Central charge of self-dual strings from holography

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With John Estes, Darya Krym, Andy O'Bannon, and Brandon Robinson

Based on 1811.12375, 1812.00923

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Science & Technology Facilities Council

Outline

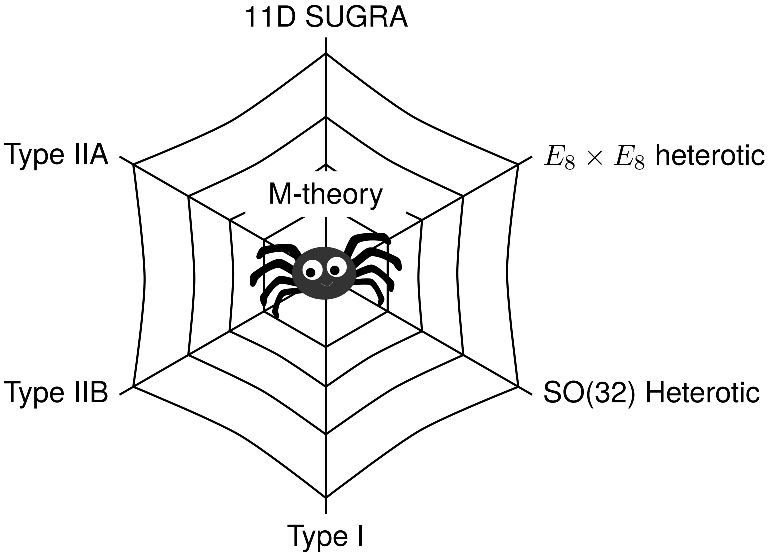
Background and motivation

- M-theory
- Central charges
- Entanglement entropy

Results

Summary and outlook

M-theory


11D SUGRA
Type IIA
$E_8 \times E_8$ heterotic
M-theory
Type IIB
SO(32) Heterotic
Type I

11D supergravity

$$16\pi G_N S = \int \mathrm{d}^{11}x\sqrt{-G}\left(R - \frac{1}{2}|F_4|^2\right) - \frac{1}{6}\int C_3 \wedge F_4 \wedge F_4$$

$F_4 = \mathrm{d}C_3$, $F_7 = \star F_4 = \mathrm{d}C_6 - \frac{1}{2}\mathrm{d}C_3 \wedge C_3$

$C_3 \Rightarrow$ M2-branes

$C_6 \Rightarrow$ M5-branes

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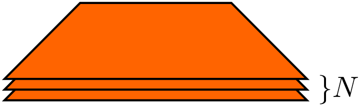
Stack of D-branes: Yang-Mills

What theory describes a stack of M-branes?

Stacks of M2-branes: ABJM



2+1 dimensions, $\mathcal{N} = 6$ or 8 superconformal symmetry

$U(N)_k \times U(N)_{-k}$ Chern-Simons

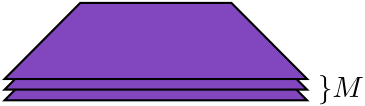
[Aharony, Bergman, Jafferis, Maldacena, 0806.1218]

$N \gg k^5$: dual to 11D SUGRA on $AdS_4 \times S^7/\mathbb{Z}_k$

Degrees of freedom $\sim N^{3/2}$

[Klebanov, Tseytlin, hep-th/9604089]

Stacks of M5-branes



5+1 dimensions, $\mathcal{N} = (2,0)$ superconformal symmetry

$\mathfrak{su}(M)$ gauge algebra

Tensor multiplet:

- Self-dual two-form gauge field A_2, $F_3 = \star F_3$
- One pair of symplectic Majorana-Weyl fermions
- Five real scalar fields

$M \gg 1$: dual to 11D SUGRA on $AdS_7 \times S^4$

Degrees of freedom $\sim M^3$

[Klebanov, Tseytlin, hep-th/9604089]

Why study $\mathcal{N} = (2, 0)$ theory?

Important part of M-theory – interesting for quantum gravity

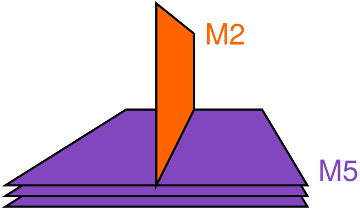
6 dimensions = maximal number with superconformal symmetry

6D SCFTs can serve as “master theories”

4D $\mathcal{N} = 2$ theories from compactification on Riemann surface
[Alday, Gaiotto, Tachikawa 0906.3219]

$\mathcal{N} = 4$ SYM obtained by compactifying $\mathcal{N} = (2, 0)$ on T^2
Modular transformations $\Rightarrow$ S-duality in $\mathcal{N} = 4$ SYM
[Vafa, hep-th/9707131]

Self-dual strings



Two-dimensional intersection: “string”

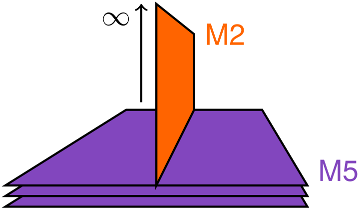
Electric charge = magnetic charge:

$$Q_E = \int_{S^3} \star F_3 = \int_{S^3} F_3 = Q_B, \quad \text{“self-dual”}$$

Tension $\sim$ length of M2-brane

[Howe, Lambert, West, hep-th/9709014]

Self-dual strings



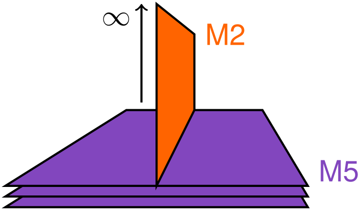
Infinite tension: length of M2 to infinity

M5-brane perspective: Insertion of half-BPS “Wilson surface” into M5-brane theory

$$\langle W_{\mathcal{R}} \rangle = \mathrm{tr}_{\mathcal{R}} \exp\left(i \int A_2 + \ldots \right)$$

Depends on representation $\mathcal{R}$ of Wilson surface

Self-dual strings

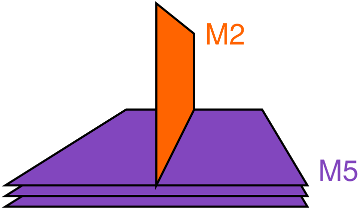


Infinite tension: length of M2 to infinity

M2-brane perspective: ABJM with a boundary

Boundary degrees of freedom: [Niarchos 1509.0767]

Self-dual strings



Planar intersection

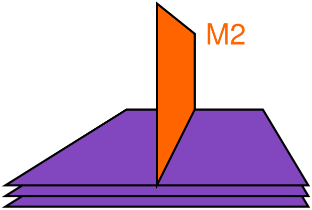
	0	1	2	3	4	5	6	7	8	9	10
M5	•	•	•	•	•	•	–	–	–	–	–
M2	•	•	–	–	–	–	•	–	–	–	–

Preserves 2D large $\mathcal{N} = (4,4)$ SUSY

M5-brane perspective:

$$SO(6,2) \times SO(5)_R \to SO(2,2) \times SO(4)_R \times SO(4)_R$$

Self-dual strings



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M2-brane perspective:

$$SO(3,2) \times SO(8)_R \to SO(2,2) \times SO(4)_R \times SO(4)_R$$

Self-dual strings in holography

M5-branes: $M \gg 1$, asymptotically $AdS_7 \times S^4$

M2-branes: $N \gg 1$, asymptotically $(AdS_4/\mathbb{Z}_2) \times S^7$

SUGRA solutions with flux

- Partition of M2-branes on M5-branes
- Young tableau

Representation depends on configuration of fluxes

boxes in Young tableau $= N$

[Lunin, 0704.3442; D'Hoker, Estes, Gutperle, Krym, 0806.0605;...]

Central charge: 2D CFT

2D CFTs are characterised by their central charge c.

Virasoro algebra:

$$[L_m, L_n] = (m-n)L_{m+n} + \frac{c}{12}(m^3 - m)\delta_{m,-n}$$

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Conformal anomaly: $\langle T^\mu{}_\mu \rangle = \frac{c}{24\pi} R$

Stress-tensor OPE: $T(z)T(0) = \frac{c}{2z^4} + \ldots$

Thermal entropy: $S = \frac{\pi}{3} cLT$

Entanglement entropy: $S_{\mathrm{EE}} = \frac{c}{3} \log\left(\frac{2\ell}{\epsilon}\right) + \ldots$

Central charge counts degrees of freedom.

Central charge: 2D defects

Higher dimensional CFT with 2D defect or boundary

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Central charge: 2D defects

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$$S_{\rm EE}^{\rm defect} = \frac{b}{3}\log\left(\frac{2\ell}{\epsilon}\right) + \dots$$

Central charge: 2D defects

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Entanglement entropy:

$$S_{\mathrm{EE}}^{\mathrm{defect}} = \frac{b}{3}\log\left(\frac{2\ell}{\epsilon}\right) + \ldots$$

Conformal anomaly:

$$\langle T^{\mu}{}_{\mu}\rangle = \text{(bulk terms)} + \delta(x_\perp)\frac{1}{24\pi}\left(c\hat{R} + d_1\mathring{\Pi}^2 - d_2 W_{ab}{}^{ab}\right)$$

Plan:

- Build brane intersections in SUGRA
- Compute entanglement entropy
- Extract b

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Background and motivation

- M-theory
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- **Entanglement entropy**

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Entanglement entropy

Factorize Hilbert space $\mathcal{H} = \mathcal{A} \otimes \mathcal{B}$

Reduced density matrix on $\mathcal{A}$:

$$\rho_{\mathcal{A}} = \sum_{|\psi\rangle \in \mathcal{B}} \langle\psi|\, \rho \,|\psi\rangle$$

EE = von Neumann entropy of reduced density matrix:

$$S_{\mathrm{EE}} = -\mathrm{Tr} \rho_{\mathcal{A}} \log \rho_{\mathcal{A}}$$

Entanglement entropy

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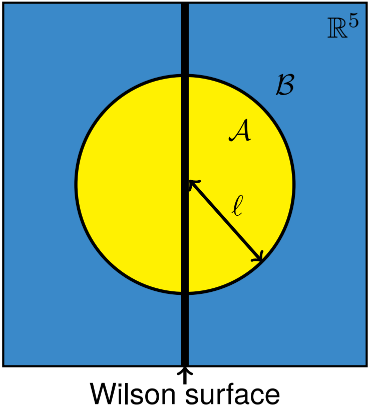
For a QFT, natural splitting of $\mathcal{H}$ into spatial subregions.

2D CFT: [Cardy, Calabrese, hep-th/0405152]

$$S_{\mathrm{EE}} = \frac{c}{3} \log \left(\frac{2\ell}{\epsilon} \right)$$

Entanglement entropy with 2D defect

Entangling region: sphere of radius ℓ



Defect contribution to EE:

$$S_{\mathrm{EE}} = S_{\mathrm{EE}}^{d=6} + S_{\mathrm{EE}}^{\mathrm{d}=2}$$

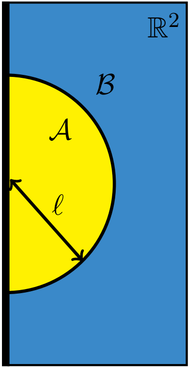
Determined by central charge:

$$S_{\mathrm{EE}}^{\mathrm{d}=2} = \frac{b}{3} \log\left(\frac{2\ell}{\epsilon}\right) + \dots$$

$$b = 3\ell \frac{\mathrm{d}}{\mathrm{d}\ell} S_{\mathrm{EE}}^{d=2}$$

Entanglement entropy with boundary

Entangling region: semicircle of radius ℓ



Defect contribution to EE:

$$S_{\rm EE} = \frac{1}{2} S_{\rm EE}^{d=3} + S_{\rm EE}^{\rm d=2}$$

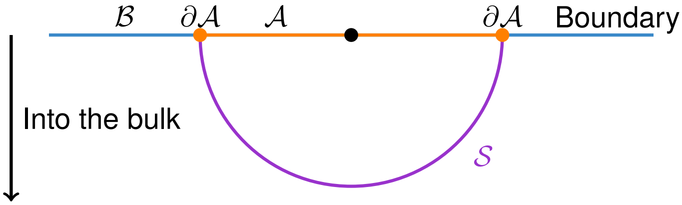
Determined by central charge:

$$S_{\rm EE}^{\rm d=2} = \frac{b}{3} \log\left(\frac{2\ell}{\epsilon}\right) + \ldots$$

$$b = 3\ell \frac{\rm d}{{\rm d}\ell} S_{\rm EE}^{d=2}$$

Holographic entanglement entropy

Minimal surface extending into bulk:



$$S_{\mathrm{EE}} = \frac{\mathrm{Area}[\mathcal{S}]}{4G_N}$$

[Ryu, Takayanagi, hep-th/0603001 & hep-th/0605073]

[Lewkowycz, Maldacena, 1304.4926]

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Parameterisation of solutions

M_a M5-branes that have N_a M2-branes ending on them:

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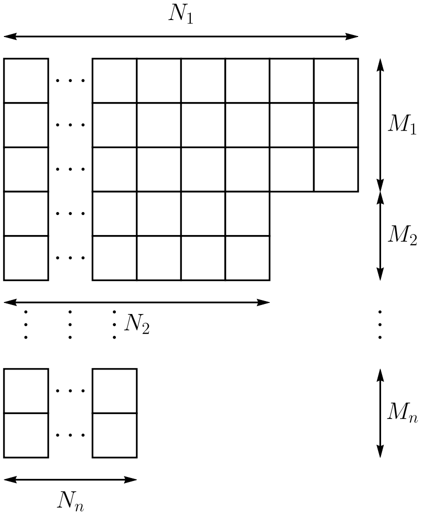


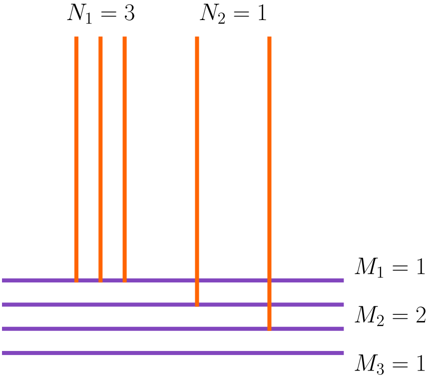
$$M = \sum_{a=1}^{n+1} M_a$$

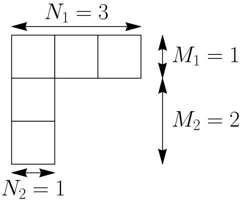
$$N = \sum_{a=1}^{n} M_a N_a$$

$$N_{n+1} = 0$$

Parameterisation of solutions

M_a M5-branes that have N_a M2-branes ending on them:





$$M = \sum_{a=1}^{n+1} M_a, \quad N = \sum_{a=1}^{n} N_a M_a, \quad N_{n+1} = 0$$

Central charge: M5-branes

$$
\begin{aligned}
b_{\mathrm{M5}} &= 3\ell \frac{\mathrm{d}}{\mathrm{d}\ell}\left[S_{\mathrm{EE}} - S_{\mathrm{EE}}^{d=6}\right] \\
&= \frac{3}{5}\left[8MN + \frac{N^2}{M} + \sum_{a=1}^{n}\left(8M_a^2 N_a - M_a N_a^2 - 16\sum_{b=1}^{a} M_a M_b N_a\right)\right]
\end{aligned}
$$

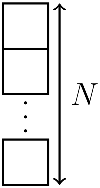
In terms of highest weight λ and Weyl vector ρ,

$$
b_{\mathrm{M5}} = \frac{48}{5}(\rho, \lambda) - \frac{3}{5}(\lambda, \lambda)
$$

Central charge: M5-branes

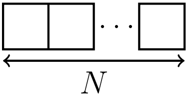
$$b_{\mathrm{M5}} = \frac{48}{5}(\rho, \lambda) - \frac{3}{5}(\lambda, \lambda)$$

Antisymmetric representation



$$b_{\mathrm{M5}} = \frac{24}{5} N(M - N)$$

Symmetric representation



$$b_{\mathrm{M5}} = \frac{24}{5} N(M - N/8)$$

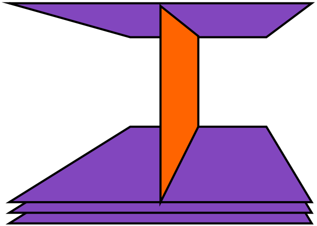
Central charge

Compare to result of [Berman, Harvey, hep-th/0408198]?

R-symmetry anomaly from inflow calculation

Coulomb branch:

$$SU(M+1) \to SU(M) \times U(1)$$



Central charge:

$$b_{\mathrm{BH}} = \frac{1}{4} N M$$

Central charge

Compare to result of [Niarchos, Siampos, 1206.2935]?

Blackfold approach: thermodynamic entropy

$$S = \frac{\pi}{3} b_{\mathrm{NS}} L T + \ldots$$

$$b_{\mathrm{NS}} \approx 1.2 \frac{N^2}{M}$$

Central charge: M2-branes

$$b_{\mathrm{M2}} = 3\ell\frac{\mathrm{d}}{\mathrm{d}\ell}\left[S_{\mathrm{EE}} - \frac{1}{2}S_{\mathrm{EE}}^{d=3}\right]$$
$$= 3\sum_{a=1}^{n}\left[-M_a^2 N_a + \frac{1}{2}M_a N_a^2 + 2\sum_{b=1}^{a} N_a M_a M_b\right]$$

Symmetric partition: $b_{\mathrm{M2}} = 3\left(N^2 + \frac{1}{2}N\right)$

Antisymmetric partition: $b_{\mathrm{M2}} = 3\left(\frac{1}{2}N^2 + N\right)$

What is field theory interpretation of Young tableau?

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Summary

Despite much progress on M5-brane physics, fundamental questions remain

We have studied Wilson surfaces in M5-brane theory holographically

Entanglement entropy provides a definition of a self-dual string central charge

Extends previous M5-brane central charge calculations, shows similar scaling with number of branes

Central charge in terms of group theory quantities

$$b_{\mathrm{M5}} = \frac{48}{5}(\rho,\lambda) - \frac{3}{5}(\lambda,\lambda)$$

Outlook

Does this central charge show up in other quantities?

- Entanglement entropy related to $\langle T^{\mu}{}_{\mu} \rangle$
  [Jensen, O'Bannon, Robinson, RR, 1812.xxxxx]
- Does this explain differences with other calculations?

Can we compute all of the trace anomaly coefficients?

- Different geometries for brane intersections?

How do these quantities behave along RG flows? Can we understand the results from field theory?

- AGT? [Alday, Gaiotto, Tachikawa, 0906.3219]
- Quantities in compactified theories that tell you about the central charge?

Thank you!

Backup slides

Bubbling geometries

[Lunin, 0704.3442; D'Hoker, Estes, Gutperle, Krym, 0806.0605; Estes, Feldman, Krym, 1209.1845; Bachas, D'Hoker, Estes, Krym, 1312.5477]

11D SUGRA solutions with $AdS_3 \times S^3 \times S^3$ isometry

Remaining two coordinates on Riemann surface, Σ_2

$$\mathrm{d}s^2 = f_1^2 \mathrm{d}s^2_{AdS_3} + f_2^2 \mathrm{d}s^2_{S^3_2} + f_3^2 \mathrm{d}s^2_{S^3_3} + 2\rho^2_{w\bar{w}} \mathrm{d}w \mathrm{d}\bar{w}$$

$$C_{(3)} = b_1 \, \mathrm{d}s_{AdS_3} + b_2 \, \mathrm{d}s_{S^3_2} + b_3 \, \mathrm{d}s_{S^3_3}$$

Bubbling geometries

[Lunin, 0704.3442; D'Hoker, Estes, Gutperle, Krym, 0806.0605;
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$$C_{(3)} = b_1 \, \mathrm{d}s_{AdS_3} + b_2 \, \mathrm{d}s_{S_2^3} + b_3 \, \mathrm{d}s_{S_3^3}$$

f_i, b_i, $\rho_{w\bar{w}}$ real functions on Σ_2, express in terms of $\{\gamma, h, G\}$

Super algebra $D(2,1;\gamma) \oplus D(2,1;\gamma)$
[Sevrin, Troost, Van Proeyen, Phys. Lett. B, **208**, 447 (1988)]

$$\partial\bar{\partial}h = 0, \quad \partial G = \frac{1}{2}(G + \bar{G})\partial \log h, \quad h > 0, \quad |G| < 1$$

$$h = 0, \quad G = \pm i \quad \text{on } \partial\Sigma_2$$

Bubbling geometries

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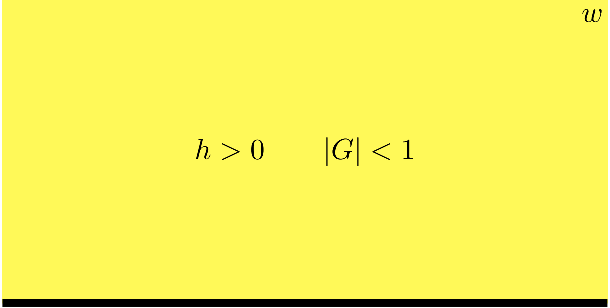

$h = 0 \qquad G = \pm i$

$AdS_7 \times S^4$ asymptotics

$$\gamma = -\frac{1}{2}, \quad h = -i(w - \bar{w}), \quad G = -i\left(1 + \sum_{j=1}^{2n+2} (-1)^j \frac{w - \xi_j}{|w - \xi_j|}\right)$$

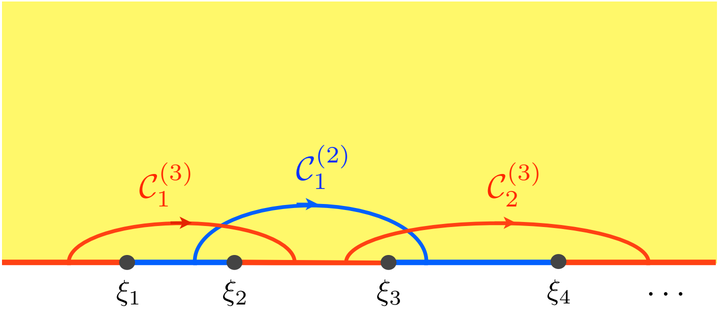

Figure from Bachas, D'Hoker, Estes, Krym 1312.5477

$AdS_4 \times S^7$ asymptotics

$$\gamma < 0, \quad h = -i(w - \bar{w}), \quad G = -i \sum_{j=1}^{2n+1} (-1)^j \frac{w - \xi_j}{|w - \xi_j|}$$

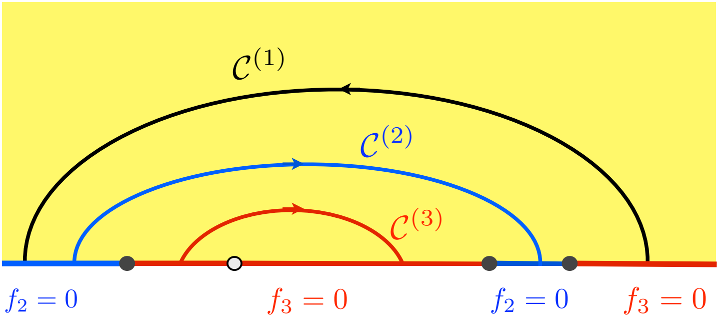

Figure from Bachas, D'Hoker, Estes, Krym 1312.5477

Entanglement entropy

Ryu-Takayanagi surface known for any geometry dual to a conformal defect: [Jensen, O'Bannon, 1309.4523]

Hemisphere in AdS_3, wrapping $S^3_{2,3}$ and Σ_2

$$\mathrm{d}s^2 = f_1^2 \mathrm{d}s^2_{AdS_3} + f_2^2 \mathrm{d}s^2_{S^3_2} + f_3^2 \mathrm{d}s^2_{S^3_3} + 2\rho^2_{w\bar{w}} \mathrm{d}w \mathrm{d}\bar{w}$$

Entanglement entropy

$$\begin{aligned} S_{\mathrm{EE}} &= \frac{\mathrm{Area[RT]}}{4G_N} \\ &= \frac{2(2\pi^2)^2}{4G_N} \int \mathrm{d}w \mathrm{d}\bar{w} \left(\rho^2_{w\bar{w}} f_1 f_2^3 f_3^3\right) \underbrace{\int_\epsilon^\ell \frac{\mathrm{d}z}{z} \frac{\ell}{\sqrt{\ell^2 - z^2}}}_{\log(2\ell/\epsilon) + \mathcal{O}(\epsilon)} \end{aligned}$$

[Gentle, Gutperle, Marasinou, 1506.00052]