Search for new physics through lattice simulations

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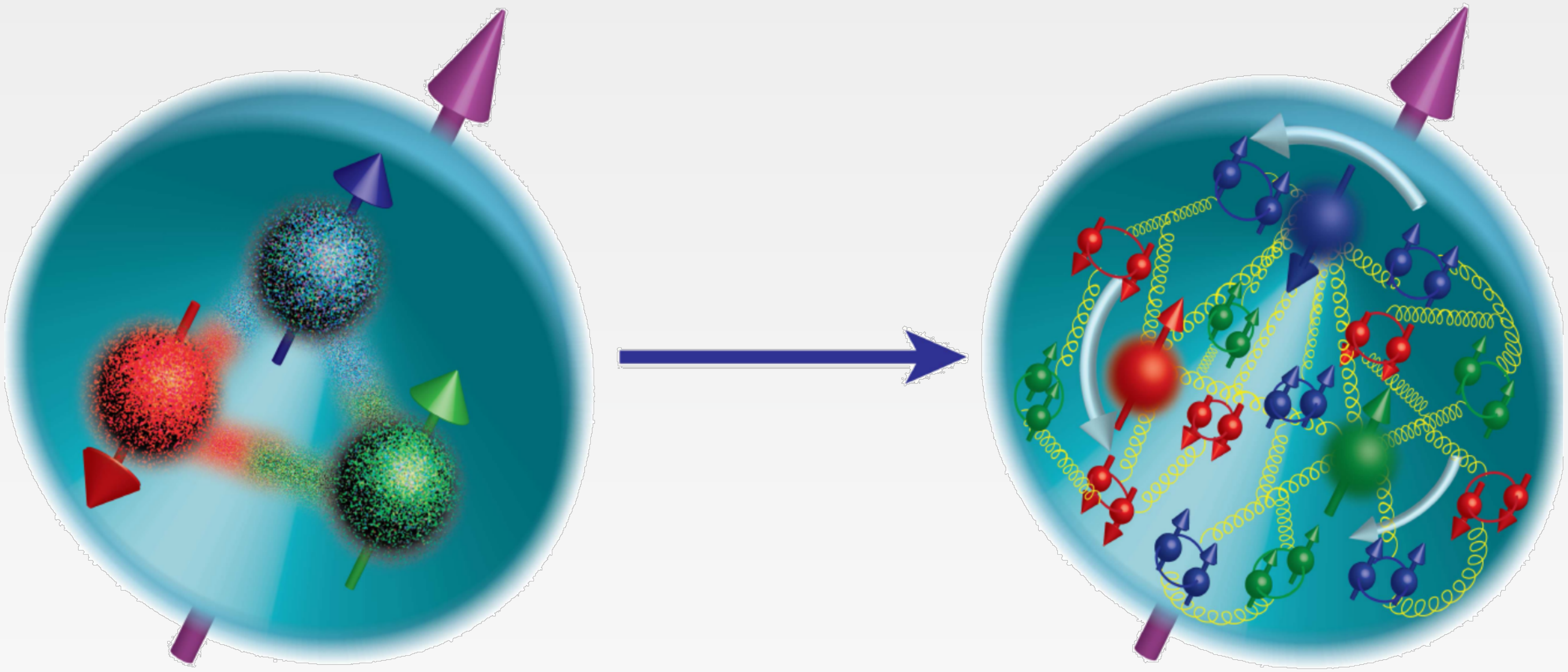
20th of February 2019

Higgs Maxwell Meeting 2019

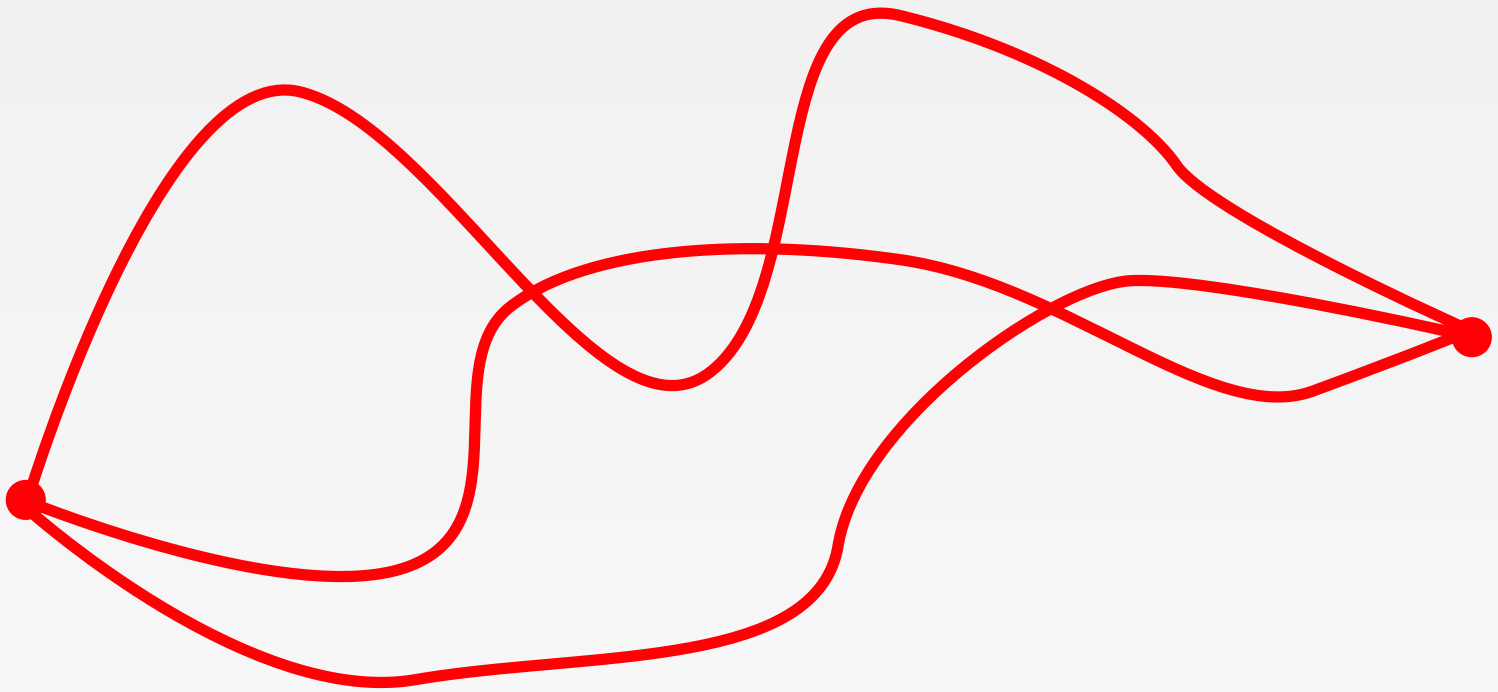
THE UNIVERSITY of EDINBURGH

- Non-perturbative quantum field theory
- The Standard Model at low energies
- Holographic cosmological models
- Outlook & perspectives



Non-perturbative quantum field theory

The path integral



$$\langle O \rangle = \frac{1}{\mathcal{Z}} \int \mathrm{D}\phi \, O[\phi] \exp(iS[\phi])$$

Perturbation theory

- Solvable exactly: Gaussian path integrals
  Free theories

- Perturbation theory:
  power expansion around the Gaussian point

- Only valid if **coupling constant small**
  and if **amplitude admits a power expansion**

Euclidean path integrals

$$\langle O \rangle = \frac{1}{\mathcal{Z}} \int \mathrm{D}\phi\, O[\phi] \exp(iS_M[\phi])$$

Imaginary time $\Big\downarrow$ $\tau = it$

$$\langle O \rangle = \frac{1}{\mathcal{Z}} \int \mathrm{D}\phi\, O[\phi] \underline{\exp(-S_E[\phi])}$$

Probability density
(at zero density)

Key idea: numerical Monte-Carlo estimation

Lattice field theory

- Numerical: discrete space-time
- Replace derivatives by finite differences
- Scalar fields: easy
- Gauge fields: ok, careful with Gauge invariance
- Fermion fields: really hard because of chiral symmetry (Nielsen-Ninomya theorem)

Sketch of a lattice calculation

Can potentially predict many things…

…assuming that:

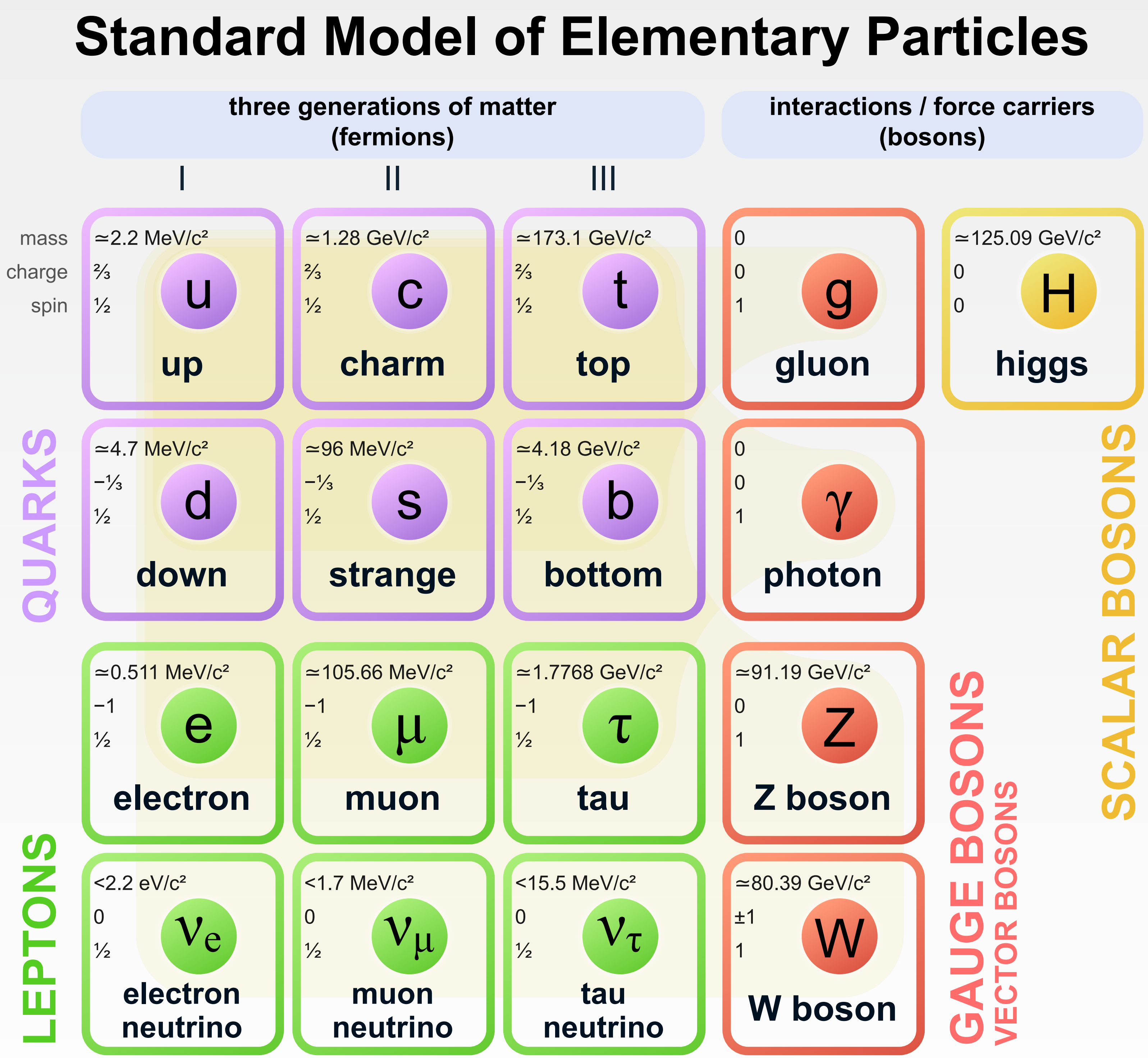
1. We can obtain the physical result from an Euclidean correlation functions.

2. Current (super)computers can do the calculation for small spacings and large volumes.

3. 1. can be applied to numerical data.

The Standard Model at low energies

The Standard Model



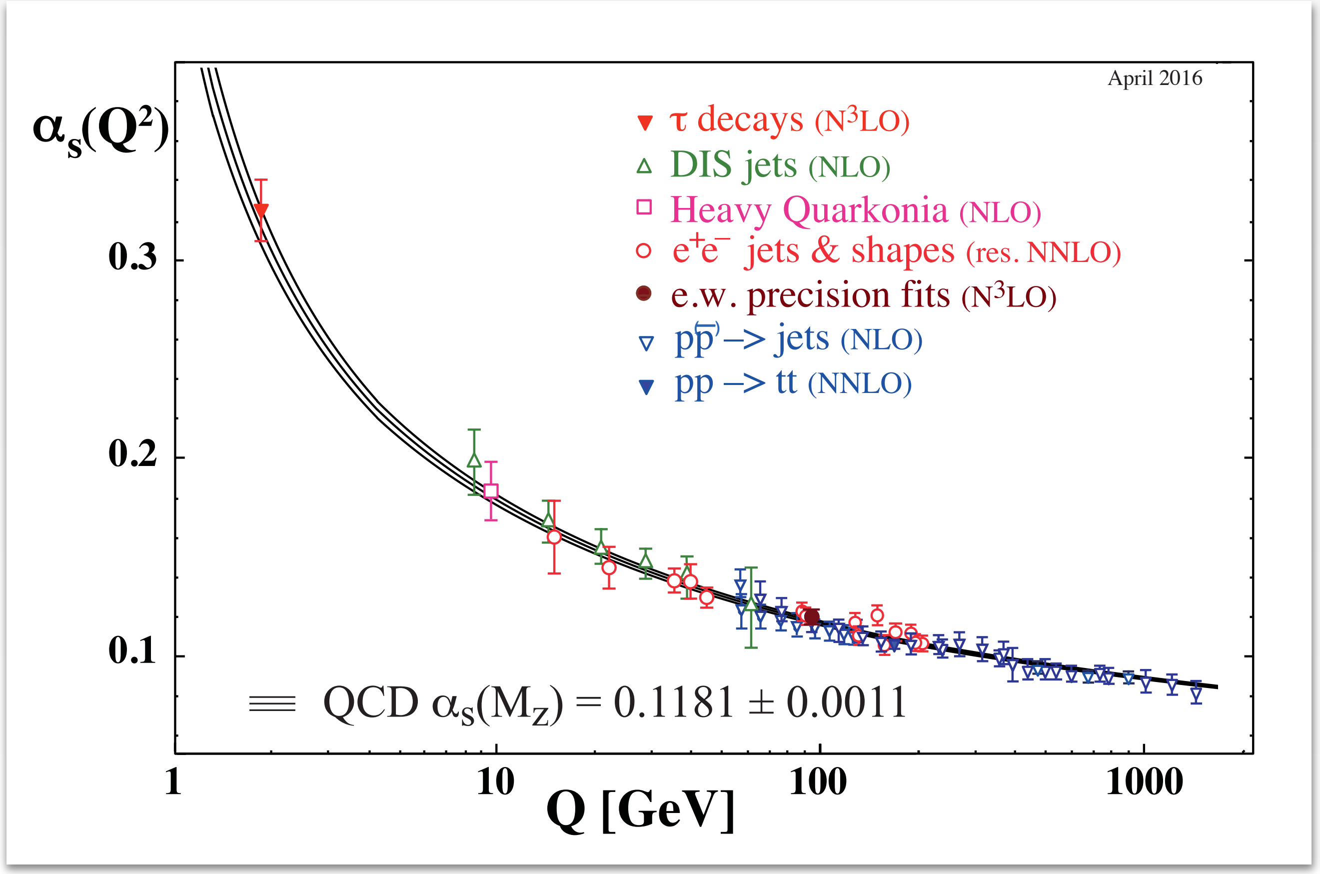
Low-energy limit of the SM

- Much below the EW scale ($\ll 100\ \mathrm{GeV}$)

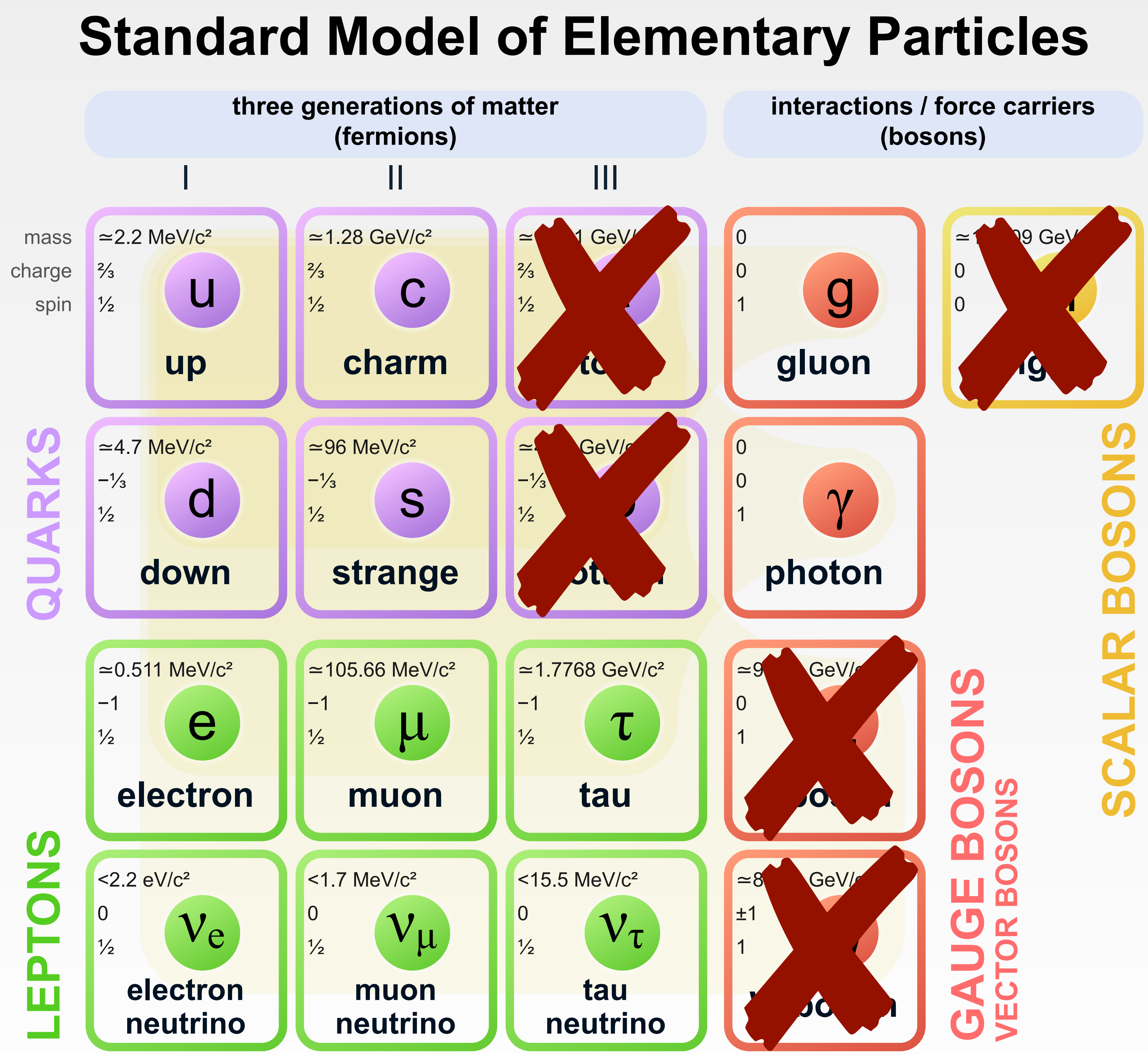
$$\mathrm{SU}(3)_c \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y \to \mathrm{SU}(3)_c \times \mathrm{U}(1)_Q$$

QCD and QED with massive fermions
+ 4-fermions effective weak interactions

- **Non-perturbative QCD**



Low-energy limit of the SM



Lattice QCD + QED

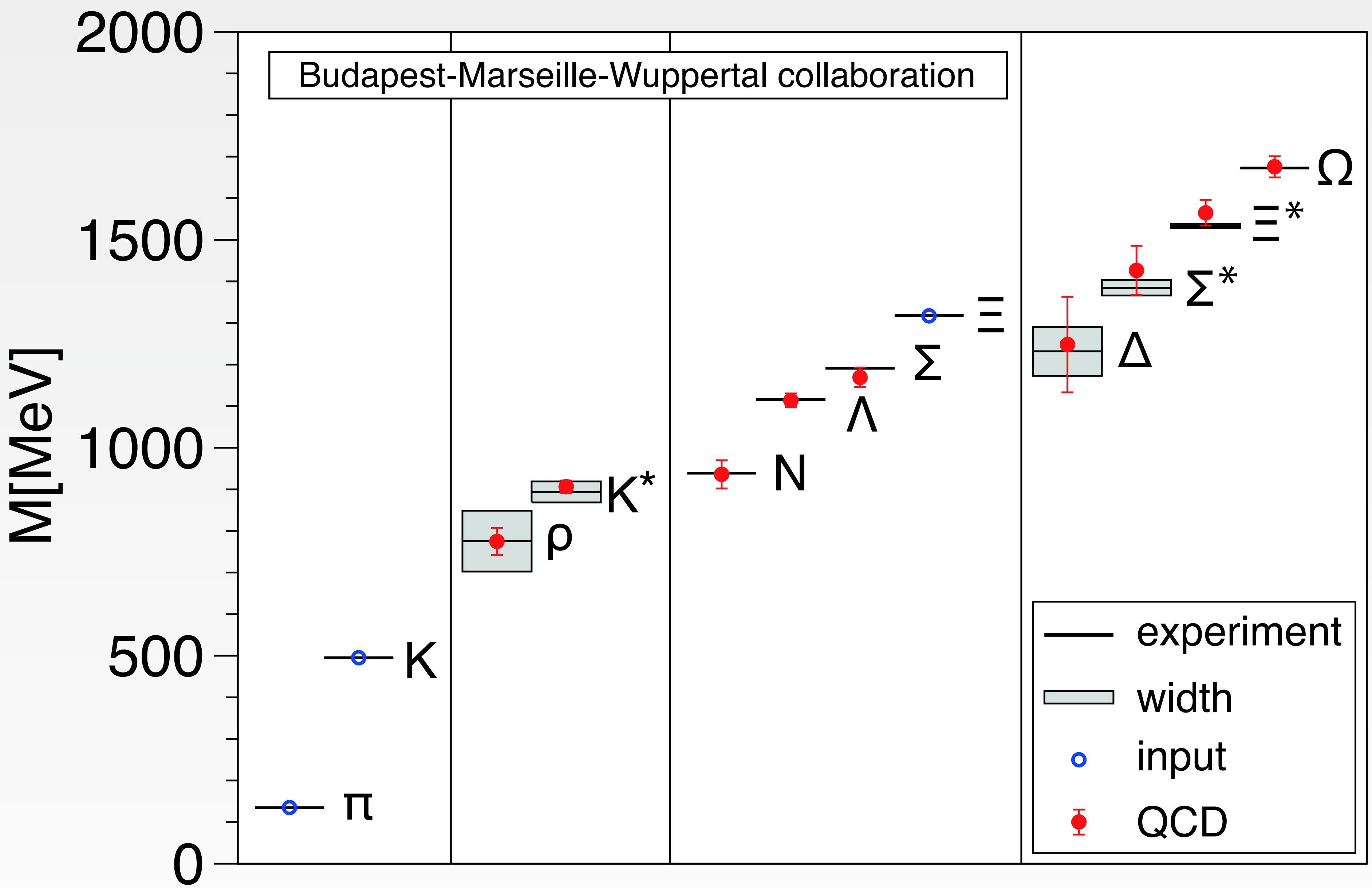
- Lattice SU(3) Yang-Mills implemented through **compact elementary gauge links**.
  [Wilson, PRD 10(8), 2445, 1974]

- Lattice EM implemented through a naive discretisation of the **Maxwell action (non-compact)**.

- **Singular photon zero-modes** need to be regularised. Popular: removal of spatial zero-modes (QED_L).

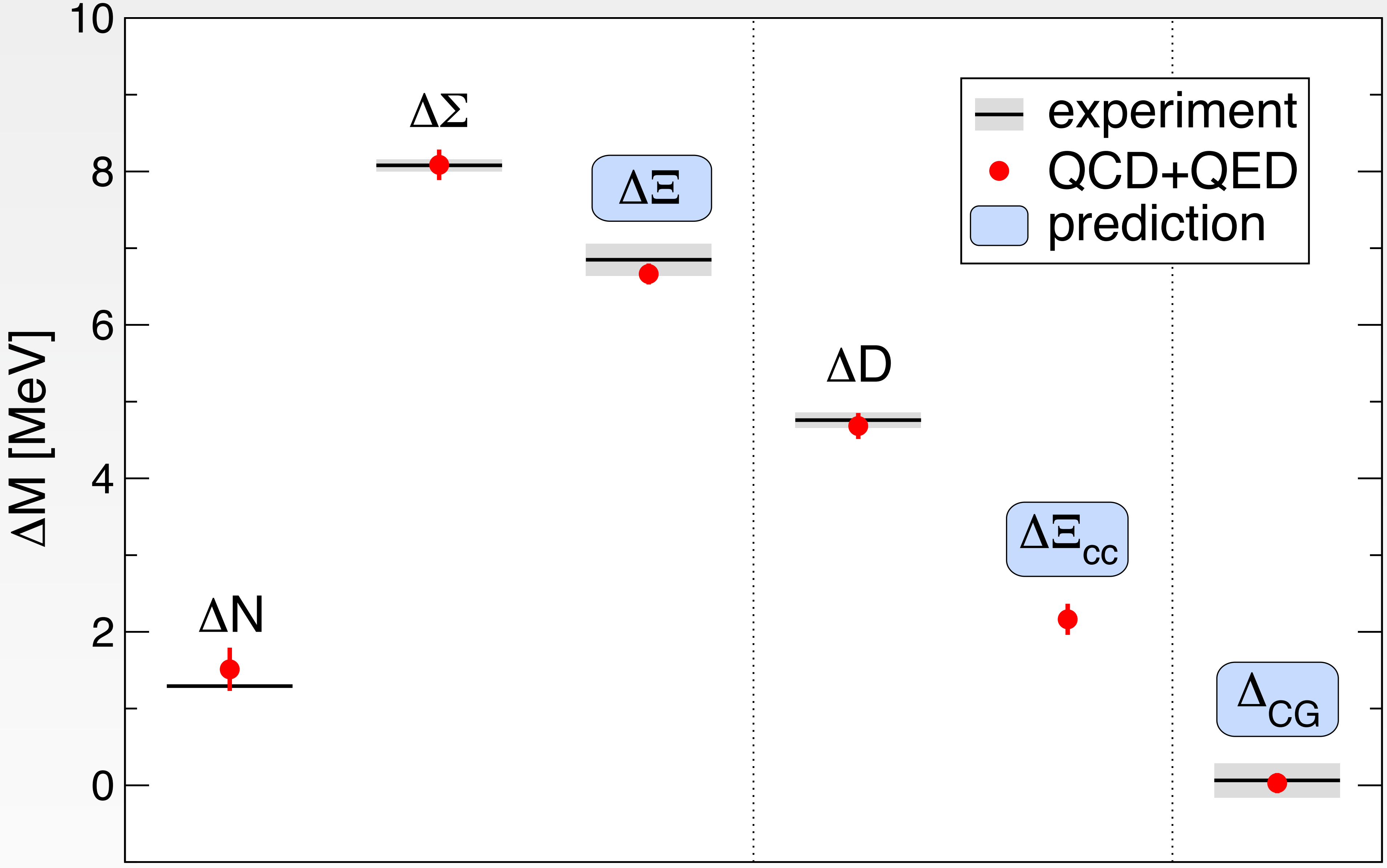
- QED_L: non-local in space, large finite-size effects.
  [Hayakawa & Uno, PTP 120(3), 431 (2008)] [Davoudi, A.P. et al., PRD (2019)]
  [S. Borsanyi, A.P. et al. (BMWc), Science 347, 1452–1455 (2015)]

The hadron spectrum



S. Dürr et al. (BMWc), Science, vol. 322, pp. 1224–1227, (2008)

The hadron spectrum



S. Borsanyi, A.P. et al. (BMWc), Science, vol. 347, pp. 1452–1455, (2015)

The muon g-2

- Deviation from the classical Landé factor

$$\boldsymbol{\mu} = g\frac{e}{2m}\mathbf{S}$$

- In QFT: given by the QED vertex function

$$\Gamma^{\mu}\left(k^{2}\right) = \gamma^{\mu}F_{1}\left(k^{2}\right) + \frac{i\sigma^{\mu\nu}k_{\nu}}{2m}F_{2}\left(k^{2}\right)$$

$$a = g - 2 = F_{2}(0)$$

- **Currently 3.5 sigmas discrepancy!**

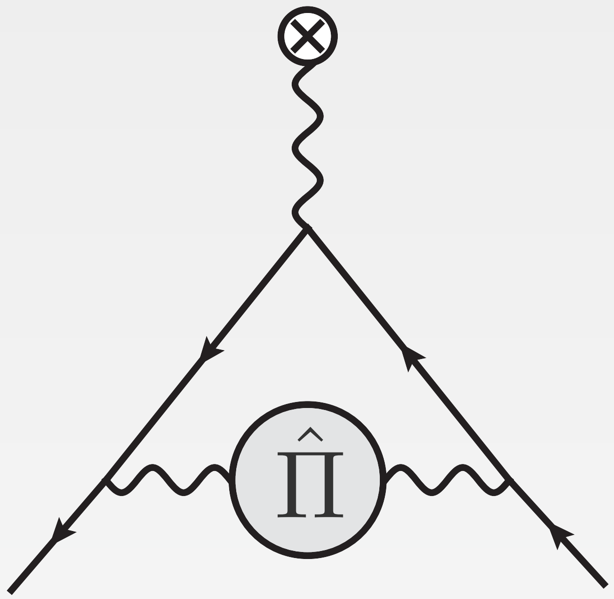
$$a_{\mu,\mathrm{exp.}} = 116592091(54)(33)\cdot 10^{-11}$$

$$a_{\mu,\mathrm{SM}} = 116591823(1)(34)(26)\cdot 10^{-11}$$

The muon g-2

- Fermilab experiment reducing exp. error by 4.
- **We should aim at something similar for theory.**

HVP contribution to the muon g-2



$$= 4\alpha^2 \int_0^{+\infty} \mathrm{d}q^2\, f(q^2, m_\mu^2)\hat{\Pi}(q^2)$$

Loop integral

Vacuum polarisation form factor

- **Purely virtual**: clean Euclidean formulation
- Main challenge comes from precision
- 4x error reduction ~ integral with 0.1% precision

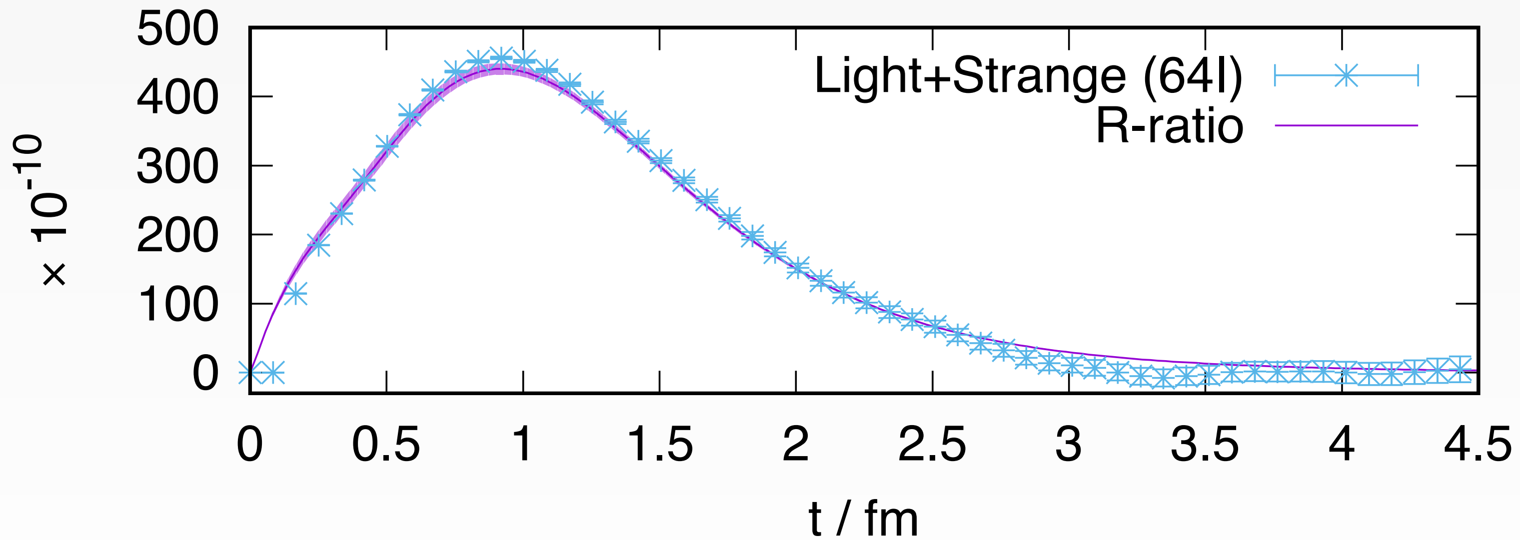
HVP contribution to the muon g-2

- **Dispersive approach**

$$\hat{\Pi}(q^2) = \frac{q^2}{48\pi^3}\int_0^{+\infty} \mathrm{d}s \, \frac{\sigma(e^+e^- \to \mathrm{had.})}{\alpha(s)^2(s+q^2)}$$

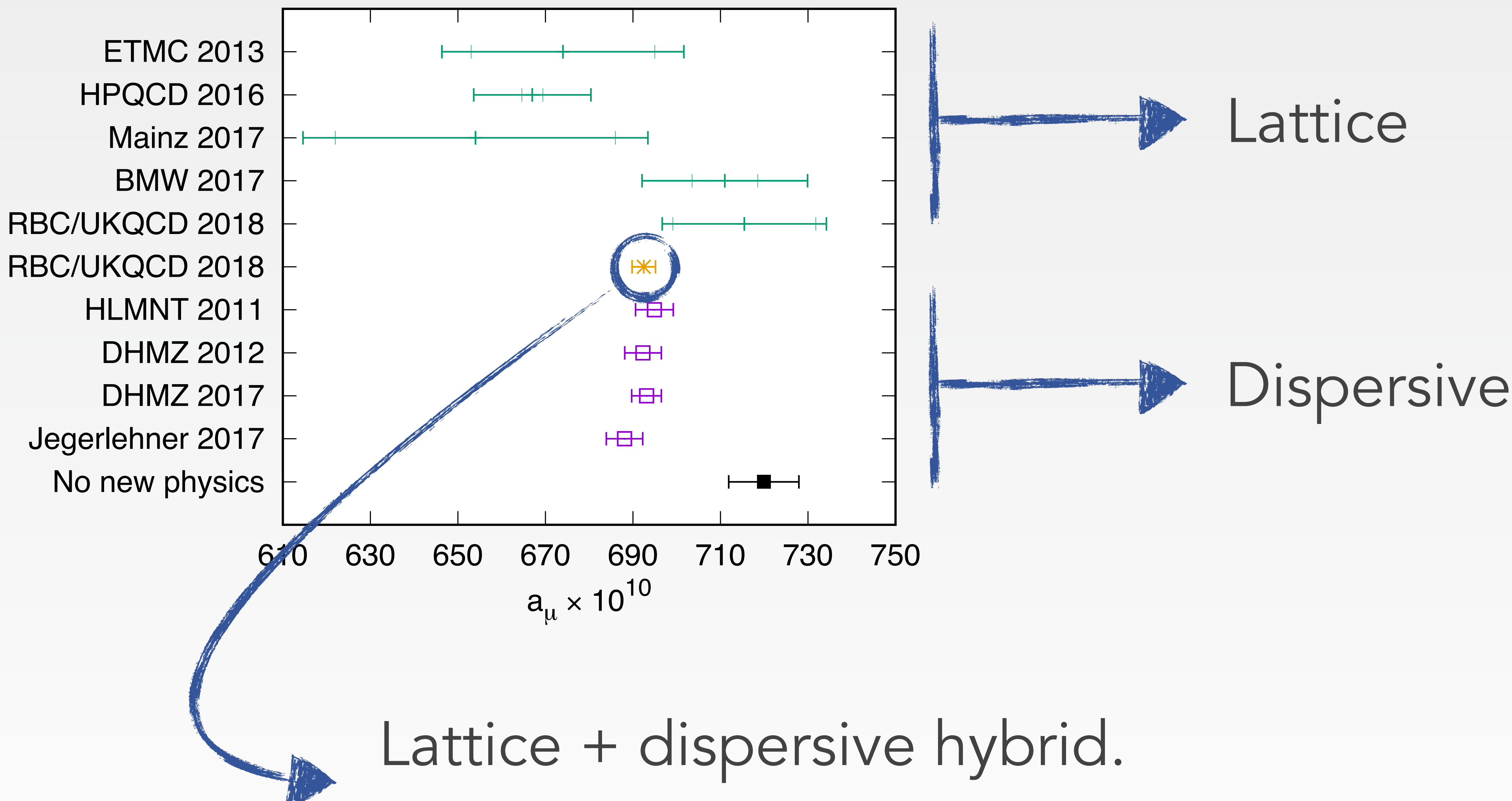
- **Lattice approach**

$$\int \mathrm{d}^4x \, \langle J_\mu(x) J_\nu(0)\rangle_{\mathrm{R}} \, e^{-iq\cdot x} = (q_\mu q_\nu - \delta_{\mu\nu} q^2)\hat{\Pi}(q^2)$$



[RBC-UKQCD, PRL 121(2), 022003 (2018)]

HVP contribution to the muon g-2

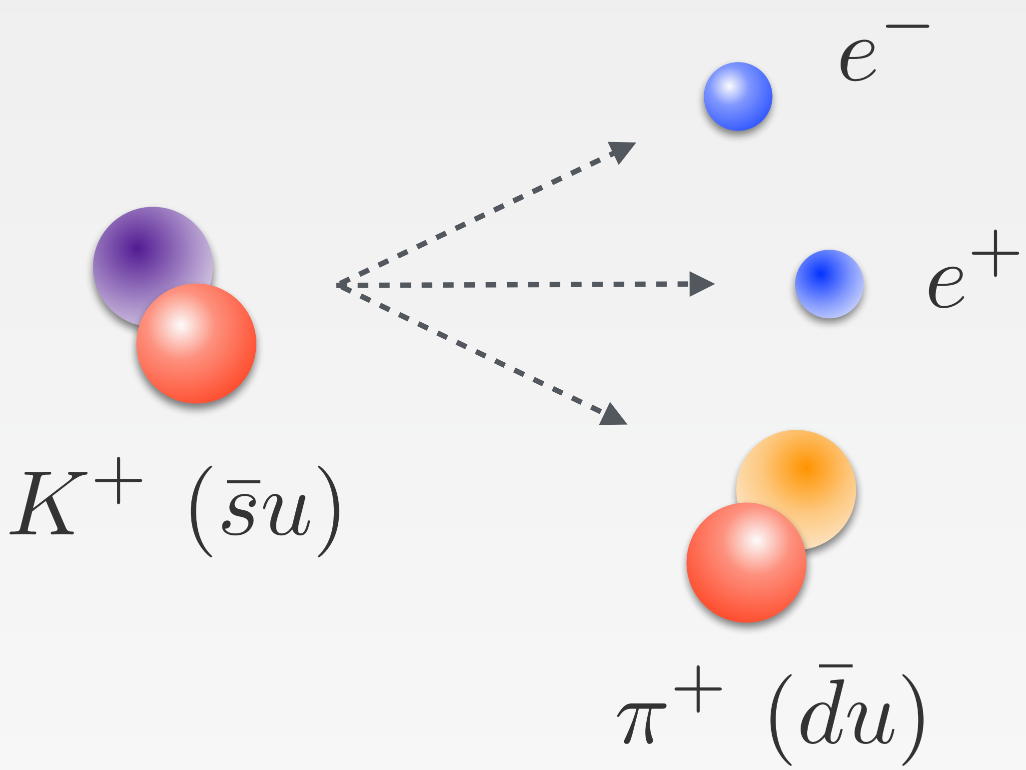


Lattice + dispersive hybrid.

Most precise prediction to date.

T. Blum, A.P., et al. (RBC-UKQCD), Phys. Rev. Lett. 121(2), 022003 (2018)

Rare kaon decays



Flavour Changing Neutral Current

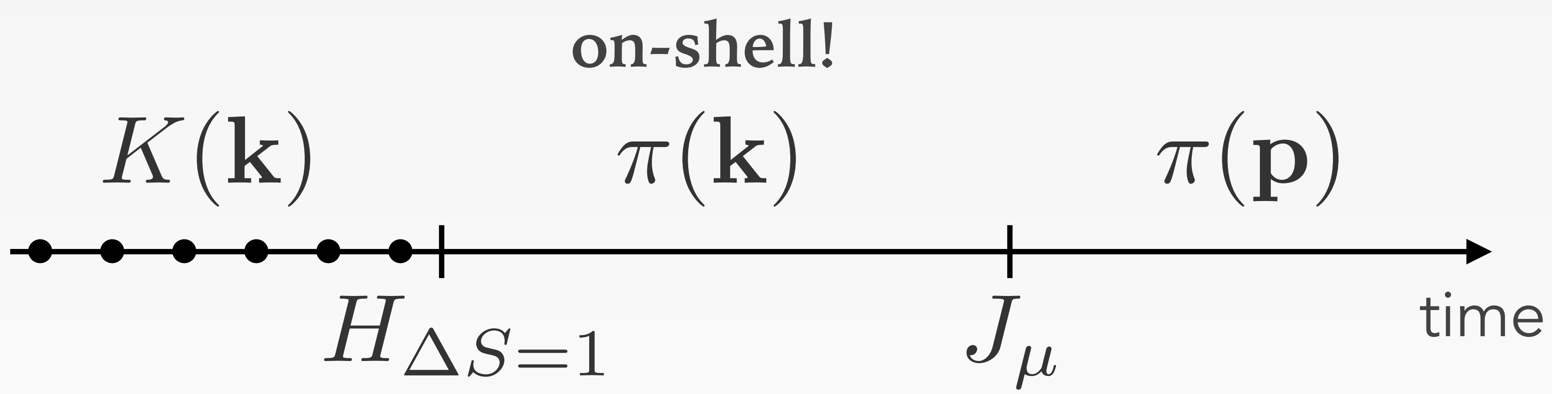
Extremely rare in the SM

$\Rightarrow$ **sensitive to new physics**

- NA62 experiment in progress at CERN.
- **Improved theory predictions are needed.**

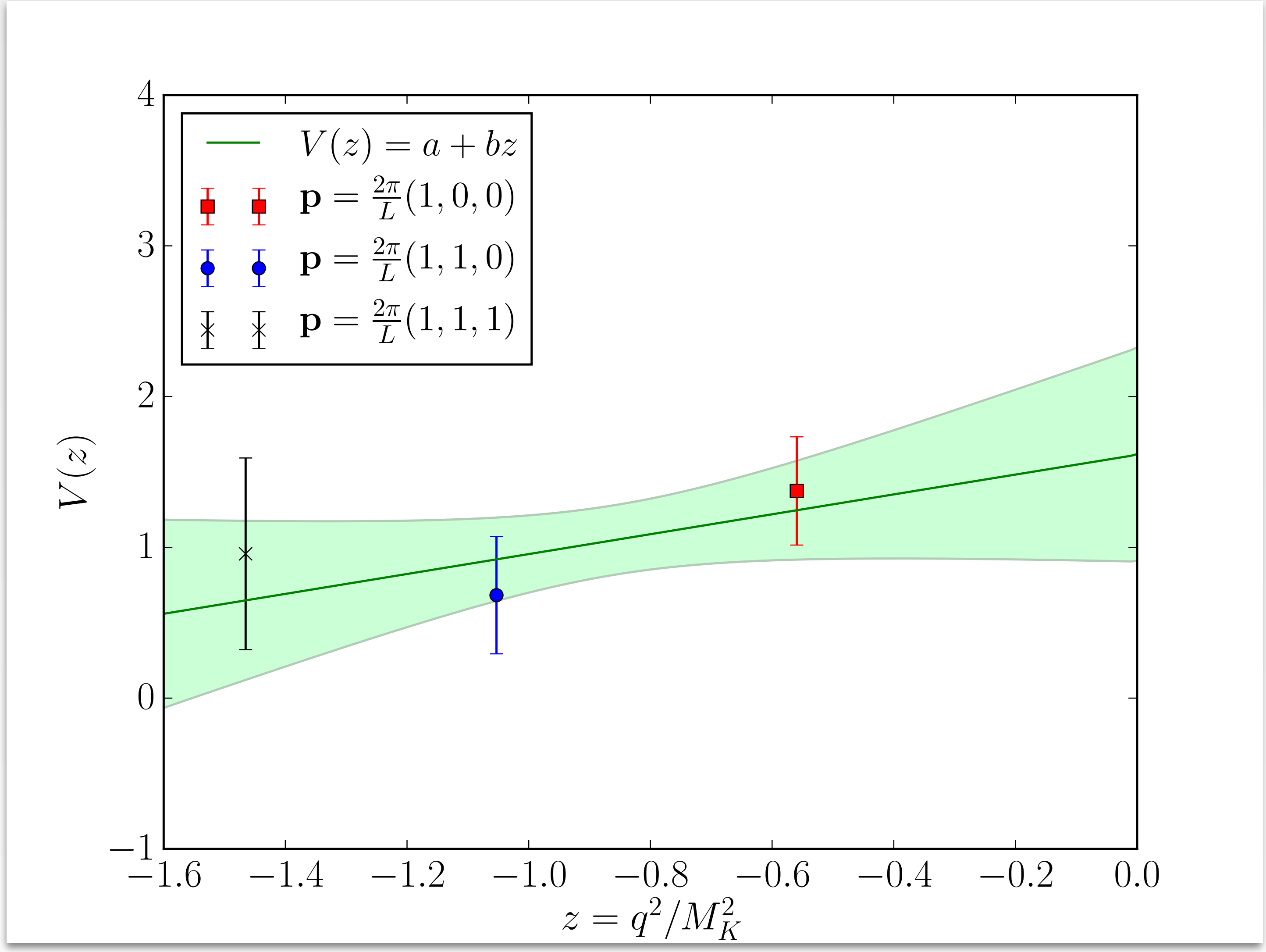
Rare kaon decays

- SM long-distance amplitude $\langle \pi(\mathbf{p}) | \, \mathrm{T}[J_\mu H_W] \, | K(\mathbf{k}) \rangle$
- **Non-trivial Wick rotation** because of on-shell intermediate states



- **General problem** of Euclidean non-local amplitudes!

Rare kaon decays

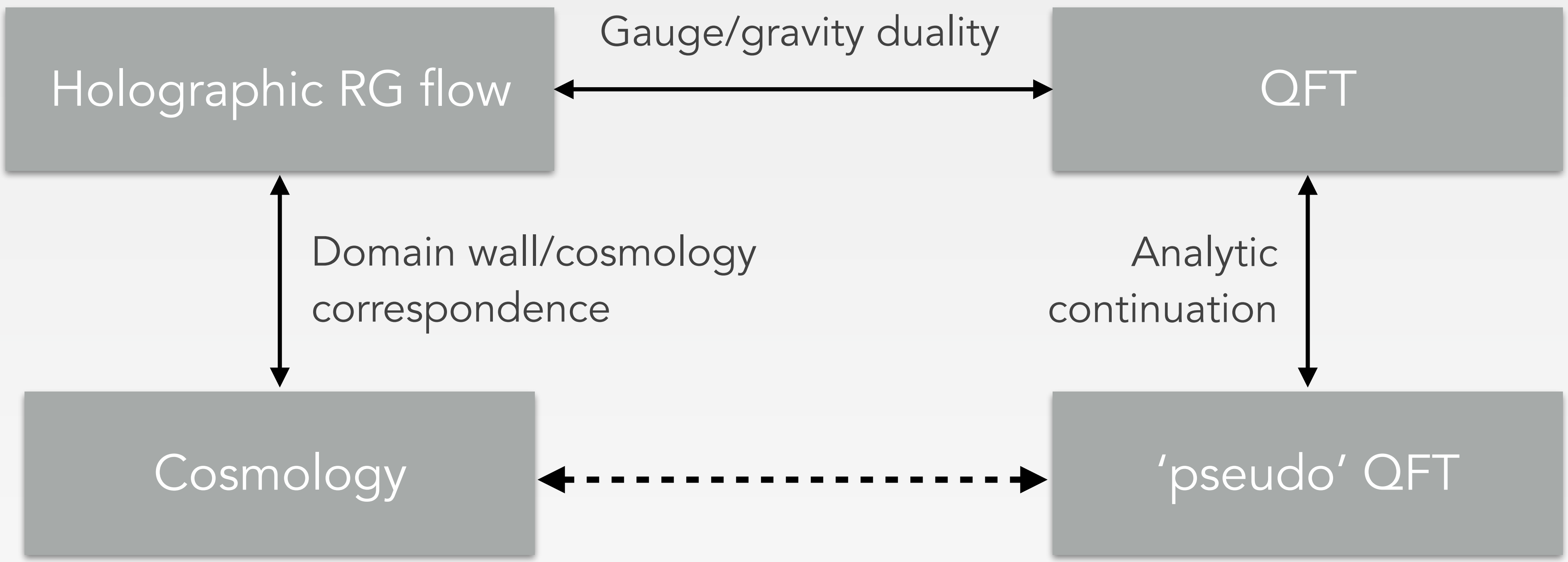


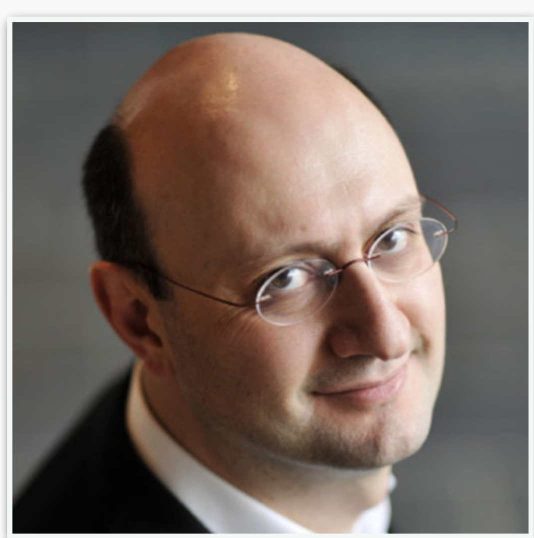
[RBC-UKQCD, PRD 92(9), 094512, 2015]
[RBC-UKQCD, PRD 93(11), 114517, 2016]
[RBC-UKQCD, PRD 94(11), 114516, 2016]
[RBC-UKQCD, PRL 118(2), 252001, 2017]
[RBC-UKQCD, PRD 98(7), 074509, 2018]

- Proof-of-concept calculations for $K \to \pi \ell^+ \ell^-$ & $K \to \pi \nu \bar{\nu}$
- Physical calculation in progress.

Holographic cosmological models

The holographic Universe





P.L. McFadden and K. Skenderis
[PRD 81(2) 2010]
[J. Phys. Conf. Ser. 222(1) 2010]
[JCAP 05 2011]

Holographic CMB spectrum

- Dual theory ansatz: **3D SU(N) gauge theory** with arbitrary content of massless scalars & fermions.
- **Super-renormalisable** theory, with a dimension 1 coupling constant g_{YM}.
- **Naturally Euclidean** because of analytical continuation.
- CMB scalar power spectrum

$$\Delta^2_{\mathcal{R}}(q) = -\frac{q^3}{4\pi^2}\frac{1}{\langle T(q)T(-q)\rangle} \quad (T = T_{\mu\mu})$$

Holographic CMB spectrum

- At 2-loop, universal form

$$\Delta^2_{\mathcal{R}}(q) = \frac{\Delta_0^2}{1 + \frac{gq_*}{q}\log|\frac{q}{\beta g q_*}|}$$

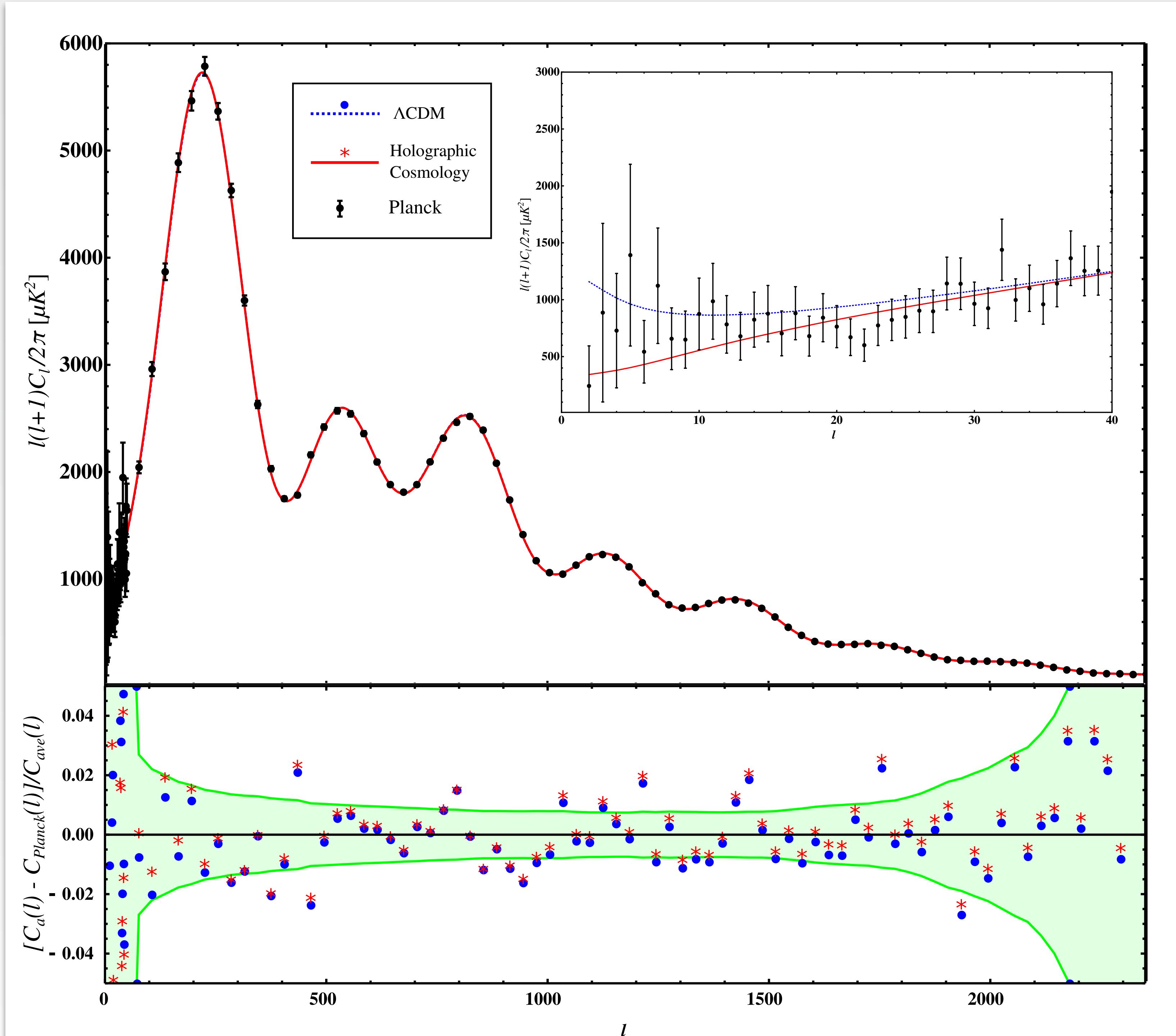
3 cosmological parameters

- Reminder: in ΛCDM

$$\Delta^2_{\mathcal{R}}(q) = \Delta_0^2 \left(\frac{q}{q^*}\right)^{n_s - 1}$$

2 cosmological parameters (3 with running)

Confrontation to Planck data



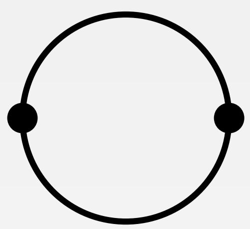
[Afshordi et al., PRL 118(4) & PRD 95(1), 2017]

Conclusions of Planck analysis

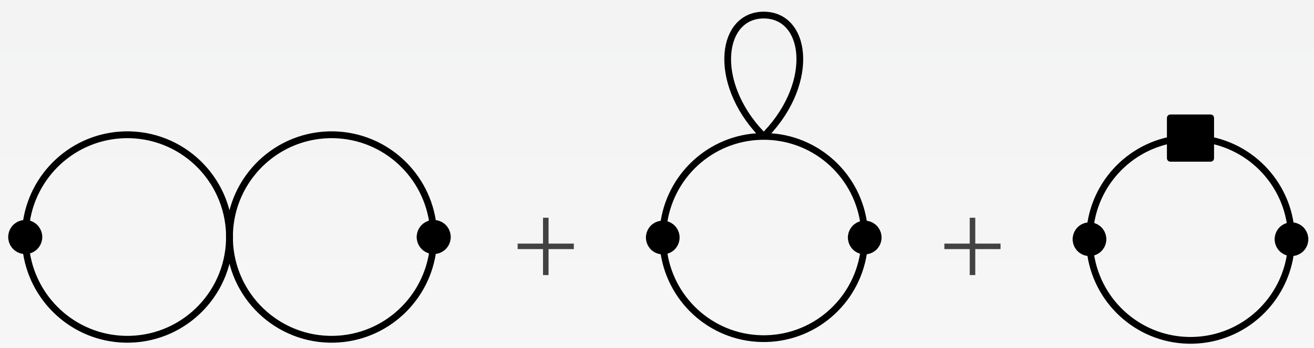
- **Competitive with ΛCDM**, 3 cosmological parameters.
- Model selection: **no fermions**, large N and large number of nearly conformal scalars.
- The dual theory become non-perturbative around $l \sim 35$ **Low-multipole region cannot be trusted**.
- The dual theory has IR divergences which are believed to be an **artefact of perturbation theory**.
- **Clear motivations for a non-perturbative calculation**.

Perturbative issues

$\mathrm{tr}(\phi^2)$ 2-pt function in $\mathfrak{su}(N)$ scalar ϕ^4 theory



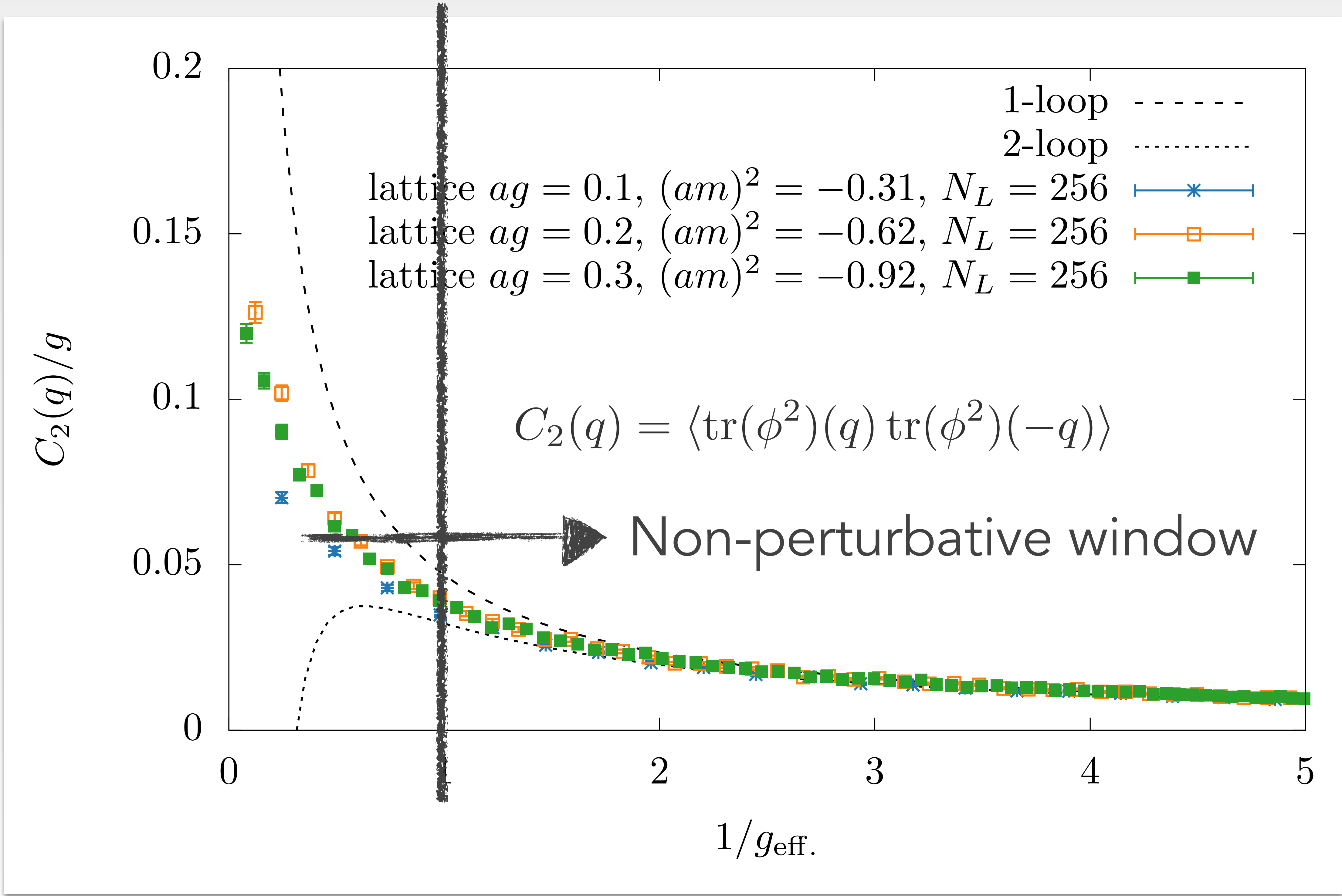
$$\sim \frac{g}{|p|}$$



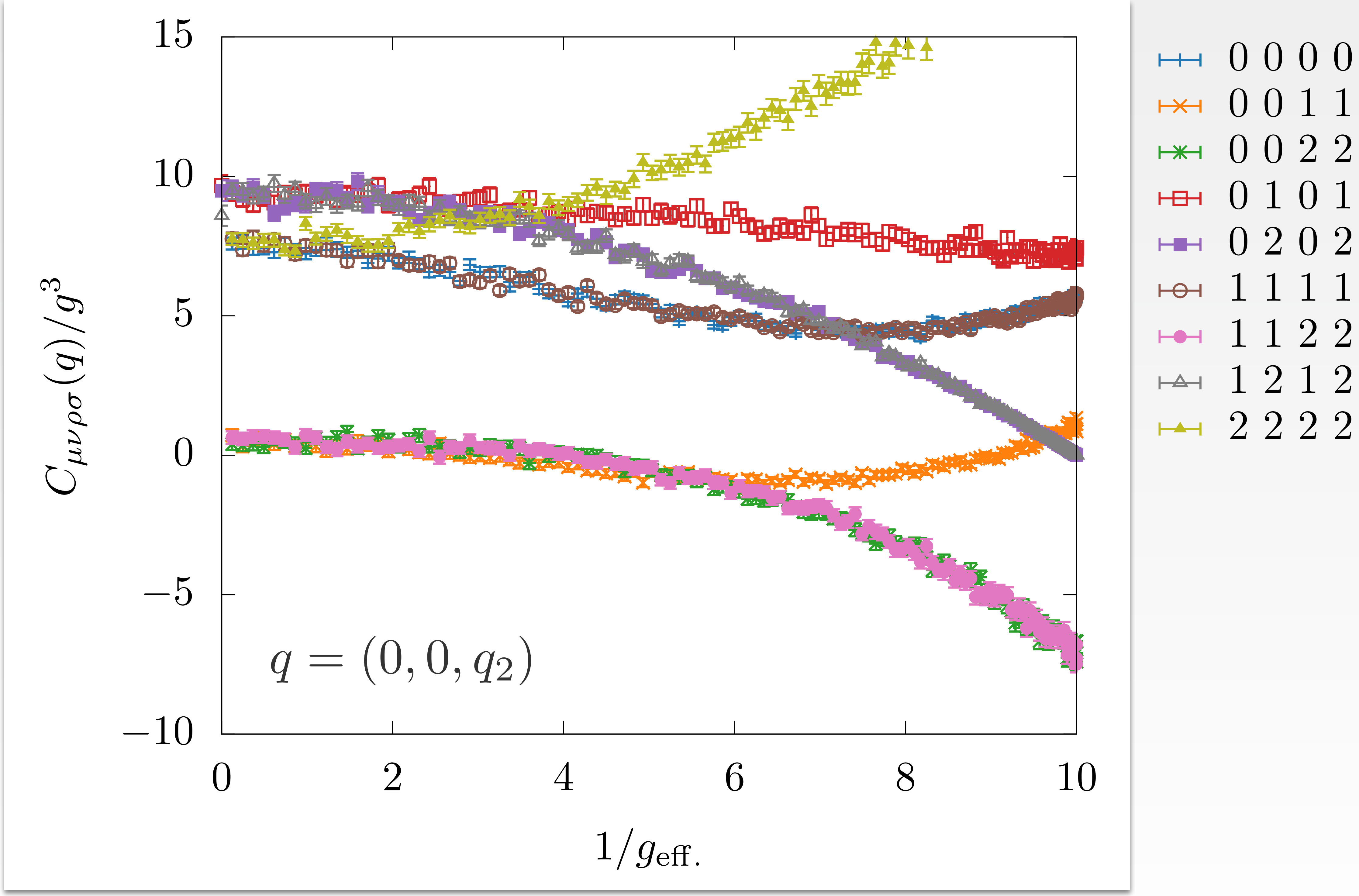
$$\sim \frac{g^2}{|p|^2}$$

- Expansion driven by $g_{\text{eff.}} = g/|p|$.
- Leading large-N corrections at $\mathrm{O}(1/N^2)$.

The non-perturbative window



Lattice EMT 2-point function



Outlook & perspectives

Hardware & software challenges

- Very large number of d.o.f.: very challenging for scalability for hardware & software.
- 1 year lattice calculations on a supercomputer ~ **1 millennium on a decent workstation**.
- 6 last months: **2PB data generated**.
- Ever-increasing complexity in computational methods.
- Ever-increasing complexity in data analysis.

Unified software framework

- Edinburgh-based data parallel library **Grid** — https://github.com/paboyle/Grid
- Cutting-edge low-level optimisations for CPU/GPU performances and network scalability.
- High-level multi-platform interface.
- Used in all the projects presented here!

Outlook

- Lattice QCD can produce reliable, high-precision predictions in hadronic physics.
- Supports experimental searches for new physics at collider experiments.
- Example here: g-2 & FCNC decays.
- Beyond the SM it allows to explore new theories based on strongly interacting systems.
- Example here: dual theory for holographic cosmology.

Perspectives

- Per-mil precision on the hadronic contribution to g-2 and light CKM matrix elements.
- Realistic calculation of non-local hadronic matrix elements — first step: rare K decays.
- Confirmation of holographic cosmological models?
- Unified HPC framework for lattice field theory.

Thank you!

RBC-UKQCD

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A. Yong
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Finite-volume QED

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