Quantum gravity and the dilaton portal

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Outline

- Plenty of reasons to want to quantize gravity; early universe cosmology, black hole singularities, dark energy etc
- We know gravity is difficult to quantize
- From the renormalization group (RG) perspective the coupling $\kappa \propto G^{\frac{1}{2}}$ is irrelevant, i.e. $[\kappa] = -1$
- As a result no naive UV complete theory of quantum gravity (QG) was ever possible, despite tricks at low loop level calculations

- This is particularly evident when looking for interactions of the graviton:

$$\mathcal{L}_{int} \supset H^n \partial H \partial H$$

- These form irrelevant operators of dimension n + 4

- In a minimal theory then the trick is to look for interacting operators of the graviton which are relevant (even though I just said they will all be irrelevant)

Outline

- The trick to getting around this impasse is the decomposition of the graviton into it's diagonal and off-diagonal parts:

$$g_{\mu\nu} = \delta_{\mu\nu} + \kappa H_{\mu\nu}$$

$$H_{\mu\nu} = h_{\mu\nu} + \tfrac{1}{2}\delta_{\mu\nu}\phi$$

$$\mathcal{L}_{EH}^{kin} = \tfrac{1}{2}(\partial_\lambda h_{\mu\nu})^2 - \tfrac{1}{2}(\partial_\lambda \phi)^2$$

- where ϕ is our 'dilaton', a non-dynamical field contributing to the background

- Typically this is brushed under the rug since it's non-dynamical and the minus sign is removed by setting $\phi \to i\phi$
- We choose not to do this and instead as part of our definition of quantization demand that the operators are square integrable under a weight $e^{+a^2\widetilde{\phi}^2}$
- This restricts the operators that are allowed within our hilbert space, Σ^-

Outline

- Typically the Sturm-Liouville weight has a negative sign for a positive kinetic sign, leading to a space of function square integrable under $e^{-a^2\widetilde{\phi}^2}$:

$$\int_{-\infty}^{\infty} d\widetilde{\phi} e^{-a^2\widetilde{\phi}^2} \mathcal{O}_n(\widetilde{\phi})\mathcal{O}_m(\widetilde{\phi}) \propto \delta_{nm}$$

- The situation is subtly but drastically different for a negative kinetic sign, leading to a change in the sign of the weight;

$$\int_{-\infty}^{\infty} d\widetilde{\phi} e^{+a^2\widetilde{\phi}^2} \mathcal{O}_n(\widetilde{\phi})\mathcal{O}_m(\widetilde{\phi}) \propto \delta_{nm}$$

- As a consequence of this change in sign we are now restricted to having our 'tower operator'

$$\delta^{(n)}_{\Lambda_0}(\phi) \equiv \frac{\partial^n}{\partial\phi^n}\delta^{(0)}_{\Lambda_0}$$

$$\text{with } \delta^{(0)}_{\Lambda_0} = \frac{1}{\sqrt{2\pi\Omega_{\Lambda_0}}} e^{-\frac{\phi^2}{2\Omega_{\Lambda_0}}} \text{ with } \Omega_{\Lambda_0} \propto \hbar\Lambda_0^2$$

- This effervescent tower operator has dimension

$$[\delta^{(n)}_{\Lambda_0}(\phi)] = -1 - n$$

- Hence it is non-perturbative in $\hbar$

- With this operator we can now account for the large dimension of our interacting graviton operators
- The combination of our tower operator with the graviton interaction operator produces a 'top term' which will be relevant, of the form $\delta_\Lambda^{(n)}(\phi)\sigma(h, \partial, \partial\phi ...)$
- From this we find we now have an infinite number of relevant operators for the interacting graviton

- Interestingly given we are non-perturbative in $\hbar$ there is no classical limit of this theory
- By it's self a dilaton theory would have issues of unitarity however when we return to Minkowski space it is non-dynamical and so this problem resolves itself
- Now to make this all about gravity we need to talk about diffeomorphism invariance...

Outline

- Let's talk about diffeomorphism invariance, regularization and how we can simultaneously apply both to this theory
- We find that such a theory requires a quantum version of diffeomorphism invariance rather than a classical one, we implement this using anti-field or Batalin-Vilkovisky formalism
- In essence we are looking for an action that is closed but not exact under a BRST symmetry such that in the limit $\kappa \to 0$ linearised diffeomorphism invariance is returned

- BRST symmetry is a global symmetry which formally introduces Fadeev-Popov ghosts rather than the original ad hoc method

- It is characterised by the action of a BRST operator on each field which is nilpotent, i.e.

$$\delta\psi = Q\psi \ , \ Q^2\psi = 0$$

- We will be interested in actions which are closed but not exact as those that are exact will amount to reparametrisations of fields and don't provide new physics

- Gauge invariance is implemented at the quantum level by demanding a BRST invariance, we introduce our 'anti-fields' as source terms for these BRST transformations:

$$\delta\Phi^A = \epsilon Q\Phi^A$$

$$S = S[\Phi] - (Q\Phi^A)\Phi^*_A$$

- where ϵ is Grassmannian and the anti-fields have opposite statistics to their corresponding fields

- We now introduce the Quantum Master Equation (QME), an action satisfying this equation will incorporate the gauge symmetry corresponding to a given BRST symmetry:

$$\frac{1}{2}(S,S) - \Delta S = 0$$

with

$$(X,Y) = \frac{\partial_r X}{\partial \Phi^A}\frac{\partial_l Y}{\partial \Phi^*_A} - \frac{\partial_r X}{\partial \Phi^*_A}\frac{\partial_l Y}{\partial \Phi^A}\ ,\ \Delta X = (-)^A \frac{\partial_l}{\partial \Phi^A}\frac{\partial_l Y}{\partial \Phi^*_A} X$$

- Combining this QME with the regularisation demanded by our work on the tower operator leads us to the introduction of a cutoff profile $C^{\Lambda}(p) = C(\frac{p^2}{\Lambda^2})$ between bilnear terms, in particular

$$S_0 = \tfrac{1}{2}\Phi^A(\Delta^{\Lambda})^{-1}_{AB}\Phi^B - (Q_0\Phi^A)(C^{\Lambda})^{-1}\Phi^*_A$$

and

$$(X,Y) = \frac{\partial_r X}{\partial \Phi^A} C^{\Lambda} \frac{\partial_l Y}{\partial \Phi^*_A} - \frac{\partial_r X}{\partial \Phi^*_A} C^{\Lambda} \frac{\partial_l Y}{\partial \Phi^A}\ ,\ \Delta X = (-)^A \frac{\partial_l}{\partial \Phi^A} C^{\Lambda} \frac{\partial_l Y}{\partial \Phi^*_A} X$$

- This combines the two concepts whilst leaving the all-important BRST invariance unaffected

Outline

- We now have the tools to quantize this theory perturbatively in κ whilst still satisfying the QME and so introducing diffeomorphism invariance in a well-defined, regularized way

- We have a series of gradings which will also further restrict what actions are permitted, e.g. the anti-ghost number

- From this we find the natural extension to the top term discussed above, one that now also includes associated BRST fields such as the ghost, anti-ghosts, auxiliary fields etc

$$\delta_{\Lambda}^{(n)}(\phi)\sigma(\partial, \partial\phi, h, \bar{c}, c, b, \Phi^{*}) + ...$$

- Continuing the extension of the earlier we demand these are square integrable under the weight

$$\mu = exp(\frac{1}{2\Omega_{\Lambda}}[\phi^{2} - h_{\mu\nu}^{2} - 2\bar{c}_{\mu}c_{\mu}])$$

Outline

- To conclude we may have been working on a minimal route to quantizing gravity
- We resolve issues of irrelevancy using our tower operator and have begun gauging the theory under the anti-field formalism
- The next step is to produce the 4th order expansion of EH and use that plus the 3rd order expansion under the action of BRST charges to investigate the structure of the theory
- In addition to this we will also be able to begin investigating the running of couplings and the nature of marginal operators

Thanks for listening, any questions?