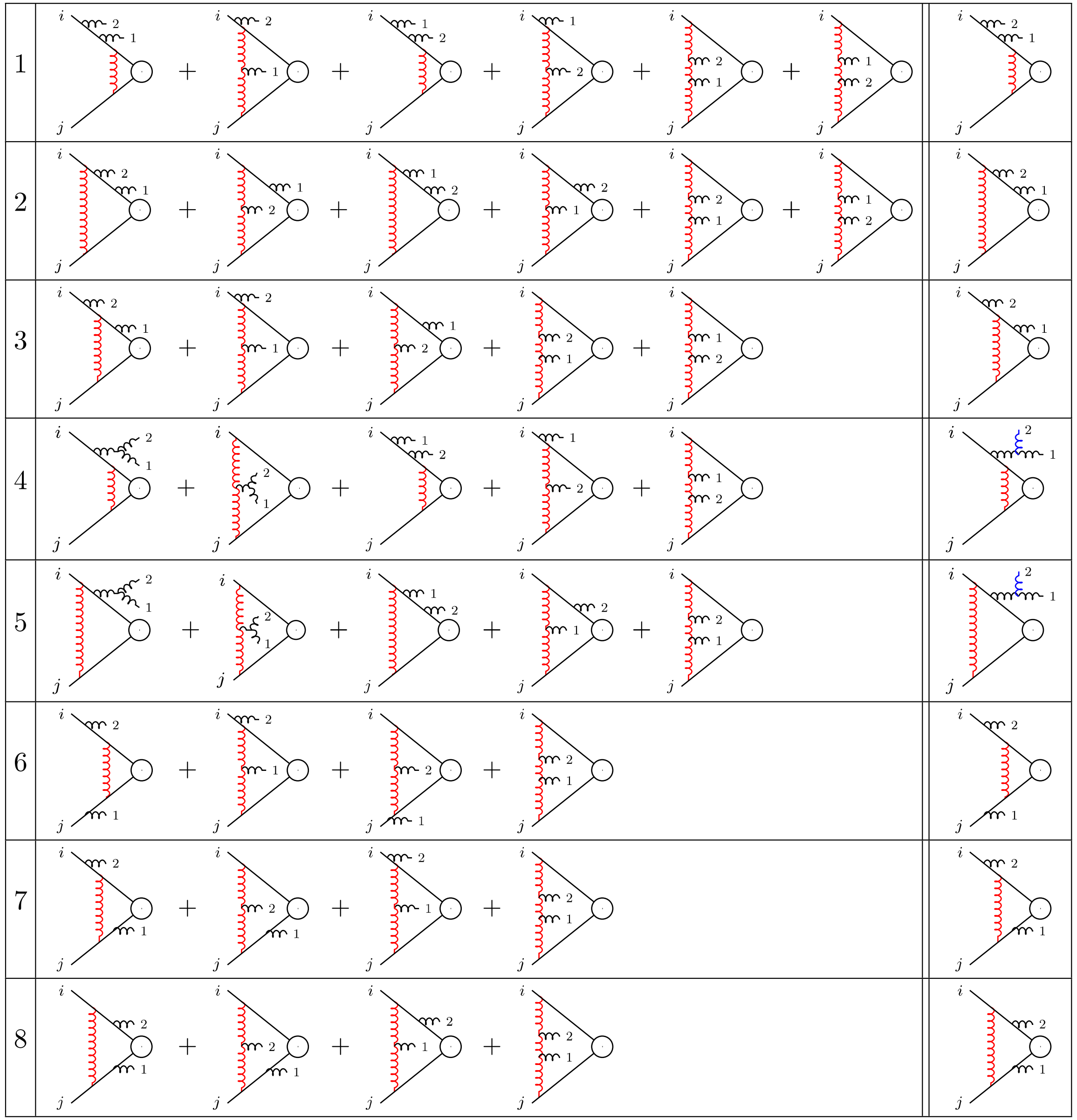
"switch"





Slide taken from Jeff Forshaw's 2017 presentation at Lund



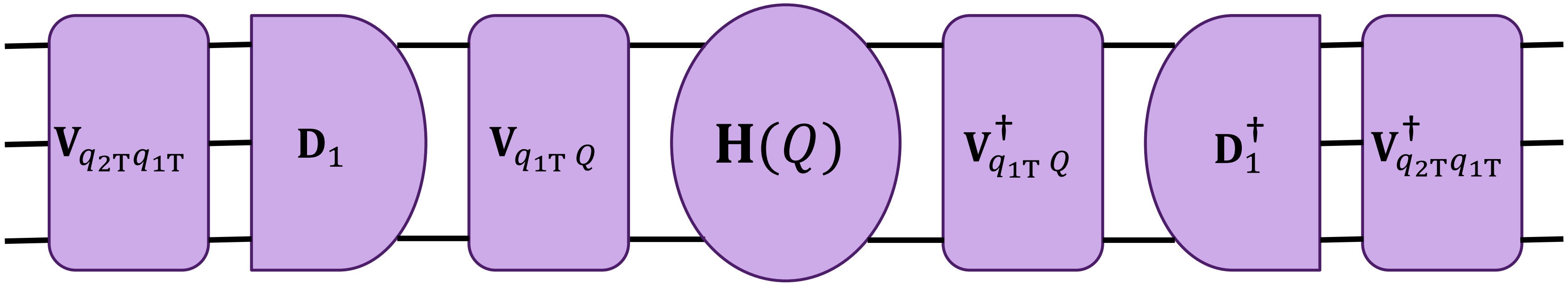


René Ángeles-Martínez: PhD thesis
Ángeles-Martínez , JRF, Seymour: arXiv:1510.07998

Slide taken from Jeff Forshaw's 2017 presentation at Lund

Our algorithm

J. Forshaw, M. Seymour, R. Angeles, M. De Angelis, S. Plätzer, J. Holguin, and more.



$$\sigma_0 = \mathrm{Tr}\left(\mathbf{V}_{\mu,Q}\mathbf{H}(Q)\mathbf{V}^\dagger_{\mu,Q}\right) \hat{=} \mathrm{Tr}\mathbf{A}_0(\mu),$$

$$\mathrm{d}\sigma_1 = \mathrm{Tr}\left(\mathbf{V}_{\mu,q_{\perp 1}}\mathbf{D}_1\mathbf{V}_{q_{\perp 1},Q}\mathbf{H}(Q)\mathbf{V}^\dagger_{q_{\perp 1},Q}\mathbf{D}_1^\dagger\mathbf{V}^\dagger_{\mu,q_{\perp 1}}\right)\mathrm{d}\Pi_1 \hat{=} \mathrm{Tr}\mathbf{A}_1(\mu)\mathrm{d}\Pi_1,$$

$$\mathrm{d}\sigma_n \hat{=} \mathrm{Tr}\mathbf{A}_n(\mu)\prod_{i=1}^{n}\mathrm{d}\Pi_i,$$

where

$$\mathbf{A}_n(q_\perp) = \mathbf{V}_{q_\perp,q_{\perp n}}\mathbf{D}_n\mathbf{A}_{n-1}(q_{\perp n})\mathbf{D}_n^\dagger\mathbf{V}^\dagger_{q_\perp,q_{\perp n}}\Theta(q_\perp \le q_{\perp n}).$$

$$\Sigma(\mu) = \int\sum_n \mathrm{d}\sigma_n u_n(q_1, ..., q_n),$$

$$= \int\sum_n\left(\prod_{i=1}^{n}\mathrm{d}\Pi_i\right)\mathrm{Tr}\mathbf{A}_n(\mu)u_n(q_1, ..., q_n).$$

Our algorithm

$$\mathbf{V}_{a,b} = \exp\left[-\frac{\alpha_s}{\pi}\int_a^b \frac{\mathrm{d}k_\perp}{k_\perp}\sum_{i<j}(-\mathbb{T}_i^g\cdot\mathbb{T}_j^g)\left\{\int\frac{\mathrm{d}y\mathrm{d}\phi}{4\pi}k_\perp^2\frac{p_i\cdot p_j}{(p_i\cdot k)(p_j\cdot k)} - i\pi\tilde{\delta}_{ij}\right\}\right.$$
$$\left.-\frac{\alpha_s}{\pi}\int_a^b\frac{\mathrm{d}k_\perp}{k_\perp}\sum_i \mathbb{T}_i^{g\,2}\sum_k\int\frac{\mathrm{d}z\mathrm{d}\phi}{8\pi}\overline{\mathcal{P}}_{ki}^{\circ}\right].$$

$$\mathrm{Tr}(\ldots\mathbf{D}_i\mathcal{O}\mathbf{D}_i^\dagger\ldots) = \mathrm{Tr}(\ldots\mathbf{S}_i\mathcal{O}\mathbf{S}_i^\dagger\ldots) + \mathrm{Tr}(\ldots\mathbf{C}_i\mathcal{O}\mathbf{C}_i^\dagger\ldots).$$

$$\mathrm{d}\Pi_i = \frac{\alpha_s}{\pi}\frac{\mathrm{d}q_{\perp i}}{q_{\perp i}}\frac{\mathrm{d}z_i}{z_i(1-z_i)}\frac{\mathrm{d}\phi_i}{2\pi},$$

$$\mathbf{S}_i = \sum_j\frac{q_{\perp i}}{2p_j\cdot q_i}\mathbb{T}_j^g\otimes(p_j\cdot\epsilon_+(q_i)\mathbb{S}^{1_i} + p_j\cdot\epsilon_-(q_i)\mathbb{S}^{-1_i}), \quad \mathbf{C}_i = \sum_j\Delta_{ij}\overline{\mathbf{P}}_{ij},$$

Our algorithm

$$
\begin{aligned}
\mathbf{P}_{ij} = \delta_{s_j-\frac{1}{2}}\delta_j^{\text{final}} \Bigg(& \sqrt{\frac{\mathcal{P}_{qq}}{\mathcal{C}_\text{F}(1+z_i^2)}}\frac{1}{\langle q_i p_j\rangle}(\mathbb{T}_j^g \otimes \mathbb{S}^{+1_i}) \\
& + \sqrt{\frac{z_i^2\mathcal{P}_{qq}}{\mathcal{C}_\text{F}(1+z_i^2)}}\frac{1}{[p_j q_i]}(\mathbb{T}_j^g \otimes \mathbb{S}^{-1_i}) + \sqrt{\frac{\mathcal{P}_{gq}}{\mathcal{C}_\text{F}(2-2z_i+z_i^2)}}\frac{1}{\langle p_j q_i\rangle}\mathbb{W}^{ij}(\mathbb{T}_j^g \otimes \mathbb{S}^{+1_i}) \\
& + \sqrt{\frac{(1-z_i)^2\mathcal{P}_{gq}}{\mathcal{C}_\text{F}(2-2z_i+z_i^2)}}\frac{1}{[q_i p_j]}\mathbb{W}^{ij}(\mathbb{T}_j^g \otimes \mathbb{S}^{-1_i}) \Bigg) \\
+ \delta_{s_j\frac{1}{2}}\delta_j^{\text{final}} \Bigg(& \sqrt{\frac{\mathcal{P}_{qq}}{\mathcal{C}_\text{F}(1+z_i^2)}}\frac{1}{[p_j q_i]}(\mathbb{T}_j^g \otimes \mathbb{S}^{-1_i}) \\
& + \sqrt{\frac{z_i^2\mathcal{P}_{qq}}{\mathcal{C}_\text{F}(1+z_i^2)}}\frac{1}{\langle q_i p_j\rangle}(\mathbb{T}_j^g \otimes \mathbb{S}^{+1_i}) + \sqrt{\frac{\mathcal{P}_{gq}}{\mathcal{C}_\text{F}(2-2z_i+z_i^2)}}\frac{1}{[q_i p_j]}\mathbb{W}^{ij}(\mathbb{T}_j^g \otimes \mathbb{S}^{-1_i}) \\
& + \sqrt{\frac{(1-z_i)^2\mathcal{P}_{gq}}{\mathcal{C}_\text{F}(2-2z_i+z_i^2)}}\frac{1}{\langle p_j q_i\rangle}\mathbb{W}^{ij}(\mathbb{T}_j^g \otimes \mathbb{S}^{+1_i}) \Bigg) \\
+ \delta_{s_j-1}\delta_j^{\text{final}} \Bigg(& \sqrt{\frac{\mathcal{C}_\text{A}(1-z_i)^2\mathcal{P}_{qg}}{\mathcal{C}_\text{F}(1-2(1-2z_i))}}\frac{1}{[p_j q_i]}(\mathbb{T}_j^q \otimes \mathbb{P}_j^2\mathbb{S}^{+\frac{1}{2}_i}) \\
& + \sqrt{\frac{\mathcal{C}_\text{A}z_i^2\mathcal{P}_{qg}}{\mathcal{C}_\text{F}(1-2(1-2z_i))}}\frac{1}{[p_j q_i]}(\mathbb{T}_j^q \otimes \mathbb{P}_j^1\mathbb{P}_j^2\mathbb{S}^{-\frac{1}{2}_i}) \\
& + \sqrt{\frac{\mathcal{P}_{gg}}{2\mathcal{C}_\text{A}(1-z_i+z_i^2)}}\frac{1}{\langle q_i p_j\rangle}(\mathbb{T}_j^g \otimes \mathbb{S}^{+1_i}) \\
& + \sqrt{\frac{z_i^4\mathcal{P}_{gg}}{2\mathcal{C}_\text{A}(1-z_i+z_i^2)}}\frac{1}{[q_i p_j]}(\mathbb{T}_j^g \otimes \mathbb{S}^{-1_i}) + \sqrt{\frac{\mathcal{P}_{gg}(1-z_i)^4}{2\mathcal{C}_\text{A}(1-z_i+z_i^2)}}\frac{1}{[p_j q_i]}(\mathbb{T}_j^g \otimes \mathbb{P}_j^1\mathbb{S}^{+1_i}) \Bigg) \\
+ \delta_{s_j1}\delta_j^{\text{final}} \Bigg(& \sqrt{\frac{\mathcal{C}_\text{A}(1-z_i)^2\mathcal{P}_{qg}}{\mathcal{C}_\text{F}(1-2(1-2z_i))}}\frac{1}{\langle q_i p_j\rangle}(\mathbb{T}_j^q \otimes \mathbb{P}_j^2\mathbb{S}^{-\frac{1}{2}_i}) \\
& + \sqrt{\frac{\mathcal{C}_\text{A}z_i^2\mathcal{P}_{qg}}{\mathcal{C}_\text{F}(1-2(1-2z_i))}}\frac{1}{\langle q_i p_j\rangle}(\mathbb{T}_j^q \otimes \mathbb{P}_j^1\mathbb{P}_j^2\mathbb{S}^{+\frac{1}{2}_i}) + \sqrt{\frac{\mathcal{P}_{gg}}{2\mathcal{C}_\text{A}(1-z_i+z_i^2)}}\frac{1}{[p_j q_i]}(\mathbb{T}_j^g \otimes \mathbb{S}^{-1_i}) \\
+ \delta_{s_j1}\delta_j^{\text{final}} \Bigg(& \sqrt{\frac{\mathcal{C}_\text{A}(1-z_i)^2\mathcal{P}_{qg}}{\mathcal{C}_\text{F}(1-2(1-2z_i))}}\frac{1}{\langle q_i p_j\rangle}(\mathbb{T}_j^q \otimes \mathbb{P}_j^2\mathbb{S}^{-\frac{1}{2}_i}) \\
& + \sqrt{\frac{\mathcal{C}_\text{A}z_i^2\mathcal{P}_{qg}}{\mathcal{C}_\text{F}(1-2(1-2z_i))}}\frac{1}{\langle q_i p_j\rangle}(\mathbb{T}_j^q \otimes \mathbb{P}_j^1\mathbb{P}_j^2\mathbb{S}^{+\frac{1}{2}_i}) + \sqrt{\frac{\mathcal{P}_{gg}}{2\mathcal{C}_\text{A}(1-z_i+z_i^2)}}\frac{1}{[p_j q_i]}(\mathbb{T}_j^g \otimes \mathbb{S}^{-1_i}) \\
& + \sqrt{\frac{z_i^4\mathcal{P}_{gg}}{2\mathcal{C}_\text{A}(1-z_i+z_i^2)}}\frac{1}{\langle p_j q_i\rangle}(\mathbb{T}_j^g \otimes \mathbb{S}^{+1_i}) + \sqrt{\frac{\mathcal{P}_{gg}(1-z_i)^4}{2\mathcal{C}_\text{A}(1-z_i+z_i^2)}}\frac{1}{\langle q_i p_j\rangle}(\mathbb{T}_j^g \otimes \mathbb{P}_j^1\mathbb{S}^{-1_i}) \Bigg)
\end{aligned}
\tag{A.1}
$$

+emissions from incoming

Our algorithm

With this algorithm we have:

1. Reproduced collinear factorisation theorems.
2. Reproduced several analytic resummations
   - Thrust
   - BMS evolution
   - DGLAP evolution
   - Gaps between Jets
   - Jet hemisphere mass
   - Fragmentation evolution
3. Performed fixed order cross checks, over 1000 diagrams.
4. Made substantial progress producing a functioning code for this algorithm. CVolver.
5. Performed numerical studies of the sub-leading colour. More coming.
6. Derived several alternative algorithms, each suited to different tasks (i.e. global observables, simplifying collinear colour structures, super-leading log insensitive observables).
7. Currently analysing and mapping onto other work with spin.
8. Hopefully several more papers coming soon with more details.

Martínez et al **arXiv:1802.08531** (2018)

Martínez et al **arXiv:1510.07998** (2015)