Translational Symmetry Breaking in Holographic Zero Sound and Conductivity

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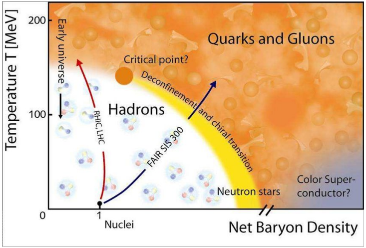
Why use holography to study strongly coupled systems?



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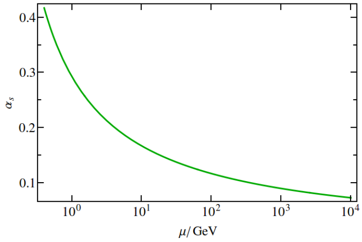

QCD running

- Transport properties of strongly coupled systems
- High temperature superconductors
- QCD phase diagram
- Neutron star equation of state

Outline

Taster

Sound Modes and Conducitivty from Green's Functions

AdS/CFT and Green's Functions

Landau Zero Sound

Translational SB in AdS/CFT

Conclusions and Outlook

Taster

In a classic Drude model the conductivity without translational symmetry breaking (TSB) is given by:

$$\sigma(\omega) \propto \frac{1}{i\omega} \tag{1}$$

This is remedied by introducing TSB that allows momentum to relax at a timescale τ:

$$\sigma(\omega) \propto \frac{1}{\frac{1}{\tau} - i\omega} \tag{2}$$

Taster

Sound modes dispersion relation can be given as:

$$\omega = v_s k - i\Gamma k^2 + \mathcal{O}(k^3) \tag{3}$$

If we include momentum relaxation then this modes dispersion relation, at long times compared to the relaxation timescale, becomes [1710.08425]:

$$\omega \approx -i v_s \tau k^2 = -iDk^2 \tag{4}$$

- TSB is important for sound modes and conductivity.
- We can find sound modes and conductivity via the gauge/gravity duality - what is the effect of TSB?

Sound Modes and Conducitivty from Green's Functions

$$Z \supset e^{\int d^4x \phi \mathcal{O}} \tag{5}$$

The Greens functions are obtained in the usual way:

$$G_R^{\mu\nu}(\omega, k) = \frac{\delta^2 logZ}{\delta\phi^a \delta\phi^b} = \frac{\delta\langle\mathcal{O}^a\rangle}{\delta\phi^b} \tag{6}$$

For example, if we have a global $U(1)$ external field A_μ we would have:

$$Z \supset e^{\int d^4x A_\mu J^\mu} \tag{7}$$

$$G_R^{\mu\nu}(\omega, k) = \frac{\delta^2 logZ}{\delta A_\mu \delta A_\nu} = \frac{\delta\langle J^\mu\rangle}{\delta A_\nu} \tag{8}$$

- If we have some sort of propagating mode via the perturbation, it isn't too surprising to expect it to show up in the Greens function.

Sound Modes and Conducitivty from Green's Functions

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If we calculate this Greens function for a given action under some perturbation of we get:

$$G_R^{\mu\nu}(\omega, k) \propto \frac{1}{\omega - \epsilon(k)} \tag{9}$$

The pole of this Greens Function will give a dispersion relation that might look like:

$$\omega = v_s k - i\Gamma k^2 + \mathcal{O}(k^3) \tag{10}$$

$$\omega = -iDk^2 + ... \tag{11}$$

- The pole of the Greens function gives a dispersion relation related to some propagating mode.

Sound Modes and Conducitivty from Green's Functions

To relate the conductivity to Greens functions we look at Kubo's formula. Conducitivty is defined by:

$$\langle J^x \rangle = \sigma E_x \tag{12}$$

Where we can write the electric field in terms of the gauge field:

$$E_x = \partial_t \delta A_x \to -i\omega\delta\tilde{A}_x \qquad i.e. \qquad \langle J^x \rangle = -i\omega\sigma\delta\tilde{A}_x \tag{13}$$

But we have:

$$\tilde{G}_R^{xx}(\omega, k) = \frac{\delta\langle J^x \rangle}{\delta\tilde{A}_x} = -i\omega\sigma \tag{14}$$

So the Kubo formula for conducitivty is:

$$\sigma(\omega) = \frac{Im\tilde{G}_R^{xx}}{\omega} \tag{15}$$

AdS/CFT and Green's Functions

- We can describe the same physics using two different theories and therefore expect all of the physics of one theory to be reproduced by the other.

$$\mathcal{N} = 4 \; SYM \text{ with } SU(N) \Longleftrightarrow Type\ IIB\ string\ theory\ on\ AdS_5 x S^5$$

$$Strong \Longleftrightarrow Weak$$

$$e^{\int d^4x \phi_{(0)} \mathcal{O}} \Longleftrightarrow e^{-S_{SUGRA}[\phi|\partial]}$$

- A field in the bulk sources an operator at the boundary. Therefore we can write the Green's functions as:

$$G_R^{\mu\nu} = \frac{\delta^2 S_{SUGRA}[\phi|\partial]}{\delta\phi^a_{(0)}\delta\phi^b_{(0)}} \rightarrow \frac{\delta^2 S_{SUGRA}[A^\mu|\partial]}{\delta A^\mu_{(0)}\delta A^\nu_{(0)}} \quad (16)$$

The D3/D7 Probe Brane Model

So far we only have adjoint degrees of freedom on both sides. To introduce fundamental, i.e quarks, we do the following:

N_f D7 branes in bulk $\Longleftrightarrow \mathcal{N} = 2$ hypermultiplets under $U(N_f)$

$$S_{D7} = -T \int d^8\xi \sqrt{-(det(g_{ab} + F_{ab}))} \tag{17}$$

We then take the probe limit: These additional branes do not back-react on the bulk. i.e. $S = S_{Bulk} + S_{D7} \approx S_{Bulk}$. In field theory terms this means $N_f \ll N_{Adjoint}$.

AdS/CFT and Green's Functions

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- Can we get sound modes and conductivity? Yes. We vary the $U(1)$ gauge field on the D7 brane and take its boundary value. We get a pole in the Greens Function at **low temperature**:

$$\omega = \pm vk - i\Gamma k^2 + \mathcal{O}(k^3) \tag{18}$$

- What kind of mode do we have? It turns out this dispersion relation is similar to that of Landau zero sound. Analysis of the attenuation has shown that this Holographic zero sound is similar to that of LZS [1807.11327].
- However it is not the same. There are other discrepancies such as the speed of propagation, and LZS relies on a Fermi surface description. There is no evidence of a Fermi surface in HZS and the microscopic phenomena giving rise to this HZS is unknown.

Landau Zero Sound

How do we describe a system of interacting fermions? Take help from non-interacting Fermi gas:

$$\text{Ground state non-interacting} \Longrightarrow \text{Ground state interacting}$$
$$\text{Excitation} \Longrightarrow \text{Quasiparticle}$$
$$\text{Fermi surface of particles} \Longrightarrow \text{Fermi surface of quasiparticles}$$

- What type of collective excitations do we get from this quasiparticle description?

$$n(t, x, p) = n_0(p) + \delta n(t, x, p) \tag{19}$$

$$\frac{dn}{dt} = I(n) \tag{20}$$

Landau Zero Sound

- If collisions dominate, $L_{mfp} \ll \lambda$, we get normal hydrodynamic sound and the Fermi surface swells and contracts
- If quasiparticle interactions dominate, $L_{mfp} \gg \lambda$, we can get another type of collective excitaiton - zero sound. Here the Fermi surface deforms but does not change its actual size.

$\implies$ Zero sound is a low temperature collective vibration of the quasiparticle ground state around the Fermi surface with dispersion relationship $\omega = vk - i\Gamma k^2 + ...$

But In AdS/CFT we cannot rely on a Fermi surface argument. HZS is not well described by LFL nor like a hydrodynamic sound - it is something new.

Translational SB in AdS/CFT

- So how do we go about incorporating TSB of the CFT? Well, lets look at the symmetries of the theories:

$$\text{Superconformal of SYM} \Longleftrightarrow \text{Isometry of gravity theory}$$
$$SO(4,2) + SO(6) \Longleftrightarrow SO(4,2) + SO(6)$$

- Example: Breaking the AdS_5 diffeomorphisms along the spatial directions only results in breaking conservation of momentum, but not energy.
- This modifies our conserved currents:

$$\partial_a T^{ai} \neq 0 \backsim \tau_{rel}^{-1} \tag{21}$$

Translational SB in AdS/CFT

How to incorporate into a model?

- Add a potential to bulk action that gives the graviton a mass and breaks diffeomorphisms [1301.0537]
- Introduce spatially dependant scalar sources into action. These will then couple to a operator in the CFT and give a non-zero contribution to the Ward identity.[1311.5157]

Translational SB in AdS/CFT

$$S_{Bulk} = \frac{1}{2\kappa^2} \int d^4x \sqrt{-g} \left[R - 2\Lambda - \frac{1}{2}(\partial\psi_{x_1})^2 - \frac{1}{2}(\partial\psi_{x_2})^2 - \frac{1}{4}\mathcal{F}^2 + \tilde{m}^2 \sum_{i=1}^{4} c_i \mathcal{U}_i \right] \tag{22}$$

$$S_{DBI} = -T \int d^8\xi V[(\partial\psi)^2] \sqrt{det(g_{ab} + W[(\partial\psi)^2]F_{ab})} \tag{23}$$

$$ds^2 = \frac{1}{z^2} \left(\frac{dz^2}{f(z)} - f(z)dt^2 + dx^2 + dy^2 \right) \tag{24}$$

$$f(z) = 1 + \alpha_1 z + \alpha_2 z^2 - mz^3 + \frac{\mu^2 z^4}{4z_0^2} \tag{25}$$

Translational SB in AdS/CFT

The TSB conductivity has already been studied in the models above and gives a result of [1306.5792]:

$$\sigma(\omega) \backsim \frac{\sigma_{DC}}{1 - i\omega\tau_{rel}} + \text{corrections} \tag{26}$$

- These corrections are in terms of the additional parameters added by the TSB terms above and vanish when TS is restored.
- The effect of the corrections is to pull spectral weight away from the Drude peak to higher frequency.

TSB and Holographic zero sound

As outlined above to compute the holographic sound mode we vary the gauge field on the DBI action and compute the Greens function:

$$A_\mu(r) \rightarrow A_\mu(r) + a_\mu(r, t, \overline{x}) \tag{27}$$

$$G_R^{\mu\nu}(\omega, k) = \frac{\delta^2 S_{DBI}}{a_{0,\mu} a_{0,\mu}} \tag{28}$$

- The pole of this Greens Function will give the translationally broken holographic sound mode.

Conclusions and Outlook

- Gauge/Gravity duality is useful in probing strongly coupled field theories.
- We can calcualte conductivity, sound modes and other aspects of CMP using the duality via Greens functions.
- These calculations could be pivatol in understanding how systems behave at strong coupling.
- This project will focus on the response of HZS when TS is broken.