Shedding Light on the Initial Conditions of Inflation with the Eisenhart Lift

arXiv:1812.07095

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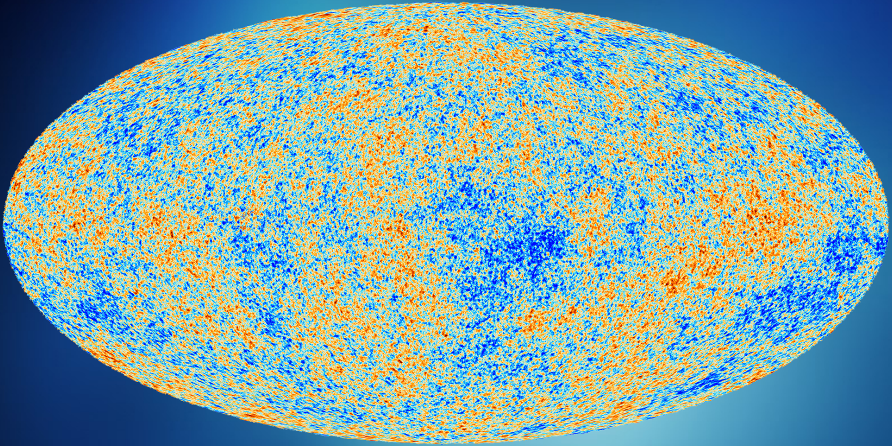
with Sotirios Karamitsos

The University of Manchester

YTF, Durham, December 19th, 2018



ESA and the Planck Collaboration

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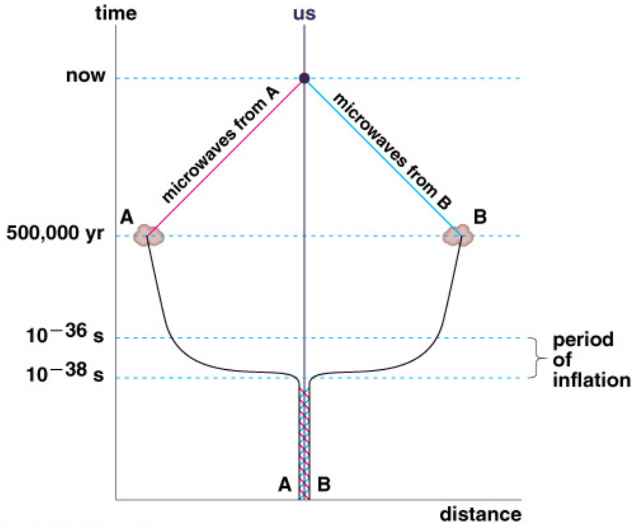
Inflation


time
us
now
microwaves from A
microwaves from B
A
B
500,000 yr
10^{-36} s
10^{-38} s
period of inflation
A
B
distance

Inflation

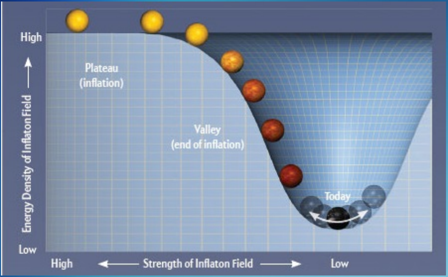
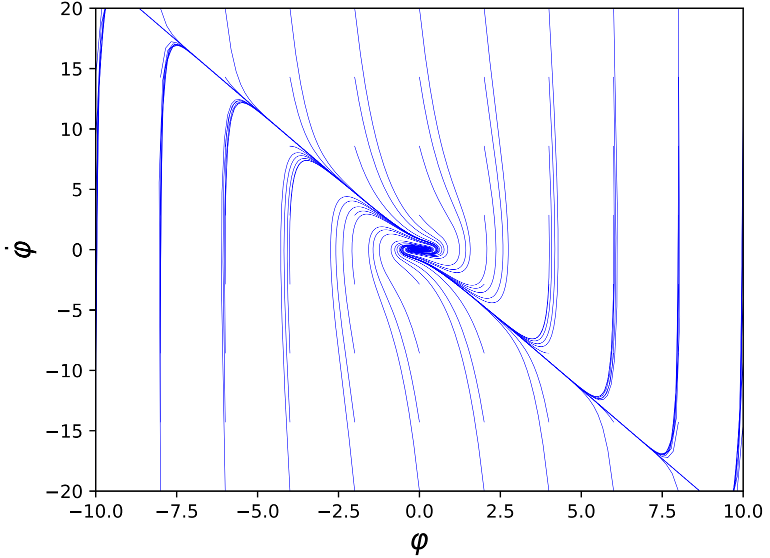


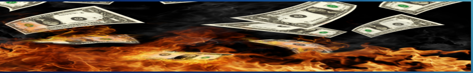
Image: Paul Steinhardt 2011

Requirements for Inflation

- Slowly rolling scalar field
- Potential with the right shape to match observations
- Graceful exit
- Large enough homogeneous patch
- Initial field values that allow sufficient inflation

The Inflationary Attractor





The Eisenhart Lift

L.P. Eisenhart: 1928

KF, S. Karamitsos and A. Pilaftsis: 2018 (arXiv:1806.02431)

Original Theory

- $\mathcal{L} = \frac{1}{2} k_{ij}(\varphi)\dot{\varphi}^i \dot{\varphi}^j - V(\varphi)$
- $\ddot{\varphi}^i + \Gamma^i_{jk}\dot{\varphi}^j \dot{\varphi}^k = -k^{ij} V_{,j}$
- $\Gamma^i_{jk} = \frac{1}{2} k^{il} \left(k_{jl,k} + k_{kl,j} - k_{jk,l}\right)$

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Lifted Theory

- $\mathcal{L} = \frac{1}{2} k_{ij}(\varphi)\dot{\varphi}^i\dot{\varphi}^j + \frac{1}{2}\frac{1}{V(\varphi)}\dot{\chi}^2$
- $\ddot{\varphi}^i + \Gamma^i_{jk}\dot{\varphi}^j\dot{\varphi}^k = -\frac{1}{2}\left(\frac{\dot{\chi}}{V(\varphi)}\right)^2 k^{ij} V_{,j}$
- $\frac{d}{dt}\left(\frac{\dot{\chi}}{V(\varphi)}\right) = 0 \quad \Rightarrow \quad \frac{\dot{\chi}}{V(\varphi)} = A$

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Lifted Theory

- $\mathcal{L} = \frac{1}{2} k_{ij}(\varphi)\dot{\varphi}^i \dot{\varphi}^j + \frac{1}{2}\frac{1}{V(\varphi)}\dot{\chi}^2$
- $\ddot{\varphi}^i + \Gamma^i_{jk}\dot{\varphi}^j \dot{\varphi}^k = -\frac{A^2}{2}\, k^{ij} V_{,j}$
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The Field Space Manifold

- $\mathcal{L} = \frac{1}{2} k_{ij}(\varphi)\dot{\varphi}^i\dot{\varphi}^j + \frac{1}{2}\frac{1}{V(\varphi)}\dot{\chi}^2$
- $\phi^A = \{\varphi^i, \chi\}$
- $\mathcal{L} = \frac{1}{2} G_{AB}\dot{\phi}^A\dot{\phi}^B$
- $G_{AB} = \begin{pmatrix} k_{ij}(\varphi) & 0 \\ 0 & \dfrac{1}{V(\varphi)} \end{pmatrix}$
- $\ddot{\phi}^A + \Gamma^A_{BC}\dot{\phi}^B\dot{\phi}^C = 0$
- $\Gamma^A_{BC} = \frac{1}{2} G^{AD}\left(G_{BD,C} + G_{CD,B} - G_{BC,D}\right)$

Application to Inflation

- $S = \int d^4x \sqrt{-g} \left(-\frac{1}{2} R + \frac{1}{2} (\partial^\mu \varphi)(\partial_\mu \varphi) - V(\varphi) \right)$
- $g_{\mu\nu} = \mathrm{diag}\left(1, -a^2, -a^2, -a^2\right)$
- $L = -3a\dot{a}^2 + \frac{1}{2} a^3 \dot{\varphi}^2 - a^3 V(\varphi)$

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- $L = -3a\dot{a}^2 + \frac{1}{2} a^3 \dot{\varphi}^2 + \frac{1}{2} \frac{1}{a^3 V(\varphi)} \dot{\chi}^2$
- $G_{AB} = \mathrm{diag}\left(-6\, a, a^3, \frac{1}{a^3 V(\varphi)} \right)$

The Phase Space Manifold

- Initial conditions live on phase space manifold = field space tangent bundle

Metric Wish List

1. Reduces to the field space metric when all $\dot{\phi}$s are zero
2. Leaves all tangent spaces flat
3. The line element is reparametrisation invariant

The Phase Space Manifold

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Metric Wish List

1. Reduces to the field space metric when all $\dot{\phi}$s are zero
2. Leaves all tangent spaces flat
3. The line element is reparametrisation invariant

- Sasaki metric (S. Sasaki: 1958)
- $ds^2 = G_{AB} d\phi^A d\phi^B + G_{AB} D\dot{\phi}^A D\dot{\phi}^B$
- $D\dot{\phi}^A = d\dot{\phi}^A + \Gamma^A_{BC} \dot{\phi}^B d\phi^C$

Invariant Volume Element

- $d\Omega = \det(G_{AB}) d^n\phi d^n\dot{\phi}$

Symmetries and Constraints

Hamiltonian Constraint

- Variation of the Action wrt lapse N_L
- $-3a\dot{a}^2 + \frac{1}{2}a^3\dot{\varphi}^2 + \frac{1}{2}\frac{1}{a^3 V(\varphi)}\dot{\chi}^2 = 0$

Symmetries

$$L = -3a\dot{a}^2 + \frac{1}{2}a^3\dot{\varphi}^2 + \frac{1}{2}\frac{1}{a^3 V(\varphi)}\dot{\chi}^2$$

- χ translation:
  - $\chi \to \chi + c$
- Spatial dilation:
  - $a \to ca, \quad \chi \to c^3\chi, \quad \dot{a} \to c\dot{a}, \quad \dot{\chi} \to c^3\dot{\chi}$
- Time dilation:
  - $\dot{a} \to c\dot{a}, \quad \dot{\varphi} \to c\dot{\varphi}, \quad \dot{\chi} \to c\dot{\chi}$

Manifold of Initial Conditions for Inflation

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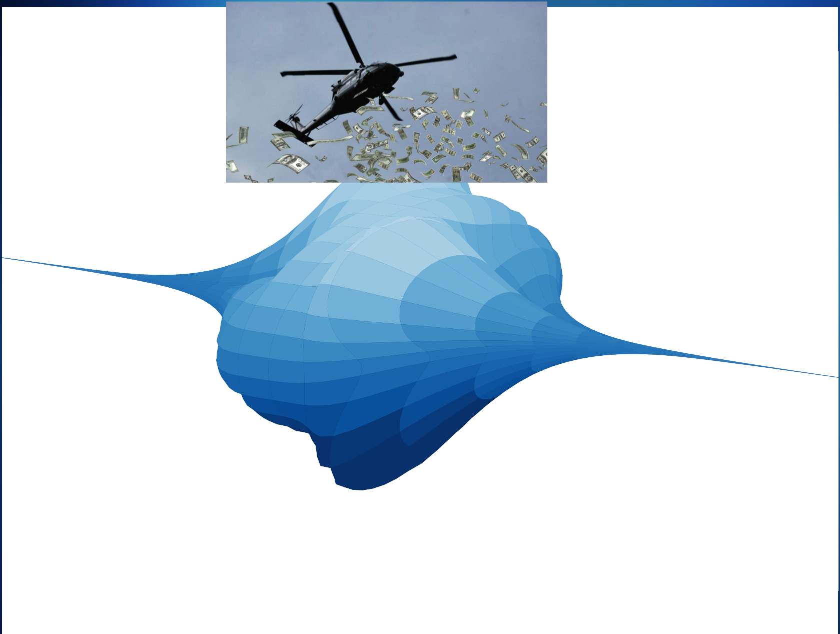
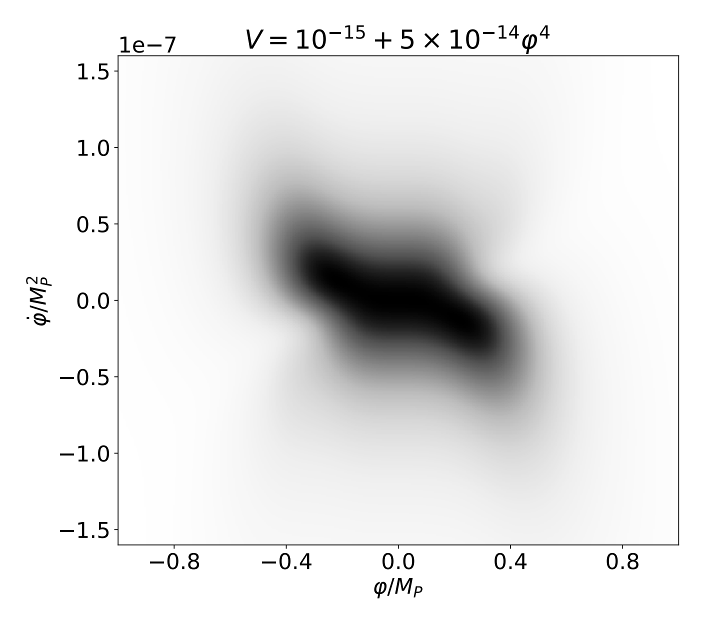


Distribution of the Inflaton Field



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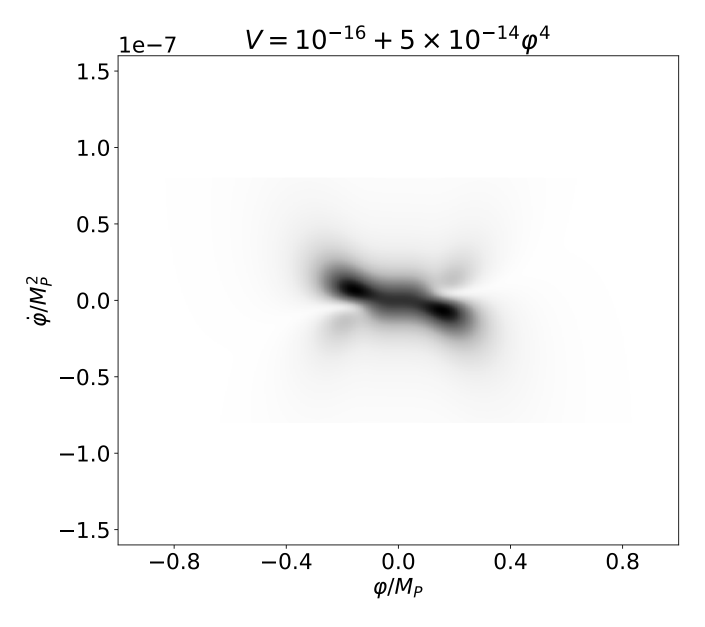
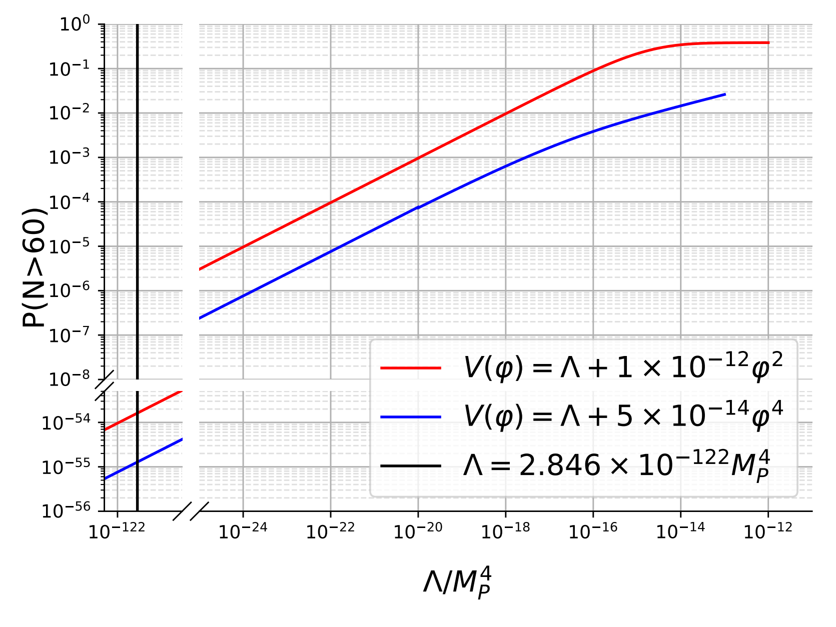


Fraction of Phase Space that allows Inflation



Summary

- The Eisenhart lift allows us to interpret inflation as geodesic motion on a field-space manifold
- We extend this to a phase space manifold with the Sasaki metric
- The total volume of the manifold is finite
- The fraction of phase space that leads to sufficient inflation is tiny