The orientifold quotient of giant gravitons

Christopher Lewis-Brown
Supervised by Sanjaye Ramgoolam

Queen Mary University of London

YTF, 19 December 2018

Based on chapter 4 of "BPS operators in $\mathcal{N} = 4$ $SO(N)$ super Yang-Mills theory: plethysms, dominoes and words" arxiv 1804.11090[hep-th]

Outline

AdS/CFT

$\mathcal{N} = 4$ super Yang-Mills $\iff$ String Theory

$U(N)$ gauge group $\iff$ Type IIB on $AdS_5 \times S^5$

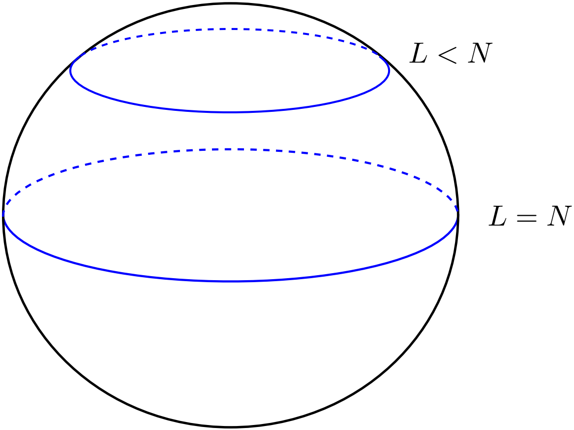
$\downarrow$ [Witten, 98]

$SO(N)$ gauge group $\iff$ Type IIB on $AdS_5 \times \mathbb{R}P^5$

Giant Gravitons

- Giant gravitons were first introduced by McGreevy, Susskind and Toumbas in 2000
- They arise in type IIB string theory on $AdS_5 \times S^5$ by considering a graviton with angular momentum L of order $N \gg 1$.
- This state can be described by a D3-brane wrapped around an S^3. The radius of the S^3 increases with increasing L, and the D3-brane rotates with angular momentum L.
- There are two classes of giants, called sphere giants and AdS giants, depending on whether the S^3 is in the S^5 or the AdS_5.
- Sphere giants have a bound on the angular momentum arising from the maximum size of an S^3 inside an S^5. This bound is $L \leq N$.

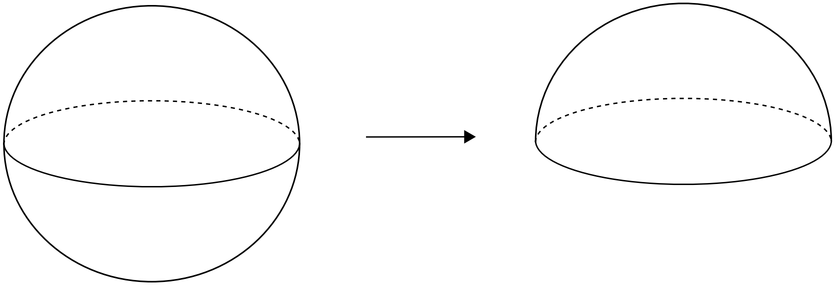
Giant Gravitons



Orientifold quotient

- Consider taking a $\mathbb{Z}_2$ quotient of the S^5 in type IIB string theory on $AdS_5 \times S^5$.
- The $\mathbb{Z}_2$ relates antipodal points in the S^5, replacing the S^5 factor with $\mathbb{R}P^5$, the real projective 5-plane.
- In addition, the $\mathbb{Z}_2$ reverses worldsheet orientation. More explicitly, in going around a non-contractible loop in $\mathbb{R}P^5$, the string orientation is reversed.
- The AdS/CFT dual of this orientifold theory is $\mathcal{N} = 4$ SYM with an orthogonal gauge group (or symplectic, depending on topological factors).
- We aim to examine the effect of this quotient on the operators in $\mathcal{N} = 4$ SYM dual to giant gravitons.

Orientifold quotient

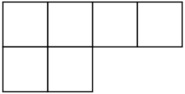


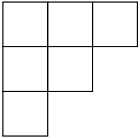
AdS/CFT dual of giant gravitons

- The AdS/CFT dual of giant gravitons are half-BPS operators in $\mathcal{N} = 4$ super Yang-Mills with gauge group $U(N)$.
- $\mathcal{N} = 4$ SYM contains 6 real scalars, or equivalently 3 complex scalars, in the adjoint of the gauge group. The half-BPS sector of the theory is spanned by gauge-invariant combinations of one of these complex scalars, X.
- The operators dual to giant gravitons contain $O(N)$ copies of X.

Partitions

- A partition p of m (written $p \vdash m$) is a set of positive numbers that sum to m. We write $p = [p_1, p_2, ...]$, where $\sum_i p_i = m$.
- A useful way of visualising partitions is with Young diagrams. These are a collection of boxes where each row length corresponds to the components of p. For example

$p = [4, 2] =$ 

$p = [3, 2, 1] =$ 

Partitions

- Conjugacy classes (cycle types) in S_m are labelled by partitions $p \vdash m$.
- Irreducible representations R of S_m are labelled by partitions $R \vdash m$.
- Irreducible representations R of $U(N)$ are labelled by partitions R with fewer than N rows ($l(R) \leq N$).

Half-BPS sector of $U(N)$ theory

- The adjoint of $\mathfrak{u}(N)$ consists of anti-hermitian matrices, so the real scalar fields are anti-hermitian and the complex scalars are generic matrices.
- Half-BPS, gauge-invariant operators are multi-traces of one of the complex matrices (X), labelled by partitions $p \vdash m$

$$\mathcal{O} = \prod_i \mathrm{Tr} X^{p_i}$$

- There are two problems with this basis
  - Multi-trace operators are orthogonal in the large N limit but the two point functions have $\frac{1}{N}$ corrections.
  - When $m < N$ they no longer form a basis, only a spanning set.

Orthogonal basis

- An orthogonal basis for the half-BPS sector is labelled by $R \vdash m$ with $l(R) \leq N$

$$\mathcal{O}_R^{U(N)} = \sum_{p \vdash n} \frac{\chi_R(p)}{z_p} \prod_i \mathrm{Tr} X^{p_i}$$

- These have two-point function

$$\left\langle \mathcal{O}_R^{U(N)} \overline{\mathcal{O}}_S^{U(N)} \right\rangle = \delta_{RS} f_R$$

Example

At $m = 4$, the $U(N)$ half-BPS operators are

$$\mathcal{O}^{U(N)}_{(4)} = \frac{1}{4}\mathrm{Tr}X^4 + \frac{1}{8}\left(\mathrm{Tr}X^2\right)^2 + \frac{1}{4}\left(\mathrm{Tr}X^2\right)(\mathrm{Tr}X)^2 + \frac{1}{3}\left(\mathrm{Tr}X^3\right)(\mathrm{Tr}X) + \frac{1}{24}(\mathrm{Tr}X)^4$$

$$\mathcal{O}^{U(N)}_{(3,1)} = -\frac{1}{4}\mathrm{Tr}X^4 - \frac{1}{8}\left(\mathrm{Tr}X^2\right)^2 + \frac{1}{4}\left(\mathrm{Tr}X^2\right)(\mathrm{Tr}X)^2 + \frac{1}{8}(\mathrm{Tr}X)^4$$

$$\mathcal{O}^{U(N)}_{(2,2)} = \frac{1}{4}\left(\mathrm{Tr}X^2\right)^2 - \frac{1}{3}\left(\mathrm{Tr}X^3\right)(\mathrm{Tr}X) + \frac{1}{12}(\mathrm{Tr}X)^4$$

$$\mathcal{O}^{U(N)}_{(2,1,1)} = \frac{1}{4}\mathrm{Tr}X^4 - \frac{1}{8}\left(\mathrm{Tr}X^2\right)^2 - \frac{1}{4}\left(\mathrm{Tr}X^2\right)(\mathrm{Tr}X)^2 + \frac{1}{8}(\mathrm{Tr}X)^4$$

$$\mathcal{O}^{U(N)}_{(1,1,1,1)} = -\frac{1}{4}\mathrm{Tr}X^4 + \frac{1}{8}\left(\mathrm{Tr}X^2\right)^2 - \frac{1}{4}\left(\mathrm{Tr}X^2\right)(\mathrm{Tr}X)^2 + \frac{1}{3}\left(\mathrm{Tr}X^3\right)(\mathrm{Tr}X) + \frac{1}{24}(\mathrm{Tr}X)^4$$

Giant graviton interpretation

- A Young diagram with $m = O(1)$ boxes can be interpreted as a low energy excitation of the gravitational field.
- A single column Young diagram $R = [1^k]$, where $k = O(N)$, is dual to a giant graviton wrapped on the S^5 with angular momentum k. So the bound on column length ($l(R) \leq N$) is dual to the bound on angular momentum for sphere giants.
- A single row Young diagram $R = [k]$ is dual to a giant graviton wrapped on AdS_5.
- Multi-column/row diagrams are dual to multiple giants or a single giant wrapped multiple times around the S^3.
- The $SO(N)$ half-BPS also contain baryonic operators.

Half-BPS sector of $SO(N)$ theory

- The adjoint of $\mathfrak{so}(N)$ consists of anti-symmetric matrices. This is a linear condition under complexification, so both the real and complex scalar fields are anti-symmetric matrices.
- Half-BPS, gauge-invariant operators are multi-traces of one of the complex matrices (X). Since odd powers of X have vanishing trace, these are labelled by partitions $p \vdash \frac{m}{2}$

$$\mathcal{O} = \prod_i \mathrm{Tr} X^{2p_i}$$

- This suffers the same issues at finite N as the $U(N)$ multi-trace basis.

Orthogonal basis

- An orthogonal basis for the half-BPS sector is labelled by $t \vdash \frac{m}{2}$ with $l(t) \leq \frac{N}{2}$

$$\mathcal{O}_t^{SO(N)} = \sum_{q \vdash \frac{m}{2}} \frac{1}{z_{2q}} \chi_t(q) \prod_i \mathrm{Tr} X^{2q_i}$$

- These have two-point function

$$\left\langle \mathcal{O}_t^{SO(N)} \overline{\mathcal{O}}_{t'}^{SO(N)} \right\rangle = \delta_{tt'} f_t^{SO(N)}$$

- These have a similar giant graviton interpretation as the $U(N)$ case.

Example

At $m = 4$, the $SO(N)$ half-BPS operators are

$$\mathcal{O}^{SO(N)}_{\square\square} = \frac{1}{4}\mathrm{Tr}X^4 + \frac{1}{8}\left(\mathrm{Tr}X^2\right)^2$$
$$\mathcal{O}^{SO(N)}_{\substack{\square\\\square}} = -\frac{1}{4}\mathrm{Tr}X^4 + \frac{1}{8}\left(\mathrm{Tr}X^2\right)^2$$

Orientifold quotient on CFT side

- On the CFT side, the $\mathbb{Z}_2$ quotient replaces the generic X in the $U(N)$ theory with the anti-symmetric X of the $SO(N)$ theory.
- In terms of traces, any multi-trace containing an odd order single trace vanishes, while those multi-traces containing only even single traces remain unchanged.

$$\mathrm{Tr}X^4, \left(\mathrm{Tr}X^2\right)^2 \xrightarrow{\mathbb{Z}_2} \text{themselves}$$
$$\left(\mathrm{Tr}X^3\right)\left(\mathrm{Tr}X\right), \left(\mathrm{Tr}X^2\right)\left(\mathrm{Tr}X\right)^2, \left(\mathrm{Tr}X\right)^4 \xrightarrow{\mathbb{Z}_2} 0$$

- In terms of our orthogonal bases, we can write

$$\mathcal{O}_R^{U(N)} \xrightarrow{\mathbb{Z}_2} \sum_{t \vdash \frac{n}{2}} \alpha_R^t \mathcal{O}_t^{SO(N)}$$

Quotient example

Using our examples at $m = 4$, we have

$$\mathcal{O}^{U(N)}_{\ydiagram{4}} \xrightarrow{\mathbb{Z}_2} \mathcal{O}^{SO(N)}_{\ydiagram{2}}$$

$$\mathcal{O}^{U(N)}_{\ydiagram{3,1}} \xrightarrow{\mathbb{Z}_2} -\mathcal{O}^{SO(N)}_{\ydiagram{2}}$$

$$\mathcal{O}^{U(N)}_{\ydiagram{2,2}} \xrightarrow{\mathbb{Z}_2} \mathcal{O}^{SO(N)}_{\ydiagram{2}} + \mathcal{O}^{SO(N)}_{\ydiagram{1,1}}$$

$$\mathcal{O}^{U(N)}_{\ydiagram{2,1,1}} \xrightarrow{\mathbb{Z}_2} -\mathcal{O}^{SO(N)}_{\ydiagram{1,1}}$$

$$\mathcal{O}^{U(N)}_{\ydiagram{1,1,1,1}} \xrightarrow{\mathbb{Z}_2} \mathcal{O}^{SO(N)}_{\ydiagram{1,1}}$$

Plethysms

- $t \vdash \frac{m}{2}$ labels an irrep of $U(N)$ with representation space V_t.
- $V_t \otimes V_t$ carries a reducible representation of $U(N)$, given by

$$D_{t^{\otimes 2}}(U)\,(v \otimes w) = [D_t(U)v] \otimes [D_t(U)w]$$

- This has invariant subspaces given by $S^2(V_t)$, the symmetric part of $V_t \otimes V_t$ and $\Lambda^2(V_t)$, the anti-symmetric part of $V_t \otimes V_t$.
- Can decompose these spaces into irreps of $U(N)$. These will be labelled by $R \vdash m$ with $l(R) \leq N$

$$S^2(V_t) = \bigoplus_{\substack{R \vdash m \\ l(R) \leq N}} \mu_{t,S^2,R} V_R$$

$$\Lambda^2(V_t) = \bigoplus_{\substack{R \vdash m \\ l(R) \leq N}} \mu_{t,\Lambda^2,R} V_R$$

Plethysms

- The multiplicities in these decompositions are called plethysm coefficients

$$\mathcal{P}\left(t, S^2, R\right) = \mathrm{Dim}\left(V^{mult}_{t,S^2,R}\right) \qquad \mathcal{P}\left(t, \Lambda^2, R\right) = \mathrm{Dim}\left(V^{mult}_{t,\Lambda^2,R}\right)$$

- The projection coefficients can be written in terms of these plethysms

$$\alpha^t_R = \mathcal{P}\left(t, S^2, R\right) - \mathcal{P}\left(t, \Lambda^2, R\right)$$

Littlewood-Richardson coefficients

- The Littlewood-Richardson coefficient $g_{t,t;R}$ is defined to be the multiplicity of R in $t \otimes t$ (as $U(N)$ irreps), so

$$g_{t,t;R} = \mathcal{P}\left(t, S^2, R\right) + \mathcal{P}\left(t, \Lambda^2, R\right)$$

- There is a well-known combinatorial rule for finding the Littlewood-Richardson coefficients. They are given by the number of Littlewood-Richardson tableaux of shape T/R and evaluation S (equivalently shape T/S and evaluation R).
- We give an example of finding $g_{R,S;T} = 4$ with $R = [5, 2, 2, 1]$, $S = [4, 3, 2]$ and $T = [7, 4, 4, 3, 1]$

Littlewood-Richardson example

$$R = [5,2,2,1]\ ,\ \ S = [4,3,2]\ ,\ \ T = [7,4,4,3,1]$$

					1	1
		1	1			
		2	2			
	2	3				
3						

					1	1
		1	1			
		2	2			
	3	3				
2						

					1	1
		1	2			
		2	3			
	1	3				
2						

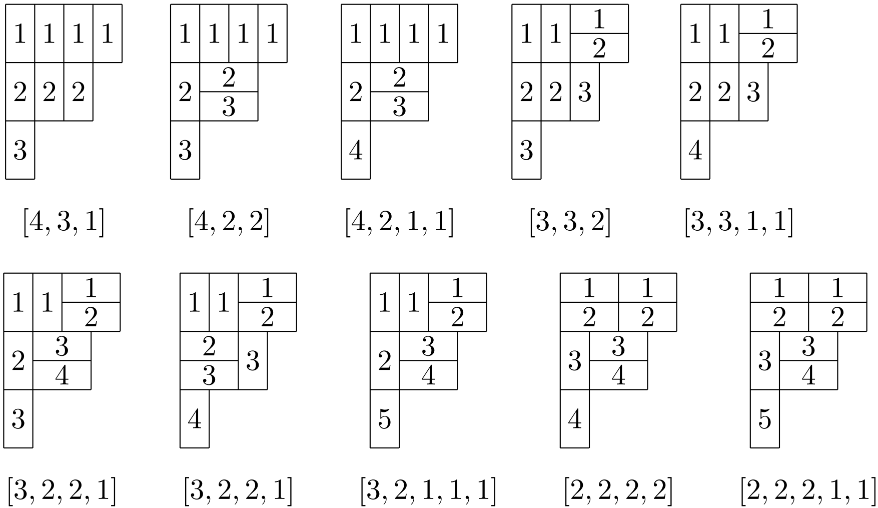
					1	1
		1	2			
		2	3			
	2	3				
1						

Domino combinatorics

- The plethysm coefficients $\mathcal{P}\left(t, S^2, R\right)$, $\mathcal{P}\left(t, \Lambda^2, R\right)$ also have combinatorial formulae in terms of similar objects to Littlewood-Richardson tableaux, called Yamanouchi domino tableaux [Carre, Leclerc, 95].
- These are domino tableux rather than Young tableaux, so rather than 1×1 boxes, we instead have 2×1 dominoes, arranged horizontally or vertically.
- The same rules (suitably generalised) apply for the placement of numbers as in Littlewood-Richardson tableaux.

Yamanouchi domino tableaux

All Yamanouchi domino tableaux of shape $R = [4, 4, 3, 3, 1, 1]$



Domino combinatorics

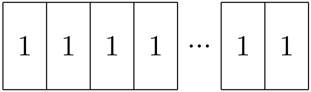
- Let D_t^R be the number of Yamanouchi domino tableaux of shape R and evaluation t.
- Then we can give a combinatorial formula for the projection coefficients:

$$\alpha_R^t = \mathcal{P}\left(t, S^2, R\right) - \mathcal{P}\left(t, \Lambda^2, R\right) = \varepsilon_2(R) D_t^R$$

- $\varepsilon_2(R)$ relates to the orientation of the dominoes in the relevant domino tableaux.

Giant graviton interpretation

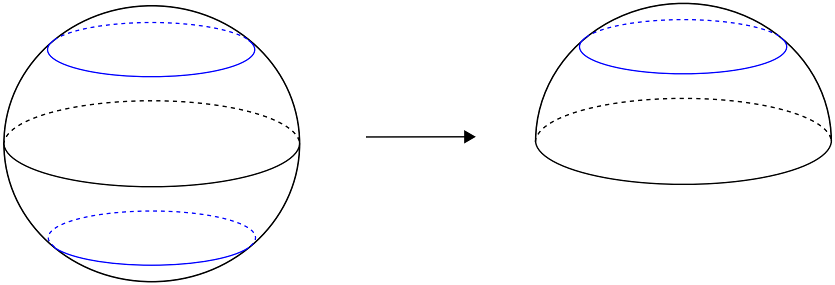
Consider two giant gravitons of the same angular momentum $2k$, or equivalently in the $U(N)$ gauge theory take $R = [2k, 2k]$. There are two obvious ways to arrange the dominoes. Firstly



for $\quad t = [2k]$

This can be interpreted as placing two giant gravitons anti-podally and identifying their world-volumes under the quotient.

Giant graviton interpretation



Giant graviton interpretation

Secondly

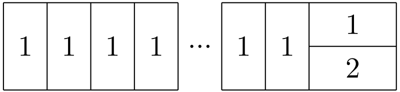
1	1	...	1
2	2		2

for $t = [k, k]$

Each giant graviton is identified with itself. This is understood by taking a quantum superposition of each D3-brane with its antipodal partner.

Giant graviton interpretation

The domino tableau approach allows interpolation between these two possibilities. For example



for $t = [2k-1, 1]$

The giant graviton interpretation of such tableaux is not so clear cut. We require some way of changing the quotient from identifying the two branes with each other to identifying each brane with a quantum superposition of itself.

Further questions

- How to interpret the mixed domino tableaux (those with both horizontal and vertical dominoes).
- How to interpret the domino tableaux for two giant gravitons of different angular momentum, and therefore different radius.
- Giant gravitons wrapped on an $\mathbb{R}P^3$ instead of an S^3 correspond to baryonic operators in the $SO(N)$ theory. It's not obvious how these appear in the quotient from the gauge theory side.