Global fixed points of scalar theories in 3D

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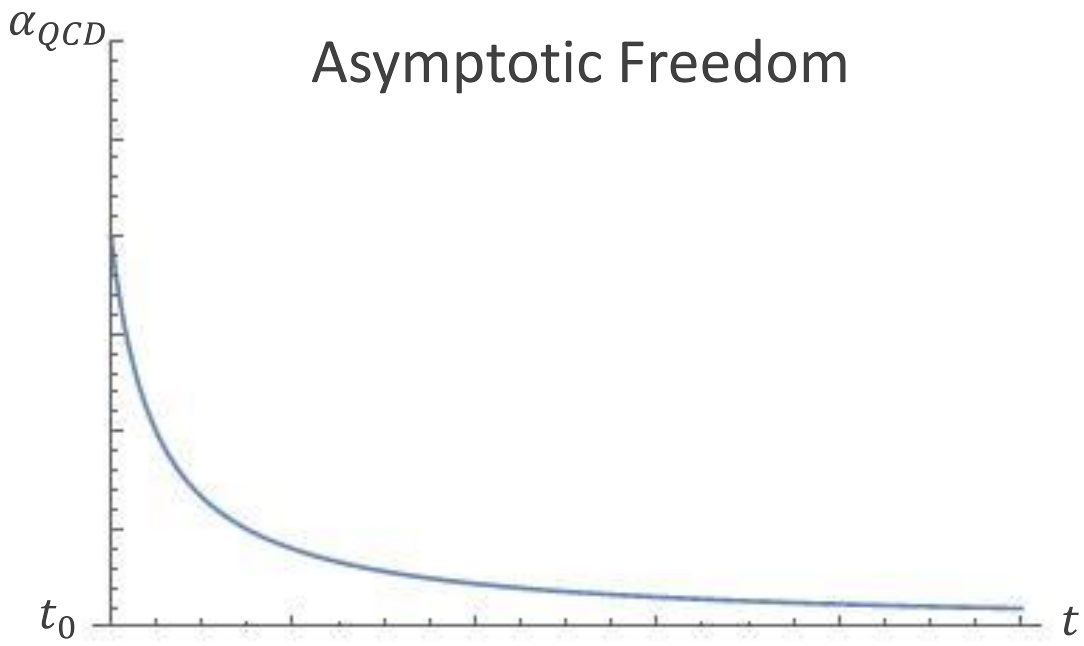
Fixed Points

Running of couplings: $\beta\ (\lambda) = \frac{\partial \lambda}{\partial t}$

Fixed points occur when $\beta(\lambda) = 0$



Non-interacting UV Fixed Point

$t = \ln k.$

D.J. Gross; F. Wilczek (1973) H. Politzer (1973) S. Weinberg (1978)

$O(N)$ Symmetric Scalar Model in 3D.

The **effective average action** Γ_k can be approximated by

$$\Gamma_k = \int d^3x \sum_{i=1}^{N} \left[\frac{1}{2}\partial_\mu \phi_i \partial^\mu \phi_i + V_k\left(\phi_i^2\right)\right].$$

Introduce **dimensionless** field and potential

$$\rho = \frac{1}{2}\phi_i^2 k^{-1} \qquad\qquad u(\rho) = V_k(\phi)\, k^{-3}$$

The **FRG** and **Wetterich Equation** yields

$$\partial_t u = -3u + \rho u' + \frac{(N-1)}{(1+u')} + \frac{1}{(1+u'+2\rho u'')}.$$

Fixed point solutions are the functions $u(\rho)$ that satisfy $\partial_t u = 0$. **Not** analytically solvable.

Fixed Point Solutions

In the **limit** $N \to \infty$

$$\partial_t u' = -2u' + \rho u'' - \frac{u''}{(1+u')^2}$$

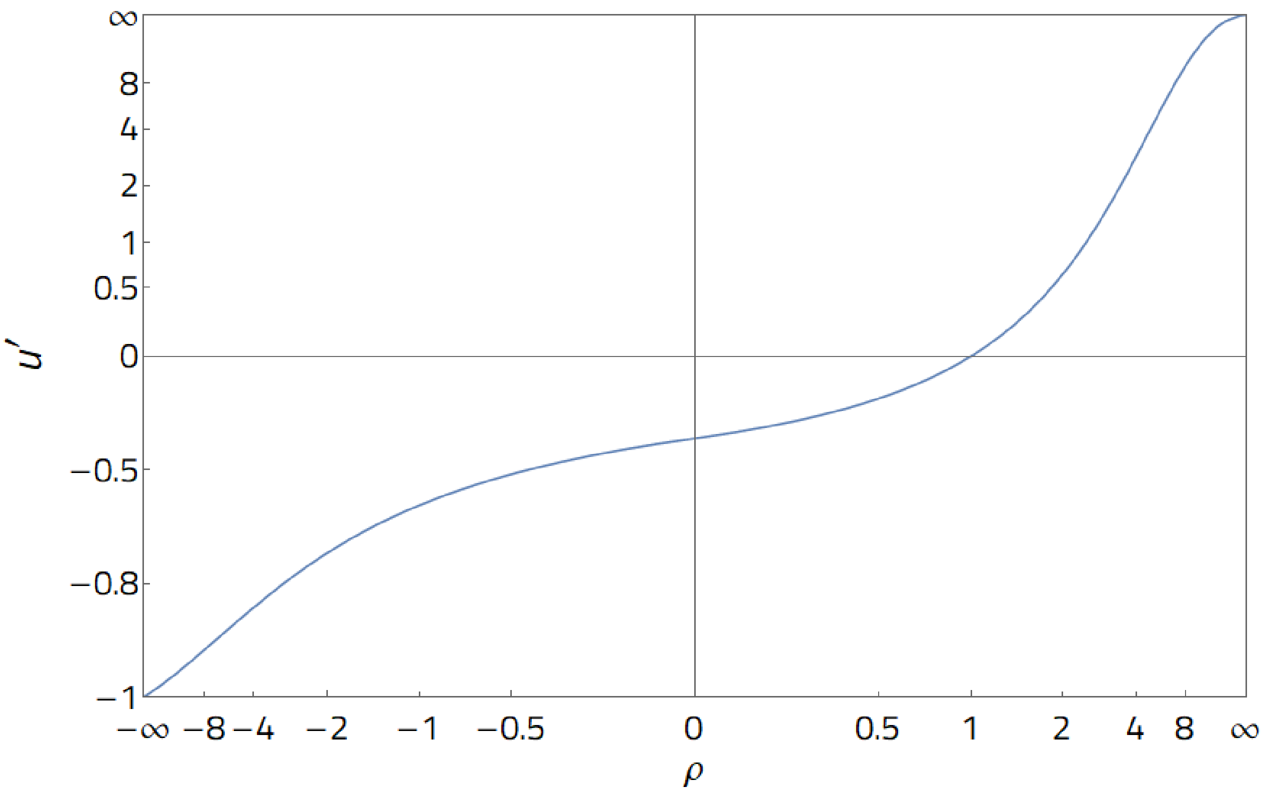
one can obtain a **family** of global analytic fixed point solutions.

One can use a **polynomial expansion** to find a given solution 'locally'

$$u(\rho) = \sum_{n=1}^{M} \frac{\lambda_n}{n!}\rho^n = \lambda_1 \rho + \frac{\lambda_2}{2!}\rho^2 + \frac{\lambda_3}{3!}\rho^3 + \cdots$$

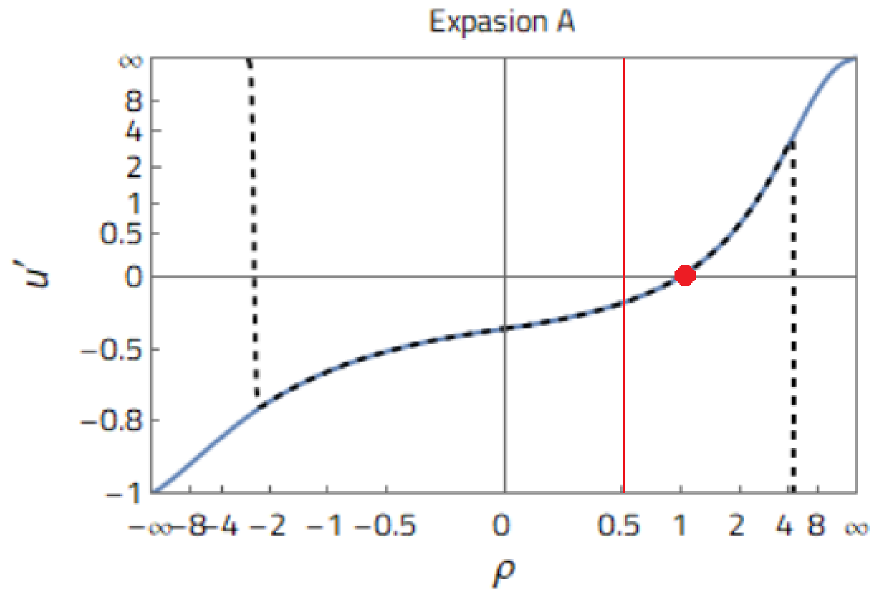
One can solve **recursively** for λ_n's.

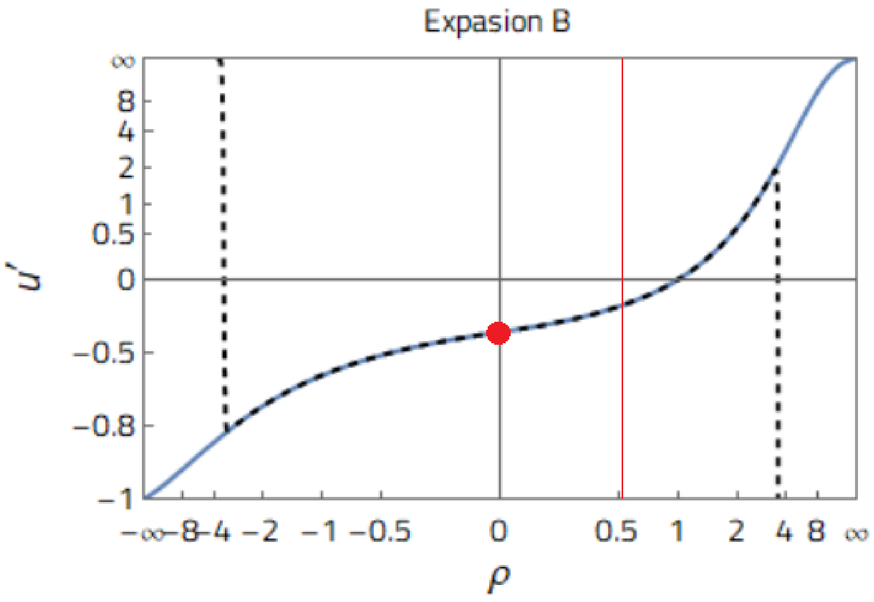
Wilson-Fisher FP Solution



Polynomial Expansions (testing ground)

The problem with polynomial expansions: They have a **finite radius of convergence** due to singularities that occur in the complexified solution.





Singularities in complexified Wilson-Fisher fixed point solution at $u''(0.51 \pm 3.14i)$

Solution? Padé Approximants!

A Padé approximant is a **rational function** of order [n/m] of the form

$$f(x) \approx \text{Padé}_m^n\left[\sum_{i=0}^{M} c_i x^i\right] = \frac{\sum_{i=0}^{n} a_i x^i}{1 + \sum_{i=0}^{m} b_i x^i}.$$

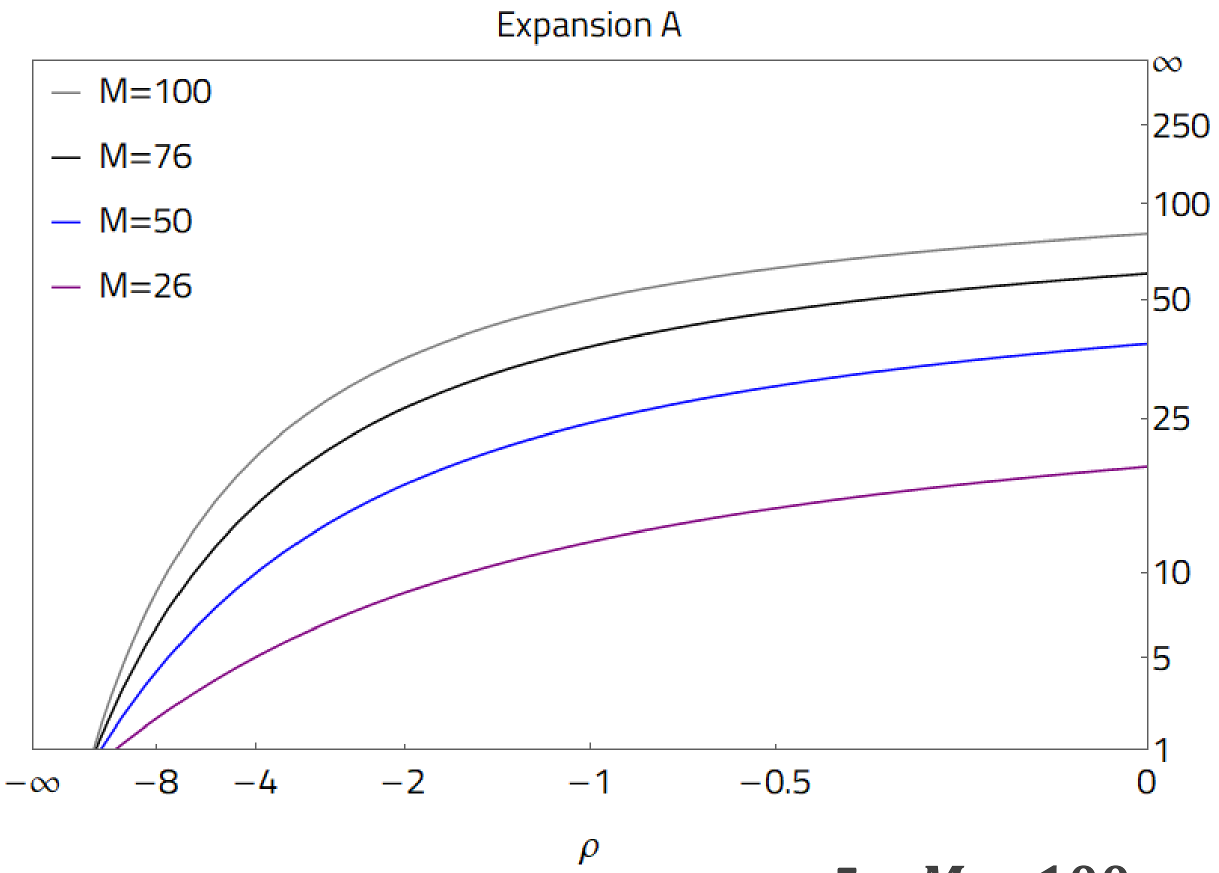
Known to **extent** the radius of converge of traditional Taylor expansions.

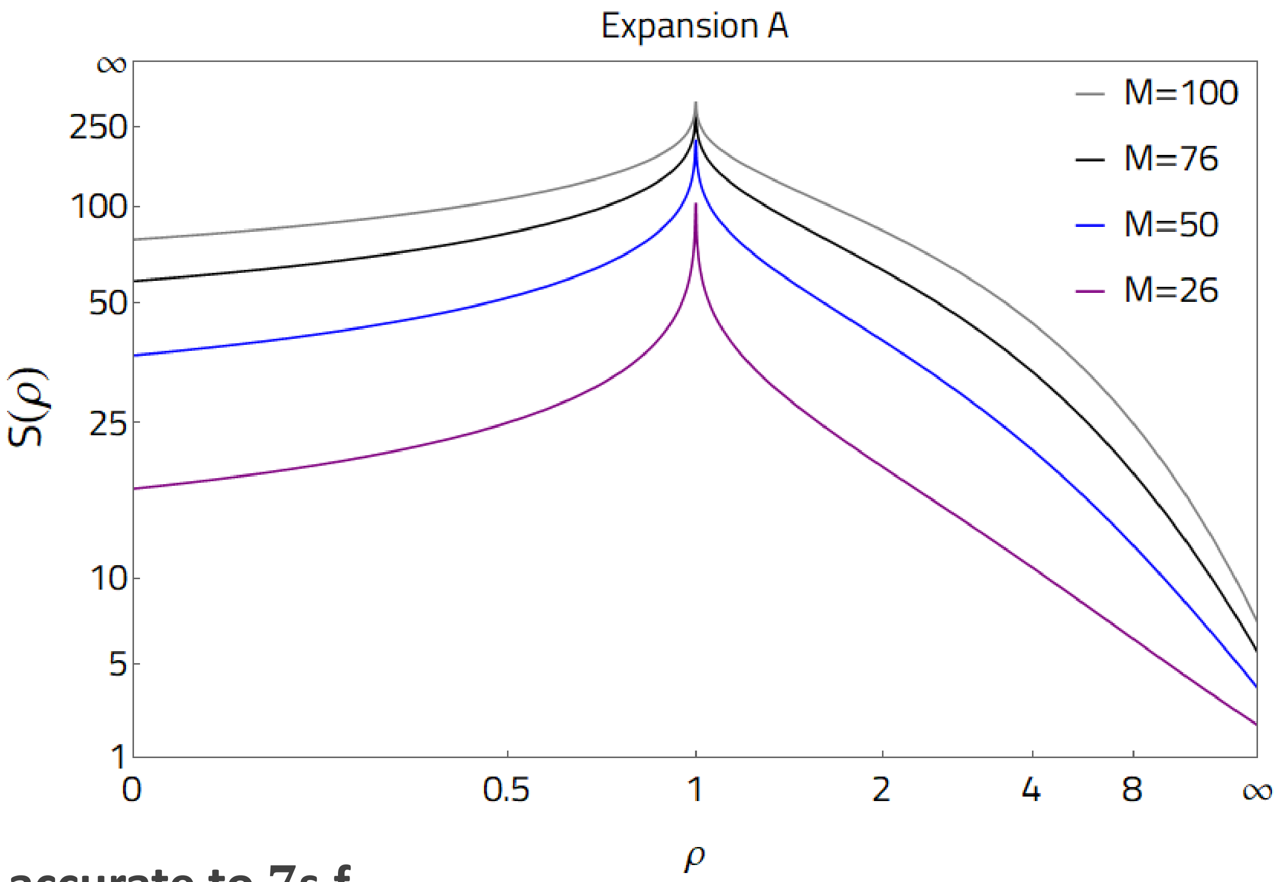
Benefit: Can be modified to factor in the behaviour of $f(x \to \pm\infty)$

$$u'(\rho \to \infty) = \gamma\rho^2 + subleading \qquad u'(\rho \to -\infty) = -1 + subleading$$

$$u'(\rho) \approx \text{Padé}_{M/2}^{M/2}\left[\frac{u'^{[M]}(\rho)}{\rho^2}\right]\rho^2$$

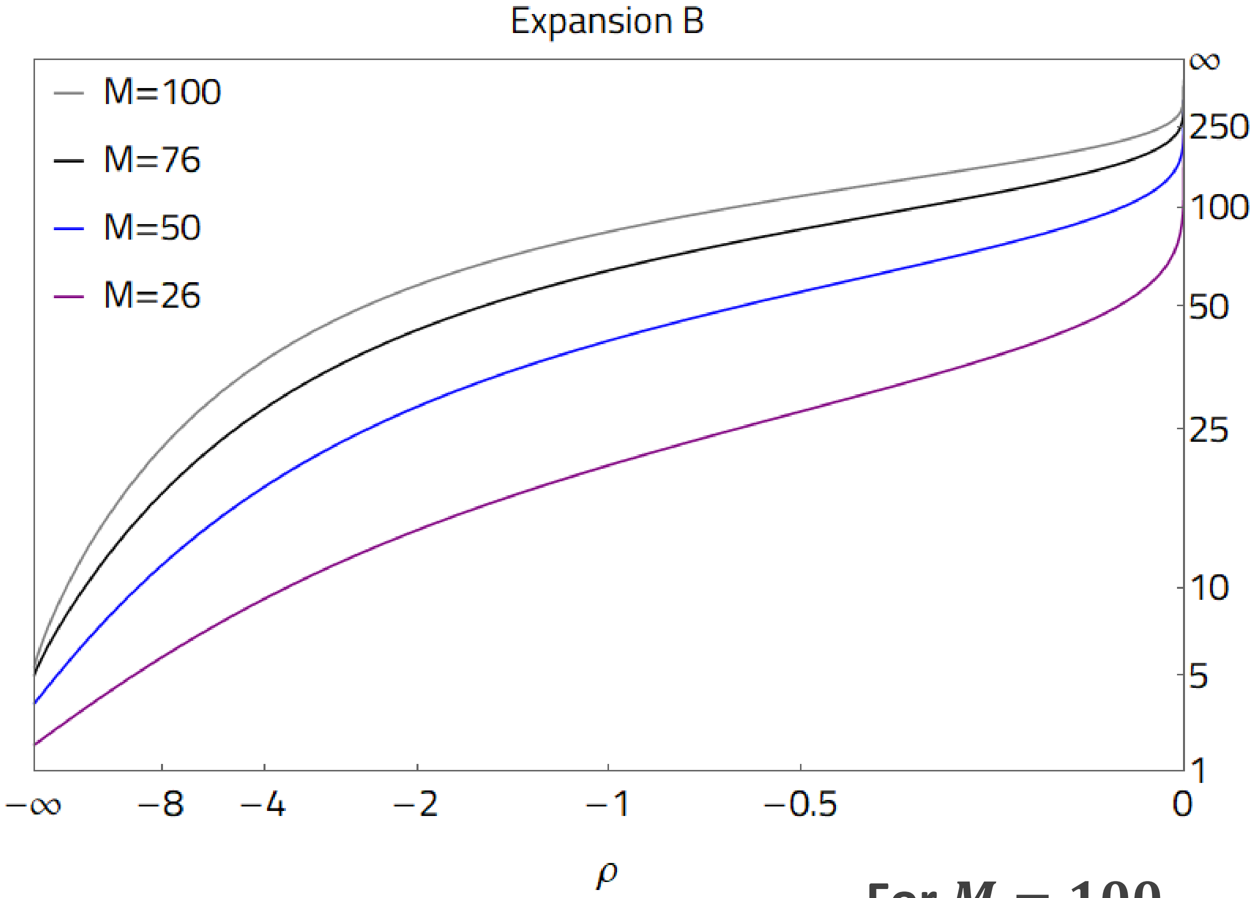
Expansion A $(N \to \infty)$

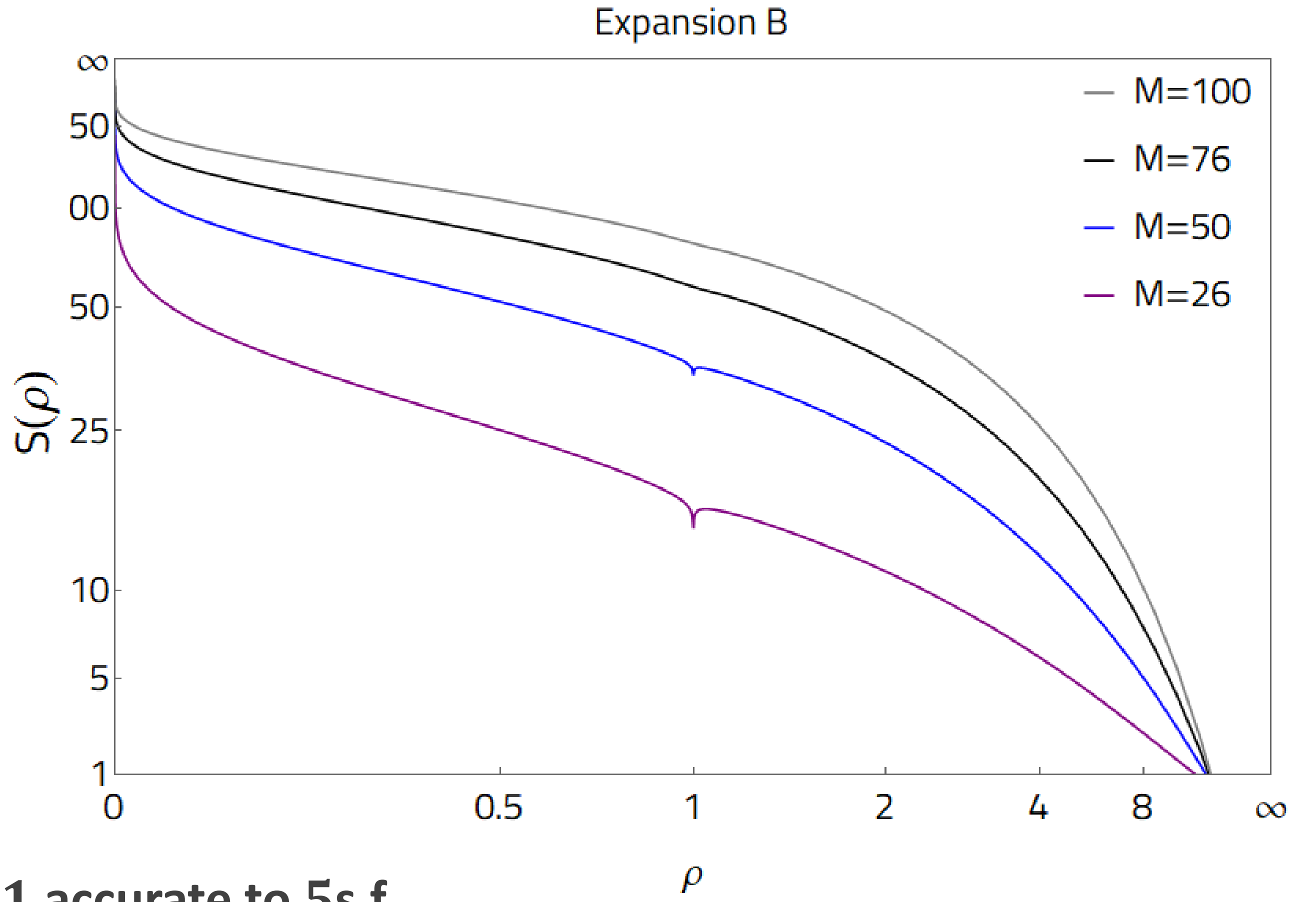




For $M = 100$, γ accurate to 7s.f

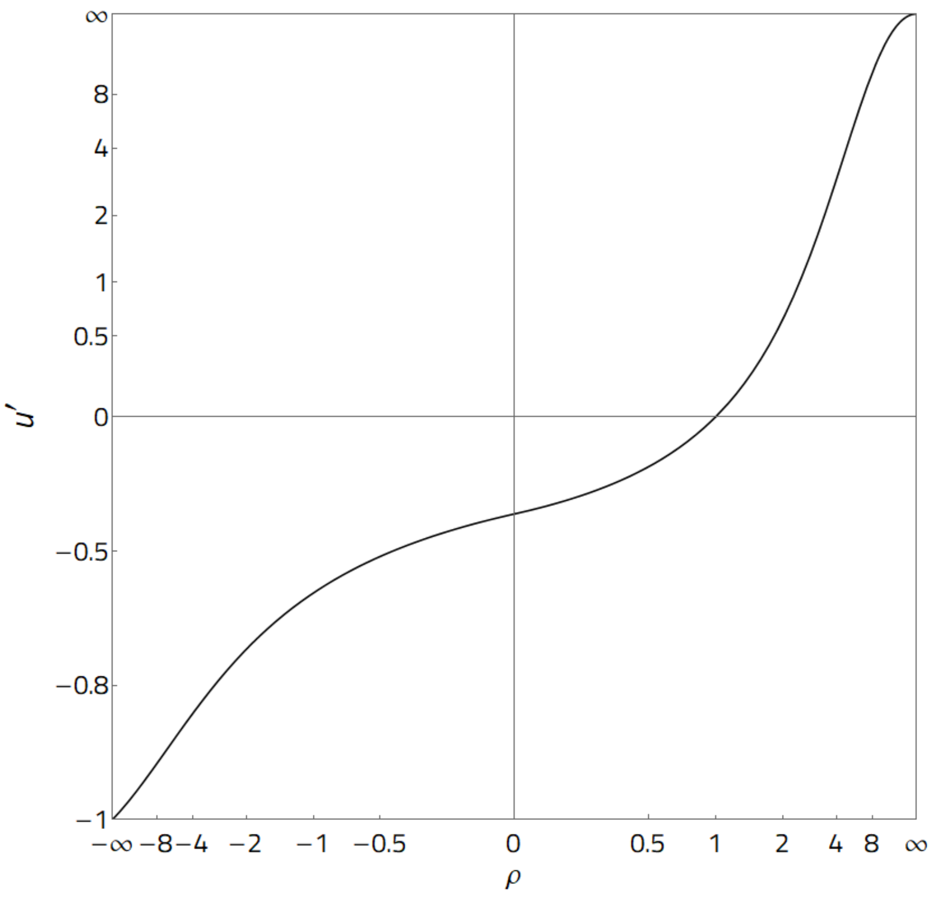
Expansion B $(N \to \infty)$

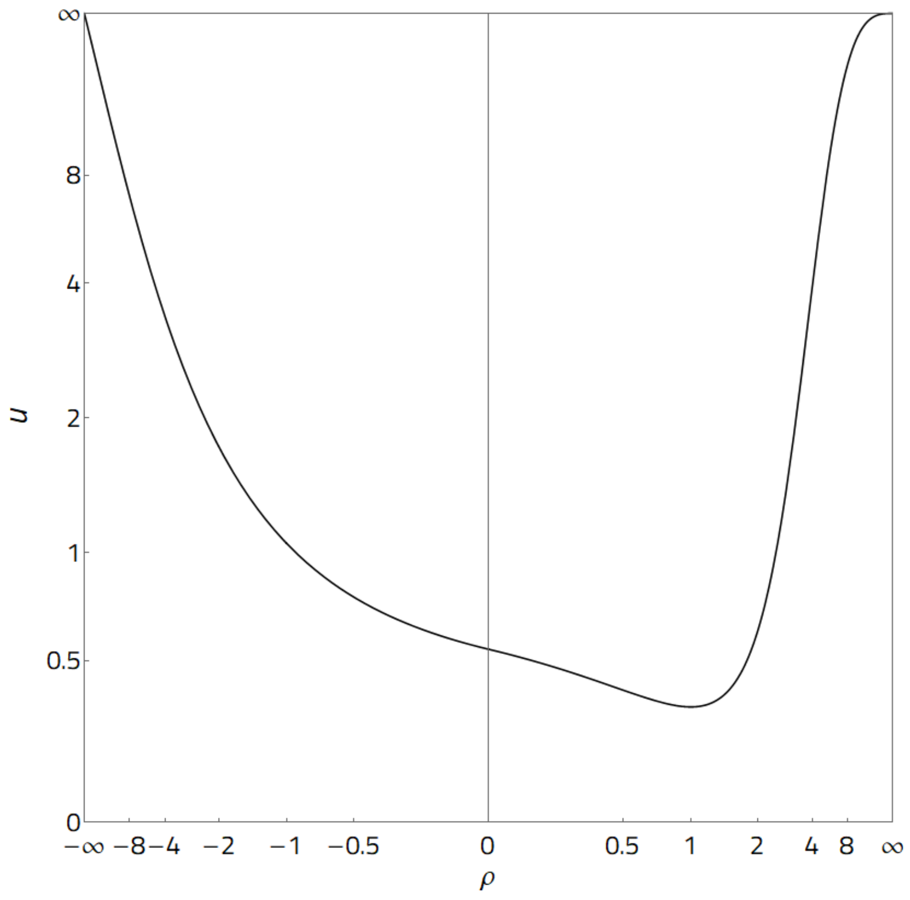




For $M = 100$, -1 accurate to 5s.f

Wilson-Fisher FP Solutions ($M = 100$)



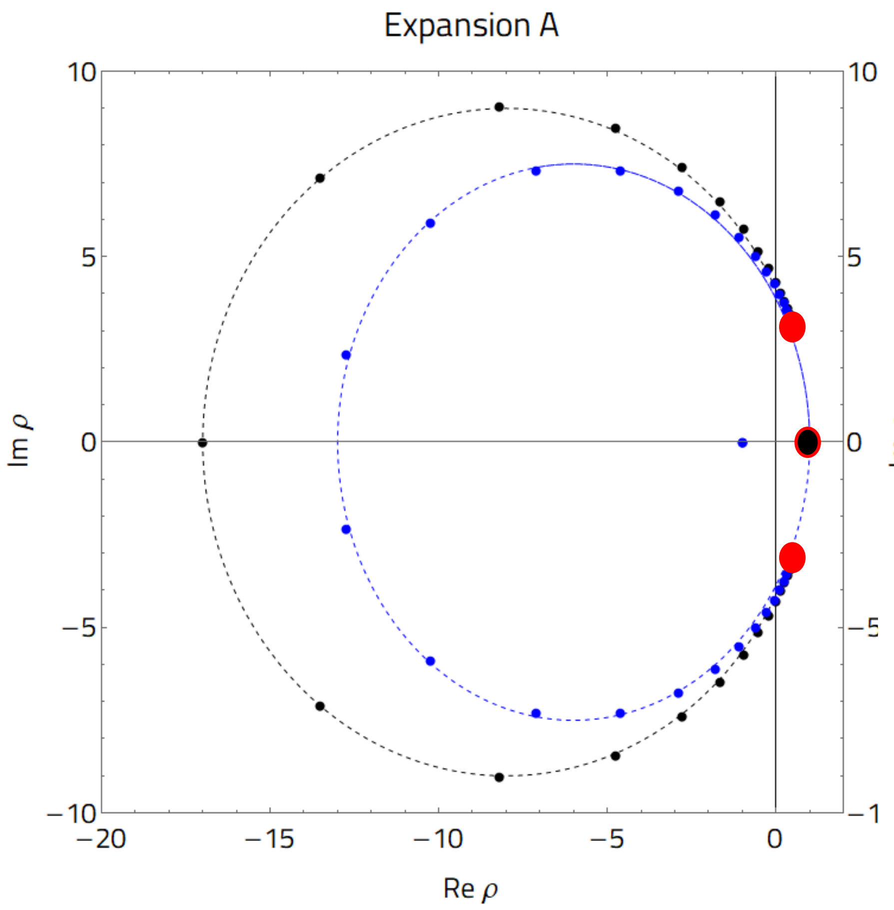


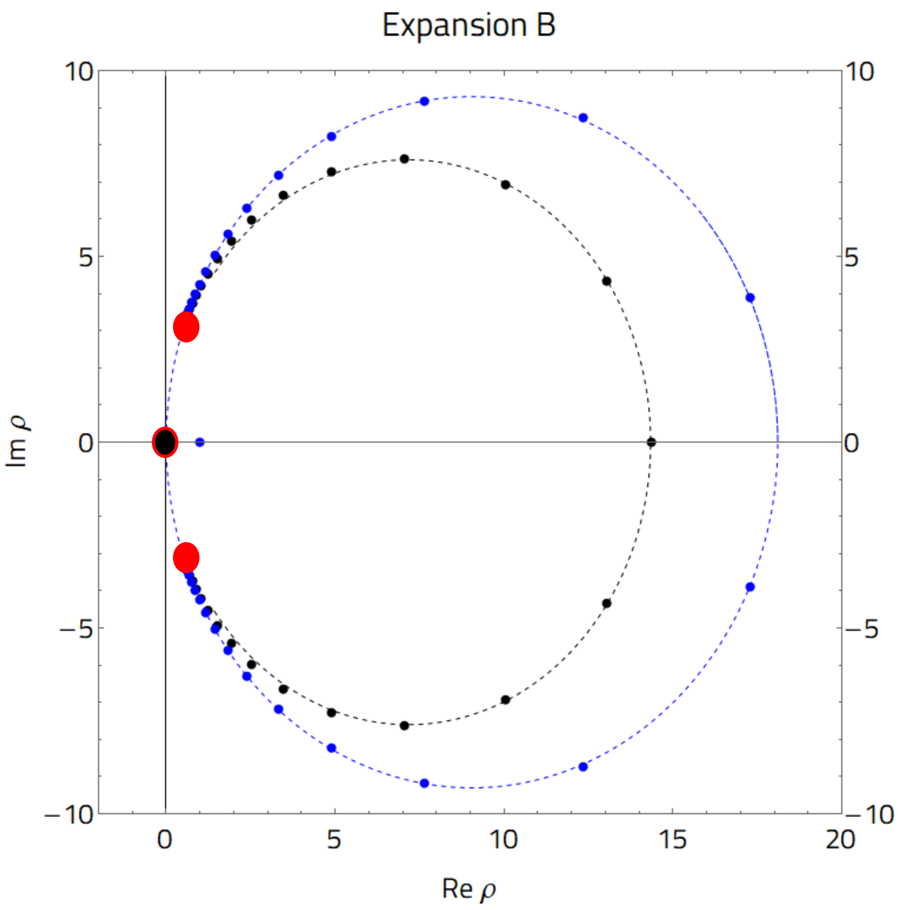
Defects and Convergence.

Padé Approximants exhibit many **spurious** zeros and poles.

Spurious zeros and poles create patterns in complexified field plane.

Defects are pairs of spurious poles and zeros and are known to limit convergence.





Further Applications

1. **Finite** N scalars in 3D.

C. Bervillier, A. Jüttner, and D. F. Litim (2007)
A. Jüttner; D. Litim; E. Marchais (2017)

2. Analogous **Fermionic** models in 3D – Infinite and Finite N_F.

A. Jakovacl; A. Patkos; P. Posfay (2014)

3. Fixed point solutions in **4D QFT's** via the FRG.

4. Asymptotically safe **Quantum Gravity**.

K. Falls, D. Litim, K. Nikolakopoulos, and C. Rahmede (2016)

Thank You

Questions?

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