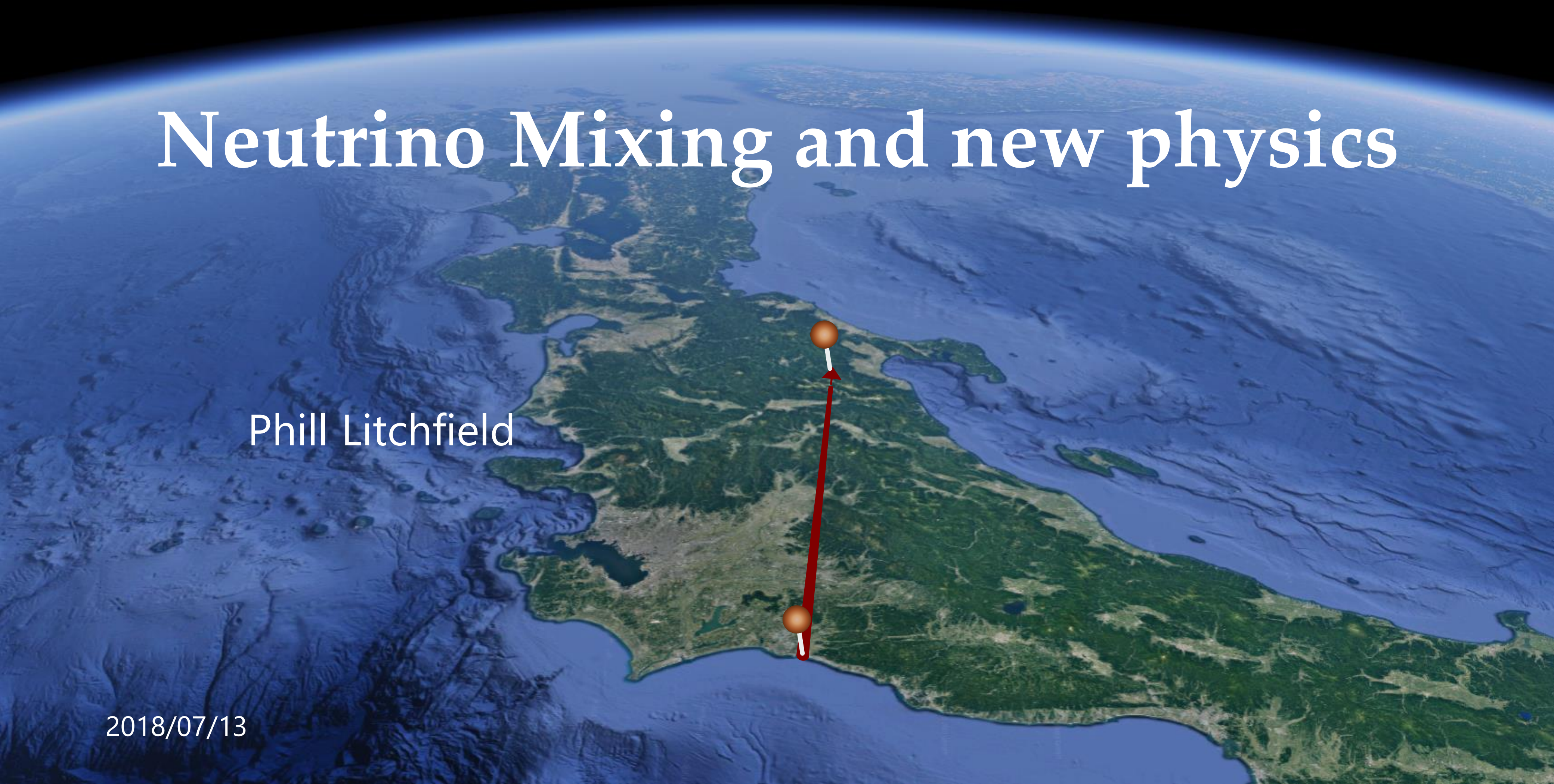
Neutrino Mixing and new physics



Phill Litchfield

2018/07/13

Outline

- Neutrino oscillations

Where we are

- The T2K experiment
- Search for CP violation

Where we are going

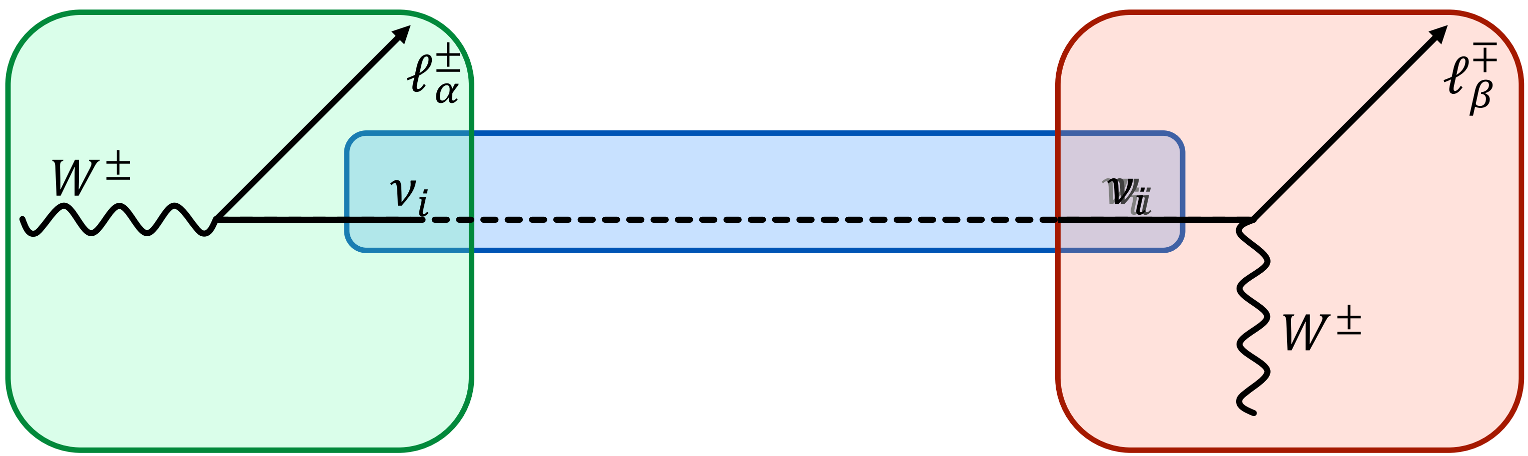
- Measurement of δ_{CP}

Neutrino oscillations

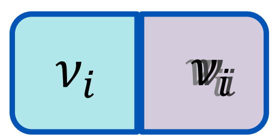
Neutrino mixing

Neutrinos always **produced** and **detected** as weak states $\nu_\alpha = \{\nu_e, \nu_\mu, \nu_\tau\}$ which is (very) different from **propagation** basis $\nu_i = \{\nu_1, \nu_2, \nu_3\}$

- In vacuum, **propagation basis ≡ mass basis**



Propagation states eventually get out of phase



- The superposition resolves as a different weak *flavor*

Angle parameterisation

Oscillation is the combination of a mixing matrix (U) and the different phase advances of the three mass states

$$P_{\mathrm{vac}}(\nu_\alpha \to \nu_\beta) = \delta_{\alpha\beta} - 4\sum_{i>j} Re\left[U_{\beta i}U^*_{\alpha i}U^*_{\beta j}U_{\alpha j}\right]\sin^2\left(\frac{\Delta m^2_{ij}L}{4E}\right)$$
$$+ 2\sum_{i>j} Im\left[U_{\beta i}U^*_{\alpha i}U^*_{\beta j}U_{\alpha j}\right]\sin\left(\frac{\Delta m^2_{ij}L}{2E}\right)$$

Mixing must be unitary, so decompose in terms of $\{c,s\}_{ij} = \{\cos,\sin\}\,\theta_{ij}$

$$U_{PMNS} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Angle parameterisation

Have 2 independent mass² scales, and 4 unitary mixing parameters:

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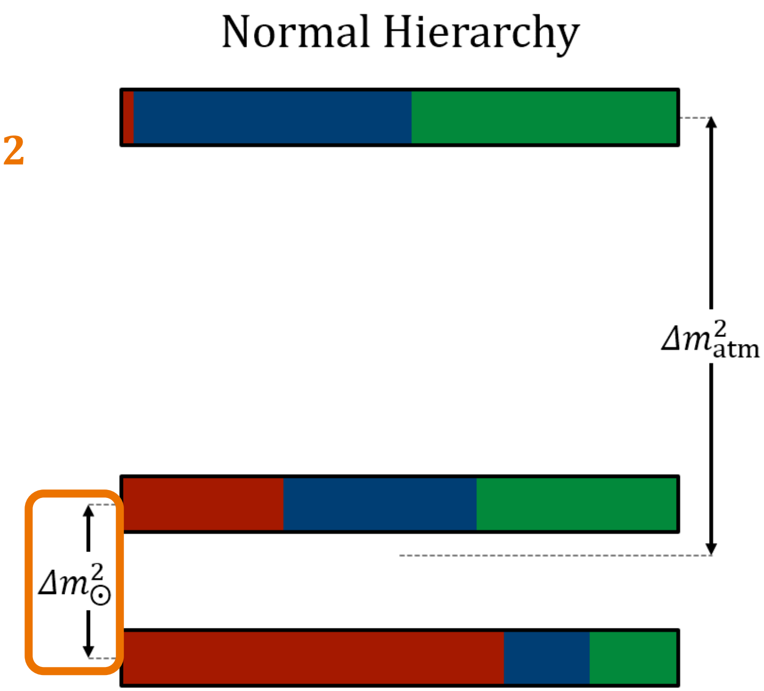
Historically useful:

- ν_e disappearance in **solar** neutrinos from $\boldsymbol{\theta_{12}}$ mixing and splitting $\boldsymbol{\Delta m^2_{21} = m^2_2 - m^2_1}$



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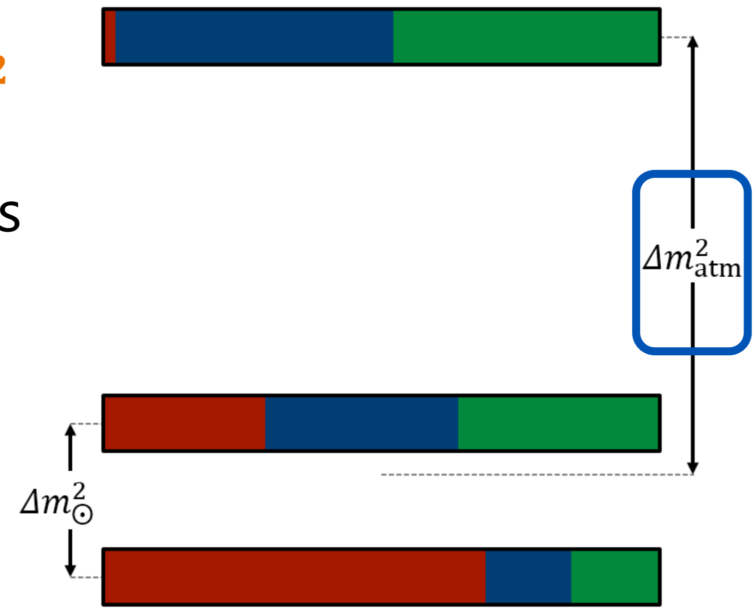
- ν_e disappearance in **solar** neutrinos from $\boldsymbol{\theta_{12}}$ mixing and splitting $\boldsymbol{\Delta m^2_{21} = m^2_2 - m^2_1}$
- $\nu_\mu \leftrightarrow \nu_\tau$ oscillations in **atmospheric** neutrinos from $\boldsymbol{\theta_{23}}$ mixing and splitting $\boldsymbol{\Delta m^2_{3i}}$

Normal Hierarchy



Angle parameterisation

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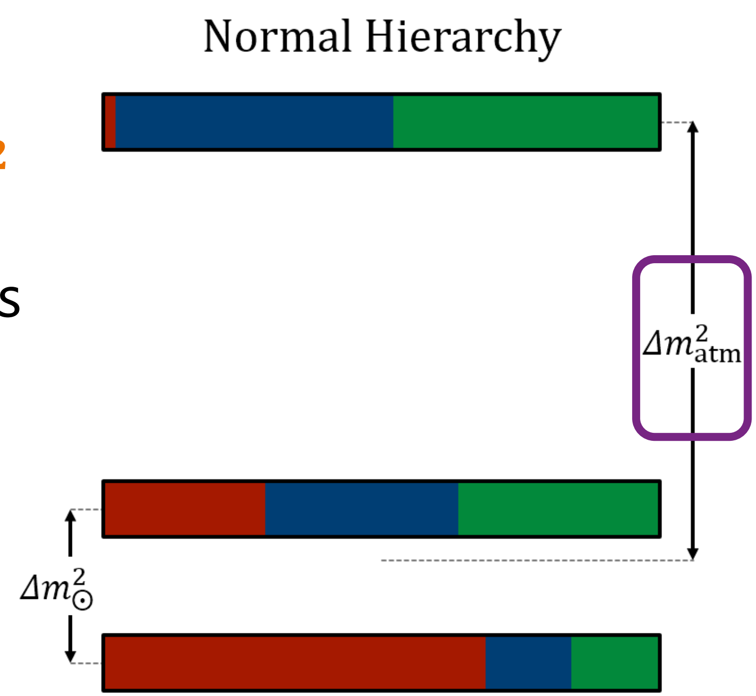
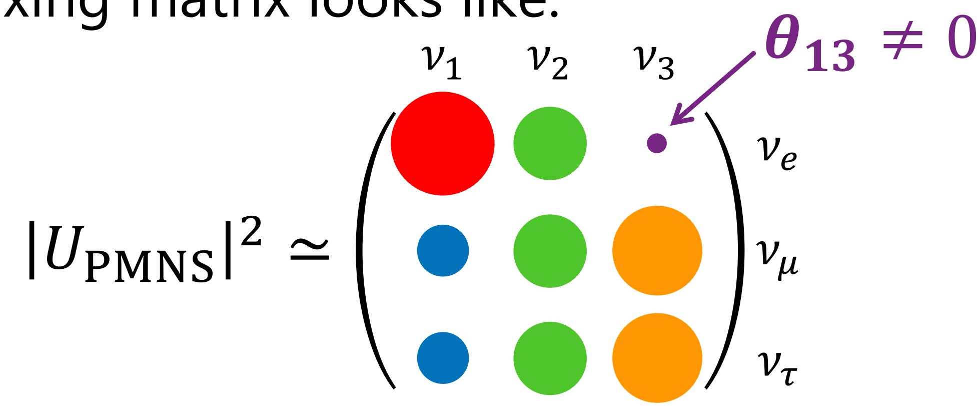
It also works out that **reactor** neutrinos at 1km are sensitive to $\boldsymbol{\theta_{13}}$ and $\boldsymbol{\Delta m^2_{3i}}$



Standard neutrino picture

Now: precision measurement — can't approximate as a single sub-matrix.

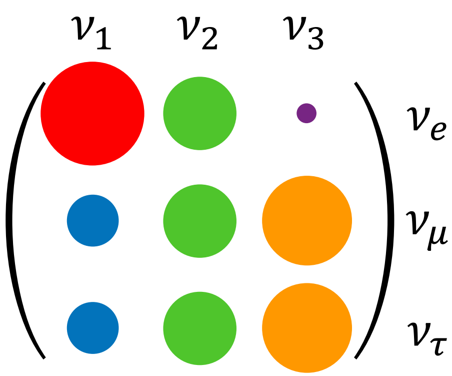
- We know fairly well what the mixing matrix looks like:



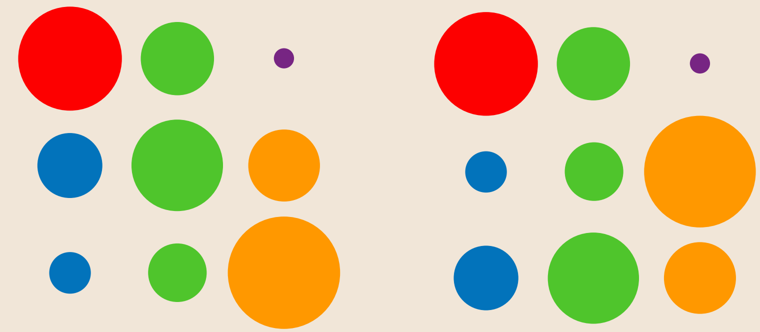
Open questions

Now: precision measurement — can't approximate as a single sub-matrix.

- We know *fairly* well what the mixing matrix looks like:

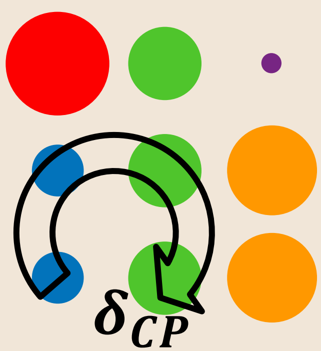
$|U_{\mathrm{PMNS}}|^2 \simeq$ 

Octant deceneracy



Lower ($\theta_{23} < 45°$) Upper ($\theta_{23} > 45°$)

CP Violation

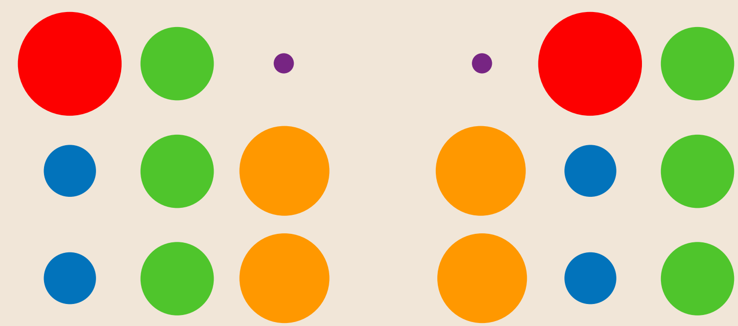


Complex mixing of these 4 elements causes

$P(\nu_\alpha \to \nu_\beta) \neq P(\bar{\nu}_\alpha \to \bar{\nu}_\beta)$

Key parameter: $\boldsymbol{\delta_{CP}}$

Mass Ordering (Hierarchy)



Normal (NO) Inverted (IO)

University of Glasgow

VIA VERITAS VITA

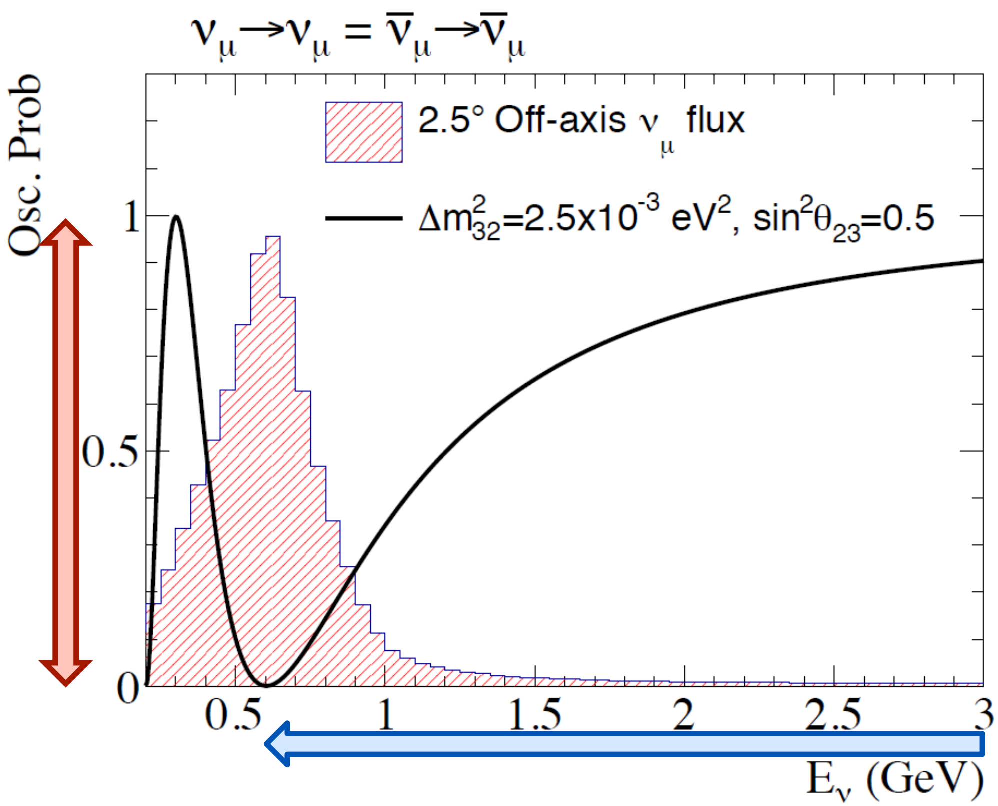
T2K

The T2K experiment & the search for CP violation

T2K measurements

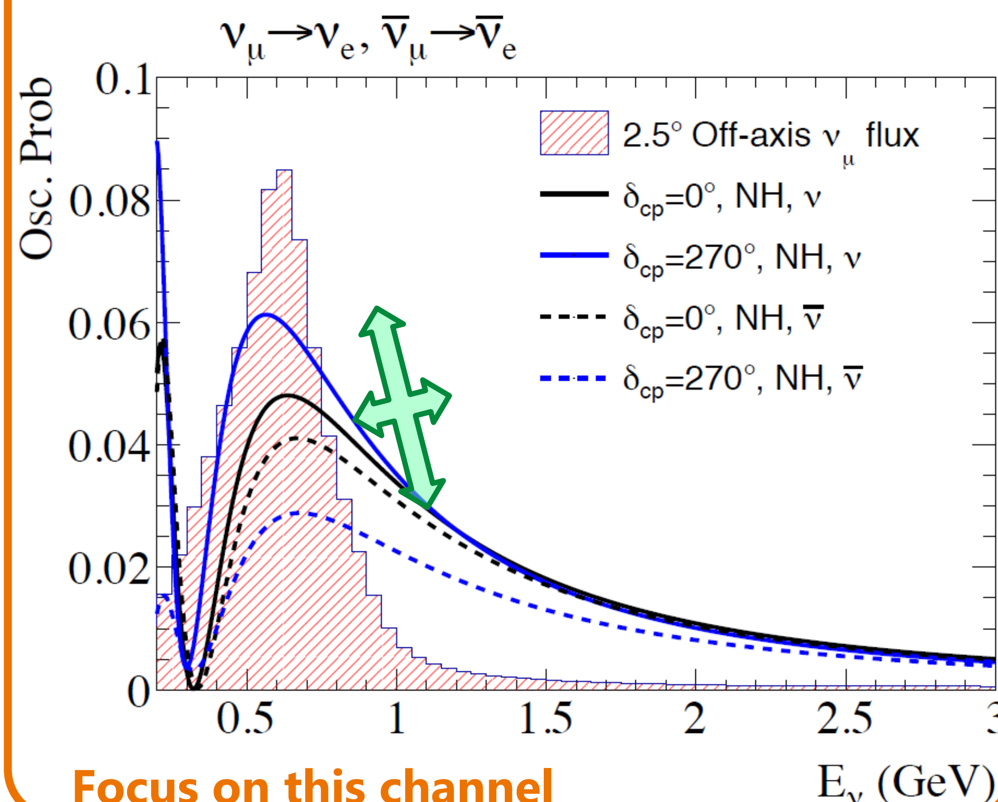
ν_μ Disappearance:

- Precision measurement of $\sin^2 2\theta_{23}$ and Δm^2_{32}



ν_e Appearance:

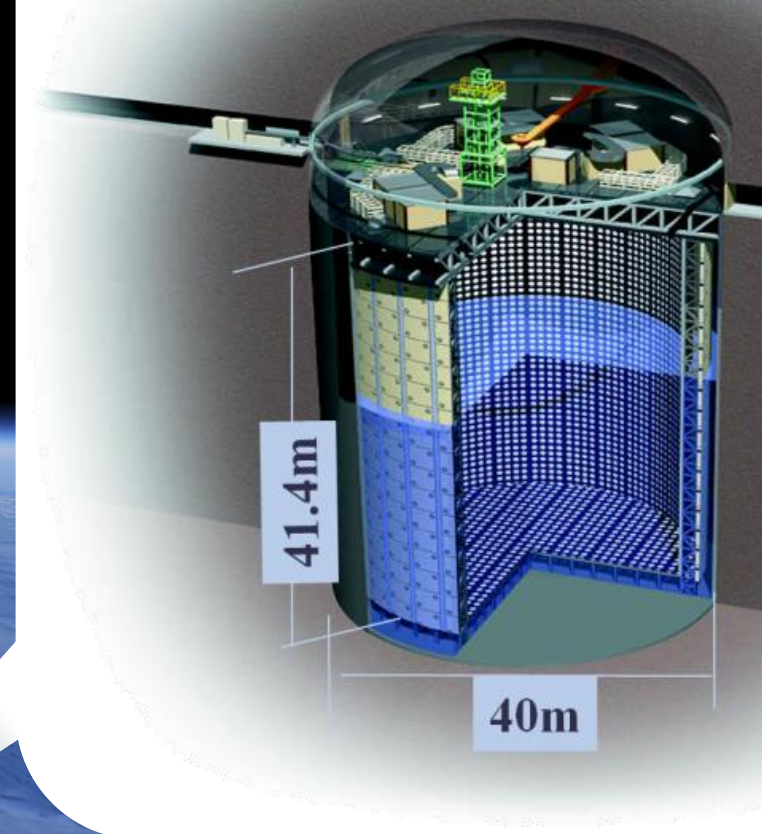
- Enhanced for ν if $-\pi < \delta_{CP} < 0$
- NO/NH also enhances ν

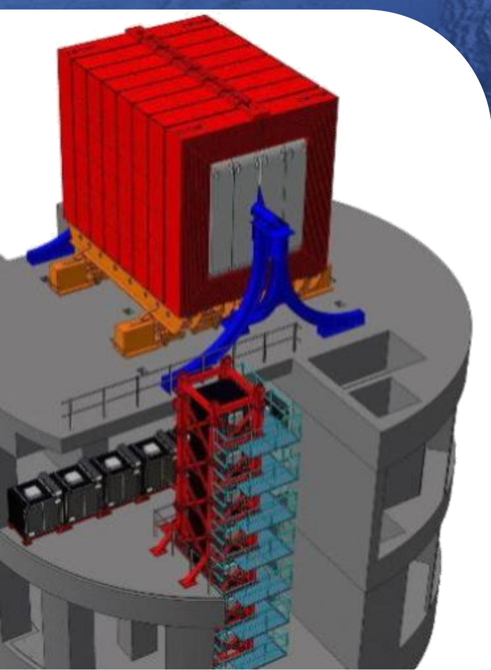


Focus on this channel

Muon neutrino beam produced at J-PARC on Japan's east coast

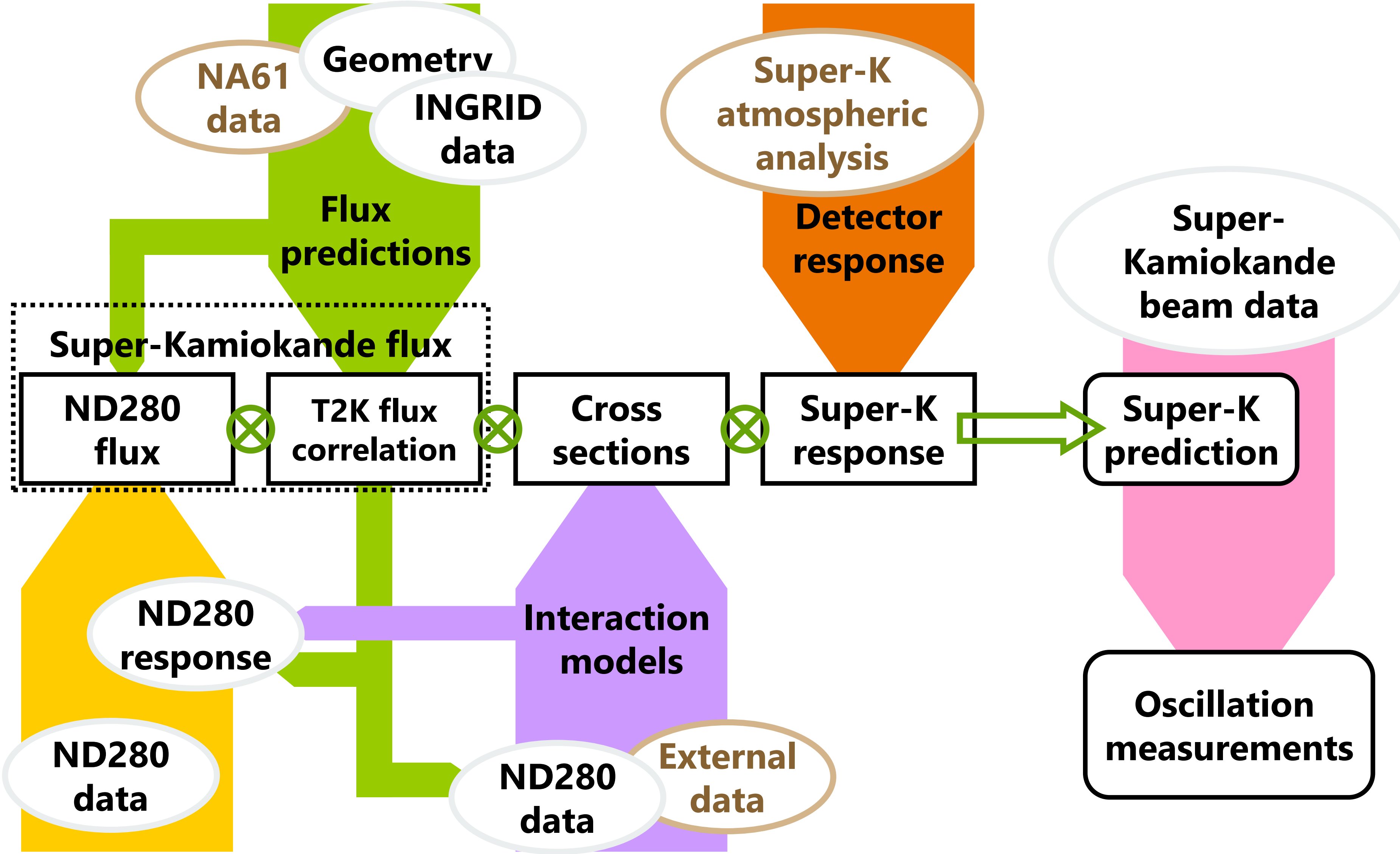
- Directed 'towards' Super-Kamiokande, 295km away.
- Near Detector complex at 280m (ND280) to study beam and interactions





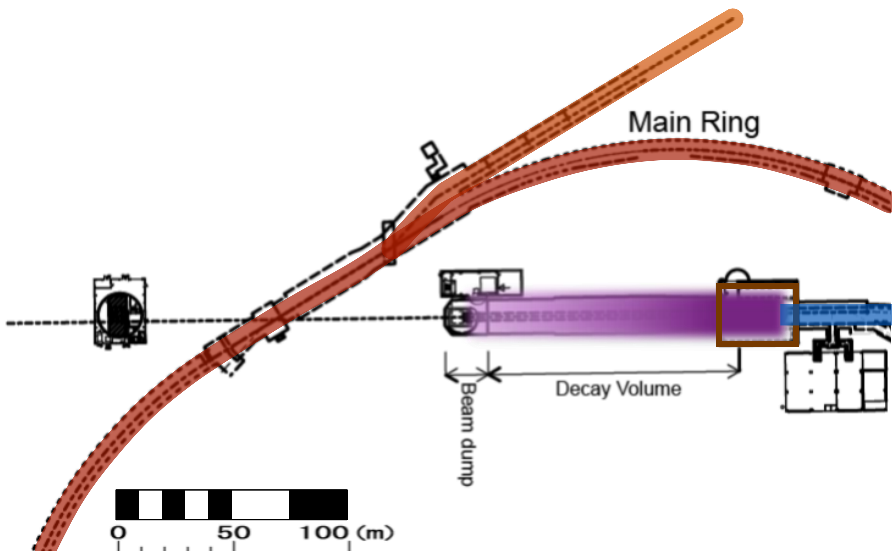
The T2K Experiment

Analysis strategy



J-PARC Accelerators

- 30 GeV/c proton beam
- Fast-extracted onto carbon target to produce (mostly) pions
- 8 bunches/spill at ~580ns spacing.
- 2.48s between spills



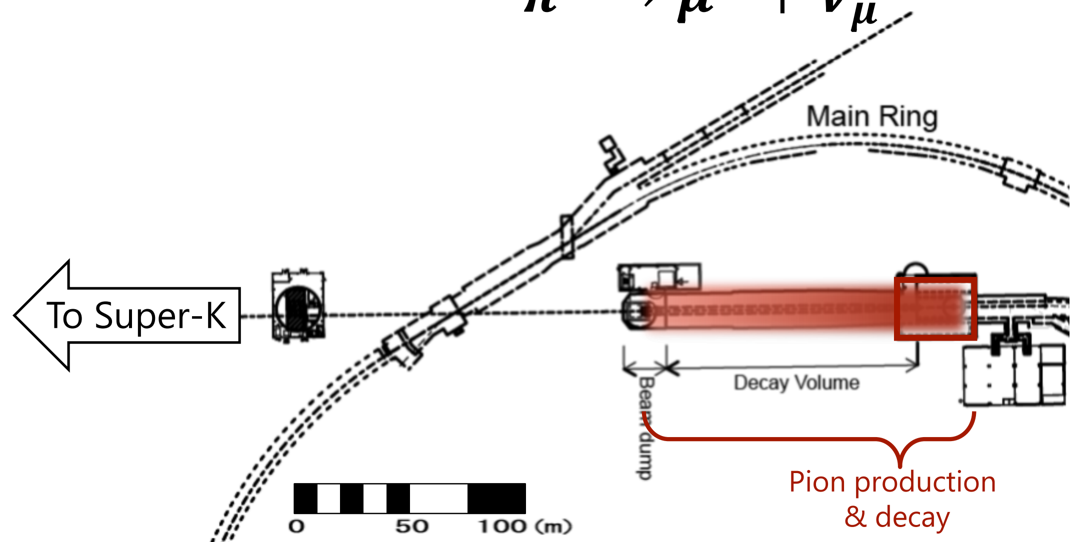
The T2K experiment

Many pions produced at target:

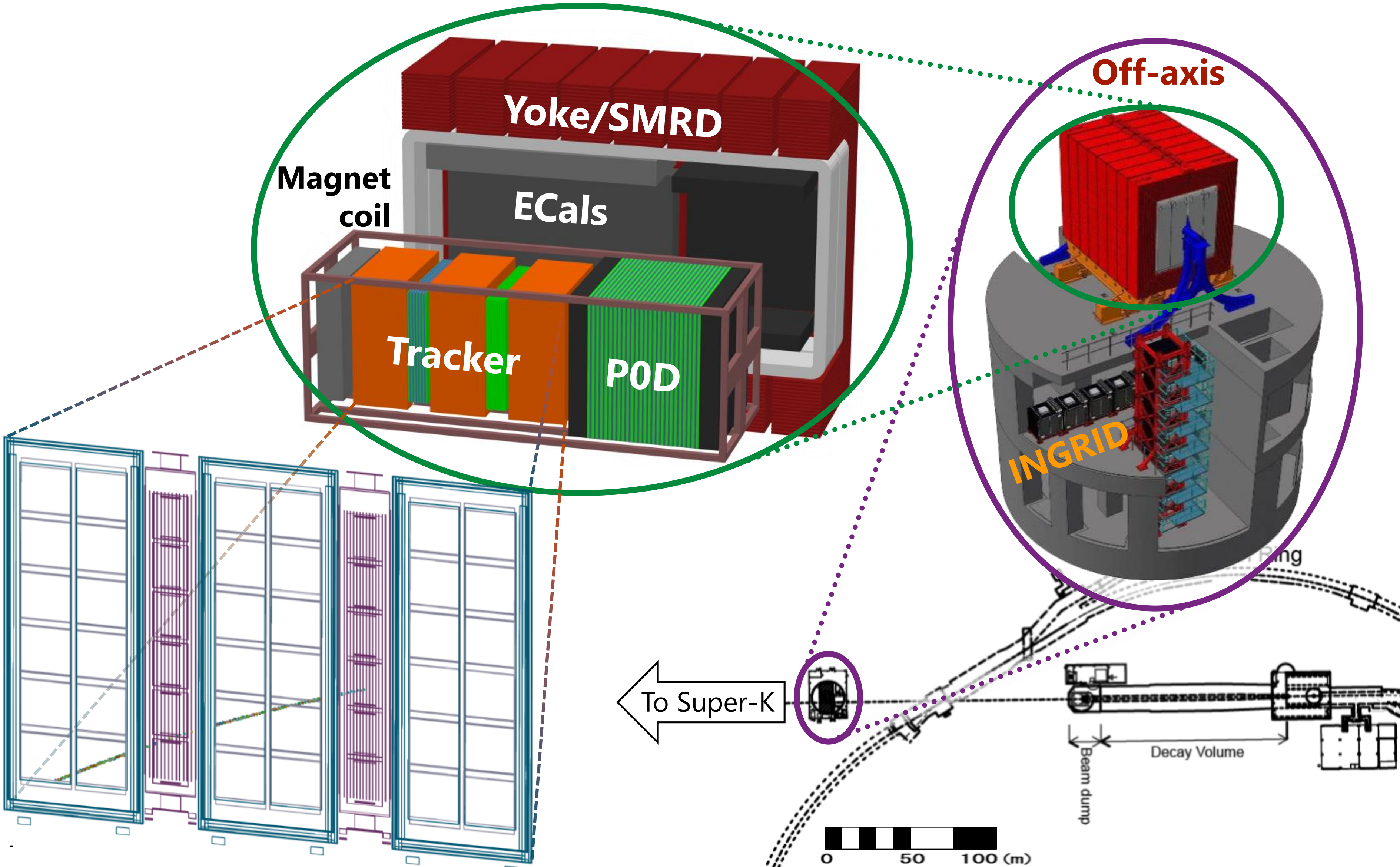
$$\pi^{+} \rightarrow \mu^{+} + \nu_{\mu}$$

or

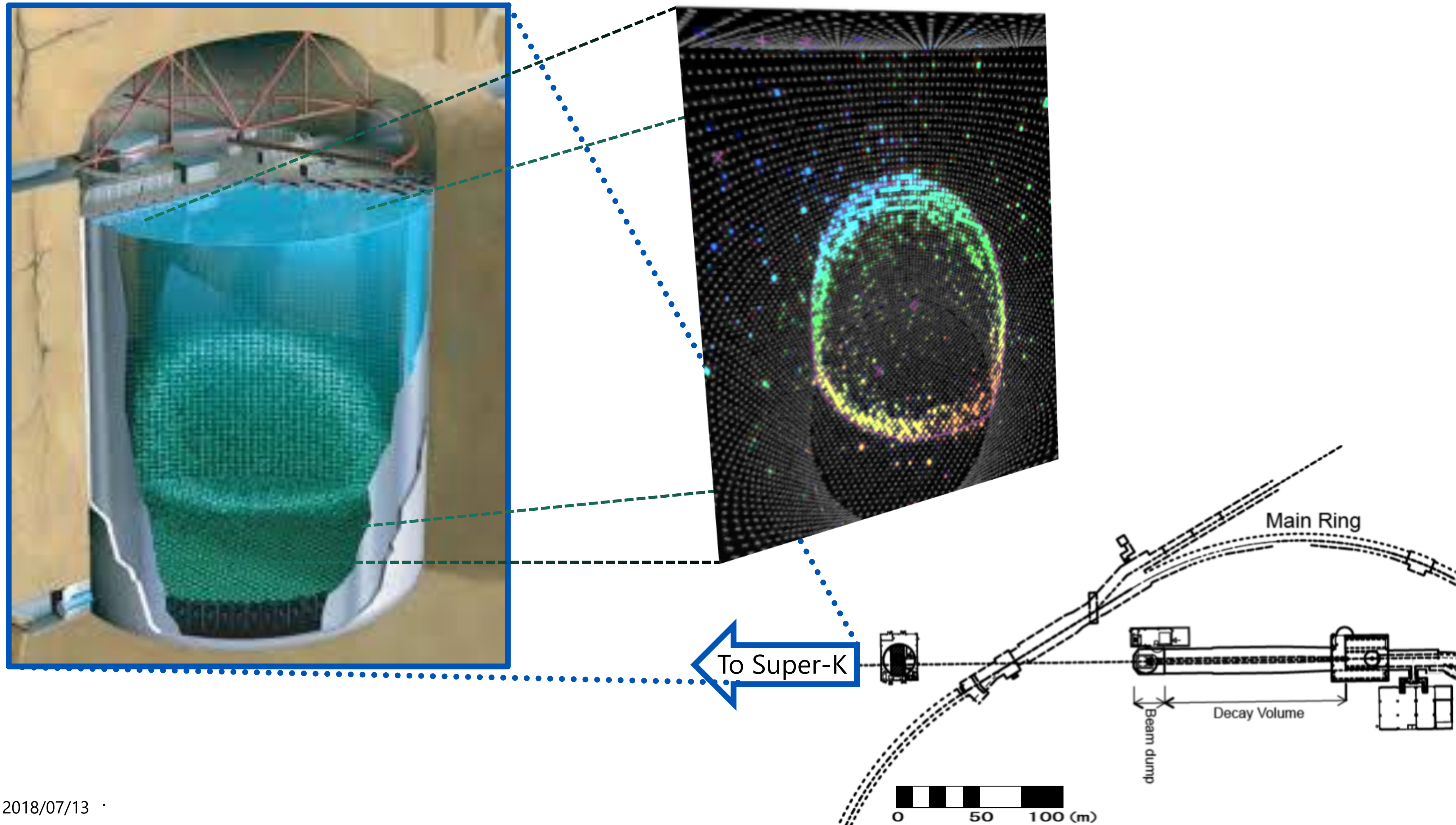
$$\pi^{-} \rightarrow \mu^{-} + \bar{\nu}_{\mu}$$



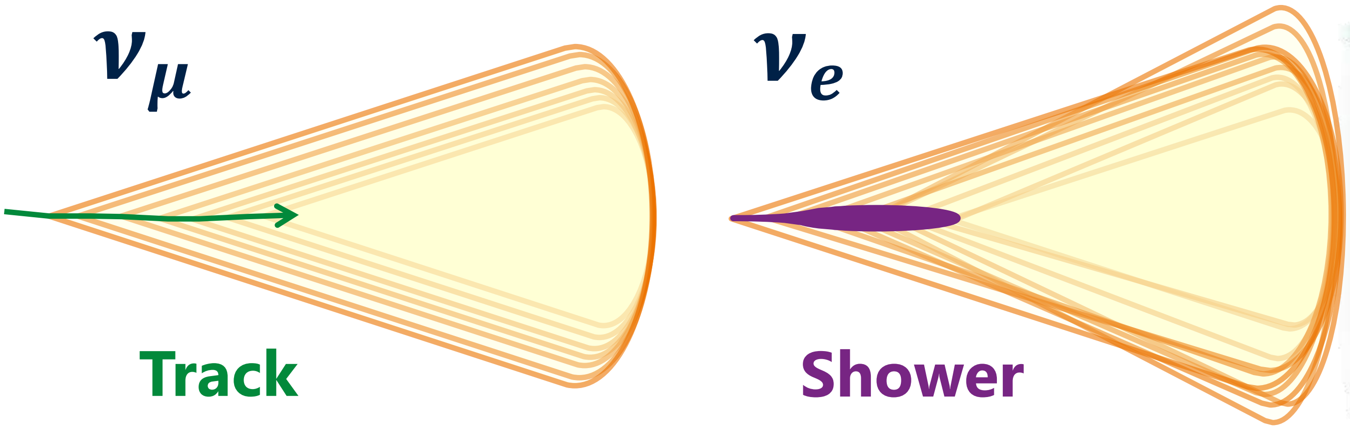
The T2K experiment: ND280

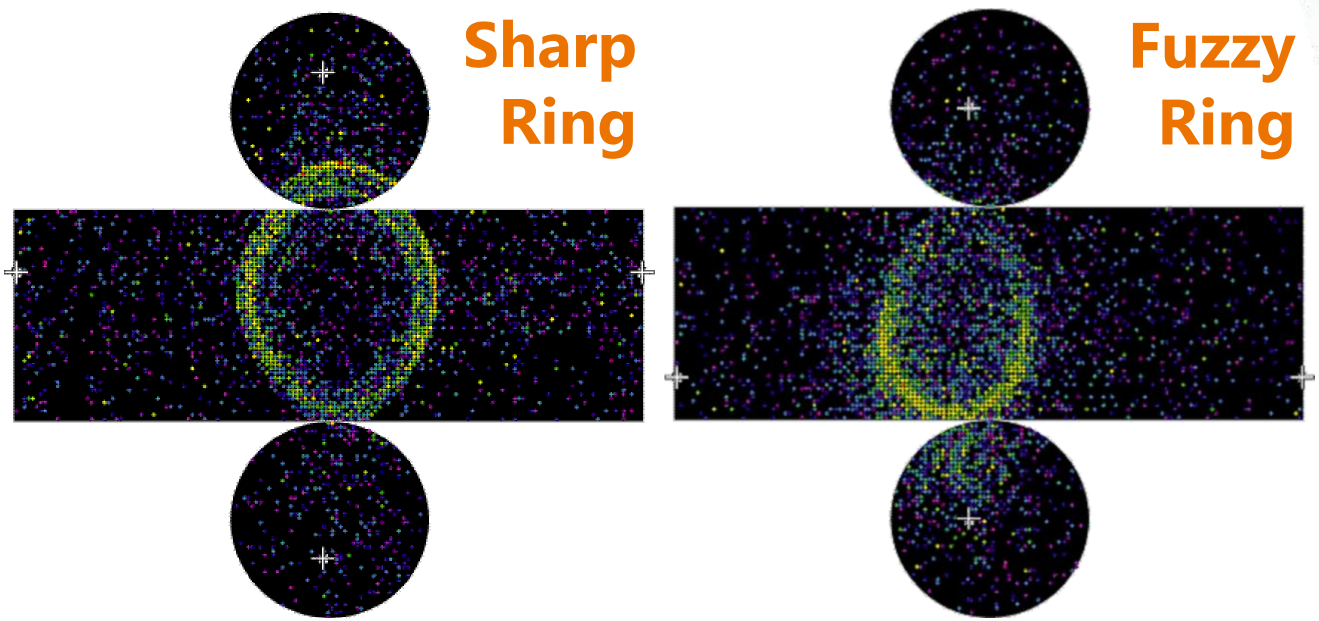


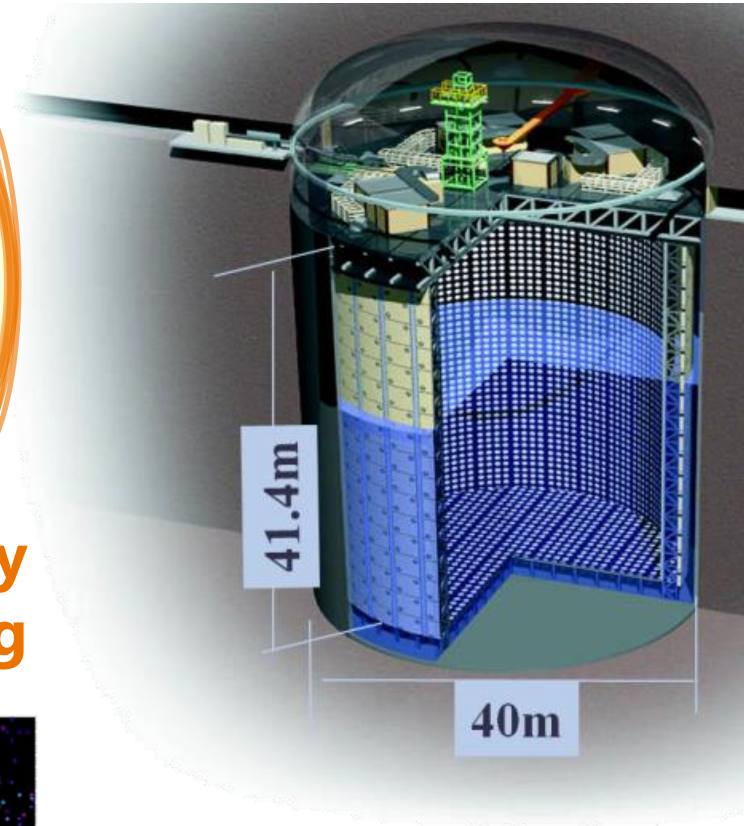
The T2K experiment: SK



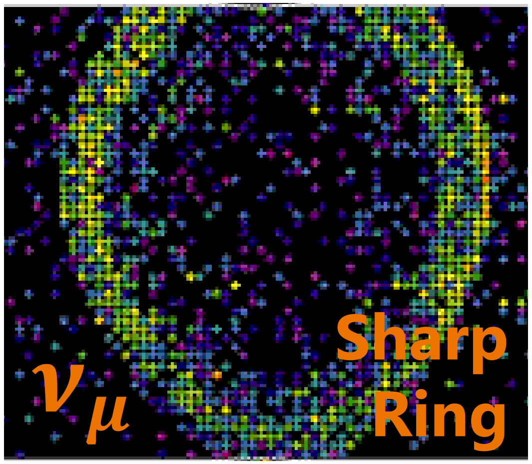
Super-Kamiokande events



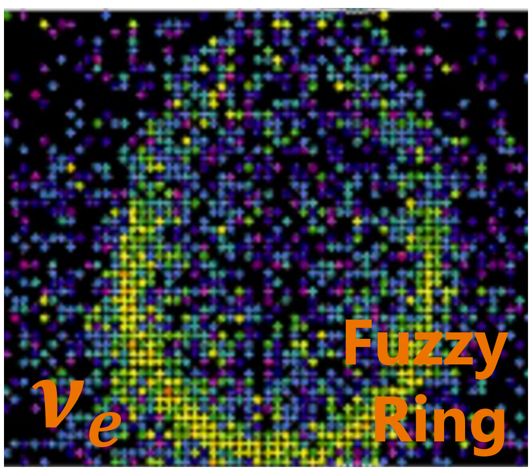




Super-Kamiokande samples



1R-μ



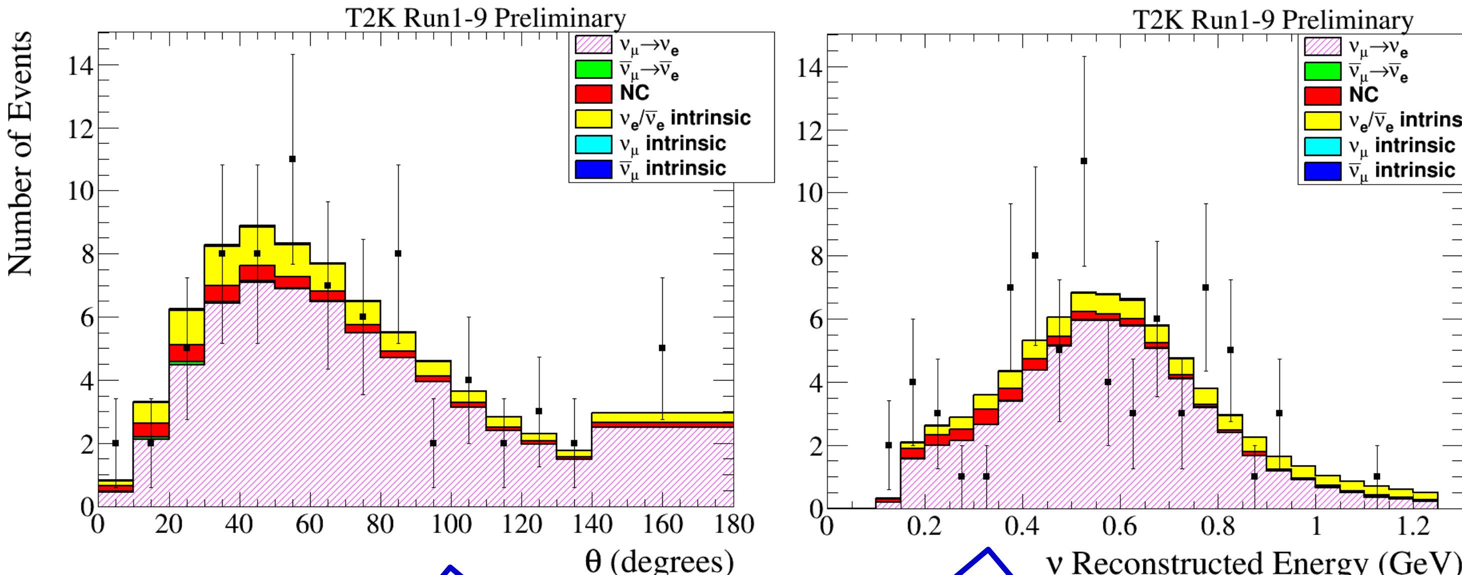
1R-e

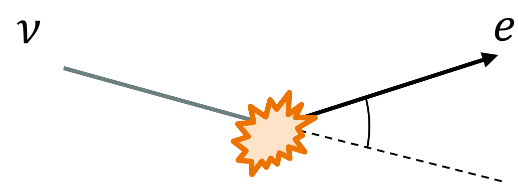
1R-e + d.e.

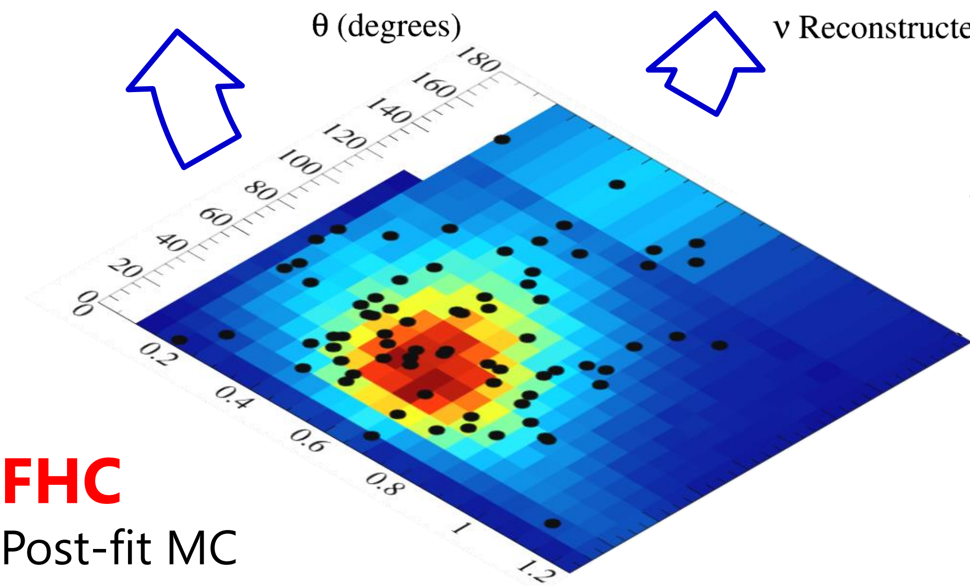
FHC	RHC
FHC sample, expect: **94%+6% ν_μ + $\bar{\nu}_\mu$**	RHC sample, expect: **60%+40% $\bar{\nu}_\mu$ + ν_μ**
FHC sample, expect: **81% $(\nu_\mu \rightarrow)\nu_e$, 18% beam ν_e + ν_μ**	RHC sample, expect: **45% $(\bar{\nu}_\mu \rightarrow)\bar{\nu}_e$, 10% $(\nu_\mu \rightarrow)\nu_e$**
FHC sample, expect: **79% $(\nu_\mu \rightarrow)\nu_e$, 21% beam ν_e + ν_μ**	

New sample added. Delayed-coincidence Michel electron tags low-momentum pion (in FHC only)

Super-K electron-like samples







FHC

Post-fit MC

Electron-like samples are analysed in 2D

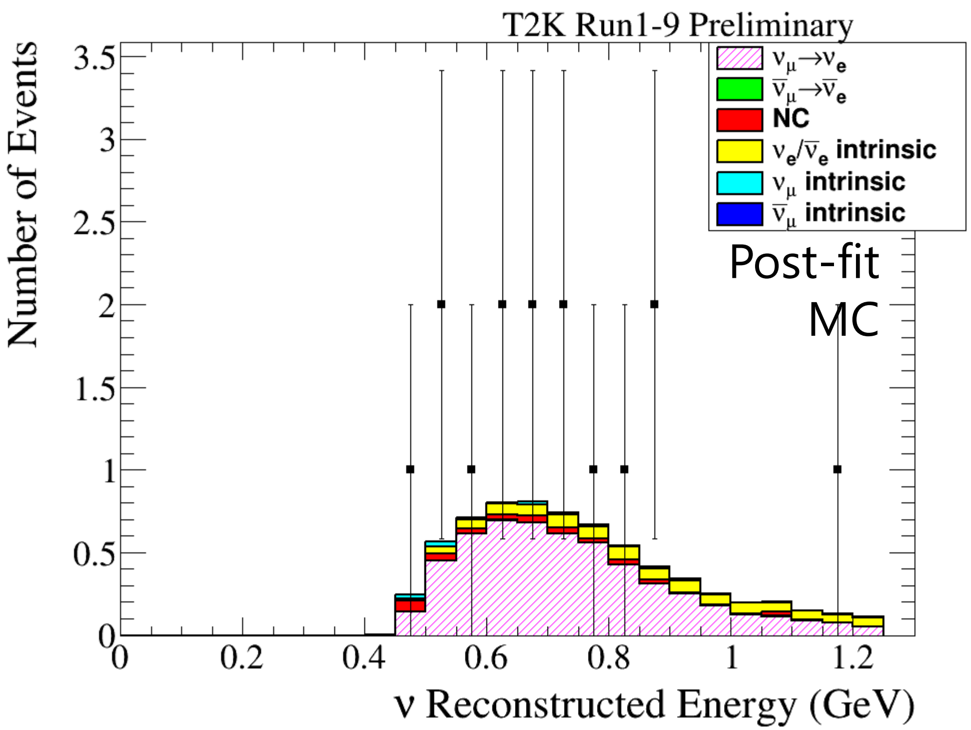
- **75 events observed**
- Signal and BG have different $\theta_{\nu e}$ distributions

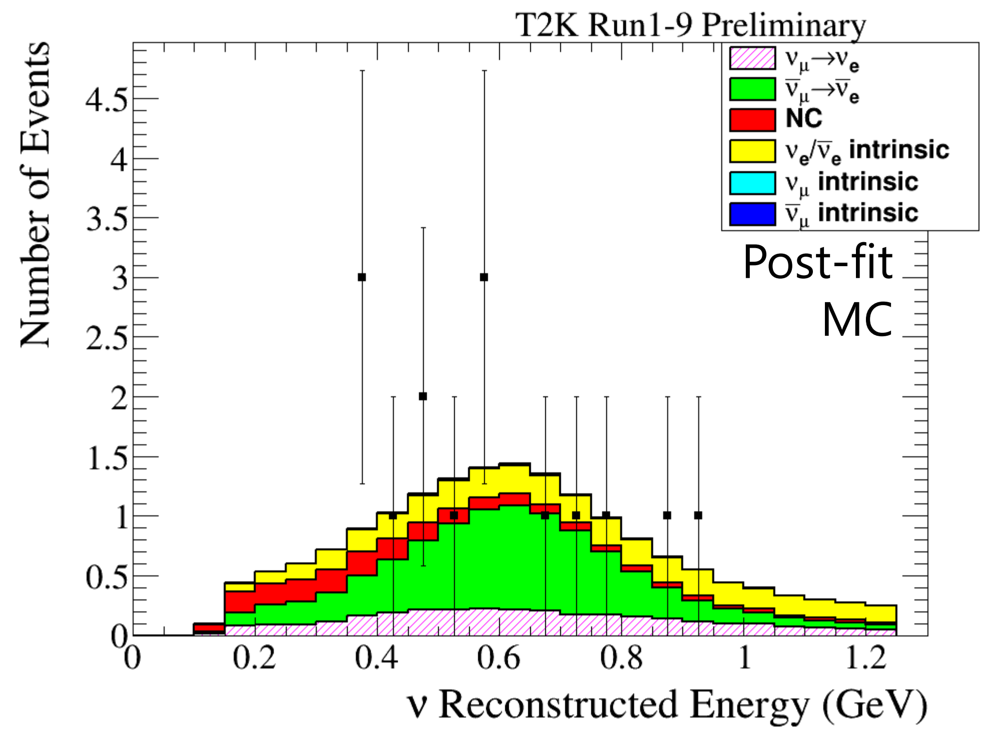
Super-K electron-like samples

FHC: 15 events observed with decay electron +75 “1Re” = **90 events total**

RHC: 15 events observed

- Flux and cross-section differences mean this should be low, but



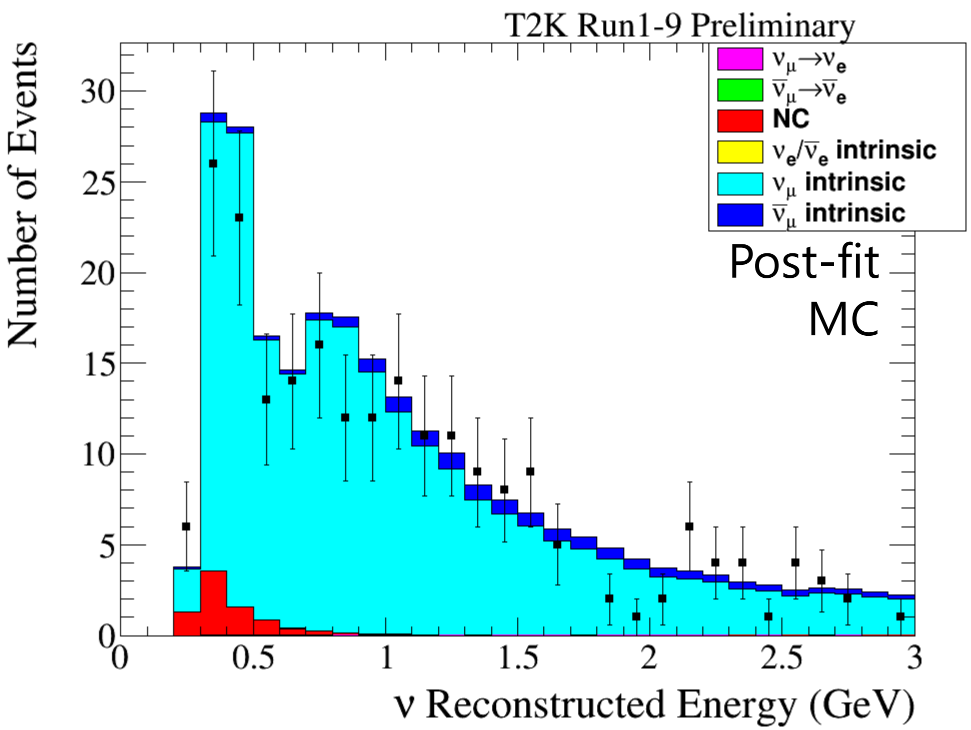


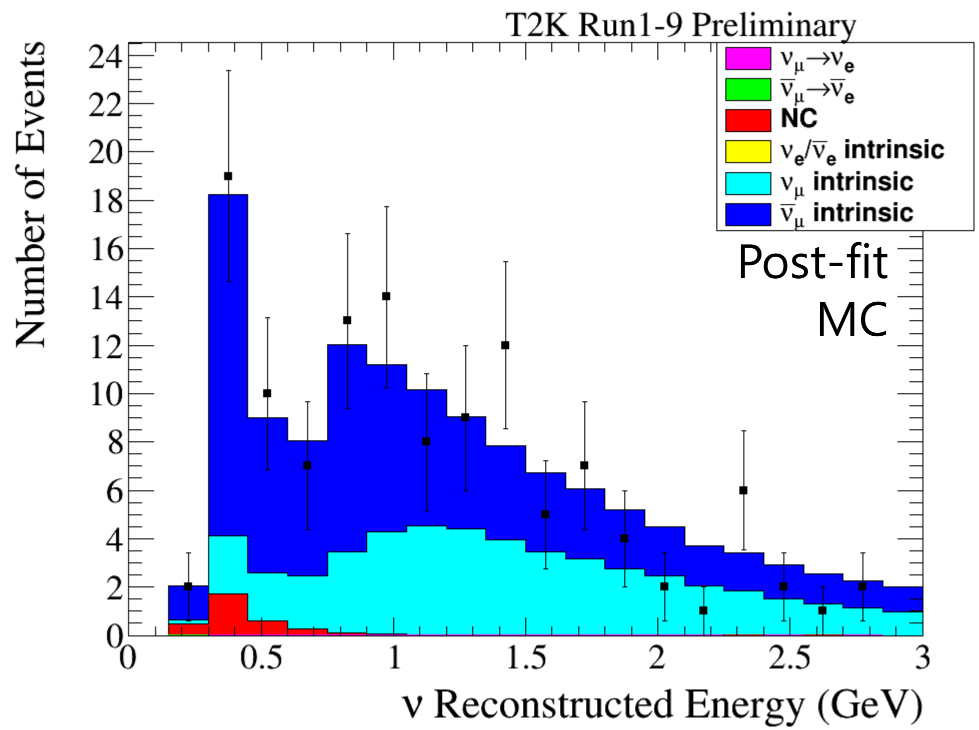
Super-K muon-like samples

FHC: 243 events observed
(~268 expected)

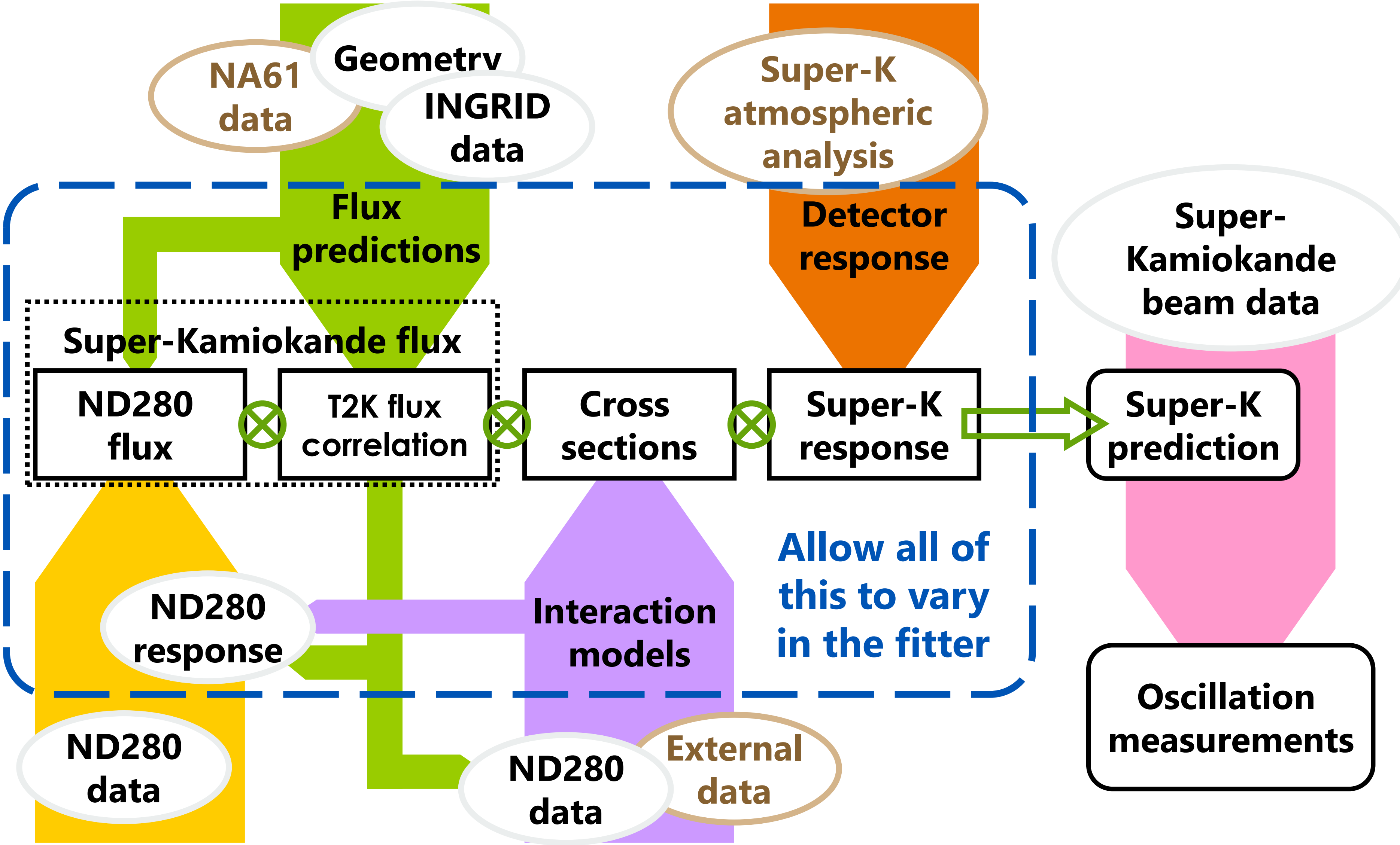
RHC: 102 event observed
(~95 expected)

- 'Wrong sign' ν_μ component is large but counts as signal





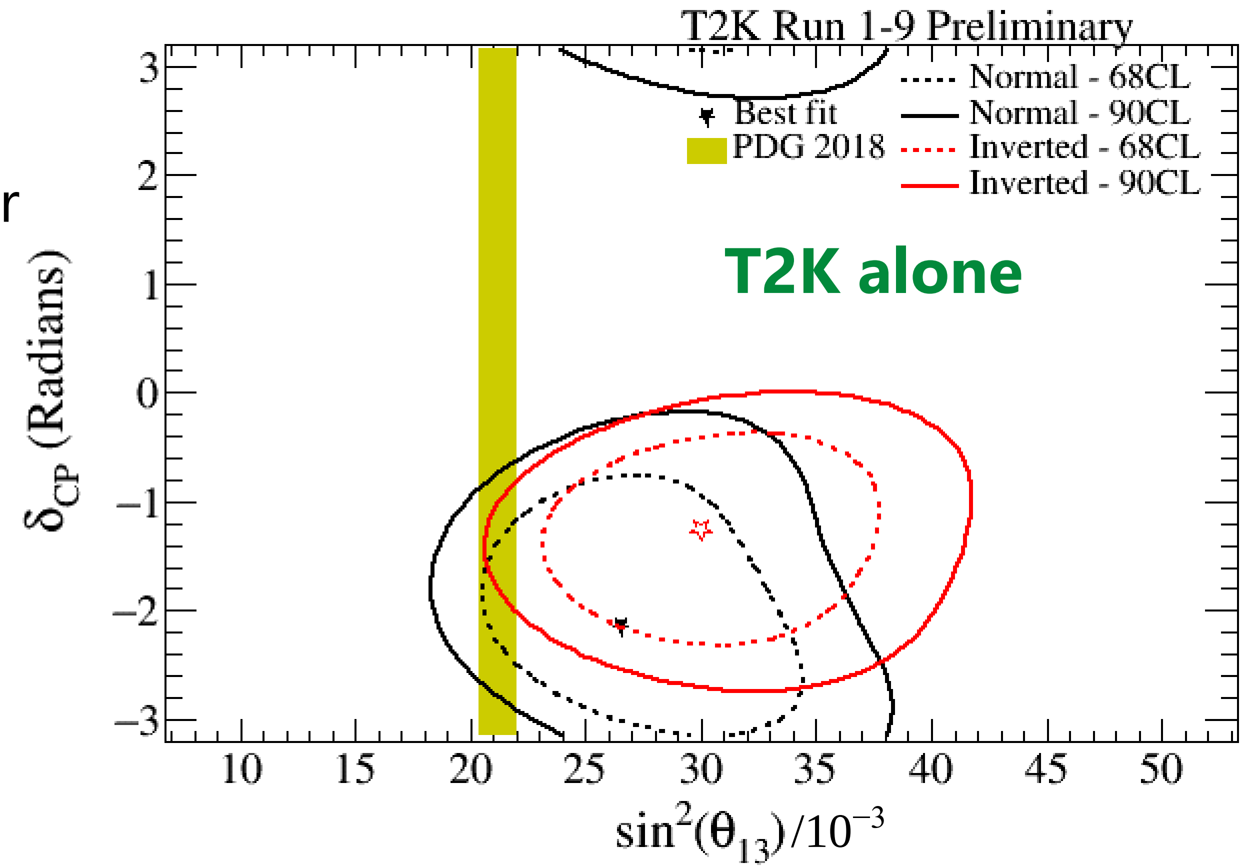
Analysis strategy (Reprise)



Appearance* Results

T2K results in $\sin^2\theta_{13} - \delta_{CP}$ plane are S-curves

- One curve for FHC, another for RHC
- New RHC data improves T2K-only constraints
- Inverted Ordering needs slightly larger $\sin^2\theta_{13}$



*Uses ν_μ disspearance data as well; marginalises over other oscillation parameters

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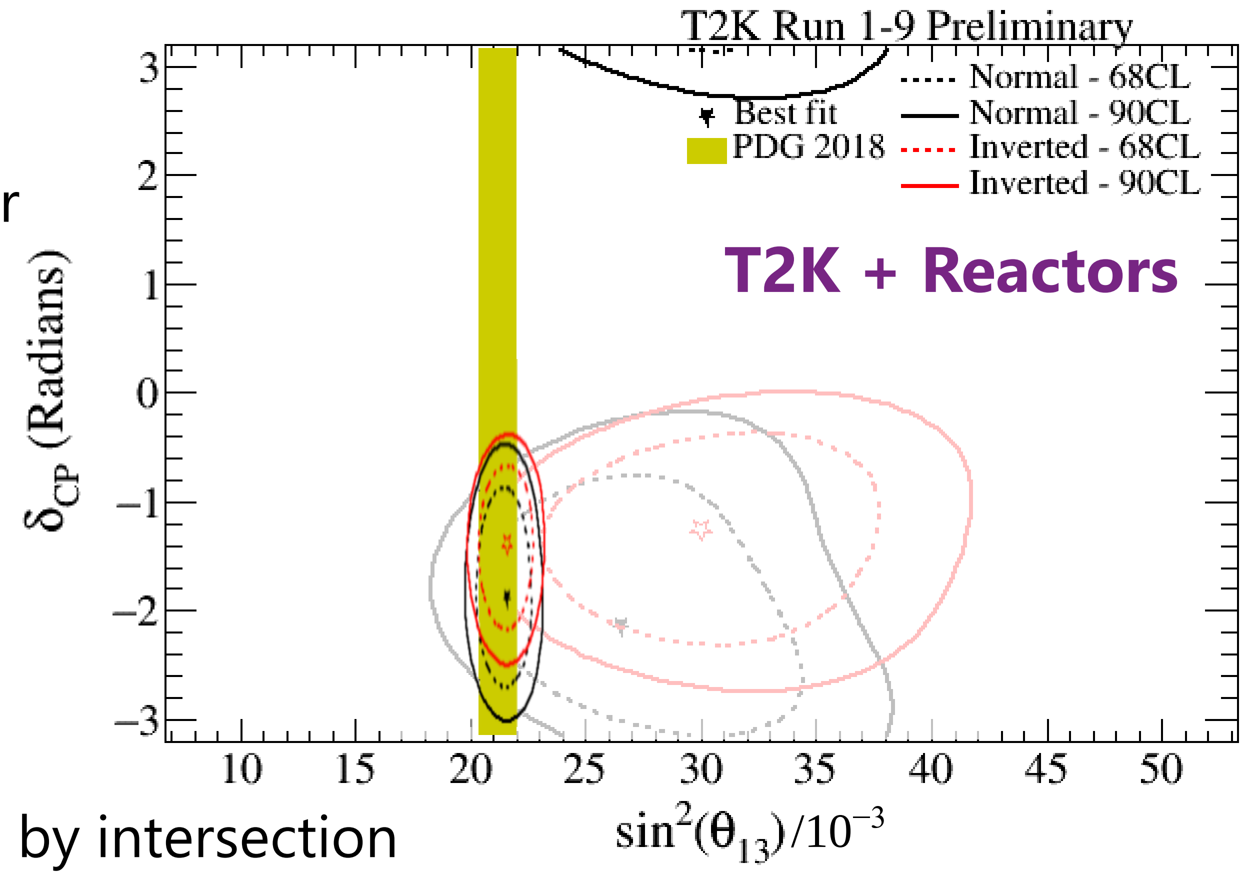
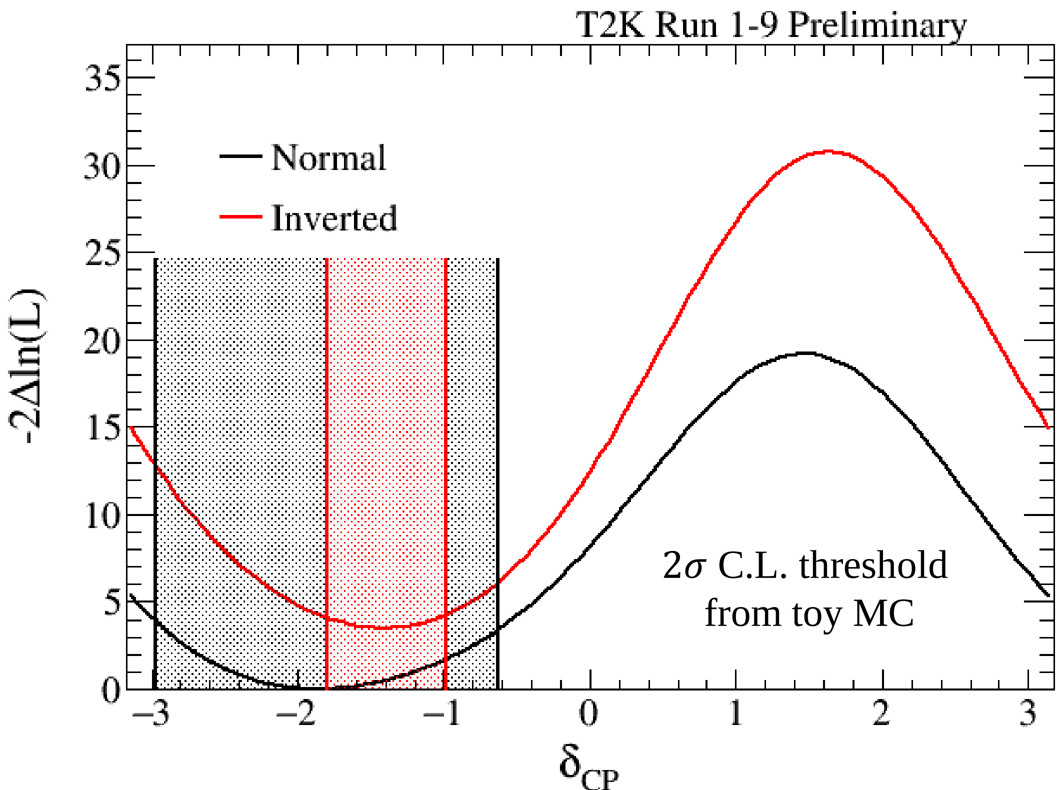


δ_{CP} constraint then improved by intersection with reactor value.

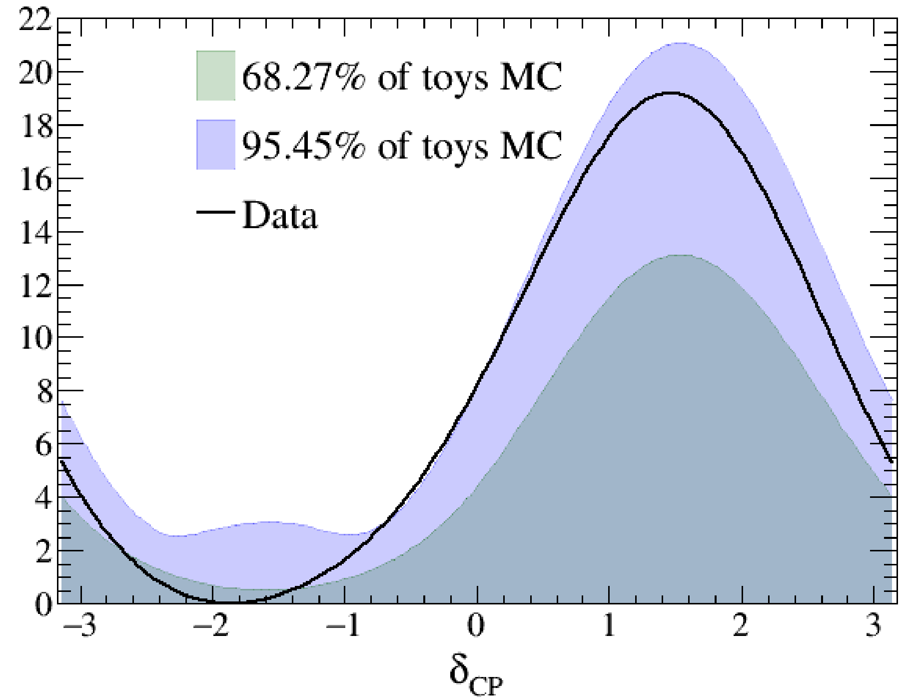
- More tension in Inverted Ordering, leading to stronger than expectation preference for Normal Ordering

Constraint on δ_{CP}

- Marginalise over everything except δ_{CP}
- Compare to frequentist critical values
- Exclude CP conservation at $> 2\sigma$ C.L.
- Inverted ordering only just $< 2\sigma$ C.L.



- Stronger than expected sensitivity



- In toy experiments at best fit, 2σ exclusion of $\delta = \{0, \pi\}$ occurs in 25% of cases

Summary numbers

Almost double exposure since last paper [Phys.Rev.D96, 092006 (2017)]

- FHC (mainly ν): $7.482\times10^{20} \rightarrow \mathbf{1.49\times10^{21}}$ POT
- RHC (mainly $\bar{\nu}$): $7.471\times10^{20} \rightarrow \mathbf{1.63\times10^{21}}$ POT

CP conserving values of δ_{CP} lie outside the 2σ interval

- Best IO point also disfavoured at nearly 2σ

Mass ordering	Best fit δ_{CP}	2σ interval
Normal	-1.89 (-0.58π)	[-2.96, -0.63]
Inverted	-1.38 (-0.44π)	[-1.80, -0.98]

Updated constraints on other parameters:

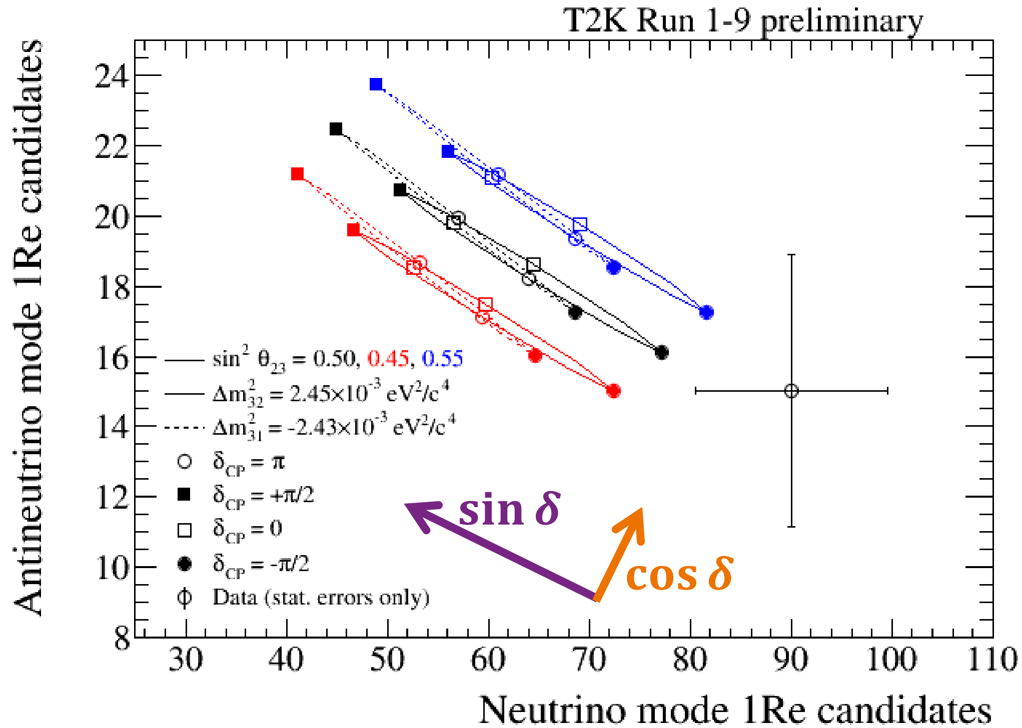
Parameter	Best fit [NO]	1σ interval	Best fit [IO]	1σ interval
$\sin^2\theta_{23}$	0.532	[0.495, 0.562]	0.532	[0.495, 0.567]
Δm^2_{32}	+2.45	[+2.38, +2.52]	−2.51	[−2.58, −2.44]
$\sin^2\theta_{13}$ (T2K only)	2.68×10^{-3}	$[2.23, 3.23]\times10^{-3}$	3.00×10^{-3}	$[2.50, 3.59]\times10^{-3}$

Another way to think about δ_{CP}

T2K measurement of δ_{CP}

T2K observes a strong CP violation signal: more ν_e and fewer $\bar{\nu}_e$ than expected

- Can present that statement graphically: the "Bi-Rate" plot



In this plot T2K data is 'extreme'

- ~1 in 4 chance even for the best fit

Usually focus on **sin δ**, which is proportional to the invariant CP-violation strength.

Mass ordering	Best fit δ_{CP}	2σ interval
Normal	(-0.58π)	$[-2.91,\ -0.64]$
Inverted	(-0.44π)	$[-1.57,\ -1.16]$

Working in sinδ

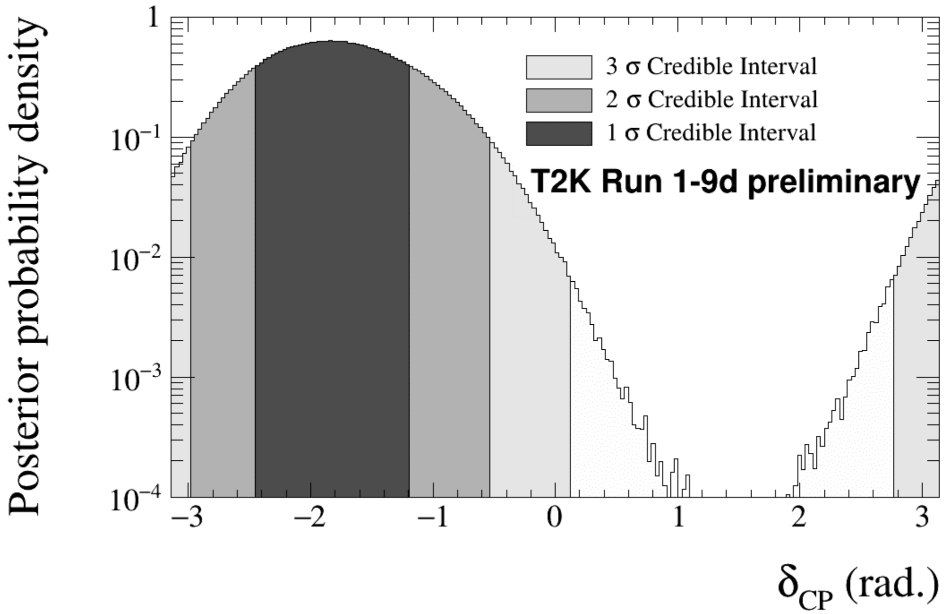
Discomfort with δ as a parameter has been expressed before:

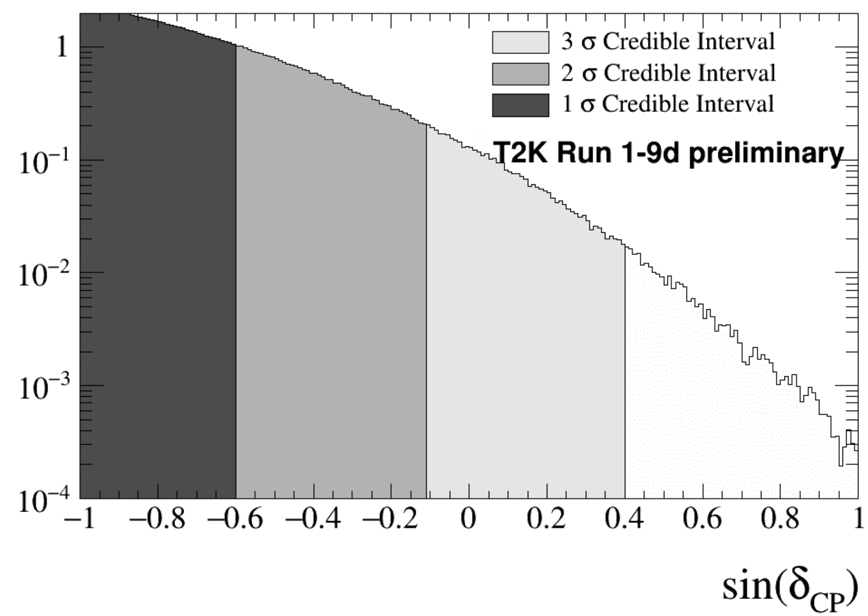
- (Discovery of) CP violation a function of $\sin\delta$, rather than δ
- In the idealised case, $\sin\delta$ is simply related to the asymmetry:

$$A_{\mathrm{CP}} = \frac{\mathrm{N}(\nu_e) - \mathrm{N}(\bar{\nu}_e)}{\mathrm{N}(\nu_e) + \mathrm{N}(\bar{\nu}_e)}$$

[In practice, the matter effect, cross-sections & wrong-sign BGs all have to be accounted for.]

T2K fit in $\sin\delta$ (right) is more Gaussian (parabolic) than when presented in δ





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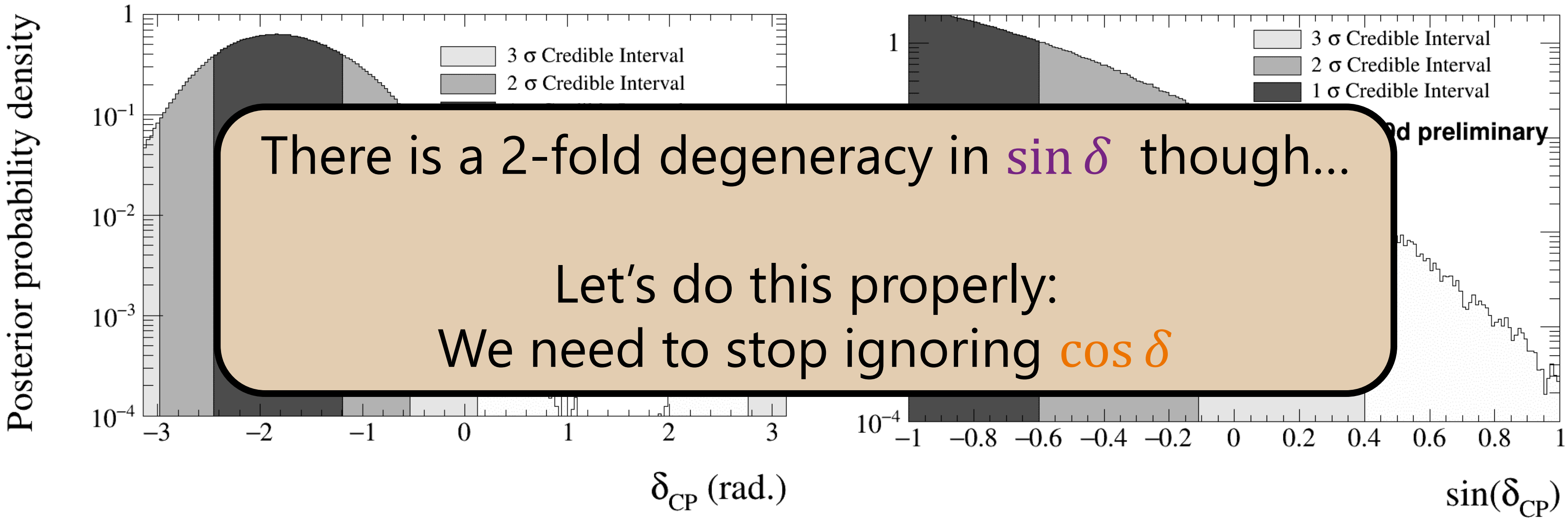


There is a 2-fold degeneracy in $\sin\delta$ though…

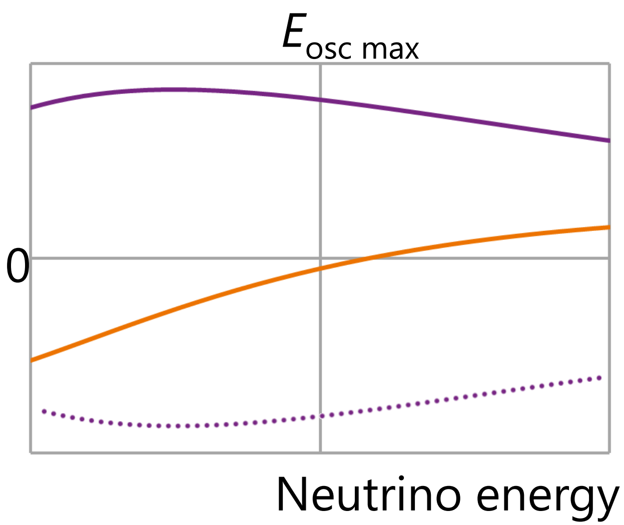
Let's do this properly:
We need to stop ignoring $\cos\delta$

Dependence on δ

The appearance probability can be approximated as:

$$P(\nu_\mu \to \nu_e) \approx T_{\theta\theta}\sin^2 2\theta_{13}\frac{\sin^2([1-A]\Delta)}{[1-A]^2} + T_{\alpha\alpha}\alpha^2\frac{\sin^2(A\Delta)}{A^2}$$
$$- T_{\alpha\theta}\alpha\sin 2\theta_{13}\frac{\sin([1-A]\Delta)}{(1-A)}\frac{\sin(A\Delta)}{A}\sin\Delta\,\boldsymbol{\sin\delta}$$
$$+ T_{\alpha\theta}\alpha\sin 2\theta_{13}\frac{\sin([1-A]\Delta)}{(1-A)}\frac{\sin(A\Delta)}{A}\cos\Delta\,\boldsymbol{\cos\delta}$$

where $\Delta = \Delta m^2_{31}L/4E$ is $\sim\pi/2$ at the oscillation maximum



◄ Terms multiplying the **$\sin\delta$** and **$\cos\delta$** parts are rather different:

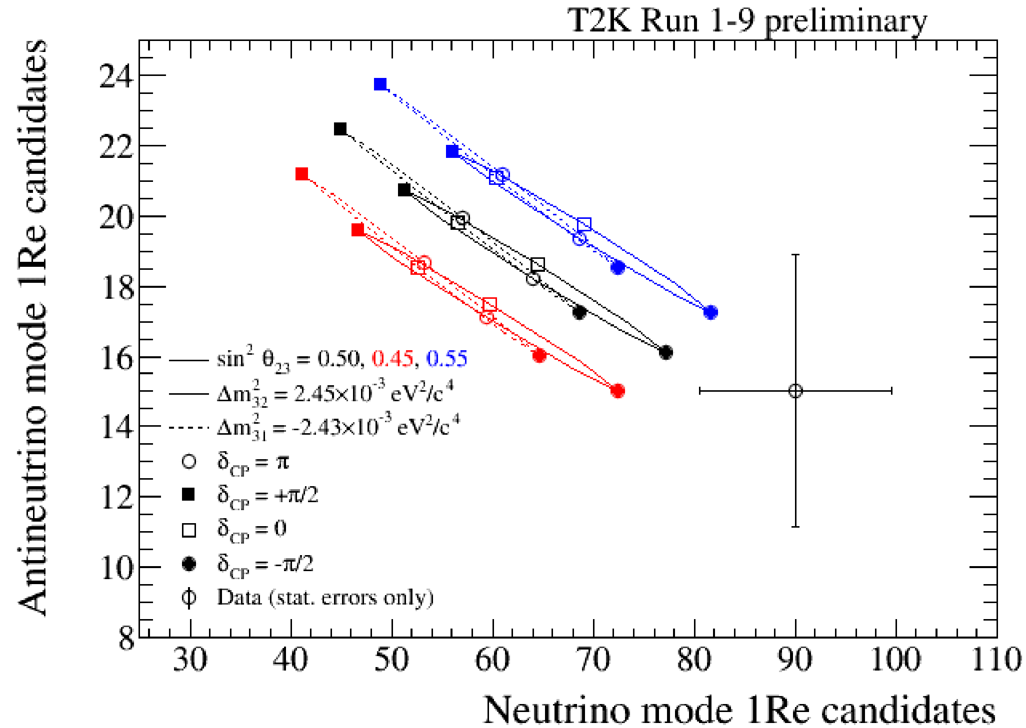
- Factor with **$\sin\delta$** is large, ~uniform and inverts for antineutrinos
- Factor with **$\cos\delta$** is changes sign either side of the oscillation maximum

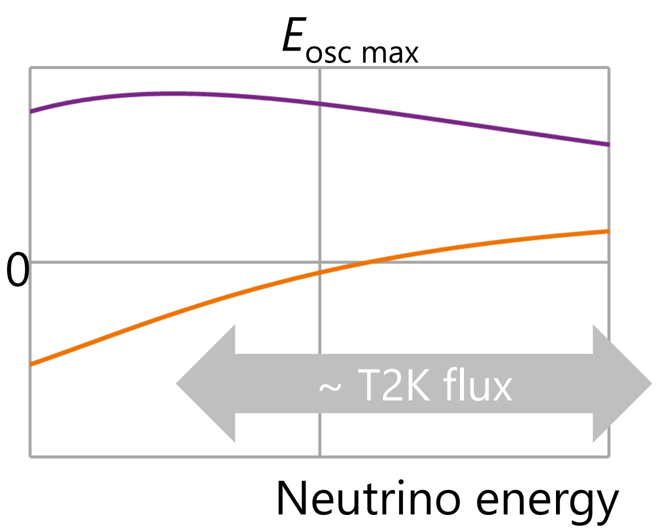
→ **Look for $\cos\delta$ as an energy distortion.**

Looking again at the Bi-rate plot

Can now understand another feature of the Bi-rate plot:

- The ellipses are highly eccentric – almost lines
- Side effect of a well-aligned neutrino spectrum





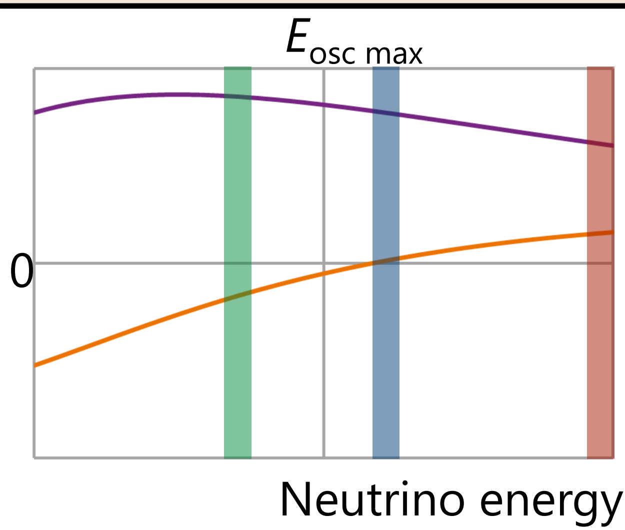
Because T2K flux is about equal above and below $E_{\text{osc max}}$ the **$\cos\delta$** dependence washes out when looking at the total rate

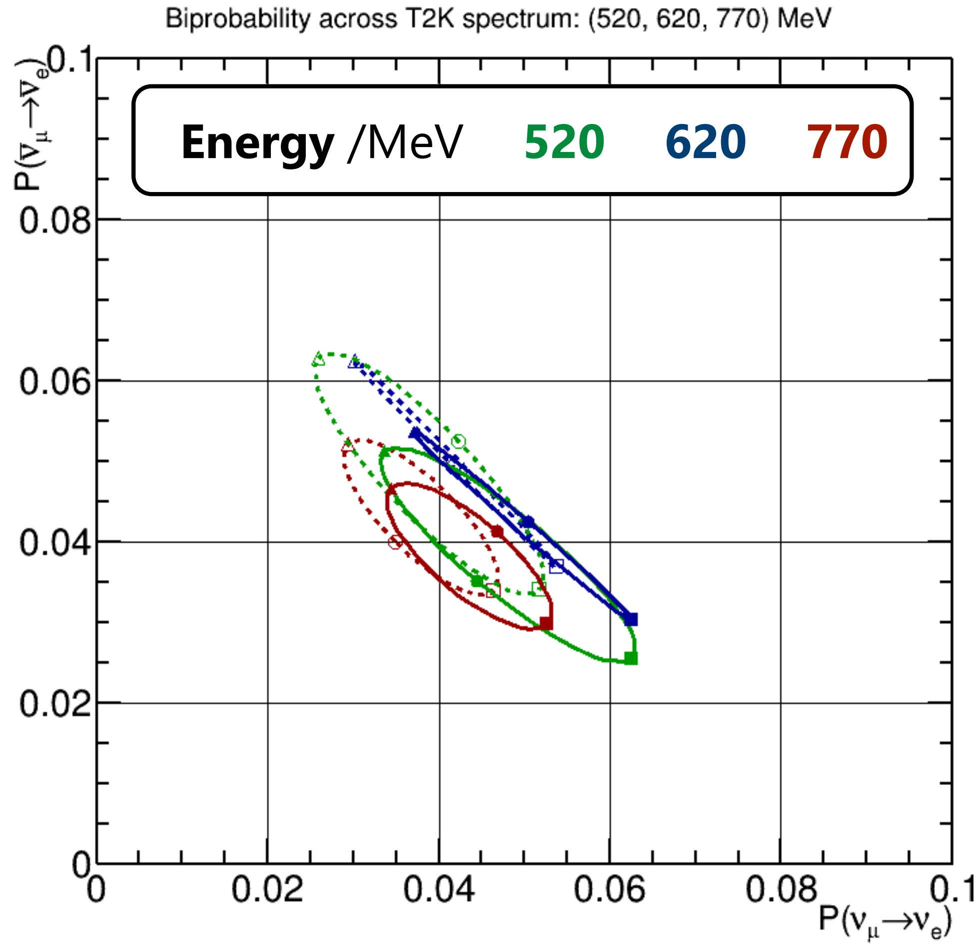
Spectral distortion

If we *don't* integrate the flux and look at fixed energies, **$\cos\delta$** dependence *is* visible.

Total rate measures **$\sin\delta$**

Spectral distortion measures **$\cos\delta$**





Separating the measurement

Goal: measure the two axes of this ellipse *independently*.

- Modify the fit function so that the explicit $\sin\delta$, $\cos\delta$ terms become *independent* free parameters: $(\sin\delta, \cos\delta) \rightarrow (X_s, X_c)$

This is easy to do with the approximate formula:

$$P(\nu_\mu \rightarrow \nu_e) \approx T_{\theta\theta}\sin^2 2\theta_{13}\frac{\sin^2([1-A]\Delta)}{[1-A]^2} + T_{\alpha\alpha}\alpha^2\frac{\sin^2(A\Delta)}{A^2}$$

$$- T_{\alpha\theta}\alpha\sin 2\theta_{13}\frac{\sin([1-A]\Delta)}{(1-A)}\frac{\sin(A\Delta)}{A}\sin\Delta\,\boldsymbol{X_s}$$

$$+ T_{\alpha\theta}\alpha\sin 2\theta_{13}\frac{\sin([1-A]\Delta)}{(1-A)}\frac{\sin(A\Delta)}{A}\cos\Delta\,\boldsymbol{X_c}$$

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$$+ T_{\alpha\theta}\alpha\sin 2\theta_{13}\frac{\sin([1-A]\Delta)}{(1-A)}\frac{\sin(A\Delta)}{A}\cos\Delta\, \boldsymbol{X_c}$$

⚠ This is a non-physical model. It will break unitarity somewhere.

- We introduced an extra degree of freedom!
- In practice this is fine, over a limited range.
- Can use it to investigate the constraints provided by the data.
- Physical solutions are recovered by imposing $X_c^2 + X_s^2 = 1$

A better implementation

Adapting the approx. formula is (almost) 'obvious'.

But T2K analyses do not use this formula.

- Instead, a numerical calculation (`Prob3++`) that is more accurate.
- We want to add the flexibility without also adding approximations.
  - So that the $\{X_c^2 + X_s^2 = 1\}$ sub-space is correct.

Two questions we must resolve:

[1] How to fit $(\Delta m_{ji}^2, \theta_{ij}, X_s, X_c)$, when calculations use $(\Delta m_{ji}^2, \theta_{ij}, \delta)$?

[2] Would a full series include other terms of $f(\delta)$?
e.g. $\sin 2\delta$, $\cos 2\delta$

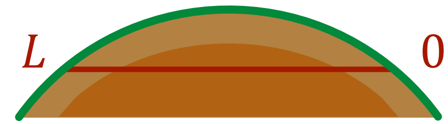
Are there other terms?

arXiv:hep-ph/0207174

Thankfully, this has been studied.

Writing $P(\nu_\alpha \to \nu_\beta) = P_{\alpha\beta}$, Yokomakura *et al.* show that if the matter density is symmetric, $\rho(x) = \rho(L-x)$, then:

$$\boldsymbol{P_{\mu e}(\delta) = A_{\mu e}\cos\delta + B_{\mu e}\sin\delta + C_{\mu e}}$$



This is exactly what we need! This answers the question 2 on the previous slide: there are no other terms to worry about.

With that, we can use the fact that:

$$-\cos(\pi + x) = \cos(x)$$
$$-\sin(\pi + x) = \sin(x)$$

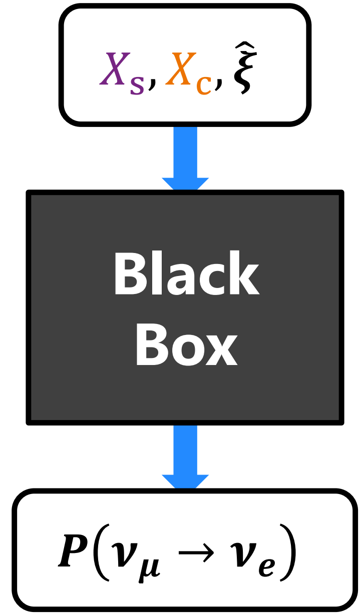
To form linear combinations that let you scale the $A_{\mu e}$, $B_{\mu e}$, $C_{\mu e}$ independently, which solves question 1

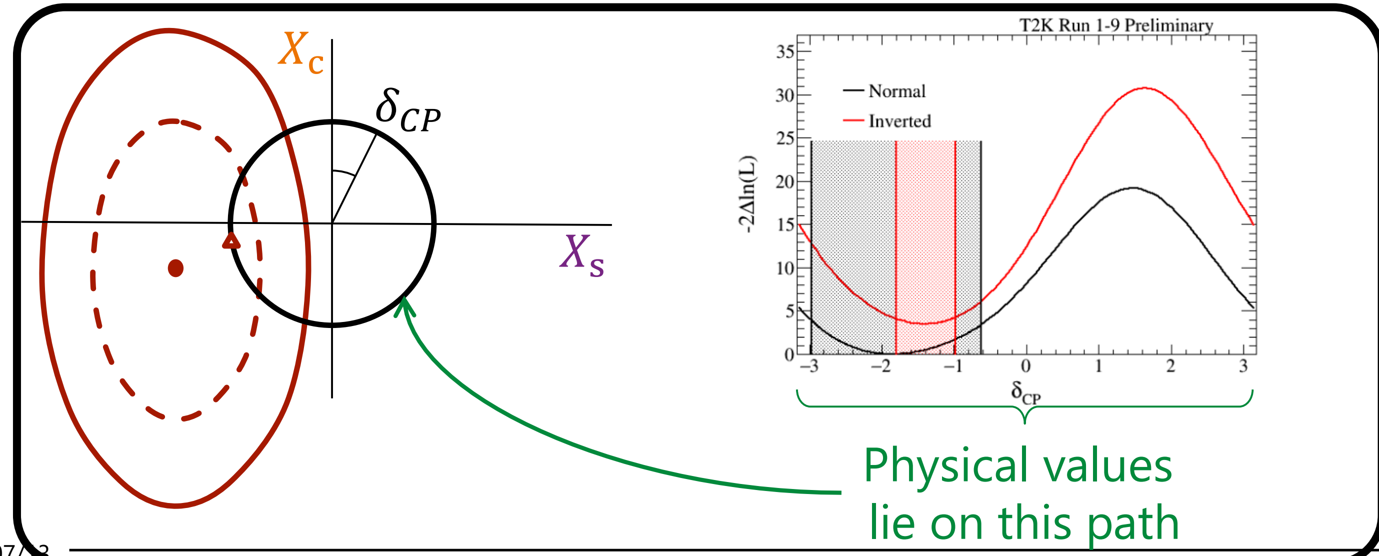
Recipe

To calculate an oscillation probability, given $(X_\mathrm{s}, X_\mathrm{c}, \hat{\vec{\xi}})$

1. Calculate $\rho = \sqrt{X_\mathrm{s}^2 + X_\mathrm{c}^2}$ and $\delta = \mathrm{sign}(X_\mathrm{s}) \arctan \frac{X_\mathrm{s}}{X_\mathrm{c}}$
2. Call the 'normal' probability calculation code at δ and $\pi + \delta$
3. Calculate the extended probability using

$$P'_{\mu e}(X_\mathrm{s}, X_\mathrm{c}, \hat{\vec{\xi}}) = \frac{1+\rho}{2} P_{\mu e}(\delta, \hat{\vec{\xi}}) + \frac{1-\rho}{2} P_{\mu e}(\pi + \delta, \hat{\vec{\xi}})$$





To the future

University of Glasgow

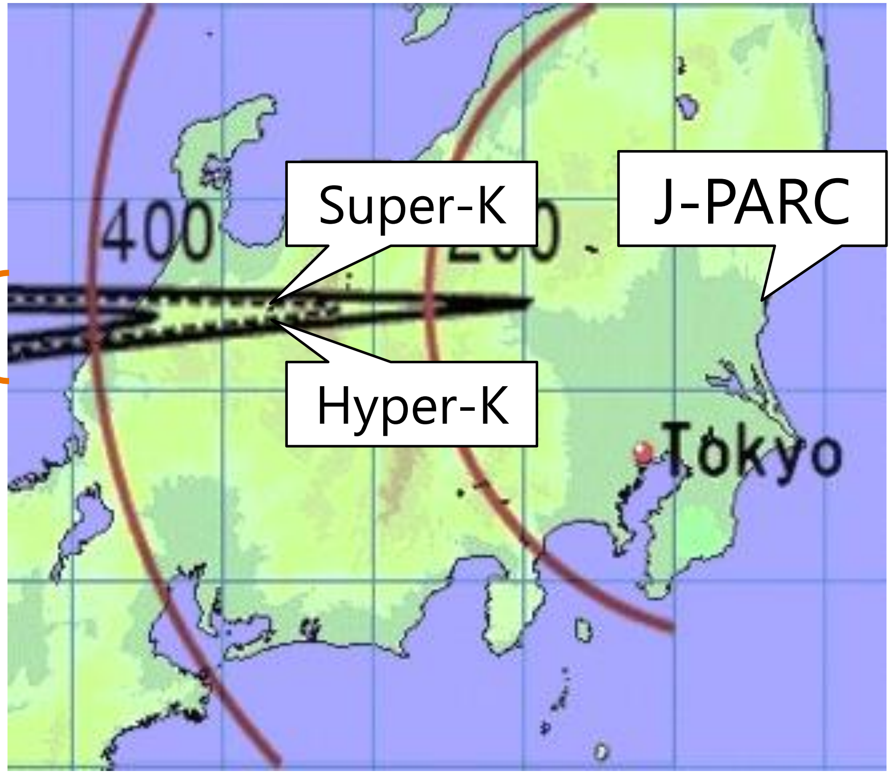
Hyper-Kamiokande

'The future' = Hyper-K

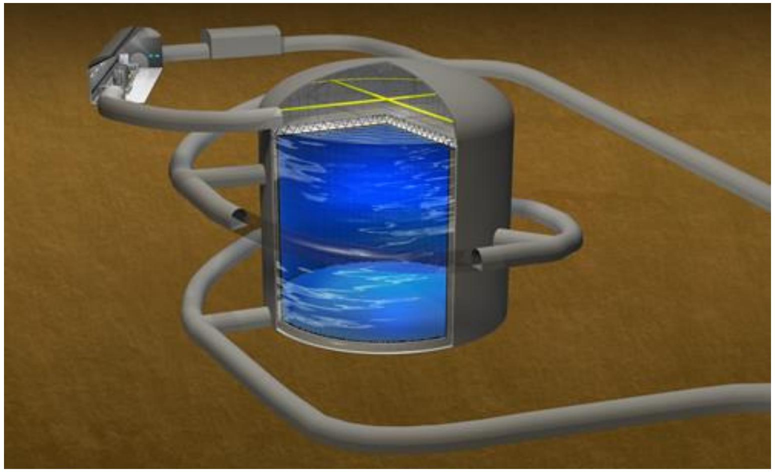
New detector ~10km south of Super-K

- Around 8 times larger (fiducial) mass
- Opposite side of the beam
  but same off-axis angle
  → same neutrino flux.

Beam cone
intersection with
earth surface



At the same time, increase beam power from 0.5 to 1.3MW



Accumulate neutrino events 20× faster than T2K

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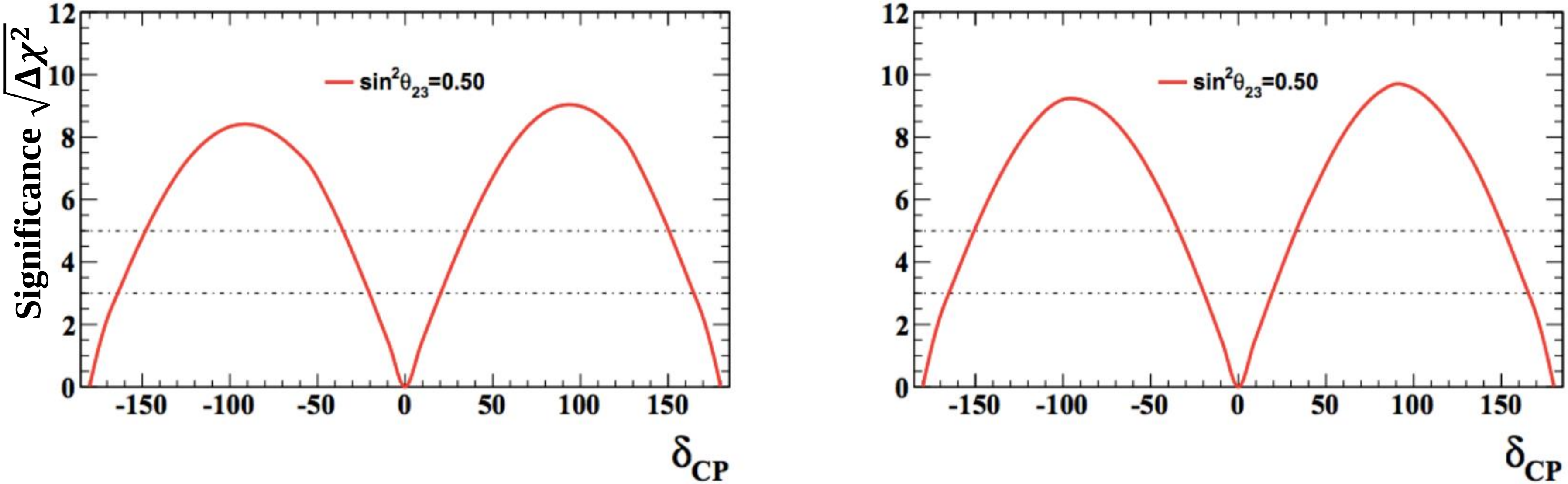
Hyper-Kamiokande

CPV discovery potential



Exclusion of $\delta_{\mathrm{CP}} = 0$ at 3σ for 76% of values

at 5σ for 57% of values

This assumes all values are equally likely (flat prior)

- Values preferred by T2K are where CPV discovery is easiest
- Reasonable to expect Hyper-K could discover CPV very early!

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If CPV is discovered, what next?

If we know $\sin\delta \neq 0$, what should we prioritise?

(In my view) two things are important:

- **Consistency**: Demonstrate PMNS mixing really fits the data
- **Precision**: Measure δ_{CP} to the best possible accuracy

Consistency → embed in models with more freedom, as before

But what about *precision*?

- Predictions of δ_{CP} by many models of neutrino mass → falsify some!
- **'Paradoxical' fact: Precision on δ_{CP} is hardest where discovery of CPV is easiest** (when $|\sin\delta| = 1$)
  - For Hyper-K $\sigma(\delta = 0°) = 7.2°$, and $\sigma(\delta = -90°) = 23°$

'Precision paradox'

Sensitivity to a δ_{CP} is depends on $\frac{dN}{d\delta} \propto \frac{d}{d\delta} P(\nu_\mu \to \nu_e)$

Oscillation probability depends on δ_{CP}, via $\sin\delta$ and $\cos\delta$ terms...

...and $\frac{d}{d\delta}\{\sin\delta, \cos\delta\} = \{\cos\delta, -\sin\delta\}$

So near $\delta_{\mathrm{CP}} = \pm\pi/2$, the $\sin\delta$ term is insensitive to δ_{CP}!

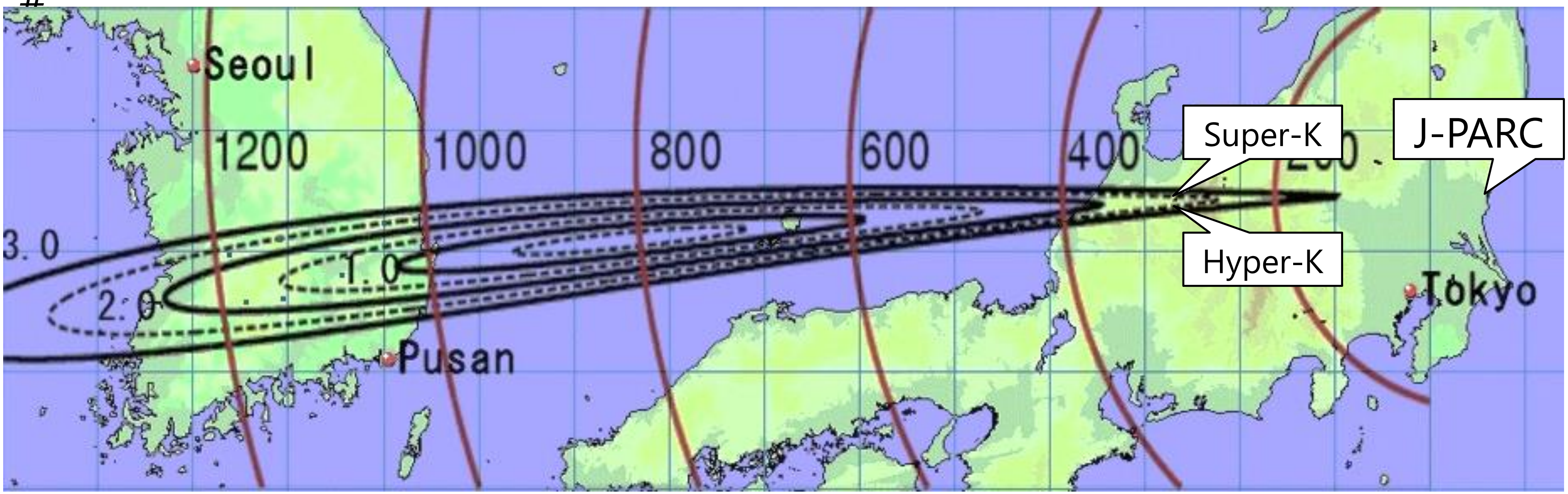
Therefore it becomes necessary to understand (and maximise!) our sensitivity to the CP conserving $\cos\delta$ term

The T2HKK proposal

J-PARC beam centre emerges in the sea between Japan and Korea

- Possible to place a detector in the beam at ~1100km baseline
- Many choices for of axis angle (& therefore beam energy)

#



Can optimise for precision measurement of δ_{CP}!

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T2HKK sensitivity

Remember, sensitivity to **$\cos\delta$** comes from spectrum distortion.

- So wide energy band configuration gives best precision on δ_{CP}
- Three configurations with Hyper-K + 1 more tank
- Not much difference around $\delta = 0$

Around $\delta = 3\pi/2$:

JD×2 simply doubles statistics:

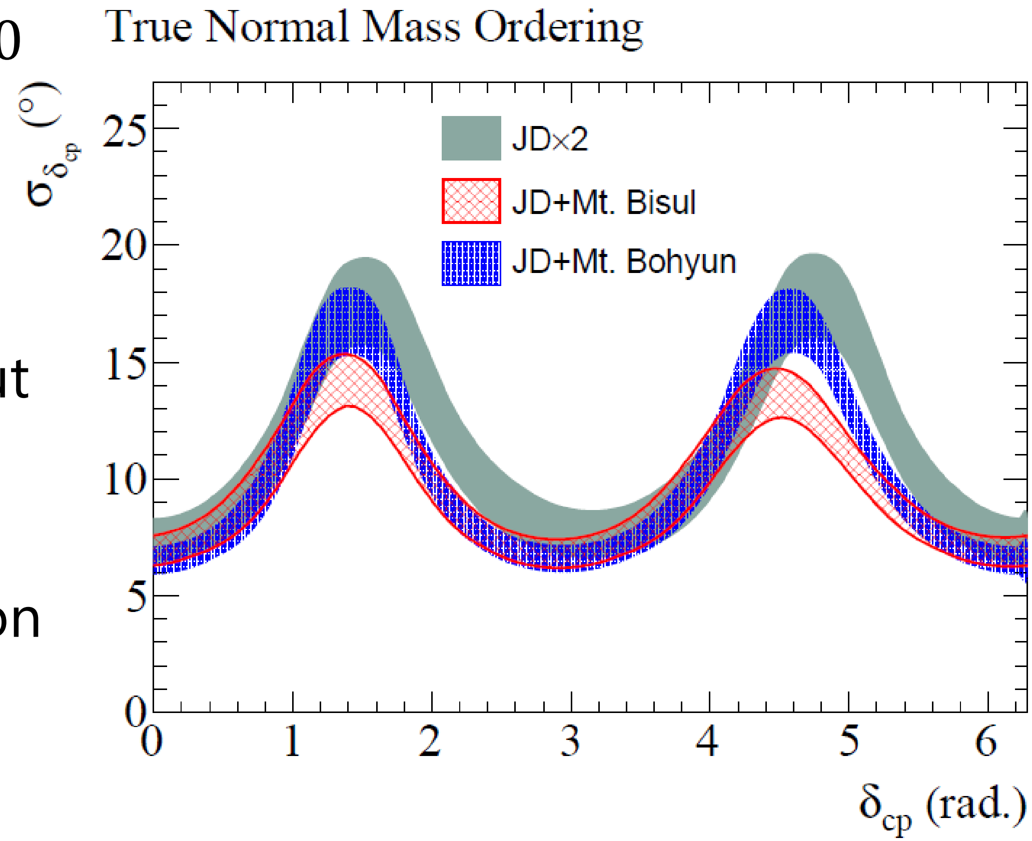
$$\sigma(\delta) \to 19° \text{ (from } 23°)$$

Mt Bohyun is similar to Kamioka, but at 2nd oscillation maximum:

$$\sigma(\delta) \to 17°$$

Mt Bisul is a wide-band configuration (less far off-axis):

$$\sigma(\delta) \to 14°$$



Summary

Measurements of the CP parameter δ are not simple.

- It only shows up as interference between 2 scales of oscillation

Cannot isolate a CP-violating 'signal' on top of a CP-conserving background…

But relaxing the unitarity constraint allow the problem to be recast in this form, along with an independent CP conserving term

Current experiment focus is **very much on the CP violating $\sin\delta$ term**

But if we are lucky, **the next generation will care more about $\cos\delta$**