Isospin breaking corrections to leptonic decay rates on the lattice

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Outline

- Lattice QCD introduction
- Isospin breaking effects
- Isospin breaking corrections to the Pion
- Isospin breaking corrections to leptonic decay rates
- Sea quark effects

Lattice QCD

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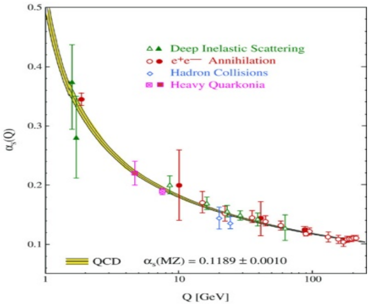
- Path integral:

$$\langle \mathcal{O} \rangle = \frac{1}{Z} \int D[\bar{\psi}, \psi, U] \mathcal{O} e^{i\mathcal{S}}$$



- Free fermion action:

$$\mathcal{S}_F = \int d^4x \sum_f \bar{\psi}_f (i\gamma^\mu \partial_\mu - m_f) \psi_f$$

- Lattice naive free fermion action:

$$\mathcal{S}_F = a^4 \sum_{n \in \Lambda} \sum_f \bar{\psi}_f(n) \left(\gamma_\mu \frac{\psi(n+\hat{\mu}) - \psi(n-\hat{\mu})}{2a} - m\psi_f(n) \right)$$

- Fermion action:

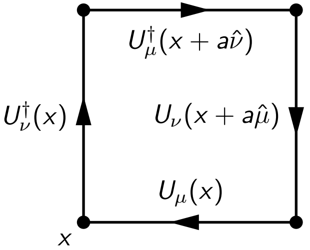
$$\mathcal{S}_F = a^4 \sum_{n \in \Lambda} \sum_f \bar{\psi}_f(n) \left(\gamma_\mu \frac{\psi(n+\hat{\mu})U_\mu(n) - U_{-\mu}(n)\psi(n-\hat{\mu})}{2a} - m\psi_f(n) \right)$$

Lattice QCD: gauge fields and the path integral

- Gauge links: $U_\mu(x) = exp(iqaA_\mu(x))$
- Gauge action:

$$U_{\mu\nu}(x) = exp(-iqa^2 F_{\mu\nu})$$

$$\mathcal{S}_g = \frac{2}{g_0^2} \sum_x \sum_{\mu \leq \nu} Tr\left[1 - \frac{1}{2}[U_{\mu\nu}(x) + U^\dagger_{\mu\nu}(x)]\right]$$



- Euclidean path integral:

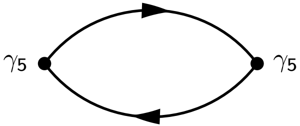
$$< \mathcal{O} >= \frac{1}{Z} \int \mathcal{D}[U, \psi, \bar{\psi}] e^{-S_{Lat}[U,\psi,\bar{\psi}]} \mathcal{O}[\psi, \bar{\psi}]$$

$$< \mathcal{O} >= \int D[U] p(U) \mathcal{O}(U) \quad , \quad p[U] = \frac{1}{Z}\left[\prod_f det(D_f[U])\right] e^{-S_g}$$

$$< \mathcal{O} >= \lim_{N \to \infty} \frac{1}{N} \sum_{n=1}^{N} \mathcal{O}(U_n) + O(N^{-1/2})$$

Example Calculation: Pion mass

- Pion correlator:



$$< 0|\mathcal{O}_{\pi^+}(x)\mathcal{O}^\dagger_{\pi^+}(y)|0 >$$
$$= \langle 0|\, \bar{d}(x)\gamma_5 u(x)[\bar{d}(y)\gamma_5 u(y)]^\dagger \,|0\rangle$$
$$= \langle 0|\, \bar{d}(x)\gamma_5 u(x)\bar{u}(y)\gamma_5 d(y) \,|0\rangle$$
$$= -tr[\gamma_5 S_d(x,y)\gamma_5 S_u(y,x)]$$

$$< 0|\mathcal{O}_\pi(t)\mathcal{O}_\pi(0)^\dagger|0 > = \frac{1}{Z}\int \mathcal{D}[U,\psi,\bar{\psi}]\left[tr[\gamma_5 S(x,0)\gamma_5 S(0,x)]\right] e^{-S[U,\psi,\bar{\psi}]}$$

$$< 0|\mathcal{O}_\pi(t)\mathcal{O}_\pi(0)^\dagger|0 > = \sum_n < 0|\mathcal{O}_\pi(t)|n >< n|\mathcal{O}_\pi(0)^\dagger|0 >$$
$$= \sum_n |< 0|\mathcal{O}_\pi(0)|n >|^2 e^{-E_n t} = |A_\pi|^2 e^{-m_\pi t}(1 + \mathcal{O}(e^{-\Delta E t}))$$

- Correlator:

$$C_\pi = |A_\pi|^2(e^{-m_\pi t} + e^{-m_\pi(T-t)})$$

- Effective mass:

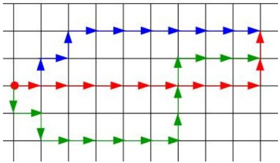
$$m_{eff} = ln\left(\frac{C_\pi[t]}{C_\pi[t+1]}\right)$$

The propagator

- Inverting the Dirac operator:

$$(\not{D} + m)\psi = \eta$$

$$[\not{D}]_{size} = (48^3_{space} \times 96_{time} \times 12_{spin/colour} \times 2_{complex} \times 8_{double})^2$$



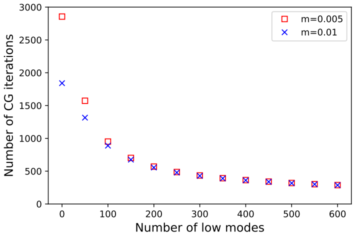
The propagator is the sum over paths of link variables.

- Low modes:

$$S_f = [\not{D}_f + m]^{-1} = [\not{D}_f + m - \sum_\lambda \lambda \left|\psi_\lambda\right\rangle \left\langle\bar{\psi}_\lambda\right|]^{-1} + \sum_\lambda \lambda^{-1} \left|\psi_\lambda\right\rangle \left\langle\bar{\psi}_\lambda\right|$$

- Deflation:

Using the low modes of the Dirac operator it is possible to form a better initial guess of the propagator.



Lattice QCD: Calculations

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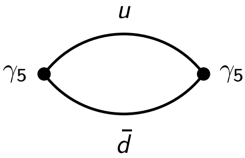
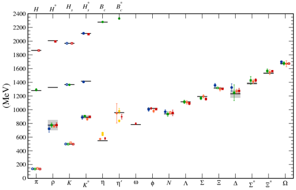
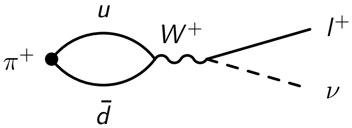
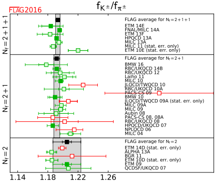
- Spectral quantities:



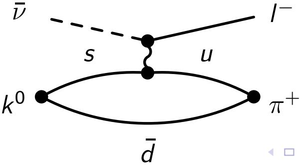


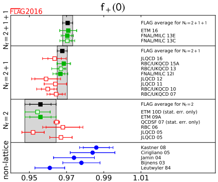
- Leptonic decays:



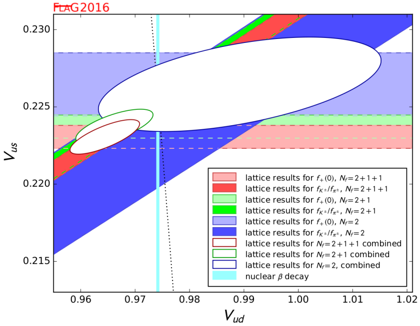


- Semi-leptonic decays:



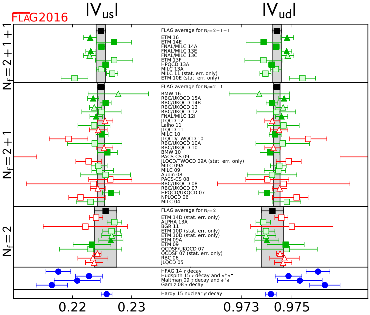


CKM matrix: Precision of Lattice QCD



Unitarity constraints on the CKM matrix are an important bound on the BSM theories.

Lattice QCD is now at the precision of one percent in the light sector:



Isospin breaking corrections

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Two sources of isospin breaking (IB) corrections:

- QED isospin breaking corrections:
  - Difference in the electromagnetic charge on the up and down type quarks.
- Strong isospin breaking corrections:
  - Difference in the up and down quark masses.
- How to include these IB effects?
  - These effects by power counting are of order 1 %:

$$\alpha \approx 1/137 \approx 1\% \quad , \quad \frac{(m_u - m_d)}{\Lambda_{QCD}} \approx 1\%$$

  - This give us an expansion parameter for each contribution.
- What makes this difficult?
  - Small effect
  - QED difficult
  - Expensive

Lattice QFT with Isospin breaking corrections

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Both of the IB corrections can be calculated in terms of a pertubative expansion:

- QED Isospin breaking:
  - Order alpha correction can be calculated by using a perturbative approach:

$$\langle \mathcal{O} \rangle = \langle \mathcal{O} \rangle_0 + \frac{e^2}{2} \frac{\partial^2}{\partial e^2} \langle \mathcal{O} \rangle \Big|_{e=0} + O(\alpha^2)$$

  - If the operator $\mathcal{O}$ is α independent then the correction has the form:

$$\langle \mathcal{O} \rangle = \langle \mathcal{O} \rangle_0 - \frac{e^2 q_f q_{f'}}{2} \langle \mathcal{O} V^c_\mu(x) V^c_\nu(y) \rangle_0 \Delta_{\mu\nu}(x-y) - \frac{(eq_f)^2}{2} \langle \mathcal{O} T_\mu(x) \rangle_0 \Delta_{\mu\mu}$$

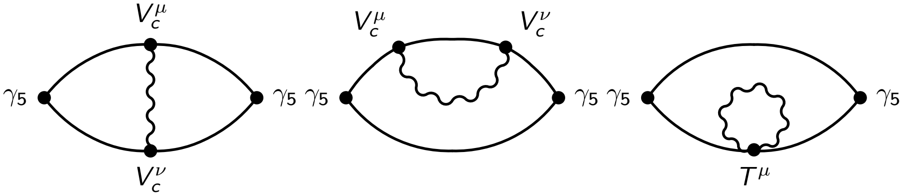
[G.M.de Divitiis et al.Phys.Rev.D87(2013)114505]

- Strong Isospin breaking at first order:

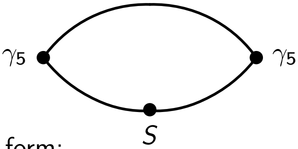
$$\langle \mathcal{O} \rangle_{m_u \neq m_d} = \langle \mathcal{O} \rangle_{m_u = m_d} + (m_d - m_u) \frac{\partial}{\partial m} \langle \mathcal{O} \rangle \Big|_{m_u = m_d} = \langle \mathcal{O} \rangle_{m_u = m_d} + (m_d - m_u) \langle \mathcal{S} \mathcal{O} \rangle \Big|_{m_u = m_d}$$

Example Calculation: IB Corrections to the M_π

- The connected corrections are the following four diagrams:
- QED IB:



- Strong IB:

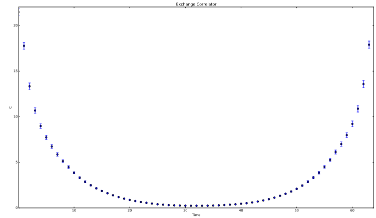


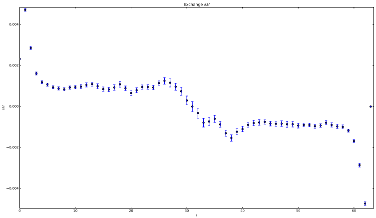
- The correlator has the form:

$$C_\pi = (A + \delta A)(e^{-(m+\delta m)t} + e^{-(m+\delta m)(T-t)})$$

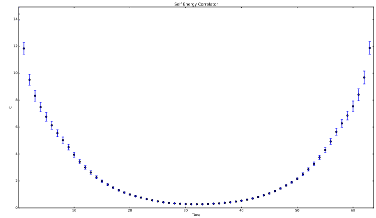
Example Calculation: IB Corrections to M_π

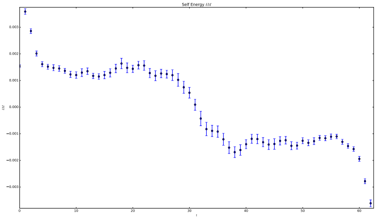
- Exchange diagram (Preliminary)



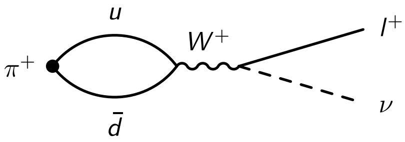


- Self energy diagram (Preliminary)





Leptonic decays



- The decay rate is:

$$\Gamma(\pi^+ \to l^+\nu) = \frac{m_\pi}{8\pi} G_F^2 |f_{\pi^+}|^2 |V_{ud}|^2 m_l^2 \left(1 - \frac{m_l^2}{m_\pi^2}\right)^2$$

$$\langle 0 | \bar{d}\gamma_\mu\gamma_5 u \,|\pi^+(p)\rangle = ip_\mu f_{\pi^+}$$

- IR finite order α decay rate:

$$\Gamma_\alpha = \Gamma_0 + \Gamma_1$$

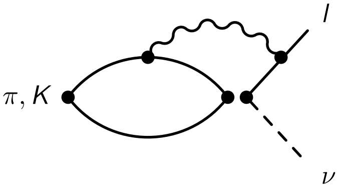
[Bloch and Nordsieck (1937)] [N.Carrasco et al, Phys.Rev.D91(2015)no.7,07450]

– Γ_0: Order alpha corrections without a final state photon.

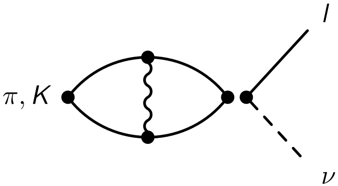
– Γ_1: Order alpha corrections with a final state photon.

IB corrections to leptonic decay rates

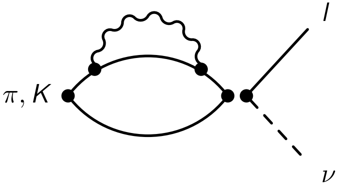
- The connected diagrams that contribute to the order α QED correction to leptonic decays:



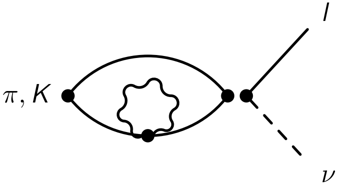
Lepton coupling diagram



Exchange diagram



Self-energy diagram



Tadpole diagram

Initial results: QED IB corrections

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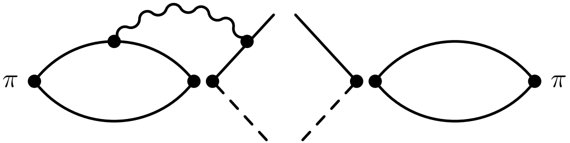

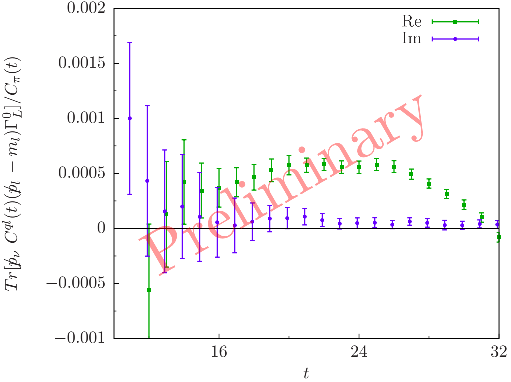

Sea quark: IB corrections to M_π

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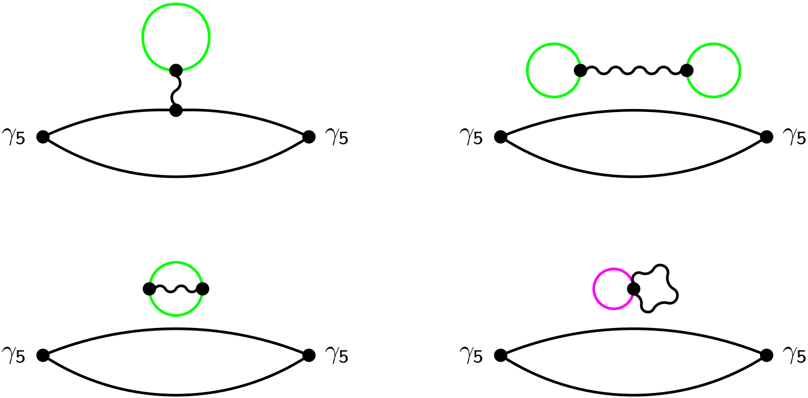
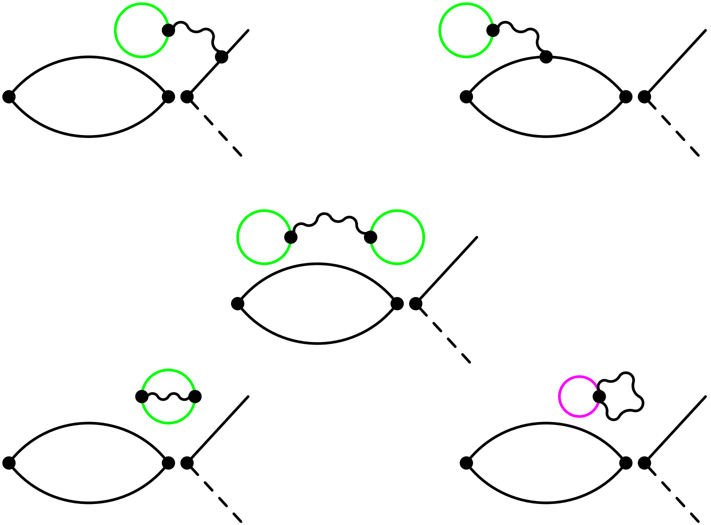


Sea quarks: IB corrections to leptonic deacys



Conclusion

- We are working towards a calculation of leptonic decay rates with isospin breaking corrections.
- Our implementation phase is concluding and a number of tests have been undertaken on ensembles with unphysical quark masses.
- Next phase is move to our physical point ensemble and begin generating data with a view towards calculating a physical result.
- In the future we want to study IB corrections to semi-leptonic decays. However some theoretical developments are required.

Back up Slides

All to All Propagator and the meson field

- All to all propagator:

$$D^{-1}_{A2A}(x,y) = \sum_{i=0}^{N_l+N_h} v_i(x) w_i^\dagger(y) = \sum_{l=0}^{N_l} v_l(x) w_l^\dagger(y) + \sum_{h=N_l}^{N_l+N_h} v_h(x) w_h^\dagger(y)$$

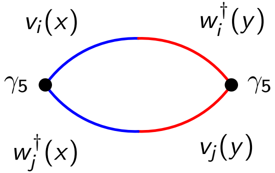
- Low modes (from eigenvectors):

$$v_l(x) = \phi_l(x)$$
$$w_l(x) = \phi_l(x)/\lambda_l$$

- High modes (from stochastic solves):

$$v_h(x) = D^{-1}\eta_h(x)$$
$$w_h(x) = \eta_h(x)$$

- Two point function:



- Meson Field:

$$\Pi_{ji}(t_x;\gamma_5) = \sum_{\vec{x}} w_j^\dagger(x)\gamma_5 v_i(x)$$

[J.Foley et al, CPC 172 (2005)0010-4655]

[M.Peardon et al,Phys.Rev.D.80.054506(2009)]

- 3,4,... pt functions can be made contracting the relevant meson fields with the correct gamma structure.

All to All: Connected leptonic decay diagrams

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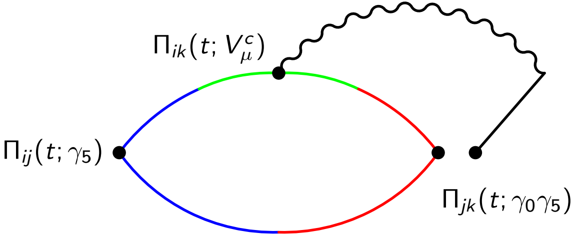
- Leptonic coupling correlator using meson fields:



- Similarly the other diagrams that contribute to the decay rate can be split into meson fields: