Supercharging Superstrata

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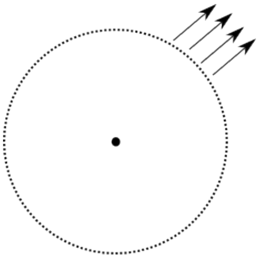
19th December 2018

Problems on the Horizon [Bekenstein, Hawking]

- Information Paradox
  - The spectrum of emitted particles is thermal
- Entropy problem
  - Assign entropy to an object with a Horizon

$$S_{BH} = \frac{c^3 A_H}{4 G_N \hbar}$$

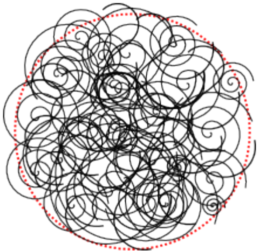
  - Microscopical description $\leftrightarrow$ No-hair theorems



Fuzzball Proposal [Mathur, Lunin,...]

At the scale of the horizon, a new fuzz state appears.

- Black hole has $e^{S_{BH}}$ microstates.
- Microstates constructed from String theory
- Some allow supergravity description
  $\rightarrow$ **Microstate Geometries**



Smooth, horizonless solutions of supergravity, valid within the supergravity approximation of string theory and have the same mass, angular momentum and charges as a given black hole

D1-D5-P Black Hole

- Supersymmetric 3-charge Black hole solution in on $\mathbb{R}_t \times \mathbb{R}^4 \times S^1_y \times T^4$
- Bekenstein–Hawking entropy

$$S_{BH} = \frac{2\pi^2 (2\pi R_y)((2\pi)^4 V)\sqrt{Q_1 Q_5 Q_p}}{4 G_{10}}$$

Consider the string theory setup

Object	Number	Charge	$\mathbb{R}_t$	$\mathbb{R}^4$	S^1_y	T^4
D1	n_1	Q_1	—	●	—	●
D5	n_5	Q_5	—	●	—	—
P	n_p	Q_p	—	●	—	●

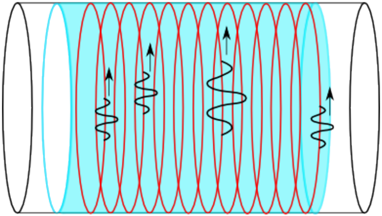
Use relations between Q and n to get

$$\boxed{S_{BH} = 2\pi\sqrt{n_1 n_5 n_p}}$$

Entropy Matching

[Strominger, Vafa, Maldacena,....]

- Number of configurations
  $=$ Partitions of $n_1 n_5 n_p \to e^{2\pi\sqrt{n_1 n_5 n_p}}$
- Entropy $S_{micro} = 2\pi\sqrt{n_1 n_5 n_p}$,



Entropy calculations match in the two different regimes

- What can we say about each individual configuration?

Entropy Matching

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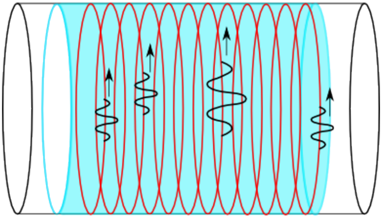


Entropy calculations match in the two different regimes

- What can we say about each individual configuration?

Find new microstate geometries of the D1-D5-P black hole using the superstratum technology.

Microstate Geometries [Giusto,Martucci, Petrini, Russo]

- Supersymmetry constraints the form of the physical fields
- Use $u = (t-y)/\sqrt{2}, \quad v = (t+y)/\sqrt{2}$.

$$ds^2_{10} = -2\frac{\sqrt{Z_1 Z_2}}{\mathcal{P}}(dv+\beta)\left[du+\omega+\frac{\mathcal{F}}{2}(dv+\beta)\right] + \sqrt{Z_1 Z_2}ds_4^2 + \sqrt{\frac{Z_1}{Z_2}}\, d\hat{s}_4^2,$$

$$C_0 = \frac{Z_4}{Z_1} \qquad B = -\frac{Z_4}{\mathcal{P}}(du+\omega)\wedge(dv+\beta) + a_4\wedge(dv+\beta)+\delta_2,$$

where Z_1, Z_2, Z_4, $\mathcal{F}$ are scalars, β, ω, a_4 one-forms, and δ_2 two-forms on $\mathbb{R}^4$ with $\mathcal{P} = Z_1 Z_2 - Z_4^2$.

- Use $\mathcal{D} \equiv d_4 - \beta\wedge\partial_v$, and combine content into new two-forms, such as $(\dot{} \equiv \partial_v)$

$$\Theta_4 \equiv \mathcal{D}a_4 + \dot{\delta}_2.$$

BPS Equations

- Zeroth Layer:
  - The base space $\mathbb{R}^4$ has a flat metric ds_4^2,
  - β is self-dual: $d\beta = *_4 d\beta$.
- First Layer: System of linear differential equations for pairs $(Z_1, \Theta_2), (Z_2, \Theta_1)$, and (Z_4, Θ_4), such as

$$\boxed{*_4\mathcal{D}\dot{Z}_4 = \mathcal{D}\Theta_4, \qquad \mathcal{D} *_4 \mathcal{D}Z_4 = -\Theta_4 \wedge d\beta, \qquad \Theta_4 = *_4\Theta_4.}$$

- Second Layer: Linear equations for $\mathcal{F}$ and ω.

$$\begin{aligned} \mathcal{D}\omega + *_4\mathcal{D}\omega + \mathcal{F}d\beta &= Z_1\Theta_1 + Z_2\Theta_2 - 2Z_4\Theta_4, \\ *_4\mathcal{D} *_4 \left(\dot{\omega} - \frac{1}{2}\mathcal{D}\mathcal{F}\right) &= \partial_v^2(Z_1 Z_2 - Z_4^2) - (\dot{Z}_1\dot{Z}_2 \\ &\quad - (\dot{Z}_4)^2) - \frac{1}{2} *_4 (\Theta_1 \wedge \Theta_2 - \Theta_4 \wedge \Theta_4). \end{aligned}$$

Seed Solution

[Lunin, Maldacena, Maoz; Kanitscheider, Skenderis, Taylor; Lunin, Mathur, Saxena]

- Two charge supertube solution with a perturbation:

$$\boxed{Z_1 = \frac{Q_1}{\Sigma}, \quad Z_2 = \frac{Q_5}{\Sigma}, \quad Z_4 = R_y\, b a^k \frac{\sin^k\theta e^{-ik\phi}}{(r^2+a^2)^{k/2}\Sigma} \quad \mathcal{F} = \Theta_4 = 0,}$$

with $\Sigma \equiv r^2 + a^2\cos^2\theta$.

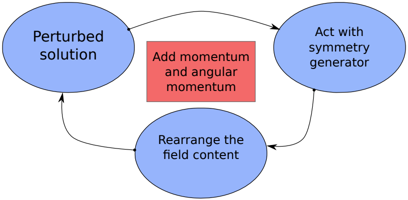
Asymptotically $\mathrm{AdS}_3 \times S^3 \times T^4 \Rightarrow$ use AdS/CFT duality:

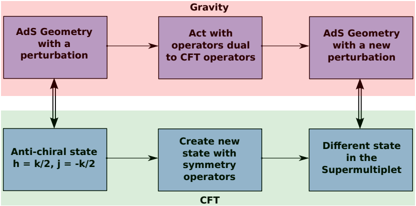
- States of the CFT $\Leftrightarrow$ Geometries in the bulk:
  Supertube with perturbation dual to an anti-chiral primary $|k/2, -k/2\rangle$
- Symmetries of the Geometry $\Leftrightarrow$ Symmetries of the CFT
  - AdS_3 symmetry $\Leftrightarrow$ Conformal symmetry
    $SO(2,2) \simeq SL(2,\mathbb{R})_L \times SL(2,\mathbb{R})_R$
  - S^3 symmetry $\Leftrightarrow$ $\mathcal{R}$–symmetry $SO(4) \simeq SU(2)_L \times SU(2)_L$

Solution Generating technique

[Bena, Giusto, Martinec,Russo, Shigemori, Turton, Warner]

- Use symmetry generators L_{-1} of $SL(2,\mathbb{R})_L$ and J_0^+ of $SU(2)_L$ to add momentum





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$$(J_0^+)^m (L_{-1})^n |k/2, -k/2\rangle \Leftrightarrow$$

$$Z_4 = b\, R_y \, \frac{\Delta_{k,m,n}}{\Sigma} \, \cos \hat{v}_{k,m,n},$$

$$\Theta_4 = -\sqrt{2} b\, \Delta_{k,m,n} \Big[\Big((m+n)\, r \sin\theta + n \Big(\frac{m}{k} - 1\Big) \frac{\Sigma}{r \sin\theta} \Big) \Omega^{(1)} \sin \hat{v}_{k,m,n} + \Big(m \Big(\frac{n}{k} + 1\Big) \Omega^{(2)} + \Big(\frac{m}{k} - 1\Big) n\, \Omega^{(3)} \Big) \cos \hat{v}_{k,m,n} \Big]$$

$$\Delta_{k,m,n} \equiv \left(\frac{a}{\sqrt{r^2+a^2}}\right)^k \left(\frac{r}{\sqrt{r^2+a^2}}\right)^n \cos^m\theta \, \sin^{k-m}\theta \,,$$

$$\hat{v}_{k,m,n} \equiv (m+n) \frac{\sqrt{2}\, v}{R_y} + (k-m)\phi - m\psi \,.$$

What about supersymmetry?

Fermionic Solution Generating Technique

- CFT generator $G^{\alpha A}_{\pm\frac{1}{2}}$ is dual to variations generated by Killing spinors of $\mathrm{AdS}_3 \times S^3 \times T^4$ (seed solution with $b = 0$)
- Procedure:
  - Solve the Killing spinor equations for empty $\mathrm{AdS}_3 \times S^3 \times T^4$ to find explicit form of these spinors
  - Identify the complex component $\zeta^{\alpha A}_{\pm}$ corresponding to $G^{\alpha A}_{\pm\frac{1}{2}}$
  - Act on the seed ($b \neq 0$) to obtain a new solution using

$$\delta'\delta C^{(0)} = \frac{1}{2} e^{-\phi}\, \epsilon^T \Gamma^{\underline{0}} (i\sigma^2) \delta'\lambda\,,$$

$$\delta'\delta B_{\mu\nu} = 2\bar{\epsilon}\,\Gamma_{[\mu}\sigma^3 \delta'\psi_{\nu]}\,.$$

Linear order in Perturbation

- The geometry dual to state $\left(G^{+1}_{-\frac{1}{2}}\, G^{+2}_{-\frac{1}{2}} + \frac{1}{k} J^{+}_{0} L_{-1}\right) |k/2, -k/2\rangle$ is

$$\boxed{\begin{aligned} & Z_4 = 0, \\ & \Theta_4 = -\sqrt{2}\, b\, \Delta_{k,1,1} \left[\frac{\Sigma}{r \sin\theta} \Omega^{(1)} \sin \hat{v}_{k,1,1} + \left(\Omega^{(2)} + \Omega^{(3)}\right) \cos \hat{v}_{k,1,1} \right] , \end{aligned}}$$

- Use this as a new seed to obtain geometry dual to $(J^{+}_{0})^{m} (L_{-1})^{n} \left(G^{+1}_{-\frac{1}{2}}\, G^{+2}_{-\frac{1}{2}} + \frac{1}{k} J^{+}_{0} L_{-1}\right) |k/2, -k/2\rangle$

$$\boxed{\begin{aligned} Z_4 &= 0\,, \\ \Theta_4 &= -\sqrt{2}\, b\, \Delta_{k,1+m,1+n} \\ &\qquad \times \left[\frac{\Sigma}{r \sin\theta} \Omega^{(1)} \sin \hat{v}_{k,1+m,1+n} + \left(\Omega^{(2)} + \Omega^{(3)}\right) \cos \hat{v}_{k,1+m,1+n} \right] . \end{aligned}}$$

Finite Deformation of $AdS_3 \times S^3 \times T^4$

- Make b finite $\rightarrow$ turn on $\mathcal{F}$ and ω. For example for $(k, m, n) = (2, 0, 0)$:

$$\mathcal{F}_{2,0,0} = -\frac{b^2}{18\left(a^2+r^2\right)^3}\left(8a^4+15a^2r^2+6r^4+a^2\left(2a^2+r^2\right)\cos 2\theta\right)$$

$$\omega_{2,0,0} = \frac{b^2 R_y}{18\sqrt{2}(a^2+r^2)^2\Sigma}\left(\left(4a^2+3r^2\right)\cos^2\theta d\psi+\left(6a^4+8a^2r^2+3r^4\right)\sin^2\theta d\phi\right) .$$

Properties of these solutions

- Asymptotically flat extension is trivial
- Conserved charges match the CFT result

Summary

- Found the explicit form of Killing spinors in $\mathrm{AdS}_3 \times S^3 \times T^4$
- New family of solutions with $Z_4 = 0$ and a simpler Θ_4
- Each solution has a well defined CFT dual
- In agreement with spectrum analysis [Deger et al,...]
- The solutions are simpler (v–independent even in the Asymptotically flat case)