Gravity amplitudes, observables and classical scattering

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With David Kosower, Donal O'Connell and Justin Vines

Amplitudes and gravity

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Somewhere we're good at precision calculations: on-shell scattering amplitudes.

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General Relativity from Scattering Amplitudes

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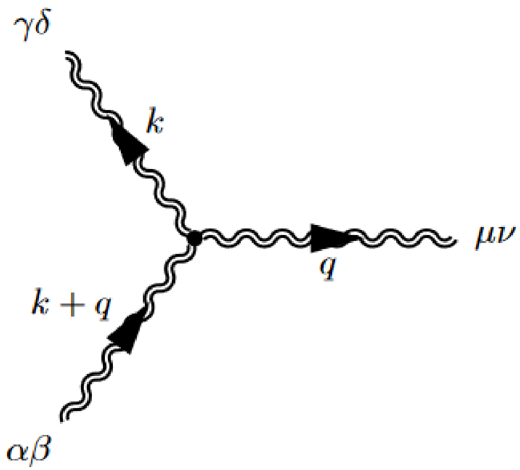
Weoutline the program to apply modern quantum field theory methods to calculate observables in classical general relativity through a truncation to classical terms of the multigraviton, two-body, on-shell scattering amplitudes between massive fields. Since only long-distance interactions corresponding to nonanalytic pieces need to be included, unitarity cuts provide substantial simplifications for both post-Newtonian and post-Minkowskian expansions. We illustrate this quantum field theoretic approach to

Gravity amplitudes = hard

Amplitudes in GR are supposed to be a nightmare:



$$
\begin{aligned}
= -\frac{i\kappa}{2}\Big\{ & P_{\alpha\beta,\gamma\delta}\Big[k^\mu k^\nu + (k+q)^\mu (k+q)^\nu + q^\mu q^\nu - \frac{3}{2}\eta^{\mu\nu}q^2\Big] \\
& +2q_\lambda q_\sigma\Big[I^{\lambda\sigma,}{}_{\alpha\beta}I^{\mu\nu,}{}_{\gamma\delta} + I^{\lambda\sigma,}{}_{\gamma\delta}I^{\mu\nu,}{}_{\alpha\beta} \\
& \qquad -I^{\lambda\mu,}{}_{\alpha\beta}I^{\sigma\nu,}{}_{\gamma\delta} - I^{\sigma\nu,}{}_{\alpha\beta}I^{\lambda\mu,}{}_{\gamma\delta}\Big] \\
& +\Big[q_\lambda q^\mu(\eta_{\alpha\beta}I^{\lambda\nu,}{}_{\gamma\delta}+\eta_{\gamma\delta}I^{\lambda\nu,}{}_{\alpha\beta}) + q_\lambda q^\nu(\eta_{\alpha\beta}I^{\lambda\mu,}{}_{\gamma\delta}+\eta_{\gamma\delta}I^{\lambda\mu,}{}_{\alpha\beta}) \\
& \quad -q^2(\eta_{\alpha\beta}I^{\mu\nu,}{}_{\gamma\delta}+\eta_{\gamma\delta}I^{\mu\nu,}{}_{\alpha\beta}) - \eta^{\mu\nu}q^\lambda q^\sigma(\eta_{\alpha\beta}I_{\gamma\delta,\lambda\sigma}+\eta_{\gamma\delta}I_{\alpha\beta,\lambda\sigma})\Big] \\
& +\Big[2q^\lambda\Big(I^{\sigma\nu,}{}_{\gamma\delta}I_{\alpha\beta,\lambda\sigma}k^\mu + I^{\sigma\mu,}{}_{\gamma\delta}I_{\alpha\beta,\lambda\sigma}k^\nu \\
& \qquad -I^{\sigma\nu,}{}_{\alpha\beta}I_{\gamma\delta,\lambda\sigma}(k+q)^\mu - I^{\sigma\mu,}{}_{\alpha\beta}I_{\gamma\delta,\lambda\sigma}(k+q)^\nu\Big) \\
& \quad +q^2(I^{\sigma\mu,}{}_{\alpha\beta}I_{\gamma\delta,\sigma}{}^{\nu} + I_{\alpha\beta,\sigma}{}^{\nu}I^{\sigma\mu,}{}_{\gamma\delta}) \\
& \quad +\eta^{\mu\nu}q^\lambda q_\sigma(I_{\alpha\beta,\lambda\rho}I^{\rho\sigma,}{}_{\gamma\delta} + I_{\gamma\delta,\lambda\rho}I^{\rho\sigma,}{}_{\alpha\beta})\Big] \\
& +\Big[(k^2+(k+q)^2)\Big(I^{\sigma\mu,}{}_{\alpha\beta}I_{\gamma\delta,\sigma}{}^{\nu} + I^{\sigma\nu,}{}_{\alpha\beta}I_{\gamma\delta,\sigma}{}^{\mu} - \frac{1}{2}\eta^{\mu\nu}P_{\alpha\beta,\gamma\delta}\Big) \\
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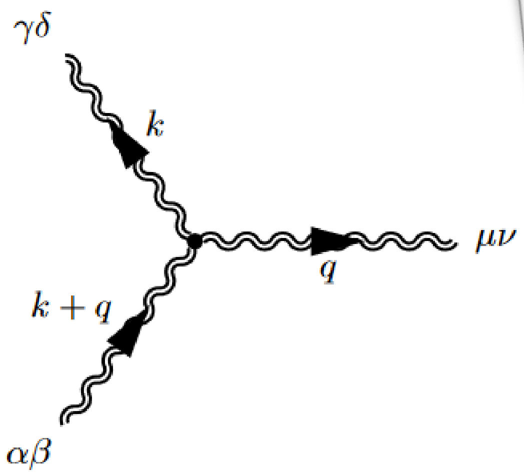

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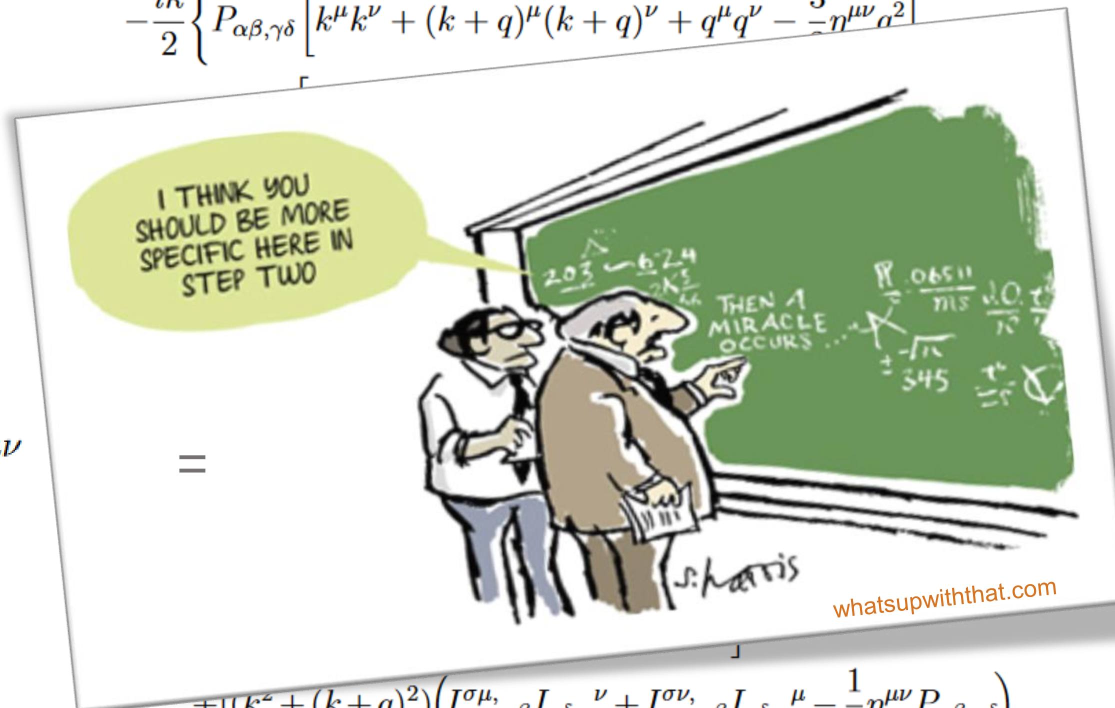

$$+\Big[(k^2+(k+q)^2)\Big(I^{\sigma\mu,}{}_{\alpha\beta}I_{\gamma\delta,\sigma}{}^{\nu} + I^{\sigma\nu,}{}_{\alpha\beta}I_{\gamma\delta,\sigma}{}^{\mu} - \frac{1}{2}\eta^{\mu\nu}P_{\alpha\beta,\gamma\delta}\Big)$$

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However they hold a surprising simplicity…

Holstein & Ross, 2008

Detour to a colourful duality

On-shell gravity amplitudes appear as "squares" of gauge theory ones:

$$\mathcal{A}_3(1^{-1}2^{-1}3^{+1}) = g\frac{\langle 1\,2\rangle^3}{\langle 1\,3\rangle\langle 2\,3\rangle}f^{abc} \qquad \mathcal{M}_3(1^{-1}2^{-1}3^{+1}) = \frac{\kappa}{2}\left(\frac{\langle 1\,2\rangle^3}{\langle 1\,3\rangle\langle 2\,3\rangle}\right)^2$$

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$$\mathcal{A}_m = g^{m-2}\sum_{i\in\text{cubic}}\frac{n_i c_i}{D_i}$$

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A powerful algebraic relation: **colour-kinematics duality**

$$c_\alpha \pm c_\beta \pm c_\gamma = 0 \Rightarrow n_\alpha \pm n_\beta \pm n_\gamma = 0$$

Bern, Carrasco & Johansson, 2008 + 10

Proven at tree level but still a conjecture at loop

Gravity amplitudes = not so bad!

If colour-kinematics duality holds we're free to replace colour with kinematics, giving

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A smörgåsbord of applications:

- Proof N=8 supergravity UV finite at 5-loops in 4d
- Constructing exact spacetimes in general relativity
- Calculating gravitational emission from black hole interactions
- Obtaining the 3PM gravitational potential

Bern, Carrasco, Chen, Edison, Johansson, Roiban, Parra-Martinez & Zeng, 2018

Monteiro, O'Connell & White, 2016

Luna, Monteiro, Nicholson & O'Connell, 2018

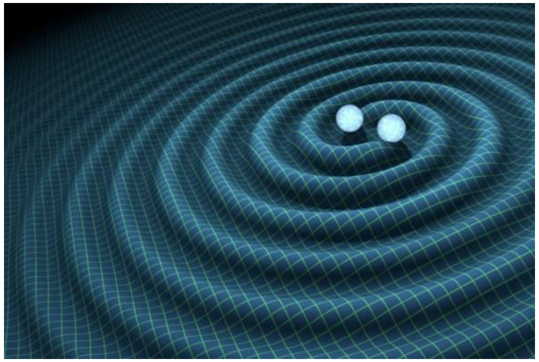
Goldberger & Ridgway, 2017
Luna, Nicholson, O'Connell & White, 2018
Shen, 2018

Bern, Cheung, Roiban, Shen, Solon, Zen & 2018

Gravity observables

Key gravitational wave observable, the phase, requires calculating:

- The **conservative potential** *V(x)*
- **Radiative flux** *F(v)*



Science in the News, Harvard

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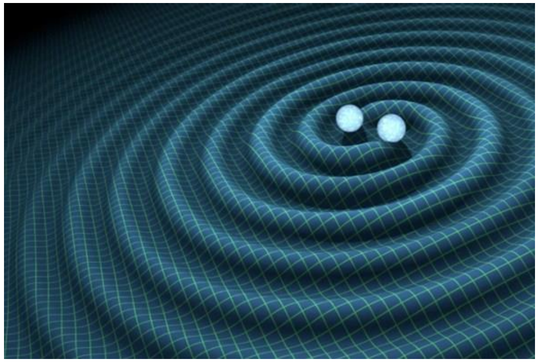

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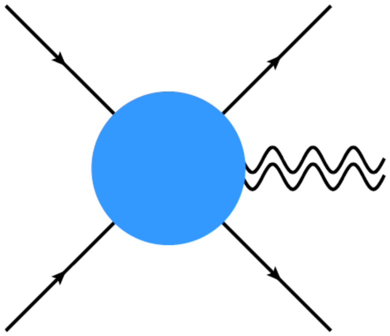
On-shell successes demand on-shell observables

- **Impulse** Δp
- Total **radiated momentum** *R*



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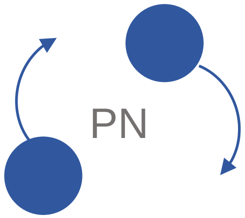

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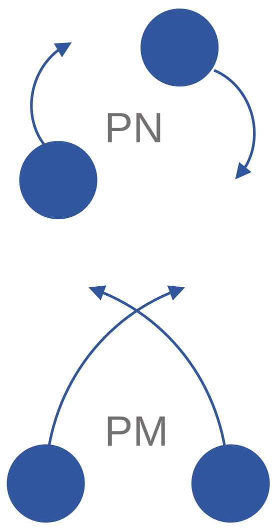

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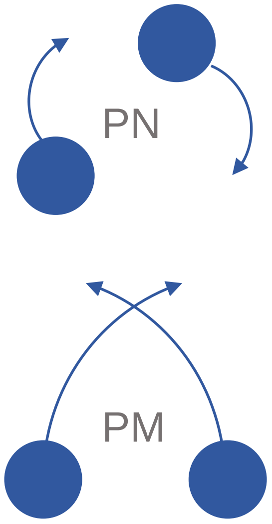

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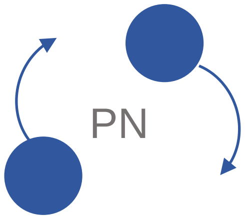

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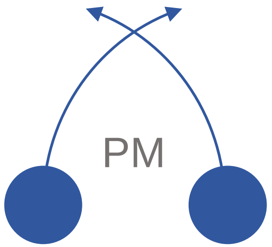

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Can treat black holes as point particles, so perfect for amplitudes methods!

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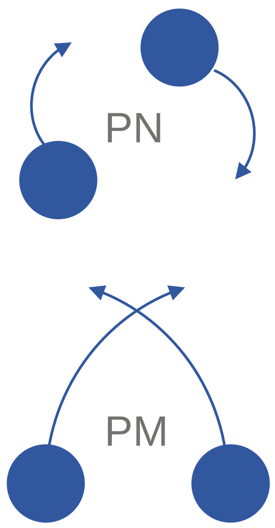

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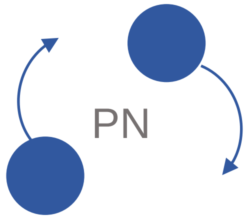

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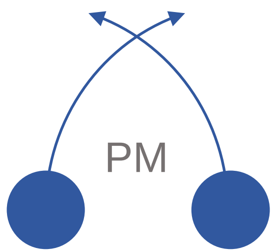

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For corresponding QFT calculations must use in-state wavepackets:

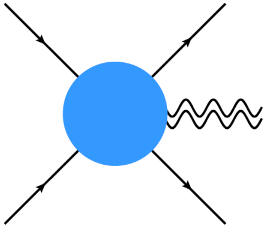
$$|\psi\rangle_{\text{in}} = \int d\Phi(p_1) d\Phi(p_2)\, \phi(p_1)\phi(p_2) e^{ib\cdot p_1/\hbar} |p_1\, p_2\rangle_{\text{in}}$$

The impulse

Kosower, **BM** and O'Connell,
arXiv:1811.10950

The S-matrix is a time evolution operator; use to take difference between far future and far past momenta:

$$\langle p_1^\mu \rangle_{\text{in}} = \langle \psi | \hat{P}_1^\mu | \psi \rangle \qquad \langle p_1^\mu \rangle_{\text{fin}} = \langle \psi | \hat{S}^\dagger \hat{P}_1^\mu \hat{S} | \psi \rangle$$



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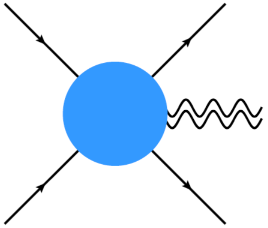

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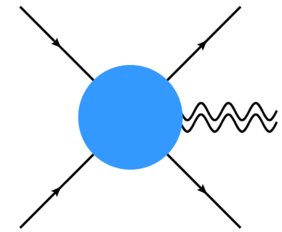

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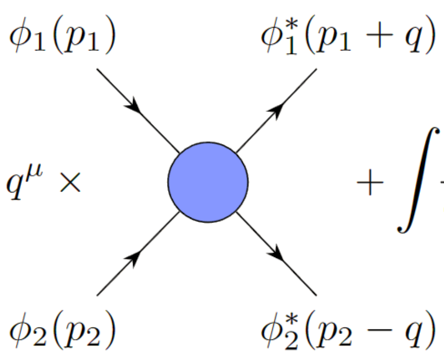
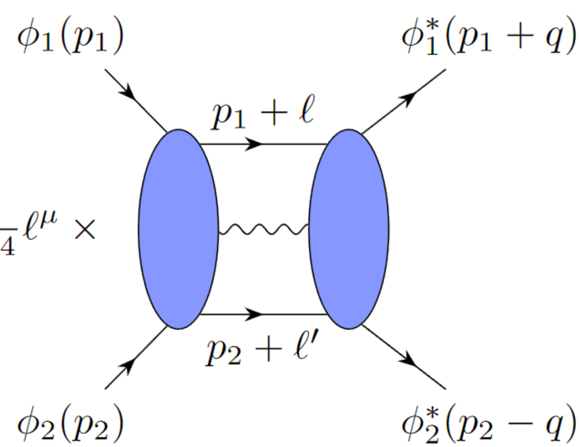
All orders definition:

$$\langle \Delta p_1^\mu \rangle = \int_{\text{on-shell}} e^{-ib\cdot q} \Bigg(q^\mu \times$$



$$+ \int \frac{d^4\ell}{(2\pi)^4} \ell^\mu \times$$



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The classical limit

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Point particle description
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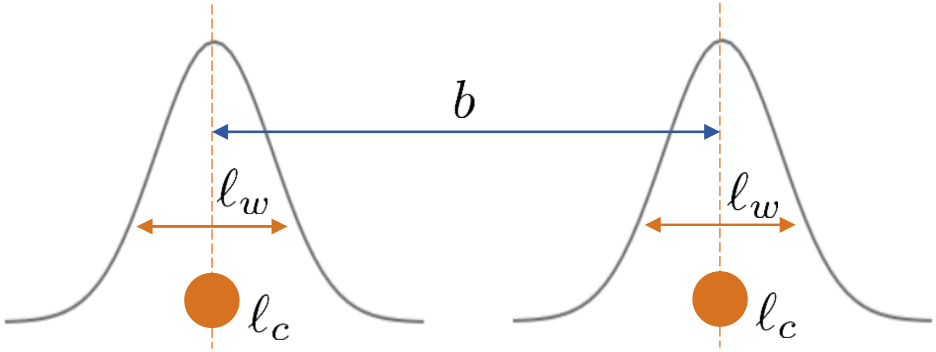

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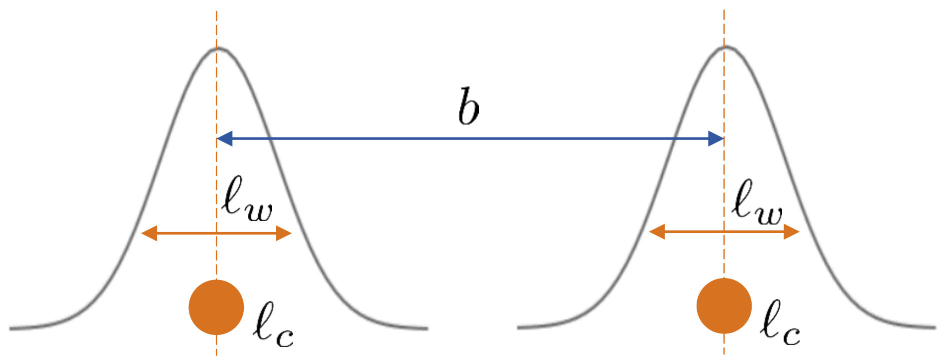

There are more $\hbar$'s:

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Stationary phase approximation intuitively suggests

$$q \equiv \hbar \bar{q}$$

The classical limit

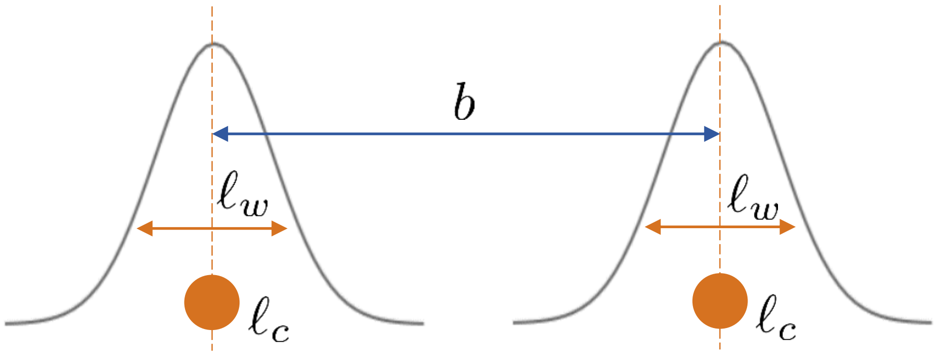
Kosower, **BM** and O'Connell,
arXiv:1811.10950

$\hbar \neq 1$!!!

Need to reinstate $\hbar$ via dimensional analysis.
In amplitude just modifies coupling constants.

Point particle description requires "Goldilocks" zone

$$\ell_c \ll \ell_w \ll b$$



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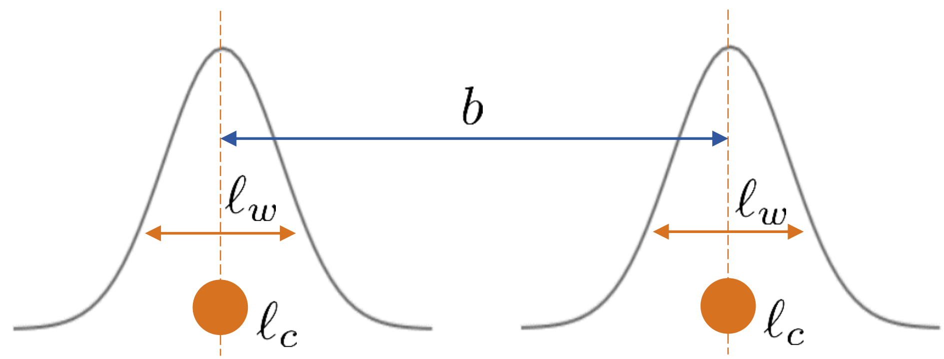

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Wavenumber

For careful derivations see paper. Demonstrates that the <u>$\hbar$ expansion is not always the loop expansion</u>, as more sources of $\hbar$ in this observable!

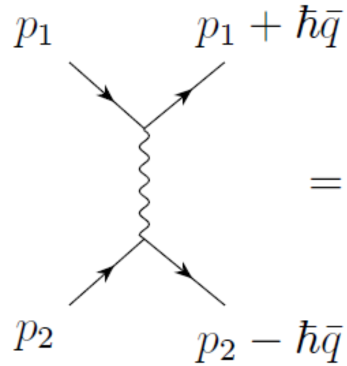
Donoghue & Holstein, 2004
Holstein & Ross, 2008
Bjerrum-Bohr *et al*, 2018

Examples

Kosower, **BM** and O'Connell,
arXiv:1811.10950

Tree level QED:

$$i\mathcal{A}^{(0)}(p_1 p_2 \to p_1 + \hbar\bar{q}\,, p_2 - \hbar\bar{q}) =$$



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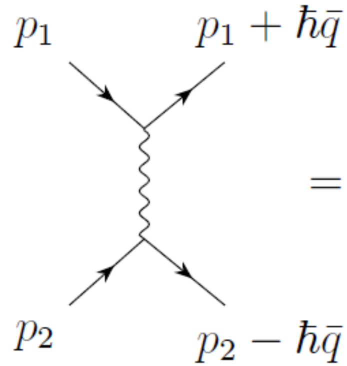

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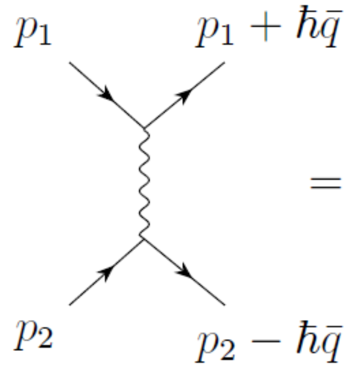

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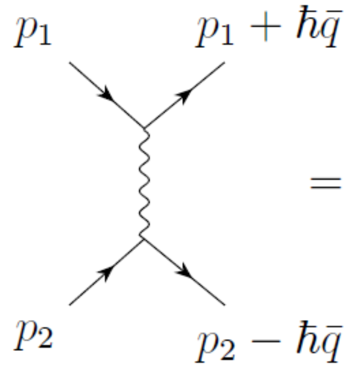

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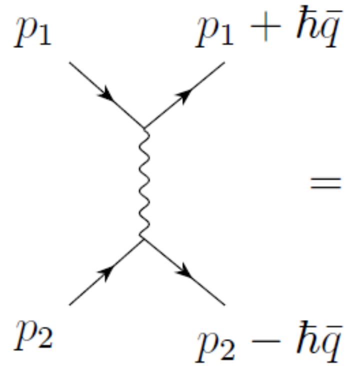

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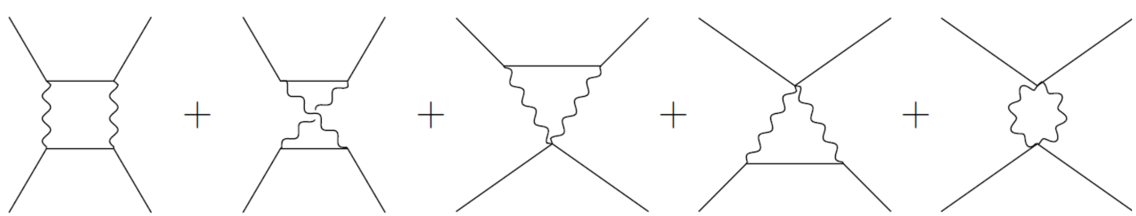
Obtain **identical** result by taking classical point-particle action and finding leading order solution to Lorentz force! At next iterative order amplitude is

$$iA^{(1)}(p_1 p_2 \to p_1' p_2') =$$



Radiated momenta

Kosower, **BM** and O'Connell,
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Can apply same ideas to radiation. Assuming no radiation in initial states,

$$R^\mu = \langle\psi|\hat{T}^\dagger \hat{K}^\mu \hat{T}|\psi\rangle = \sum_X \int d\Phi(k) d\Phi(r_1) d\Phi(r_2)\, k_X^\mu \,|\langle k\, r_1\, r_2\, X|\hat{T}|\psi\rangle|^2$$

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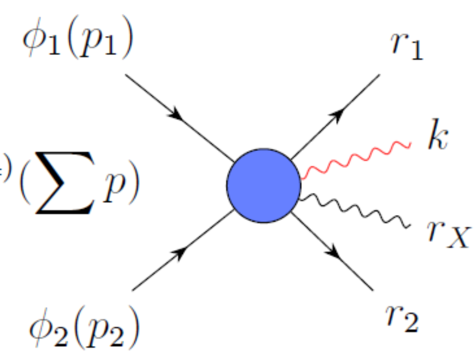

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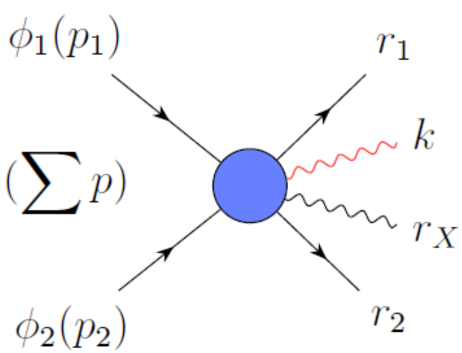

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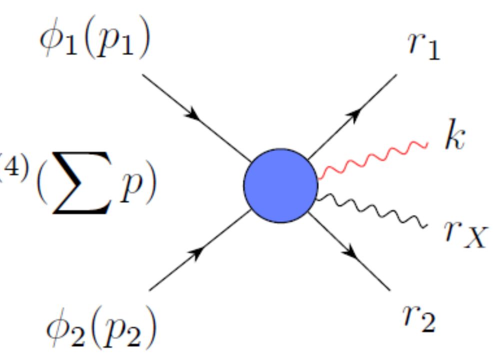

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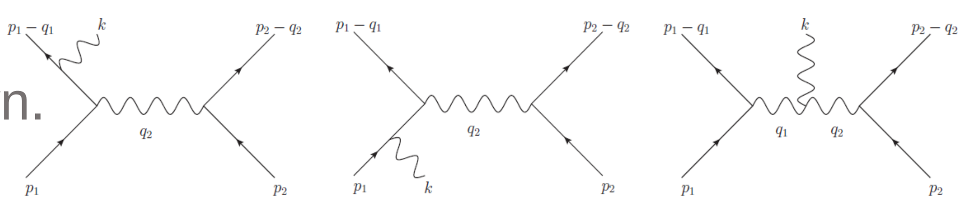
Immediately applicable to gravity!
1PM Schwarzschild amplitude known.

Luna, Nicholson, O'Connell & White, 2018



Conclusions

Kosower, **BM** and O'Connell,
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- Amplitudes methods emerging for calculations in perturbative GR.
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Problems to tackle

- How do we include spin? Can we define a similar on-shell observable?
- To what extent does the Kerr multipole expansion correspond to quantum amplitudes?
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