Sp($2N$) Gauge Theories and the Composite Higgs

~~Arthur Weasley~~
Jack Holligan

Swansea University

20$^{\text{th}}$ December 2018

Outline

1. Problems with the Higgs
2. Composite Higgs
3. Goldstone's Theorem
4. Finding the Composite Higgs

Problems with the Higgs

Like other particles in QFT, the Higgs boson acquires loop corrections to its mass:

[1] Panico and Wulzer, *The Composite Nambu-Goldstone Higgs*, 2015

Problems with the Higgs

Like other particles in QFT, the Higgs boson acquires loop corrections to its mass:

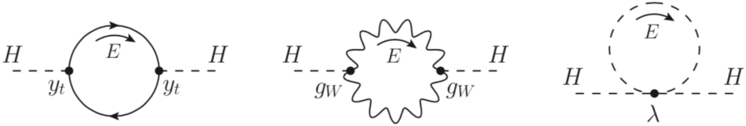

Figure 1: Loop Corrections to the Higgs Mass[1].

Left to right: fermion loop (dominated by the top quark), weak boson loop, self coupling loop.

[1] Panico and Wulzer, *The Composite Nambu-Goldstone Higgs*, 2015

Problems with the Higgs

If we insert an ultraviolet cutoff, Λ_{UV}, the aforementioned loop corrections give rise to a quadratic divergence in the correction to the Higgs mass:

$$\delta m_H^2 = \frac{3\Lambda_{\mathrm{UV}}^2}{8\pi^2}\left[y_t^2 - g_W^2\left(\frac{1}{4} + \frac{1}{8\cos^2\theta_W}\right) - \lambda\right]$$

where y_t, g_W and λ are the Higgs couplings to the top quark, W bosons and Higgs respectively[2].

[2] Panico and Wulzer, *The Composite Nambu-Goldstone Higgs*, 2015

Problems with the Higgs

If we insert an ultraviolet cutoff, Λ_{UV}, the aforementioned loop corrections give rise to a quadratic divergence in the correction to the Higgs mass:

$$\delta m_H^2 = \frac{3\Lambda_{\text{UV}}^2}{8\pi^2}\left[y_t^2 - g_W^2\left(\frac{1}{4} + \frac{1}{8\cos^2\theta_W}\right) - \lambda\right]$$

where y_t, g_W and λ are the Higgs couplings to the top quark, W bosons and Higgs respectively[2].

If the Standard Model is valid up to a finite energy then new physics emerges above Λ_{UV}. The corrections to the Higgs mass at these higher energies come from *a priori* unrelated terms. In order to produce the observed Higgs mass of 125 GeV, these two terms must cancel out correctly to 1 part in 10^{24}.

[2] Panico and Wulzer, *The Composite Nambu-Goldstone Higgs*, 2015

The Naturalness Problem

This problem of fine tuning is known as the “Naturalness Problem”. Such incredible precision between two independent terms seems unlikely to occur.

Not only is fine tuning on this scale “unnatural” but it is almost certainly impossible to measure experimentally.

A Composite Particle

A solution to the
Naturalness Problem would be to treat the Higgs not as a fundamental particle but as a composite object similar to the proton as shown in figure [2][3].

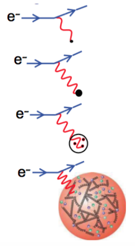

Figure 2: Proton's Internal Structure

[3] Image from Philip Clark, *Edinburgh University Lecture Notes*, 2016

A Composite Particle

A solution to the
Naturalness Problem would be to treat the Higgs not as a fundamental particle but as a composite object similar to the proton as shown in figure [2][3].

An analogy can be made
by the scattering electrons off a proton. When a soft electron collides with a proton, we may approximate the proton as being a point charge. As the electron's energy increases, the proton goes from being a point charge to being an extended charge to a triplet of quarks to a sea of partons.

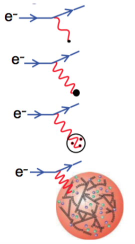

Figure 2: Proton's Internal Structure

[3] Image from Philip Clark, *Edinburgh University Lecture Notes*, 2016

Fermi Theory

Another example of such approximation is that of an effective field theory (EFT). Fermi's theory of β-decay is an example of an EFT.

Fermi Theory

Another example of such approximation is that of an effective field theory (EFT). Fermi's theory of β-decay is an example of an EFT.

The interaction term in Fermi's Theory is

$$\mathcal{L}_{\text{int}} = \frac{G_F}{\sqrt{2}} \left[\left(\underbrace{\overline{\psi}_e}_{e^-} \gamma^\mu (1-\gamma^5) \overbrace{\psi_\nu}^{\overline{\nu}_e} \right) g_{\mu\nu} \left(\underbrace{\overline{\psi}_u}_{\text{u-quark}} \gamma^\nu (1-\gamma^5) \overbrace{\psi_d}^{\text{d-quark}} \right) \right]$$

where G_F is the Fermi constant.

Fermi Theory

Another example of such approximation is that of an effective field theory (EFT). Fermi's theory of β-decay is an example of an EFT.

The interaction term in Fermi's Theory is

$$\mathcal{L}_{\text{int}} = \frac{G_F}{\sqrt{2}} \left[\left(\underbrace{\overline{\psi}_e}_{e^-} \gamma^\mu (1-\gamma^5) \overbrace{\psi_\nu}^{\overline{\nu}_e} \right) g_{\mu\nu} \left(\underbrace{\overline{\psi}_u}_{\text{u-quark}} \gamma^\nu (1-\gamma^5) \overbrace{\psi_d}^{\text{d-quark}} \right) \right]$$

where G_F is the Fermi constant.

If we examine this closely, we see that – in (3+1) dimensions – G_F has negative mass dimension which signals non-renormalisability. However this is not a problem since the theory is only a low energy approximation of β-decay which has since been replaced by our modern understanding of the Weak Force.

Fermi Theory

The Lagrangian for the Weak interaction is given by

$$\mathcal{L}_{\text{int}} = -\frac{g_W^2}{2}\left[\overline{\psi}_e \frac{1}{2}\gamma^\mu(1-\gamma^5)\psi_\nu\right]\left[\frac{g_{\mu\nu} - q_\mu q_\nu/m_W^2}{q^2 - m_W^2}\right]\left[\overline{\psi}_u \frac{1}{2}\gamma^\nu(1-\gamma^5)\psi_d\right]$$

Fermi Theory

The Lagrangian for the Weak interaction is given by

$$\mathcal{L}_{\text{int}} = -\frac{g_W^2}{2}\left[\overline{\psi}_e \frac{1}{2}\gamma^\mu(1-\gamma^5)\psi_\nu\right]\left[\frac{g_{\mu\nu} - q_\mu q_\nu/m_W^2}{q^2 - m_W^2}\right]\left[\overline{\psi}_u \frac{1}{2}\gamma^\nu(1-\gamma^5)\psi_d\right]$$

In the low energy limit ($q^2 \ll m_W^2$) this reduces to

$$\mathcal{L}_{\text{int}} = \frac{g_W^2}{8m_W^2}\left[\overline{\psi}_e \gamma^\mu(1-\gamma^5)\psi_\nu\right] g_{\mu\nu} \left[\overline{\psi}_u \gamma^\nu(1-\gamma^5)\psi_d\right]$$

giving the familiar formula $\frac{G_F}{\sqrt{2}} = \frac{g_W^2}{8m_W^2}$.

Fermi Theory

The Lagrangian for the Weak interaction is given by

$$\mathcal{L}_{\text{int}} = -\frac{g_W^2}{2}\left[\overline{\psi}_e \frac{1}{2}\gamma^\mu(1-\gamma^5)\psi_\nu\right]\left[\frac{g_{\mu\nu} - q_\mu q_\nu/m_W^2}{q^2 - m_W^2}\right]\left[\overline{\psi}_u \frac{1}{2}\gamma^\nu(1-\gamma^5)\psi_d\right]$$

In the low energy limit ($q^2 \ll m_W^2$) this reduces to

$$\mathcal{L}_{\text{int}} = \frac{g_W^2}{8m_W^2}\left[\overline{\psi}_e \gamma^\mu(1-\gamma^5)\psi_\nu\right] g_{\mu\nu}\left[\overline{\psi}_u \gamma^\nu(1-\gamma^5)\psi_d\right]$$

giving the familiar formula $\frac{G_F}{\sqrt{2}} = \frac{g_W^2}{8m_W^2}$.

The same approximation can be made in the process of muon decay: $\mu^- \to e^-\overline{\nu}_e\nu_\mu$ which is how G_F is measured experimentally.

A Resolution to the Naturalness Problem

The foregoing arguments show that a composite Higgs can be approximated as punctiform at low energies. Thus the quadratic divergence $\delta m_H^2 \propto \Lambda_{\mathrm{UV}}^2$ is merely an artifice of an approximation being pushed beyond the point of reliability.

Goldstone's Theorem

The compositeness takes care of the fine tuning but how can we ensure that the corresponding Higgs has the observed low mass?

Goldstone's Theorem

The compositeness takes care of the fine tuning but how can we ensure that the corresponding Higgs has the observed low mass?

Goldstone's Theorem

If symmetry group G is spontaneously broken to symmetry group H, there exist at least $\dim(G) - \dim(H)$ massless Nambu-Goldstone bosons (NGBs) where $\dim(X)$ is the dimension of group X.

Goldstone's Theorem

The compositeness takes care of the fine tuning but how can we ensure that the corresponding Higgs has the observed low mass?

Goldstone's Theorem

If symmetry group G is spontaneously broken to symmetry group H, there exist at least $\dim(G) - \dim(H)$ massless Nambu-Goldstone bosons (NGBs) where $\dim(X)$ is the dimension of group X.

A corollary of this is that if the symmetry breaking is explicit the corresponding bosons have a low mass. These are known as pseudo Nambu-Goldstone bosons (pNGBs). An example is chiral symmetry breaking in two-flavour ("very-low-energy") QCD.

Chiral Symmetry Breaking

In two flavour QCD, the fermionic sector is governed by

$$\mathcal{L}_{\text{ferm}} = \sum_{f=u,d} \overline{\psi}_f (i\not{D} - m_f)\psi_f$$

where f is a flavour index corresponding to the up (u) or down (d) quark. When $\{m_f\} = 0$, the theory is invariant under chiral transformations $\psi_f \to e^{i\alpha\gamma^5}\psi_f$.

Chiral Symmetry Breaking

In two flavour QCD, the fermionic sector is governed by

$$\mathcal{L}_{\text{ferm}} = \sum_{f=u,d} \overline{\psi}_f (i\not{D} - m_f)\psi_f$$

where f is a flavour index corresponding to the up (u) or down (d) quark. When $\{m_f\} = 0$, the theory is invariant under chiral transformations $\psi_f \to e^{i\alpha\gamma^5}\psi_f$.
We can separate each spinor into left- and right-handed components:

$$\begin{bmatrix} \psi_u \\ \psi_d \end{bmatrix}_L = \left(\frac{1-\gamma^5}{2}\right)\begin{bmatrix} \psi_u \\ \psi_d \end{bmatrix} \quad , \quad \begin{bmatrix} \psi_u \\ \psi_d \end{bmatrix}_R = \left(\frac{1+\gamma^5}{2}\right)\begin{bmatrix} \psi_u \\ \psi_d \end{bmatrix}$$

Chiral Symmetry Breaking

In the massless (chiral) limit, these two components are independent and, hence, can be transformed independently.

$$\begin{bmatrix} \psi_u \\ \psi_d \end{bmatrix}_L \longrightarrow U_L \begin{bmatrix} \psi_u \\ \psi_d \end{bmatrix}_L \quad , \quad \begin{bmatrix} \psi_u \\ \psi_d \end{bmatrix}_R \longrightarrow U_R \begin{bmatrix} \psi_u \\ \psi_d \end{bmatrix}_R$$

where the U's are, in general, different elements of $SU(2)$. This gives an underlying $SU(2)_L \otimes SU(2)_R$ symmetry.

Chiral Symmetry Breaking

In the massless (chiral) limit, these two components are independent and, hence, can be transformed independently.

$$\begin{bmatrix} \psi_u \\ \psi_d \end{bmatrix}_L \longrightarrow U_L \begin{bmatrix} \psi_u \\ \psi_d \end{bmatrix}_L \quad , \quad \begin{bmatrix} \psi_u \\ \psi_d \end{bmatrix}_R \longrightarrow U_R \begin{bmatrix} \psi_u \\ \psi_d \end{bmatrix}_R$$

where the U's are, in general, different elements of $SU(2)$. This gives an underlying $SU(2)_L \otimes SU(2)_R$ symmetry.

When the mass term is included, the left- and right-handed components of the spinors are mixed and the above transformation now only holds for $U_L = U_R$. Hence $SU(2)_L \otimes SU(2)_R$ has been broken to $SU(2)_V$.

By Goldstone's theorem (corollory), there should exist 3 light bosons and, indeed, there does: $\pi^{\pm}$ and π^0.

Chiral Symmetry Breaking

In the massless (chiral) limit, these two components are independent and, hence, can be transformed independently.

$$\begin{bmatrix} \psi_u \\ \psi_d \end{bmatrix}_L \longrightarrow U_L \begin{bmatrix} \psi_u \\ \psi_d \end{bmatrix}_L \quad , \quad \begin{bmatrix} \psi_u \\ \psi_d \end{bmatrix}_R \longrightarrow U_R \begin{bmatrix} \psi_u \\ \psi_d \end{bmatrix}_R$$

where the U's are, in general, different elements of $SU(2)$. This gives an underlying $SU(2)_L \otimes SU(2)_R$ symmetry.
When the mass term is included, the left- and right-handed components of the spinors are mixed and the above transformation now only holds for $U_L = U_R$. Hence $SU(2)_L \otimes SU(2)_R$ has been broken to $SU(2)_V$.
By Goldstone's theorem (corollory), there should exist 3 light bosons and, indeed, there does: $\pi^{\pm}$ and π^0.

The interpretation of the Higgs as a pNGB naturally encompasses its low mass.

A Candidate Symmetry

A gauge theory with the potential to explain a composite Higgs is the Symplectic group[4], denoted by $Sp(2N)$. A matrix, J, is in $Sp(2N)$ iff

$$J^T \Omega J = \Omega \quad \text{where } \Omega = \begin{bmatrix} 0 & \mathbb{1}_N \\ -\mathbb{1}_N & 0 \end{bmatrix}$$

and $\mathbb{1}_N$ is the $N \times N$ identity matrix. The 2N notation emphasises the fact that the group elements must be of even size though conventions differ throughout the literature.

[4] Bennett et. al, *Sp(4) Gauge Theory on the Lattice: Towards SU(4)/Sp(4) Composite Higgs and Beyond*, 2017

Making Predictions

Having justified the theory's use in describing the Higgs particle, we now use it to make predictions that can be observed in the lab. One such observable is the glueball spectrum.

Making Predictions

Having justified the theory's use in describing the Higgs particle, we now use it to make predictions that can be observed in the lab. One such observable is the glueball spectrum.

Glueballs are massive particles composed entirely of gluons or – in this case – the gauge bosons of $Sp(2N)$. By virtue of the gauge group being non-Abelian, the gauge bosons self interact and, thus, can form bound states subject to colour-confinement.

A single glueball can be uniquely identified by its spin (J), parity (P) and charge conjugation (C). These are denoted by J^{PC}.

The Glueball Spectrum

The Euclidean lattice breaks the continuous rotational symmetry of SO(3) to the discrete symmetries of a cube denoted by S_4.

The task, then, is to construct operators that live in specific representations of S_4: that is closed paths of lattice links that transform under specific representations of the cubic group. These can then be combined to form a single operator with specific parity and charge conjugation constructing a glueball of specific J^{PC}.

The Glueball Spectrum

In fact, the $Sp(2N)$ groups are pseudo-real and, thus, all glueballs have positive charge conjugation (a feature that also holds for real groups) thus the C term can be dropped for brevity.

The Glueball Spectrum

In fact, the $Sp(2N)$ groups are pseudo-real and, thus, all glueballs have positive charge conjugation (a feature that also holds for real groups) thus the C term can be dropped for brevity.

If we construct such a time-dependent operator, $\Phi(t)$, and insert a complete set of energy eigenstates, we get:

$$\begin{aligned} \langle \Phi^\dagger(t)\Phi(0)\rangle &= \sum_n \langle 0| \Phi^\dagger(t) |n\rangle \langle n| \Phi(0) |0\rangle \\ C_{\Phi\Phi}(t) &\equiv \sum_n \langle 0| e^{\hat{H}t}\Phi^\dagger(0)e^{-\hat{H}t} |n\rangle \langle n| \Phi(0) |0\rangle \\ C_{\Phi\Phi}(t) &= \sum_n | \langle n| \Phi(0) |0\rangle |^2 e^{-E_n t} \end{aligned}$$

taking $\hat{H} |0\rangle = 0 \implies \langle 0| e^{\hat{H}t} = 1$.

The Glueball Spectrum

The foregoing gives a correlator equal to a sum of decaying exponentials. As t increases, the one with the smallest mass – denoted by E_0 – dominates and the sum can be approximated as a single exponential. This gives

$$E_0 = -\lim_{t\to\infty} \frac{\ln(C_{\Phi\Phi}(t))}{t}.$$

The Glueball Spectrum

The foregoing gives a correlator equal to a sum of decaying exponentials. As t increases, the one with the smallest mass – denoted by E_0 – dominates and the sum can be approximated as a single exponential. This gives

$$E_0 = -\lim_{t\to\infty} \frac{\ln(C_{\Phi\Phi}(t))}{t}.$$

From the above, we can get an estimate of the glueball masses at different lattice spacings and then extrapolate to the continuum limit.

The Glueball Spectrum

The foregoing gives a correlator equal to a sum of decaying exponentials. As t increases, the one with the smallest mass – denoted by E_0 – dominates and the sum can be approximated as a single exponential. This gives

$$E_0 = -\lim_{t \to \infty} \frac{\ln(C_{\Phi\Phi}(t))}{t}.$$

From the above, we can get an estimate of the glueball masses at different lattice spacings and then extrapolate to the continuum limit.

This process can be repeated for $Sp(2N)$ at increasing values of N in order to obtain $N \to \infty$ extrapolations. Many physical observables based on $SU(N)$, $SO(N)$ or $Sp(2N)$ coincide in this limit and are analytically tractable.

Conclusion

- The Naturalness problem arises due to the Higgs' low mass which requires extreme fine tuning.
- A possible resolution is to treat the Higgs as a (composite) pNGB. This will remove the need for fine tuning and naturally encompasses its low mass.
- Measuring observable quantities such as the glueball spectrum will allow us to see the theory's fingerprints.