Theoretical Particle Cosmology

Lancaster University

Quasi-Degenerate Vacua in the KSVZ axion model; A minimal approach to dark energy and dark matter.

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AMY LLOYD-STUBBS

LANCASTER UNIVERSITY

SUPERVISOR: JOHN MCDONALD

Introduction and Motivation

- We focused on the minimal KSVZ axion model

- We investigated how the combination of classical and quantum effects in this model could produce an effective potential with a second quasi-degenerate minimum present.

- With an effective potential in the Higgs direction of this form it is possible to provide an explanation for dark energy without adding any extra degrees of freedom, in the form of the metastable vacuum energy of the electroweak vacuum.

What is the KSVZ axion model?

- The KSVZ axion model adds a complex singlet scalar (PQ scalar) ϕ and a heavy quark doublet Q to the Standard Model which is charged under an additional global $U(1)_{PQ}$ symmetry:

$$\mathcal{L}_Q = y_Q \bar{Q}_R Q_L \phi + h.c$$

where the PQ scalar is given by

$$\phi = \frac{\phi}{\sqrt{2}} e^{a/f_a}$$

- The angular part of this complex scalar singlet is the axion (f_a is the PQ breaking scale or axion decay constant) and it acts as the Goldstone boson when the PQ symmetry is spontaneously broken.

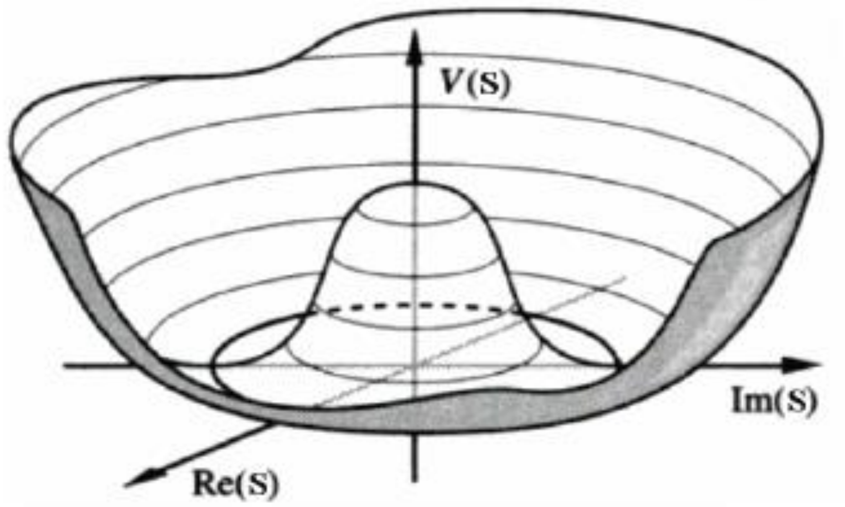


Diagram showing the broken PQ symmetry potential where S corresponds to the ϕ in our case.

The QCD Axion and the strong CP problem

- The QCD axion was originally proposed in order to provide a solution to the strong CP problem.

- The strong CP problem arises due to the fact that we do not observe as much CP violation as QCD predicts we should, in the form of a nonzero neutron electric dipole moment. This means that the theta parameter in the QCD topological term

$$\mathcal{L}_\theta = \theta \frac{{g_s}^2}{32\pi^2} Tr G_{\mu\nu}\tilde{G}^{\mu\nu}$$

must be tuned down to an extremely small number ($\theta < 10^{-10}$) which gives a naturalness problem.

The QCD Axion and the strong CP problem

- Adding the axion to the SM brings with it an axial anomaly which breaks the $U(1)_{PQ}$ symmetry and couples the axion to the topological term in the QCD Lagrangian.

- After the PQ symmetry is broken, non-perturbative effects generate the following effective potential for the axion at some scale Λ_{QCD}

$$V(a) = \Lambda_{QCD}^4 (1 - \cos \frac{a}{f_a})$$

where the topological term is zero at its minima.

- Since it provides a solution to the strong CP problem, the KSVZ axion model is a well-motivated basis for a cosmological model beyond the electroweak Standard Model.

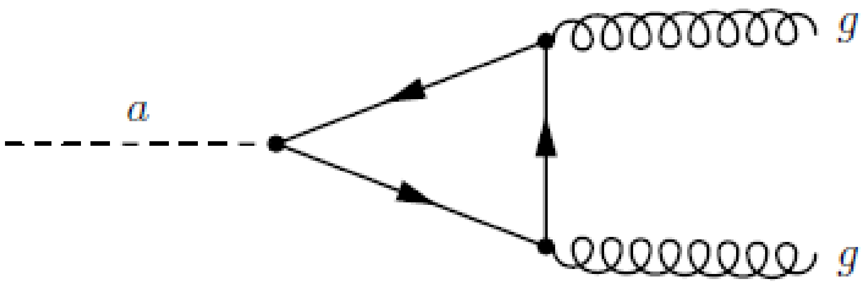


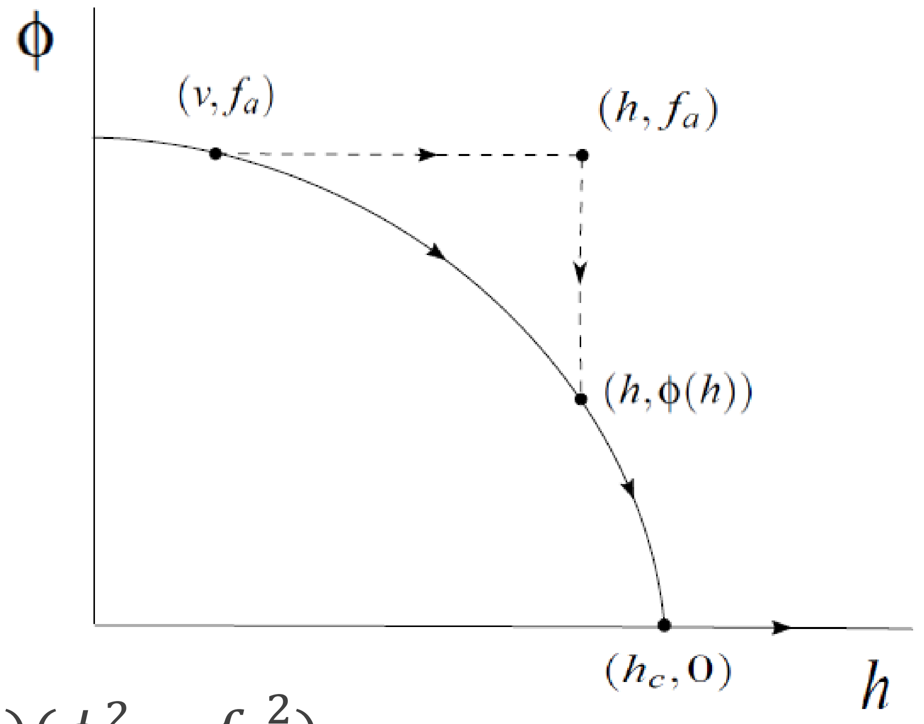
Diagram showing the gluonic triangle anomaly.

PQ Symmetry breaking and the two-field scalar potential

- Adding the PQ extension to the scalar sector of the standard model modifies the scalar potential to a function of two fields.

- The minimum of the classical potential for a given value of the Higgs field becomes a minimum trajectory in the $h - \phi$ plane.

- The form of the two-field scalar potential along the minimum trajectory is given below:



$$V(h,\phi) = \frac{\lambda_h}{4}(h^2 - v^2)^2 + \frac{\lambda_\phi}{4}(\phi^2 - {f_a}^2)^2 + \frac{\lambda_{h\phi}}{4}(h^2 - v^2)(\phi^2 - {f_a}^2)$$

arXiv: 1807.00778 [hep-ph]

PQ Symmetry breaking and the two-field scalar potential

- The point $(h_c, 0)$ is the point at which the spontaneously broken PQ symmetry is restored.

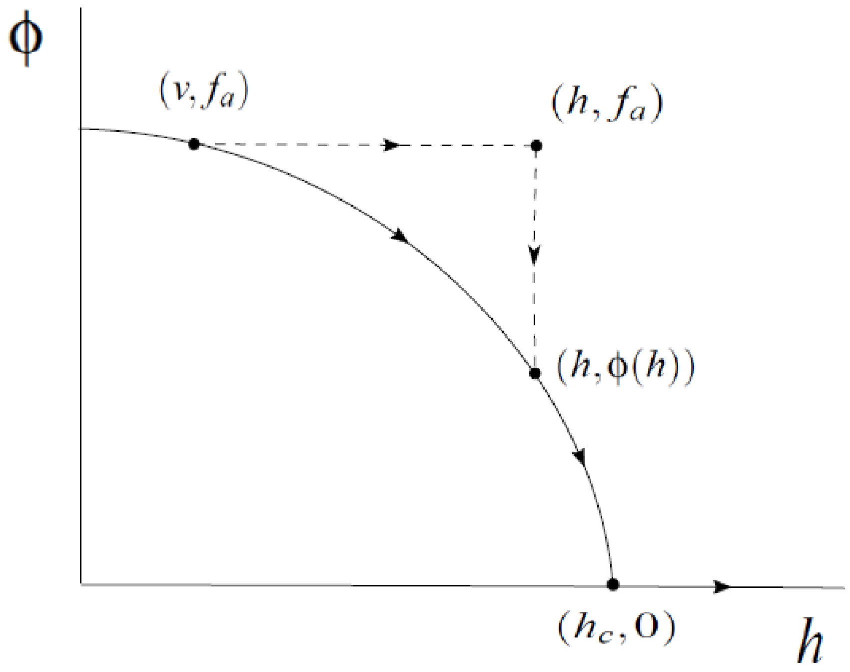
- In previous analyses, the heavy radial field ϕ is integrated out to form a single scalar effective theory in terms of h. Here the radial component is not integrated out, we use the full two-field potential.

-For $h < h_c$ the potential along the minimum trajectory has the following form

$$V(\mathrm{h}, \phi(h)) = -\frac{v^2}{2} h^2 \left(\lambda_h - \frac{{\lambda_{h\phi}}^2}{4\lambda_\phi}\right) + \frac{h^4}{4}\left(\lambda_h - \frac{{\lambda_{h\phi}}^2}{4\lambda_\phi}\right) + \frac{v^4}{4}\left(\lambda_h - \frac{{\lambda_{h\phi}}^2}{4\lambda_\phi}\right)$$

- For $h > h_c$ the potential takes the form

$$V(h, 0) = -\frac{{\mu_H}^2}{2} h^2 + \frac{\lambda_h}{4} h^4 + \frac{\lambda_h}{4} v^4 + \frac{\lambda_\phi}{4} {f_a}^4 + \frac{\lambda_{h\phi}}{4} v^2 {f_a}^2$$



arXiv: 1807.00778 [hep-ph]

PQ Symmetry breaking and the two-field scalar potential

- One effect of the two-field potential is that the observed Higgs boson is actually a mixture of the SM Higgs and the radial PQ scalar.

-The electroweak Higgs doublet quartic coupling can be larger than the SM Higgs quartic coupling.

- This phenomenon was originally explored a threshold approximation known as the Scalar Threshold Effect.

$$\lambda_{hSM} = \lambda_h - \frac{{\lambda_{h\phi}}^2}{4\lambda_\phi}$$ (Espinosa et al. arXiv: 1203.0237)

- This increase in the coupling can help to stabilize the electroweak vacuum with respect to quantum corrections.

- Additionally the change in coupling modifies the shape of the Higgs effective potential along the minimum trajectory.

Quantum Corrections and Quasi-Degenerate Minima

- So far we have considered the classical potential. In addition, quantum corrections must be included.

- SM Quantum corrections $\Longrightarrow \lambda_h(\mu)$ decreases along minimum trajectory

- At the point $(h_c, 0)$ along the minimum trajectory $\phi \longrightarrow 0$ so the h^4 coupling changes

$$\lambda_h(\mu) - \frac{{\lambda_{h\phi}}^2}{4\lambda_\phi} \longrightarrow \lambda_h(\mu)$$

- The potential kicks up at the point of PQ symmetry restoration.

2-loop SM Renormalisation Group Equations

For the gauge couplings $g_i = \{g', g, g_s\}$

$$\frac{dg_i}{dt} = \kappa g_i^3 b_i + \kappa^2 g_i^3 \left(\sum_{j=1}^{3} B_{ij} g_j^2 - d_i^t h_t^2 \right), \qquad \kappa = 1/(16\pi^2)$$

For the top Yukawa coupling:

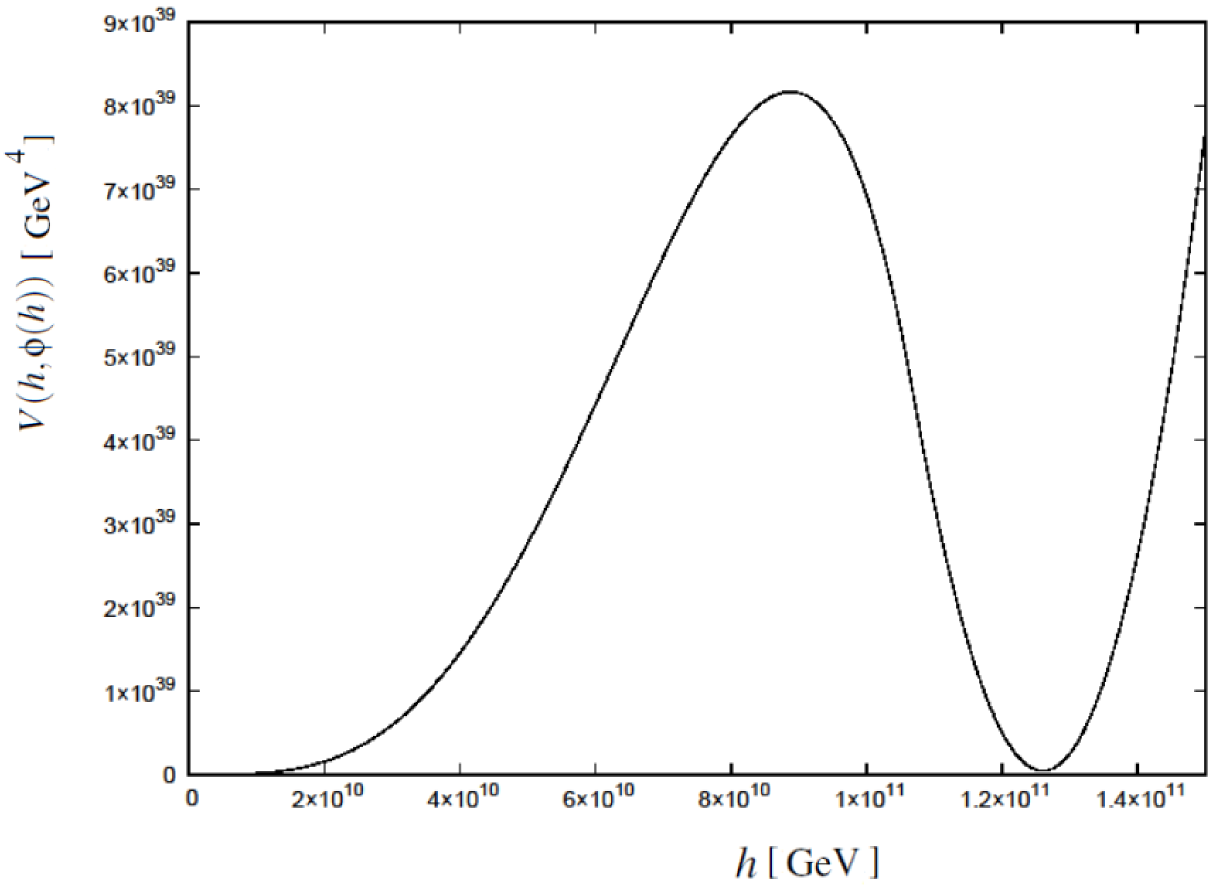
$$\frac{dh_t}{dt} = \kappa h_t \left(\frac{9}{2} h_t^2 - \sum_{i=1}^{3} c_i^t g_i^2 \right) + \kappa^2 h_t \left[\sum_{ij} D_{ij} g_i^2 g_j^2 + \sum_i E_i g_i^2 h_t^2 + 6(\lambda^2 - 2h_t^4 - 2\lambda h_t^2) \right],$$

For the Higgs quartic coupling:

$$\begin{aligned} \frac{d\lambda}{dt} = {} & \kappa \left\{ -6h_t^4 + 12 h_t^2 \lambda + \frac{3}{8} \left[2g^4 + (g^2 + g'^2)^2 \right] - 3\lambda(3g^2 + g'^2) + 24\lambda^2 \right\} \\ & + \kappa^2 \left\{ 30 h_t^6 - h_t^4 \left(32 g_s^2 + \frac{8}{3} g'^2 + 3\lambda \right) + h_t^2 \left[-\frac{9}{4} g^4 + \frac{21}{2} g^2 g'^2 - \frac{19}{4} g'^4 \right. \right. \\ & \left. + \lambda \left(80 g_s^2 + \frac{45}{2} g^2 + \frac{85}{6} g'^2 - 144\lambda \right) \right] + \frac{1}{48} \left(915 g^6 - 289 g^4 g'^2 - 559 g^2 g'^4 - 379 g'^6 \right) \\ & \left. + \lambda \left(-\frac{73}{8} g^4 + \frac{39}{4} g^2 g'^2 + \frac{629}{24} g'^4 + 108 \lambda g^2 + 36 \lambda g'^2 - 312 \lambda^2 \right) \right\} \end{aligned}$$

arXiv: 0710.2484 [hep-ph]

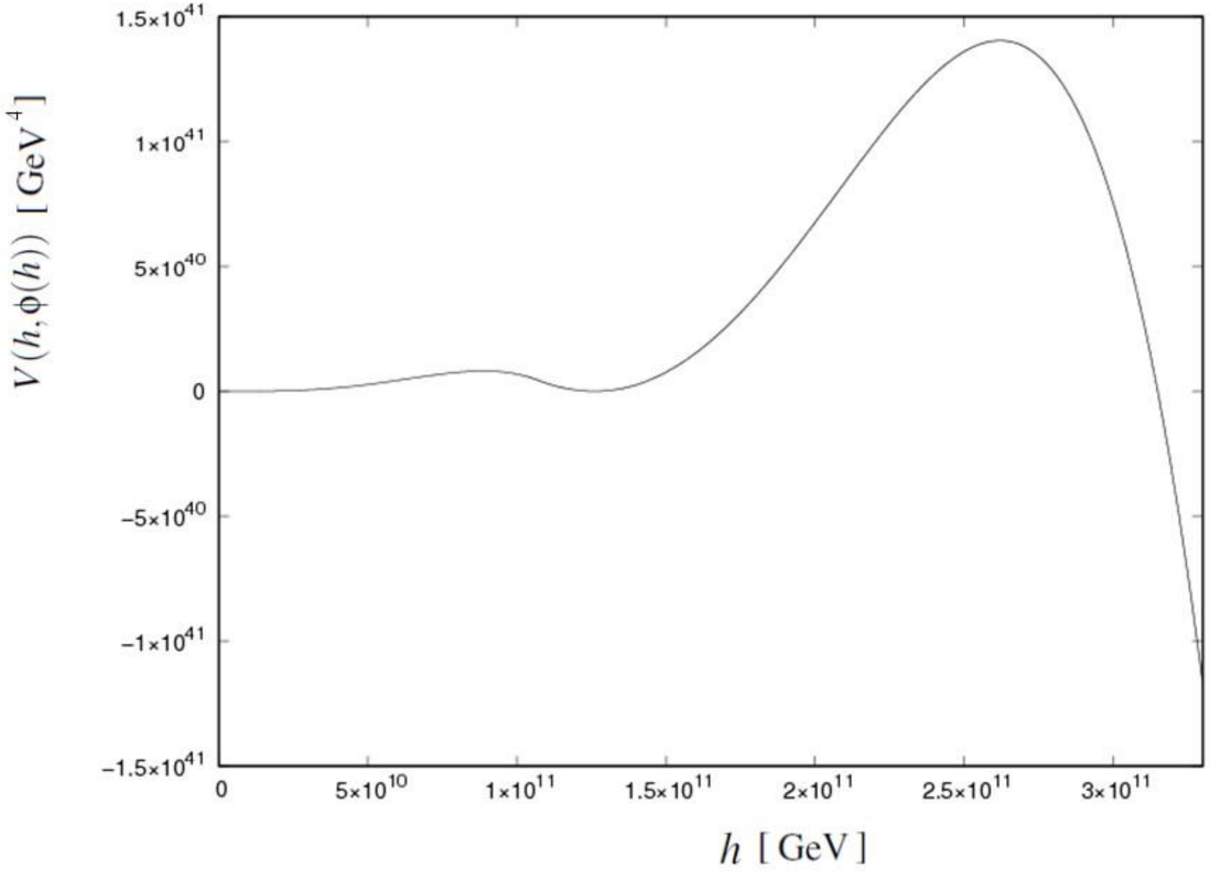
Potential with a Quasi-Degenerate Minimum at $\lambda_{h\phi}/\sqrt{\lambda_\phi} = 0.1$



Potential for $\lambda_{h\phi}/\sqrt{\lambda_\phi} = 0.1$ and $f_a = 2.39 \times 10^{10}$ GeV

arXiv: 1807.00778 [hep-ph]

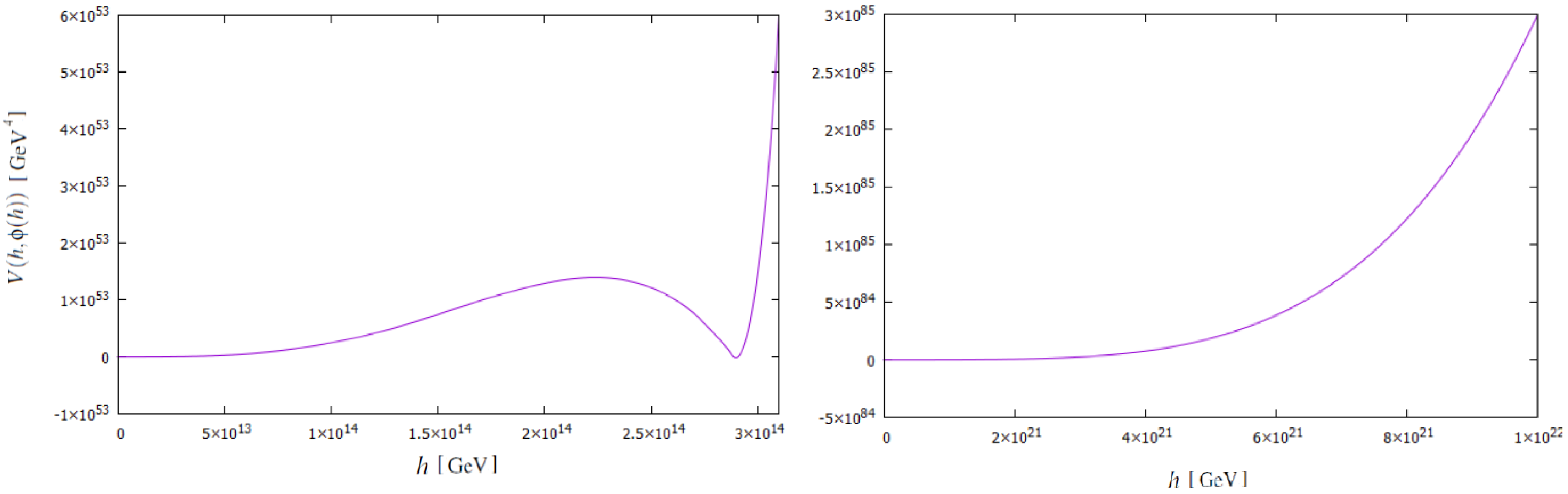
Potential with a Quasi-Degenerate Minimum at $\lambda_{h\phi}/\sqrt{\lambda_\phi} = 0.1$



Potential for $\lambda_{h\phi}/\sqrt{\lambda_\phi} = 0.1$, $f_a = 2.39 \times 10^{10}$ GeV showing the presence of a second local minimum

arXiv: 1807.00778 [hep-ph]

Potential with a Quasi-Degenerate Minimum at $\lambda_{h\phi}/\sqrt{\lambda_\phi} = 0.26$



Potential in the region of second minimum (left) and zoomed out to above Planck scale (right) for $f_a = 1.04 \times 10^{14}$ GeV and $\lambda_{h\phi}/\sqrt{\lambda_\phi} \simeq 0.26$ in the case where the potential is stabilised and the second minimum becomes a global minimum.

An Application: Dark Energy as Electroweak Vacuum Energy

- A possible application of quasi-degenerate vacua is to dark energy.

- It has been proposed that dark energy could be due to the energy density of a metastable electroweak vacuum relative to a zero energy minimum of the potential. ("Metastable Dark Energy" (Landim and Abdalla, [hep-ph]1611.00428; Landim [astro-ph] 1712.09653)).

- There exist models which can set the energy of a stable or metastable minimum to zero energy density such as vacuum sequestering (Kaloper and Padilla, [hep-th] 1309.6562), or models based on energy parity (Kaplan and Sundrum, [hep-th] 0505265).

- These models cancel any cosmological constant and so dark energy must be due to a field energy.

Bounds from the potential

- The existence of a second degenerate minimum is a non-trivial constraint on the model.

- More generally the value of f_a determines the precise form of the potential.

- It implies a lower bound on f_a and therefore on the axion dark matter density in the case where PQ symmetry breaks after inflation.

$$\Omega_a h^2 \approx (1.6 \pm 0.4) \times 10^{-2} \left(\frac{f_a}{10^{10}\,\mathrm{GeV}} \right)^{1.165}$$

Relation between axion decay constant, f_a, and physical axion dark matter density.
arXiv: [hep-ph] 1412.0789

Axion Dark Matter

- If PQ symmetry is broken during inflation then axions are produced by the misalignment mechanism.

- Axion field present during inflation $\Longrightarrow$ quantum fluctuations of the axion field can lead to isocurvature perturbations in CDM. These are strictly constrained by CMB observations.

- In this case, it can be difficult to have PQ symmetry breaking during inflation and therefore restoration, and subsequent SSB is needed.

- For PQ symmetry breaking after inflation, we get the formation of topological defects and axions are then produced by the decay of these, as well as by the misalignment mechanism.

Axion Dark Matter with Quasi-Degenerate Minima

- Lower bound on axion decay constant from presence of second minimum : $f_a \geq 2.39 \times 10^{10}$ GeV.

- Lower bound on axion dark matter falls in the range of 28-46% of total observed dark matter density.

- For 100% axion dark matter the axion decay constant falls in the range 4.7 x 10^{10} GeV to 7.2 x 10^{10} GeV.

- In this case the potential is well-defined to study the cosmology in detail.

$\lambda_{h\phi}/\sqrt{\lambda_\phi}$	$\lambda_\phi^{1/4} f_a$	h_{max}/GeV	$V_{max}^{1/4}$/ GeV	h_{min}/GeV
0.10	2.39×10^{10}	8.89×10^{10}	9.50×10^{9}	1.26×10^{11}
0.12	3.81×10^{10}	1.26×10^{11}	1.33×10^{10}	1.70×10^{11}
0.14	6.49×10^{10}	1.96×10^{11}	2.05×10^{10}	2.59×10^{11}
0.16	1.22×10^{11}	3.41×10^{11}	3.52×10^{10}	4.48×10^{11}
0.18	2.61×10^{11}	6.85×10^{11}	6.94×10^{10}	8.96×10^{11}
0.20	6.71×10^{11}	1.66×10^{12}	1.64×10^{11}	2.16×10^{12}
0.22	2.23×10^{12}	5.26×10^{12}	5.04×10^{11}	6.80×10^{12}
0.24	1.09×10^{13}	2.46×10^{13}	2.26×10^{12}	3.20×10^{13}
0.26	1.04×10^{14}	2.24×10^{14}	1.93×10^{13}	2.90×10^{14}
0.28	3.94×10^{15}	8.19×10^{15}	6.33×10^{14}	1.06×10^{16}
0.29	9.31×10^{16}	1.90×10^{17}	1.32×10^{16}	2.45×10^{17}
0.297	7.04×10^{18}	1.41×10^{19}	7.95×10^{17}	1.83×10^{19}

Table to show the location of the second minimum in the potential for different values of $\lambda_{h\phi}/\sqrt{\lambda_\phi}$ at different values of f_a (arXiv: 1807.00778 [hep-ph]).

Conclusion

The KSVZ axion model is a well-motivated framework for building a cosmological model

1. It provides a solution to the strong CP problem
2. It provides a viable dark matter candidate
3. It provides a potential which can explain dark energy without requiring additional degrees of freedom.

The requirement of the second degenerate minimum is a non-trivial condition which we can use to constrain the cosmology of the model.

Possibilities for future study

Inflation in this model

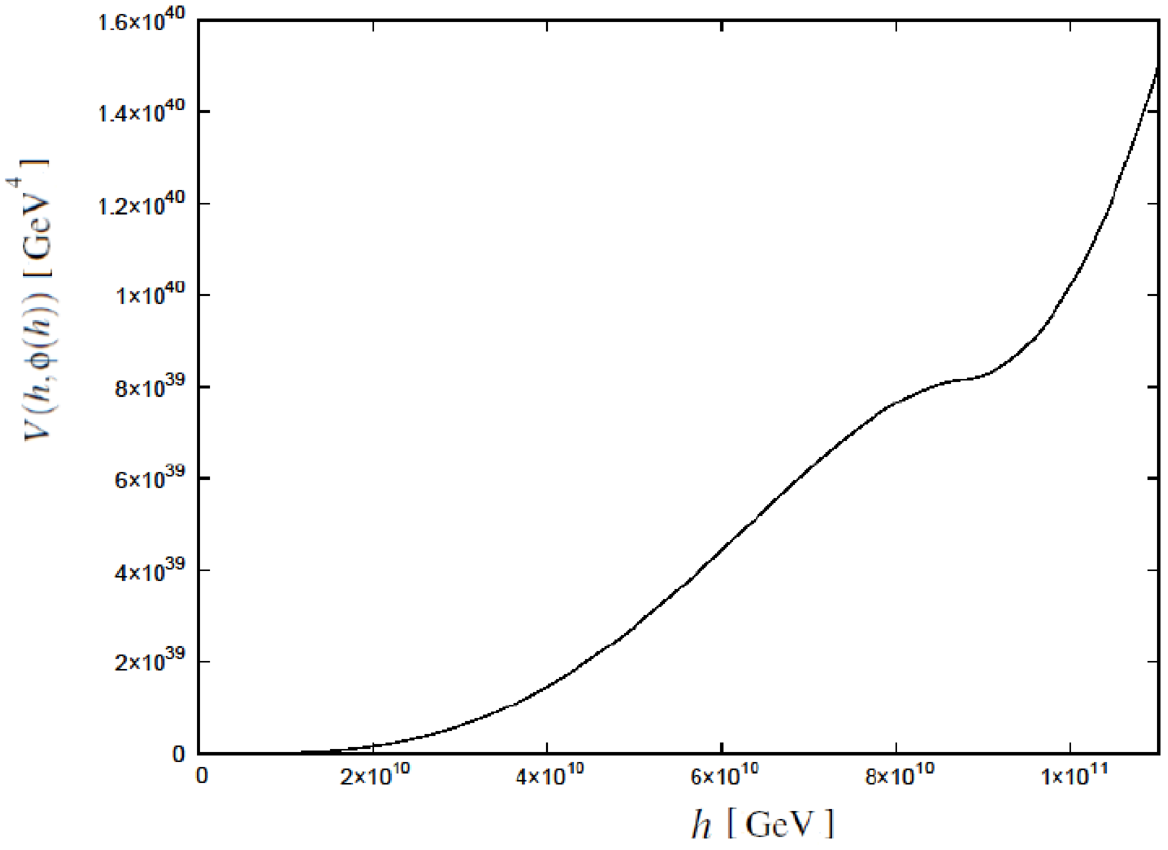
- Higgs inflation

- Non-minimally coupled PQ inflation

Other modifications to the potential; inflection points.

- Inflection point inflation (Iacobellis, arXiv: 1609.09228)

Applications to vacuum stability.



At higher values of f_a we can observe inflection point behavior in the effective potential (arXiv: 1807.00778 [hep-ph]).

Thank you for listening!

arXiv: 1807.00778 (full description of the model and what we found from this project).

Please feel free to ask me any questions!