

# Exploring Flavour Violation in an $A_4$ -Inspired SUSY GUT

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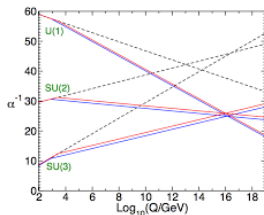
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- ▶ Introduction
- ▶ SUSY-breaking and Non-Minimal Flavour Violation
- ▶  $SU(5)$  Unification and  $A_4$
- ▶ This work - NMFV in this scenario
- ▶ Results
- ▶ Conclusions and Outlook

## Why SUSY?

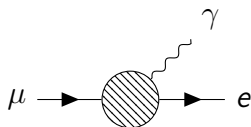
- ▶ Still (mostly) cures the hierarchy problem
- ▶ Precise gauge coupling unification
- ▶ Rich phenomenology, hints for experimentalists



Gauge couplings unify in MSSM<sup>[1]</sup>

## Why flavour physics?

- ▶ Many experimental results hint at departure from SM
- ▶ Models can predict mixing - how much?



<sup>[1]</sup>S. Martin, "A Supersymmetry primer", Adv. Ser. Direct. High Energy Phys **18** (1998), hep-ph/9709356

Viable SUSY in nature **must be broken**

General soft-breaking Lagrangian in the MSSM:

$$\begin{aligned}\mathcal{L}_{\text{soft}}^{\text{MSSM}} = & -\frac{1}{2}(M_1\tilde{B}\tilde{B} + M_2\tilde{W}\tilde{W} + M_3\tilde{g}\tilde{g} + \text{h.c.}) \\ & - M_Q^2\tilde{Q}^\dagger\tilde{Q} - M_L^2\tilde{L}^\dagger\tilde{L} - M_U^2\tilde{U}^*\tilde{U} - M_D^2\tilde{D}^*\tilde{D} - M_E^2\tilde{E}^*\tilde{E} \\ & - (A_U\tilde{U}^*H_u\tilde{Q} + A_D\tilde{D}^*H_d\tilde{Q} + A_E\tilde{E}^*H_d\tilde{L} + \text{h.c.}) \\ & - m_{H_u}^2H_u^*H_u - m_{H_d}^2H_d^*H_d - (bH_u^*H_d + \text{h.c.})\end{aligned}$$

Parameters  $M_Q$ ,  $M_L$ ,  $A_U$  etc. are **3x3 matrices** in 'flavour space'

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Parameters  $M_Q$ ,  $M_L$ ,  $A_U$  etc. are **3x3 matrices** in 'flavour space'

Minimal Flavour Violation paradigm  $\implies$  diagonal soft parameters.

$$M_Q^2 = \begin{pmatrix} (M_Q)_{11}^2 & 0 & 0 \\ \cdot & (M_Q)_{22}^2 & 0 \\ \cdot & \cdot & (M_Q)_{33}^2 \end{pmatrix} \quad A_U = \begin{pmatrix} (A_U)_{11} & 0 & 0 \\ 0 & (A_U)_{22} & 0 \\ 0 & 0 & (A_U)_{33} \end{pmatrix}$$

Assumption in most analyses, **no theory motivation**

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Relax assumption  $\implies$  **Non-Minimal Flavour Violation (NMFV)**

$$M_Q^2 = \begin{pmatrix} (M_Q)_{11}^2 & (\Delta_{12}^Q)^2 & (\Delta_{13}^Q)^2 \\ \cdot & (M_Q)_{22}^2 & (\Delta_{23}^Q)^2 \\ \cdot & \cdot & (M_Q)_{33}^2 \end{pmatrix} \quad A_U = \begin{pmatrix} (A_U)_{11} & \Delta_{12}^{AU} & \Delta_{13}^{AU} \\ \Delta_{21}^{AU} & (A_U)_{22} & \Delta_{23}^{AU} \\ \Delta_{31}^{AU} & \Delta_{32}^{AU} & (A_U)_{33} \end{pmatrix}$$

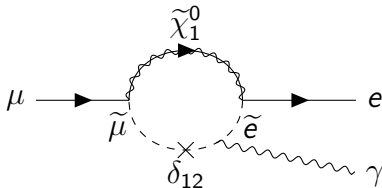
In a unified framework, flavour symmetries can generate NMFV

Reformulate NMFV by normalising to diagonal elements of soft matrices:

$$(\delta_{LL}^Q)_{ij} = \frac{(\Delta_{ij}^Q)^2}{(M_Q)_{ii}(M_Q)_{jj}}, \quad (\delta_{RR}^U)_{ij} = \frac{(\Delta_{ij}^U)^2}{(M_U)_{ii}(M_U)_{jj}}, \quad (\delta_{RR}^D)_{ij} = \frac{(\Delta_{ij}^D)^2}{(M_D)_{ii}(M_D)_{jj}},$$

$$(\delta_{RL}^U)_{ij} = \frac{v_u}{\sqrt{2}} \frac{\Delta_{ij}^{AU}}{(M_Q)_{ii}(M_U)_{jj}}, \quad (\delta_{RL}^D)_{ij} = \frac{v_d}{\sqrt{2}} \frac{\Delta_{ij}^{AD}}{(M_Q)_{ii}(M_D)_{jj}},$$

$$(\delta_{LL}^L)_{ij} = \frac{(\Delta_{ij}^L)^2}{(M_L)_{ii}(M_L)_{jj}}, \quad (\delta_{RR}^E)_{ij} = \frac{(\Delta_{ij}^E)^2}{(M_E)_{ii}(M_E)_{jj}}, \quad (\delta_{RL}^E)_{ij} = \frac{v_d}{\sqrt{2}} \frac{\Delta_{ij}^{AE}}{(M_L)_{ii}(M_E)_{jj}}$$



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Collect SM fields into irreps. of SU(5):

$$F = \bar{\mathbf{5}} = \begin{pmatrix} d_r^c \\ d_b^c \\ d_g^c \\ e^- \\ -\nu_e \end{pmatrix}_L, \quad T = \mathbf{10} = \begin{pmatrix} 0 & u_g^c & -u_b^c & u_r & d_r \\ \cdot & 0 & u_r^c & u_b & d_b \\ \cdot & \cdot & 0 & u_g & d_g \\ \cdot & \cdot & \cdot & 0 & e^c \\ \cdot & \cdot & \cdot & \cdot & 0 \end{pmatrix}_L$$

Unification gives equalities between parameters at the GUT scale:

$$\begin{aligned} M_Q^2 &= M_U^2 = M_E^2 \equiv M_T^2, & \delta_{LL}^Q &= \delta_{RR}^U = \delta_{RR}^E \equiv \delta^T, \\ M_D^2 &= M_L^2 \equiv M_F^2, & \delta_{RR}^D &= \delta_{LL}^L \equiv \delta^F, \\ A_D &= (A_E)^T \equiv A_{FT}, & \delta_{RL}^D &= (\delta_{RL}^E)^T \equiv \delta^{FT}, \\ & A_U \equiv A_{TT} & & \delta_{RL}^U \equiv \delta^{TT} \end{aligned}$$

# The $A_4 \times SU(5)$ Model

Addition of discrete symmetry unifies three families of the  $\bar{\mathbf{5}}$

Representations:

$$F = \mathbf{3}$$

$$T = \mathbf{1}$$



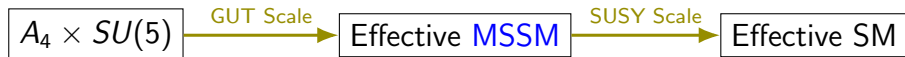
$\implies$

Unified breaking matrices:

$$M_F = \begin{pmatrix} m_F & 0 & 0 \\ 0 & m_F & 0 \\ 0 & 0 & m_F \end{pmatrix}$$

$$M_T = \begin{pmatrix} m_{T_1} & 0 & 0 \\ 0 & m_{T_2} & 0 \\ 0 & 0 & m_{T_3} \end{pmatrix}$$

Break discrete symmetry  $\implies$  NMFV patterns at the GUT scale - these incite flavour mixing at low scales



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- ▶ MFV not theoretically well motivated
- ▶ Flavour violation could place additional constraints on models
- ▶ Relax assumptions, explore phenomenology

## Question

What is the allowed flavour violation in such a scenario?

- ▶ Scan over NMFV parameters at the GUT scale simultaneously, run predictions to low scale, and determine degree of mixing permitted

## MFV Parameters

Masses:

$$m_F \quad m_{T_1} \quad m_{T_2} \quad m_{T_3}$$

$$M_1 \quad M_2 \quad M_3 \quad M_{H_u} \quad M_{H_d}$$

Couplings etc.:

$$(A_{TT})_{33} \quad (A_{FT})_{33} \quad \tan \beta \quad \mu$$

13 fixed numerical inputs, not varied in this analysis

These sets of unknowns specify MSSM SUSY-breaking entirely

## NMFV Parameters

Off-diagonal matrix elements

$$(\delta^F)_{12} \quad (\delta^F)_{13} \quad (\delta^T)_{23}$$

$$(\delta^T)_{12} \quad (\delta^T)_{13} \quad (\delta^T)_{23}$$

$$(\delta^{TT})_{12} \quad (\delta^{TT})_{13} \quad (\delta^{TT})_{23}$$

$$(\delta^{FT})_{12} \quad (\delta^{FT})_{13} \quad (\delta^{FT})_{21}$$

$$(\delta^{FT})_{23} \quad (\delta^{FT})_{31} \quad (\delta^{FT})_{32}$$

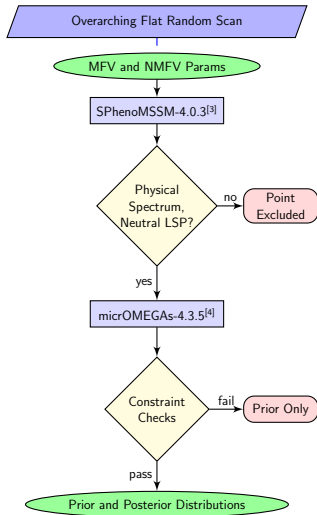
15 numerical inputs, scanned over with flat prior distributions

	Parameter/Observable	Scenario 1	Scenario 2
MFV Parameters at GUT scale	$m_F$	5000	5000
	$m_{T_1}$	5000	5000
	$m_{T_2}$	200	233.2
	$m_{T_3}$	2995	2995
	$(A_{TT})_{33}$	-940	-940
	$(A_{FT})_{33}$	-1966	-1966
	$M_1$	250.0	600.0
	$M_2$	415.2	415.2
	$M_3$	2551.6	2551.6
	$M_{H_u}$ $M_{H_d}$	4242.6 4242.6	4242.6 4242.6
	$\tan \beta$	30	30
	$\mu$	-2163.1	-2246.8

- ▶ MFV defined by flavour-conserving params
- ▶ 1 inspired by previous work<sup>[2]</sup>, 2 motivated by experimental limits
- ▶ Almost-mass-degenerate  $\tilde{\chi}_0^1$  and  $\tilde{\mu}$  to satisfy relic density through coannihilation

Table: GUT scale parameters that define MFV scenarios.

<sup>[2]</sup>A. Belyaev, S.F. King and P. Schaefer, "Muon  $g-2$  and dark matter suggest nonuniversal gaugino masses:  $SU(5) \times A_4$  case study at the LHC", Phys. Rev. D **97** (2018), 1801.00514



[3] W. Porod, "SPHeno...", Comput. Phys. Commun. **153** (2003), hep-ph/0301101

[4] G. Belanger et. al., "MicrOMEGAS...", Comput. Phys. Commun. **149** (2002), hp-ph/0112278

Observable	Constraint
$m_h$	$(125.2 \pm 2.5)$ GeV
$\text{BR}(\mu \rightarrow e\gamma)$	$< 4.2 \times 10^{-13}$
$\text{BR}(\mu \rightarrow 3e)$	$< 1.0 \times 10^{-12}$
$\text{BR}(\tau \rightarrow e\gamma)$	$< 3.3 \times 10^{-8}$
$\text{BR}(\tau \rightarrow \mu\gamma)$	$< 4.4 \times 10^{-8}$
$\text{BR}(\tau \rightarrow 3e)$	$< 2.7 \times 10^{-8}$
$\text{BR}(\tau \rightarrow 3\mu)$	$< 2.1 \times 10^{-8}$
$\text{BR}(\tau \rightarrow e^- \mu \mu)$	$< 2.7 \times 10^{-8}$
$\text{BR}(\tau \rightarrow e^+ \mu \mu)$	$< 1.7 \times 10^{-8}$
$\text{BR}(\tau \rightarrow \mu^- ee)$	$< 1.8 \times 10^{-8}$
$\text{BR}(\tau \rightarrow \mu^+ ee)$	$< 1.5 \times 10^{-8}$
$\text{BR}(B \rightarrow X_s \gamma)$	$(3.32 \pm 0.18) \times 10^{-4}$
$\text{BR}(B_s \rightarrow \mu \mu)$	$(2.7 \pm 1.2) \times 10^{-9}$
$\Delta M_{B_s}$	$(17.757 \pm 0.312)$ ps $^{-1}$
$\Delta M_K$	$(3.1 \pm 1.2) \times 10^{-15}$ GeV
$\epsilon_K$	$2.228 \pm 0.29$
$\Omega_{\text{DM}} h^2$	$0.1198 \pm 0.0042$

Table: Experimental constraints imposed on the  $A_4 \times SU(5)$  parameter space in our study.

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<i>Parameters</i>	<i>Scenario 1</i>	<i>Scenario 2</i>	<i>Principle Constraints</i>
$(\delta^T)_{12}$	[-0.015, 0.015]	[-0.12, 0.12] <sup>†</sup>	$\Omega_{\tilde{\chi}_1^0} h^2, \mu \rightarrow e\gamma$
$(\delta^T)_{13}$	[-0.06, 0.06] <sup>†</sup>	[-0.3, 0.3] <sup>†</sup>	$\Omega_{\tilde{\chi}_1^0} h^2$
$(\delta^T)_{23}$	[0, 0]*	[-0.1, 0.1] <sup>†</sup>	$\Omega_{\tilde{\chi}_1^0} h^2, \mu \rightarrow 3e, \mu \rightarrow e\gamma,$
$(\delta^F)_{12}$	[-0.008, 0.008]	[-0.015, 0.015] <sup>†</sup>	$\mu \rightarrow 3e, \mu \rightarrow e\gamma$
$(\delta^F)_{13}$	[-0.01, 0.01] <sup>†</sup>	[-0.15, 0.15] <sup>†</sup>	$\mu \rightarrow 3e, \mu \rightarrow e\gamma$
$(\delta^F)_{23}$	[-0.015, 0.015] <sup>†</sup>	[-0.15, 0.15] <sup>†</sup>	$\Omega_{\tilde{\chi}_1^0} h^2, \mu \rightarrow e\gamma, \mu \rightarrow 3e$
$(\delta^{TT})_{12}$	$[-3, 3.5] \times 10^{-5}$	$[-1, 1.5]^{\dagger} \times 10^{-3}$	prior, $\Omega_{\tilde{\chi}_1^0} h^2$
$(\delta^{TT})_{13}$	$[-6, 7]^{\dagger} \times 10^{-5}$	$[-4, 2.5]^{\dagger} \times 10^{-3}$	prior, $\Omega_{\tilde{\chi}_1^0} h^2$
$(\delta^{TT})_{23}$	$[-0.5, 4]^{\dagger} \times 10^{-5}$	$[-0.25, 0.2]^{\dagger}$	prior, $\Omega_{\tilde{\chi}_1^0} h^2$
$(\delta^{FT})_{12}$	[-0.0015, 0.0015]	$[-1.2, 1.2]^{\dagger} \times 10^{-4}$	$\mu \rightarrow 3e, \Omega_{\tilde{\chi}_1^0} h^2, \mu \rightarrow e\gamma$
$(\delta^{FT})_{13}$	[-0.002, 0.002] <sup>†</sup>	$[-5, 5] \times 10^{-4}$	$\Omega_{\tilde{\chi}_1^0} h^2, \mu \rightarrow 3e, \mu \rightarrow e\gamma$
$(\delta^{FT})_{21}$	[0, 0]*	$[-1.2, 1.2]^{\dagger} \times 10^{-4}$	$\Omega_{\tilde{\chi}_1^0} h^2, \text{prior}$
$(\delta^{FT})_{23}$	[-0.0022, 0.0022] <sup>†</sup>	$[-6, 6]^{\dagger} \times 10^{-4}$	$\mu \rightarrow 3e, \Omega_{\tilde{\chi}_1^0} h^2, \mu \rightarrow e\gamma$
$(\delta^{FT})_{31}$	[-0.0004, 0.0004] <sup>†</sup>	$[-2, 2]^{\dagger} \times 10^{-4}$	$\Omega_{\tilde{\chi}_1^0} h^2$
$(\delta^{FT})_{32}$	[0, 0]*	$[-1.5, 1.5] \times 10^{-4}$	$\Omega_{\tilde{\chi}_1^0} h^2$

Table: Estimated allowed GUT scale flavour-violation for both reference scenarios and impactful constraints.

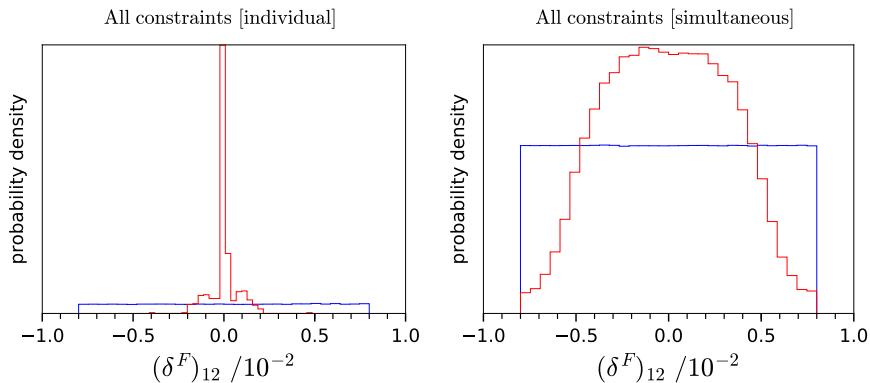
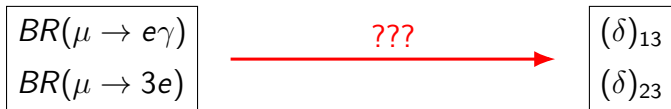
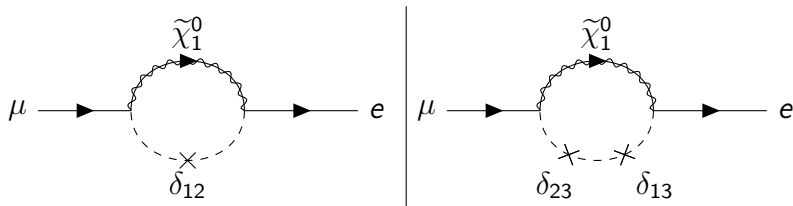


Figure: Comparison of individual VS simultaneous scan in Scenario 1 for  $(\delta^F)_{12}$ .

Blue shows prior distribution, and red shows posterior after constraints are applied



$\mu \rightarrow e\gamma$  and  $\mu \rightarrow 3e$  can have a distinctive constraining effect on  $(\delta)_{13}$  and  $(\delta)_{23}$  parameters.



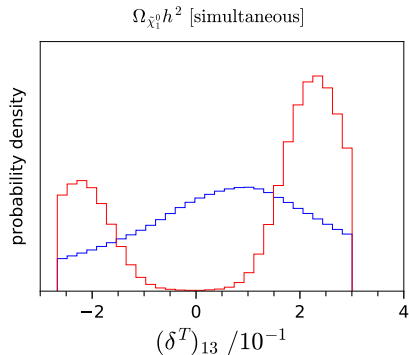


Figure: Dark matter constraint action on  $(\delta^T)_{13}$ , simultaneous scan over Scenario 2

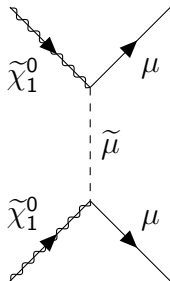


Figure: Dominant co-annihilation channel responsible for relic abundance

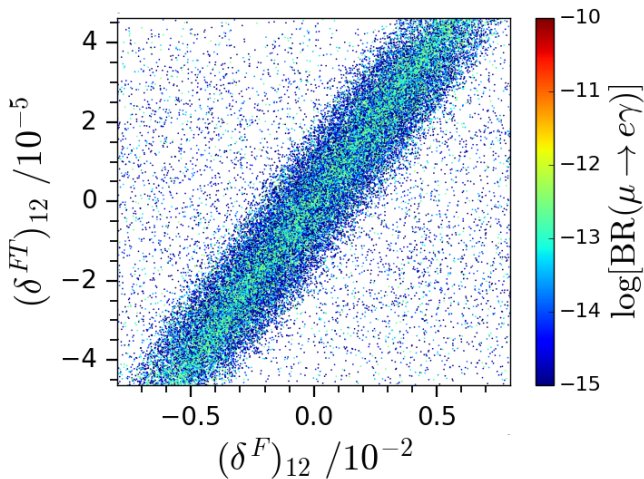


Figure: Correlations plots of  $(\delta^F)_{12}$  and  $(\delta^{FT})_{12}$  at GUT scale. Results reflect simultaneous scan around Scenario 1.

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## Outlook:

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  - ▶ Use MCMC methods to scan over MFV parameters, to determine allowed violation in pure  $SU(5)$
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**Thank you for your attention**