Semiclassical gravity in the far field of stars and black holes

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December 18, 2018

Overview

Introduction: Semiclassical gravity

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- $\langle T_{\mu\nu} \rangle$ is non-local. It requires information about global boundary conditions on the fields.
- $\langle T_{\mu\nu} \rangle$ is constructed out of products of matter field **operators**. The expectation value of such quantities is, formally speaking, infinite!

Scalar field in a spherically symmetric spacetime

- Toy model: Massless scalar field ϕ, spherically symmetric, asymptotically flat spacetime. After a Wick rotation $\tau = -it$ this has the Euclidean metric

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- Rather than $\langle T_{\mu\nu} \rangle$ we can consider the **vacuum polarization** of the scalar field, $\left\langle \phi^2 \right\rangle = \langle \phi(x)\phi(x) \rangle$.
- This is calculated from the Green's function of the field,

$$\left\langle \phi^2 \right\rangle = \lim_{x' \to x} \left\langle \phi(x)\phi(x') \right\rangle = \lim_{x' \to x} G_E\left(x, x'\right)$$

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- The Green's function G_E is the solution to the Klein-Gordon equation of motion for the field, with appropriate global boundary conditions.

$$\left[g^{\mu\nu}\nabla_\mu\nabla_\nu - \xi\mathcal{R}(x)\right] G_E\left(x, x'\right) = -\frac{\delta^4(x, x')}{\sqrt{g}}$$

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- The counterterms do not depend on the global boundary conditions.

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- Consider a quantum field in Minkowski spacetime with and without two parallel mirrors.
- The difference in the energy density of the vacuum state can be calculated

$$\Delta \left\langle T^t_t \right\rangle = \left\langle T^t_t \right\rangle_{mirror} - \left\langle T^t_t \right\rangle_{Mink} \sim \frac{\hbar c}{d^4}$$

where d is the distance between the mirrors.

Taking differences

- In general, the renormalised vacuum polarisation takes the form

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- These two scenarios could be two different quantum states, or the **same quantum state on two different spacetimes.**

Differences between spacetimes: Previous work

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$$\Delta \langle \phi^2 \rangle = \langle \phi^2 \rangle_{star}^{ren} - \langle \phi^2 \rangle_{Schw}^{ren}$$

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- The $p(r)$ and $q(r)$ are radial solutions with Dirichlet boundary conditions at the inner and outer boundaries respectively.
- C is a normalisation constant defined using p and q.

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$$\Delta \left\langle \phi^2 \right\rangle = \frac{1}{4\pi^2} \int_0^\infty d\omega \sum_{\ell=0}^{\infty} (2\ell + 1) C_{Schw} \frac{\beta}{\alpha} q_{Schw}^2$$

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- This sum and integral rapidly converge, so it is justified to look at the small ω behaviour of the $\ell = 0$ mode.

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- $\Delta \left\langle \phi^2 \right\rangle$ can then be integrated by parts to get a series expansion in r^{-1}.

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- For $\frac{M}{R} \ll 1$ at leading order, where R is the radius of the star, Fabbri and Anderson showed **universality**: $\Delta \left\langle \phi^2 \right\rangle = 0$ and $\Delta \left\langle T_{\mu\nu} \right\rangle = 0$ when $\xi = 0$ (minimal) and $\Delta \left\langle T_{\mu\nu} \right\rangle = 0$ when $\xi = \frac{1}{6}$ (conformal).

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- Question 1: Does this hold for a star model in which $\frac{M}{R}$ is not small?
- Question 2: Does this result hold beyond the leading order in r^{-1} far from the gravitational source?

Uniform density star

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- Finding q_{Schw} and p_{Schw} on the surface of the star requires the method of matched asymptotic expansions (for an analytical approximation), or numerical methods.

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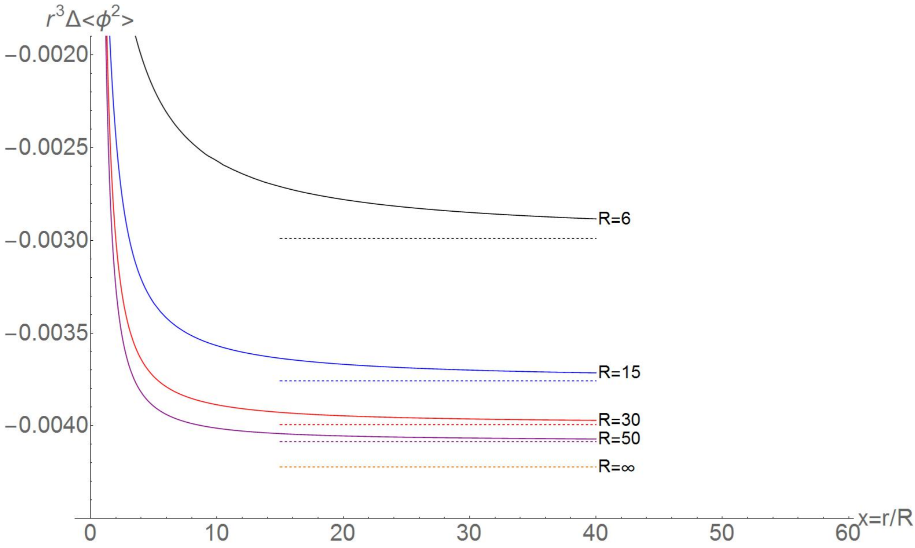
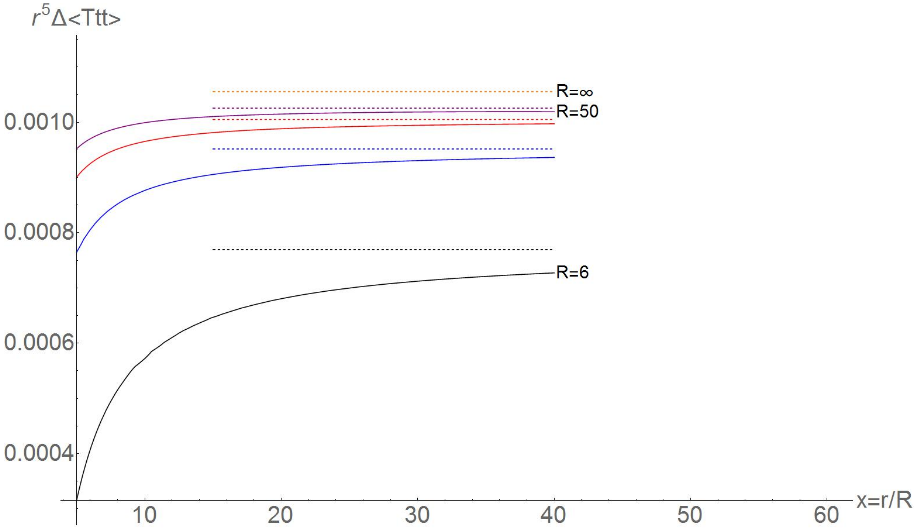
$$\Delta\left\langle T^{\mu}_{\nu}\right\rangle = -\frac{3M\xi(\xi-\frac{1}{6})}{4\pi^2 r^5}\mathcal{S}_0(\tfrac{M}{R})\mathrm{diag}\left[2,-2,3,3\right] + \mathcal{O}(r^{-6})$$

$\Delta \langle \phi^2 \rangle$, $\xi = \frac{1}{6}$



$\Delta \langle T_t^t \rangle$, $\xi = \frac{1}{12}$



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- The equations up to Y_1 can be solved for $\ell = 0$, giving us p_{Schw} and q_{Schw} to sub-leading order in ω.

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- In general, universality does not hold beyond the leading order in r^{-1}.

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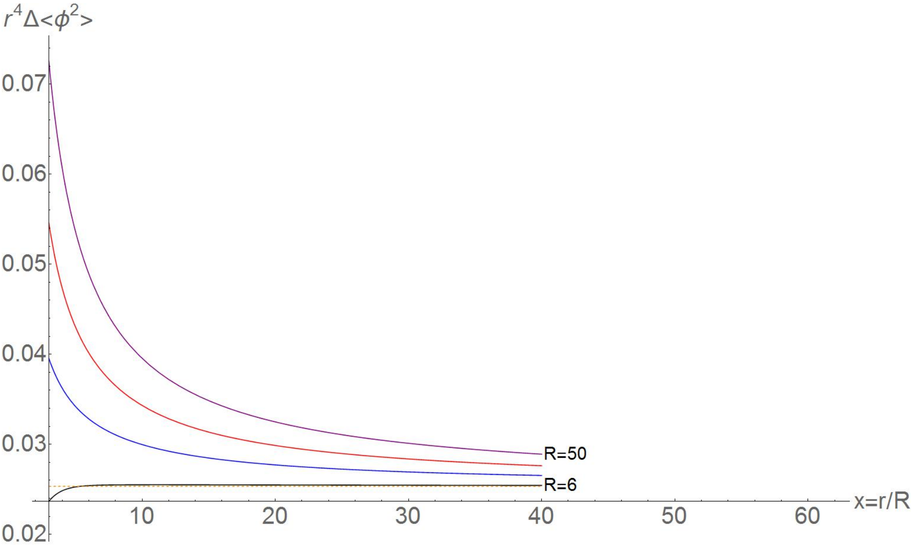

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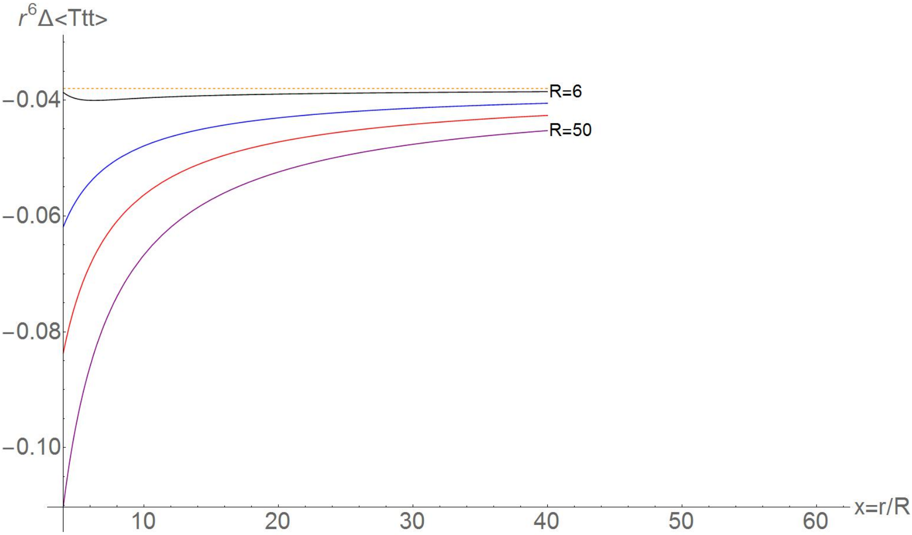

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- Universality for $\xi = 0$ does not hold beyond leading order in r^{-1}. Work for $\xi \neq 0$ is ongoing.
- Generalisations to larger spin fields and different structures of gravitational source are in principal possible, although Kerr spacetime isn't obvious due to the loss of Birkhoff's theorem.

Thanks for listening!