Lessons from the large N solution of 3 D matter Chern Simons Theories

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Based on

1110.4386 Giombi, S.M. Prakash, Trivedi, Wadia, Yin CS Fermions. Sovability at large N. Non renormalisation of spectrum. Use of lightcone gauge. Exact soln of gap equation and partition function. Proposed bulk dual. First suggestion of bose fermi duality. 1110.4382 Aharony, Gur Ari, Yacobi CS Bosons. Non renormalization of spectrum. Perturbative β function 1112.1016, 1204.3882 Maldacena Zhiboedov Solution for large N correlators using HS sym. 1207.4593 Aharony, Gur Ari, Yacobi Exact solution of 2 and 3 pt functions. Identification of MZ parameters. First concrete duality conjecture and proposal of duality map. 1207.4485 Chang, S.M., Sharma, Yin Vasiliev dual of susy theories including ABJM 1207.4750 Jain, Trivedi, Wadia, Yokoyama Susy thermal partition functions. 1207.4593 Jain, S.M. , Sharma, Takimi, Wadia, Yokoyama Partition function and phase transitions on S^2. Duality of partition functions from level rank duality. 1211.4843 Aharony, Giombi, Gur Ari, Maldacena Yacobi Correct treatment of holonomy in thermal partition function. Duality of thermal partition functions. 1305.7235 Jain, S.M., Yokoyama Flows from susy to quasi fermionic theories 1404.6373, 1505.6371 Jain, Inbasekar, Mandlik, Mazumdar, S.M. Takimi, Wadia, Umesh, Yokoyama Duality of scattering. Modification of crossing symmetry 1506.05412 Bhedotiya, Prakash Four point functions of lightest scalars, first attempt 1507.04378 Gur Ari, Yacoby Re derivation of flows from susy 1507.04546 Gur Ari, Yacoby Exact flows from quasi bosonic to quasi fermionic theories 1511.01902 Radicevic Identification of monopoles as dual Baryons 1512.00161 Aharony Precise conjecture for duality map at finite N and k 1610.08472 Giombi, Gurucharn, Kirilin, Prakash, Skvortsov, Anomalous dimensions of higher spin operators

Based on

1802.04390 Turiaci, Zhiboedov Exact 4 pt fn of lighest scalar 1804.08653 Choudhury, Dey, Halder, Jain, Janagal, S.M., Prabhakar Bosonic Large N solution in Higgs Phase 1808.03317 Aharony, Jain, S.M. Beta function of qasi bosonic theories 1808.04415 Dey, Halder, Jain, Janagal, S.M., Prabhakar Phase diagram from exact quantum effective action for $\bar{\phi}\phi$1808.04415 Dey, Halder, Jain, S.M., Prabhakar Phase diagram of susy theory from exact quantum effective action 1904.07885 Halder, SM Solution of theory in a uniform magnetic field 1906.16342 Jain, Malvimat, Mehta, Prakash, Sudhir Conjecture for anomalous dimension of lighest scalar 1907.11022 Inbasekar, Jain, Malvimat, Mehta, Nayak, Sharma 4 pt operators of light scalars in susy theory 1910.07484 Jensen, Patil Free energies and flows for $N_F > 1$

Introduction

- This talk I will describe the large N 'solution' of a class of 3 dimensional gauge theories, some lessons learnt from this solution, and hopes for the future.
- The theories we study will all be Chern Simons theories coupled to matter.
- To start let us recall that pure $SU(N)$ or $U(N)$ Chern Simons Theory is defined by

$$S_{CS} = \frac{k}{4\pi} \int d^3x \, \mathrm{Tr} \left(AdA + \frac{2}{3} A^3 \right).$$

Topological. Gauge invariant only when k is an integer. Classical equation of motion $F = 0$. No local degrees of freedom. All dof's global or boundary. Interacting but exacty solvable. And exactly solved.

- When the Chern Simons theory above is coupled to matter fields the guage field equation of motion is modified to
$$\frac{k}{2\pi}F = *J$$
where J is the matter contribution to the current. Still no propagating gluons. However 'Coulombic' gauge fields mediate nontrivial interactions between local matter dofs.
- Theories thus obtained nontrivial. Also interesting in several ways.
- First yield several 2 integer parameter families of CFTs labeled by N and k. Conformality easily achieved because the gauge coupling, $1/k$, does not multiply a local gauge invariant counterterm and so does not run.
- Second, massive deformation of these CFTs have particle like excitations which exhibit non abelian anyonic statistics. Intrinsically interesting. Also leads to connections with condensed matter physics.

Introduction

- Third (and related), pure Chern Simons theory enjoys invariance under an intriguing strong weak coupling level rank duality. $N \leftrightarrow k$. Wilson loops transposed under duality. Rows $\leftrightarrow$ columns. Symmetriic $\leftrightarrow$ Antisymmetric Couple matter. In hindsight, natural to wonder if this duality morphs into Bosons $\leftrightarrow$ fermions in matter Chrn Simons theories. We will explain, evidence the answer is yes.
- Fourth these theories admit interesting large N limits; $N \to \infty$, $k \to \infty$, N/k fixed. Atleast some matter Chern Simons theories appear have dual effective classical descriptions in this limit. Famous example: ABJM theory. Other examples: Vasiliev higher spin theories.
- Finally, because special families of these theories turn out to be solvable - and yet not completely trivial - in the large N limit. Point of this talk.

Gauge Theories at large N

- Well known that ‘vector’ like large N limits are easy to solve, but ‘matrix’ like large N limits are typically intractable. Large N $SU(N)$ gauge theories always have gauge bosons which are matrix like fields. Typically hard to solve.
- Exception. Pure Chern Simons theory in $d = 3$ discussed above. As reviewed above, theory solvable at finite N. So also at large N.
- Now consider CS theories coupled to matter in the fundamental rep. Now genuine QFT. Realized in 2011 theory still solvable at large N
- Many quantities in these theories computed leading order in $1/N$ but at all values of the t'Hooft coupling λ in these theories.

Large N solution

- Several of the computations described above have been performed using the following straightforward strategy. One adopts a non covariant lightcone gauge (analytically continued to Euclidean space).
- In this gauge the $A \wedge A \wedge A$ interaction term in the Chern Simons theory vanishes. The sum of all Feynman diagrams that contribute to a given quantity (e.g. finite temperature free energy or corrrelator or quantum effective action for lightest scalar field) is simple enough to be recursively ennumerated, and so can be shown to obey a Schwinger Dyson equation. Similar in spirit to t' Hooft's analysis of large N 2d QCD.
- Quite magically - the integral equations thus obtained always turn out to be exactly solvable. Yield analytic solutions for various quantities.
- This talk: will recall what has been calculated, what has been learnt and speculate on the future.

Theories: Quasi Fermionic

- As explained above, we study CS theory coupled to fundamental matter. Still leaves us with many choices. Best studies theories lie in three classes.
- First the so called 'quasi fermionic theories'

$$SU(N_F)_{(k_F-\frac{1}{2})} + \int \bar{\psi} D_\mu \gamma^\mu \psi + \int m_F^{\rm reg} \bar{\psi}\psi$$

$$U(N_B)_{k_B} + \int D_\mu \bar{\phi} D^\mu \phi + \int \sigma_B \left(\bar{\phi}\phi + \frac{N_B}{4\pi} m_B^{\rm cri} \right)$$

- In massless limit CFTs that reduce to the theory of free fermions / Wilson Fisher Bosons as $|k_F|$ (resp) $|k_B| \to \infty$. Single real relevant operator. Two inequivalent deformations (positive or negative mass). Two massive phases separated by CFT.

Quasi Bosonic Theories

$$U(N_B)_{k_B} + \int D_\mu \bar{\phi} D^\mu \phi + m_B^2 \bar{\phi}\phi + \frac{4\pi b_4}{\kappa_B}(\bar{\phi}\phi)^2 + \frac{(2\pi)^2}{\kappa_B^2}\left(x_6^B + 1\right)(\bar{\phi}\phi)^3, \tag{1}$$

$$S_{RF}(\psi) + \int -J_0^F \zeta + +m_F^2 \zeta - \frac{4\pi x_4}{\kappa_F}\zeta^2 + \frac{(2\pi)^2}{\kappa_F^2} x_6^F \zeta^3 \tag{2}$$

$J_0^F = \frac{4\pi\bar{\psi}\psi}{\kappa_F}$. x_6^B and x_6^F near marginal. Beta functions order $\frac{1}{N}$.

Flows of x_6

The RG flows of x_6 can be plotted as follows

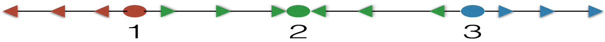


Figure: The points 2 and 1 coincide at $\lambda_B = 0$. They split up at small λ_B. At $\lambda_F = 0$, the point 2 is exactly centred between 1 and 3.

Phase Diagram

FPs 1 and 3 dont always have a stable vacuum (details depend on λ). FP 2 always has a stable vacuum and defines good CFT. 2 relevant operators. At each λ_B get 1 parameter phase diagram. Space of phases topologically a circle. Single second order and sinlge first order phase transition between un Higgsed and Higgsed phases massive phases. 2nd order phase transition governed by quasi fermionic theory.

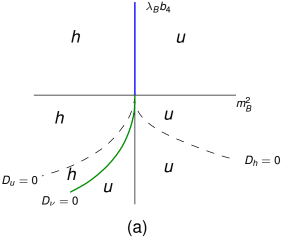
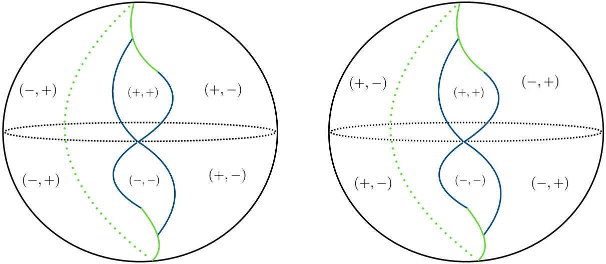


Figure: Blue curve= second order phase transition. Green curve first order phase transition.

Theories: One Boson one Fermion

- Theories describing the interaction of one fundamental boson with one fundamental fermion.
- Lagrangian long and complicated. 3 highly relevant parameters. 4 aproximately marignal parameters. Marginal at large N. Flow at finite N. Full space of flows and fixed points not worked out. Definitely includes $N = 2$ fixed point.
- The $N = 2$ theory has only 3 relevant operators. Elaborate 2 dimensional phase diagram. Recently fully worked out. Generic low energy behaviour massive (4 distinct massive phases). 2 parameter fine tuning allows for massless low energy dynamics, including quasi bosonic CFT dynamics as well as fixed points governing the interaction of one quasi fermionic and one quasi bosonic theory. N_F flavour extensions of all these theories have also been studied to some extent.

Schematic Phase diagram



$0 < \lambda < \frac{1}{2}$ $\qquad$ $\frac{1}{2} < \lambda < 1$

Exact results at the fixed points

- At fixed points: Spectrum of single trace operators. Spectrum very simple. E.g. in the case of quasi bosonic theories, at leading order in large N, the spectrum of single trace operators is given by a current at spin s and dimension $\Delta = s + 1$ for $s = 0, 1 \ldots \infty$. Anomalous dimensions at $\mathcal{O}(1/N)$ also known in most cases.
- Currents with $\Delta = s + 1$ are necessarily conserved. Anomalous dimensions - currents not quite conserved. Obey relations of schematic form

$$\partial . J = \frac{1}{N} JJ + \frac{1}{N^2} JJJ$$

  Equations can be thought of as the large N classical nonlinear equations of motion for the single trace operators J. Explicit form of these 'eoms' known. Quite simple.
- All 3 point functions and some four point functions of J also known.

Computations in massive phases

- Some correlators (mainly two point functions) also known in massive phases.
- Thermal partition function of the theory on S^2 computed in both massive phases and at conformal points. Elaborate structure of finite temperature phases involving intriguing dynamics of the holonomy.
- Really new physics in massive phases. Spectrum of non abelian particle like excitations. Elementary excitations of this sort created by the bare boson an fermion fields. The (generally non integer) spins of all such excitations known (even at finite N)
- The non abelian particles above can be scatttered against each other. Exact results for large N S matrices as function of λ. Results quite rich. Sometimes display bound state poles. Very surprising and as yet incompletely understood modification of crossing symmetry.

What has been learnt?

- Of course all these computations have taught us a great deal about the theories. Possibly the best understood class of non abelian gauge theories.
- However exact solutions are most interesting when their study yields lessons that then apply to a larger class of (typically non solvable) theories.
- In the next few slides list some lessons of this sort that have been learnt from the study of these theories.

Bose Fermi duality

- The study of these theories has led to an initially surprising discover: that of a precise QFT Bose-Fermi duality in three dimensions.
- Notation

$$\kappa_B = -\mathrm{sgn}(\mathrm{k_B})(N_B + |k_B|), \quad \lambda_B = \frac{N_B}{\kappa_B}, \quad (\text{and} \quad B \leftrightarrow F)$$

- Conjectured duality map (quasi fermionic)

$$k_B = -\mathrm{sgn}(\mathrm{k_F})\mathrm{N_F}, \quad \mathrm{N_B} = |\mathrm{k_F}|, \quad \mathrm{m_B^{cri}} = \left(\frac{|\mathrm{k_F}| + \mathrm{N_F}}{\mathrm{k_F}}\right) \mathrm{m_F^{reg}}$$

- Equivalently

$$\kappa_B = -\kappa_F, \quad \lambda_F = \lambda_B - \mathrm{sgn}(\lambda_B), \quad -\lambda_B m_B^{\mathrm{cri}} = m_F^{\mathrm{cri}}$$

How duality works

- The evidence for Bose-Fermi duality at large N is extremely detailed and overwhelming. All the quantities described above have been computed independently on the two sides of the duality. Even though the Bosonic and Fermionic computations look very different at intermediate steps, final physical answers all agree perfectly.
- Duality works in this precise and detailed way both at fixed points and in massive phases. Just to give a sense of how things work I present a brief qualitative discussion of massive phases and their excitations. For simplicity we focus on the simplest case of quasi fermion dualities.

How duality works

- Consider the $SU(N_F)_{k_F-\frac{1}{2}}$ fermion theory with $k_F m_F > 0$, dual to $U(n_B)_{k_B}$ theory with $m_B^{\rm cri} > 0$). The Fermionic/Bosonic low energy effective theories are $SU(N_F)_{k_F}$ or $U(N_B)_{k_B}$ pure CS theories which are level rank dual to each other.
- Now consider the same theories with $m_F k_K < 0$ dual to $m_B^{\rm cri} < 0$. The two low energy theories are now the $SU(N_F)_{k_F-1}$ or $U(N_B-1)_{k_B}$ pure CS theories (the bosonic rank changes because of the Higgs mechanism) which are once agian level rank dual to each other.
- While the elementary fermionic excitations in both phases are simply fermion quanta, there is a sense in which their nature changes dramatically. This is because the spin of a free mass m_F fermion is $\frac{m_F}{2}$. On the Bosonic side the change is more dramatic -the free spins change from 0 on one side to $\rm sgnk_B$ for W_μ and Z_μ bosons on the other side.

Matching of spins of Excitations

- Do spins of excitations match? Naively no. However key point. Statistical (Chern Simons) contribution to spin. Sort of like $E \times B$ Saha effect. $s_{stat} = \frac{c_2(R)}{2\kappa}$
- Physical requirement

$$s^B_{intrinsic} + s^B_{stat} = s^F_{intrinsic} + s^F_{stat} \tag{3}$$

- Group theory result:

$$s^F_{stat} - s^B_{stat} = \frac{\mathrm{sgn}(k_B)}{2} = -\frac{\mathrm{sgn}(k_F)}{2}$$

- Follows that (3) works provided

$$s^B_{intrinsic} = \frac{1}{2}\left(\mathrm{sgn}(m_F) - \mathrm{sgn}(k_F)\right).$$

But easy to see its true in both phases. Z_μ Boson...

Statistics?

- How about statistics? Bose/Fermi? Can some sort of Chern Simons 'anyonic effect' come in to convert Fermi to Bose Statistics?
- Lets first look at scattering. 4 inequivalent channels.

$$FF \to FF\,(sym), \quad FF \to FF\ as, \quad FA \to FA\,(adj), \quad FA \to FA\,(sing$$

- Effective anyonic phase $\mathcal{O}(1/N)$ in first three channels. Non anyonic channels.
- Statistics shows up in scattering of identical particles. *FF* channel. Non anyonic. Sharp puzzle?
- Detailed computation. Agreement between Fermioni FF scattering in the sym channel and Bosonic scattering in the as channel. And vica verca. Should have expected from match of Wilson lines under level rank duality.
- Lesson. Statistics is made up for by extra signs under the permutation group from gauge indices.

Duality at finite N and ...

- At large N there is now fantastic evidence for duality from detailed agreement of complicated formulae independently computed on the two sides of the duality.
- Duality believed to persist at finite N. One argument. Duality can be understood as following from Giveon Kutasov duality of the $\mathcal{N} = 2$ theory plus RG flow. Another reason: finite N matching of discrete anomalies. Yet another: matching with expectations of condensed matter physicists at $N = 1$.
- Discovery has inspired the intensive study of the so called 'web of dualities' of large classes of 3d non supersymmetric gauge theories. Much improved understanding of 3d gauge theories in general.
- Great example of a lesson learnt from exact solutions spreading beyond solvable models.
- Next couple of slides describe other less well known - and less well understood -qualitative lessons.

Qualitatively new S matrix behaviour

- Explicit computations of S matrices - computed by summing diagrams and implementing the LSZ procedure - yield a surprise. The usual rules for crossing symmetry appear to be modified.
- The puzzle happens in the fourth channel which is effectively anyonic
- We now explain why we should not have been too surpried at this.

Non Relativistic Scattering

- The S matrix for non relativistic particles interacting via Chern Simons exchange was worked out in the early 90s, most notably by Bak, Jackiw and collaborators.
- Main result. Consider the scattering of two particles in representations R_1 and R_2, in exchange channel R. It was demonstrated that the S matrix equals the scattering matrix of a $U(1)$ charged particle of unit charge scattering off a point like flux tube of magnetic field strength $\nu = \frac{c_2(R_1)+c_2(R_2)-c_2(R)}{\kappa}$. ν is the phase that is $\mathcal{O}(1/N)$ in the non anyonic channels, but is of magnitude $|\lambda|$ in the singlet channel.
- This quantum mechanical S matrix was computed originally by Aharonov and Bohm and generalized by Bak and Camillio to take account of possible point like interactions between the scattering particles.

Non Relativistic Scattering Amplitude

- $$T_{NR} = -16\pi i c_B \left(\cos\left(\pi\nu\right) - 1\right)\delta(\theta) + 8ic_B \sin(\pi\nu)\mathrm{Pv}\left(\cot\frac{\theta}{2}\right)$$
$$+ 8c_B|\sin\pi\nu|\frac{1 + e^{i\pi|\nu|}\frac{A_{NR}}{k^{2|\nu|}}}{1 - e^{i\pi|\nu|}\frac{A_{NR}}{k^{2|\nu|}}},$$
$$A_{NR} = \frac{-1}{w}\left(\frac{2}{R}\right)^{2|\nu|}\frac{\Gamma(1+|\nu|)}{\Gamma(1-|\nu|)}. \tag{4}$$

- Striking feature: term proportional to $\delta(\theta)$. Not analytic. Simple physical interpretation: interference between wave above and below the flux tube.

Relativistic Scattering: Modified Crossing

- Physical interpetation simple and clear. Makes no reference to non relativistic limit so should also apply relativistically.
- Then however have a puzzle. Delta function different in different channels. How consistent with crossing?
- Results of explicit large N computations

$$T_{sing} = \frac{\sin(\pi\lambda_B)}{\pi\lambda_B} T^{ac}_{sing} - i(\cos(\pi\lambda_B) - 1) I(p_1, p_2, p_3, p_4)$$

where T^{trial}_S is the singlet amplitude one obtaines from naive analytic continuation

- Challenge: Find the finite N version of this formula (all channels anyonic then). Perhaps by examining and correcting usual proof of crossing? Perhaps by summing IR 'divergences'?
- Likely applications to scattering in ABJM theory.

Thermal Partition Function

- Large N computations of the thermal parition function of matter CS theories lead to a final answer of the following form

$$\sum_{\lambda_i=\frac{2\pi n_i}{\kappa}} dU e^{-S_{eff}(U)}$$

- Discretized version of more usual holonomy integral. Usual holonomy integral restricts to Gauss Law singlets. What does this discretized integral do?
- Simple mathematical fact.

$$\sum_{\lambda_i=\frac{2\pi n_i}{\kappa}} dU \left(\chi_{R_1}\chi_{R_2}\cdots\chi_{R_N}\right)$$

  counts number of singlets in the WZW fusion of R_1, $R_2 \ldots R_n$. Number smaller- sometimes much smaller - than number of Gauss Law singlets.

WZW singlets?

- Suggests that there is a description of the Hilbert Space of matter CS theories which involves projecting onto something like WZW singlets. Is this true? Can it be made precise (e.g. in the context of the superconformal index)
- Is this WZW cut down the correct qualitative explanation for the $\frac{1}{\sqrt{\lambda}}$ scaling of the ABJM finite temperature partition function?
- These are some of the lessons learnt. Roughly speaking, every time we have performed a qualitatively new computation have found a new surprise. Probably lessons in store for us. One way to uncover these is to perform new innovative physically motivated computations. To end this talk I describe the results of one new computation of this sort; the Greens funtion of RF theories in a background magnetic field (coupling to $U(1)$ global symmmetry.

A recent computation

- The RF theory enjoys invariance under global rotations $\psi -> e^{i\alpha}\psi$. The thermal partition function computations performed for these theories have been generalized to include a chemical potential for this symmetry. Physically quite interesting. Fermi sea vs Bose Condensate.
- At the technical level the chemical potential is a constant background for A_0 the non dynamical gauge field that couples to the global symmetry above. Easy to perform computation with A_0. Note preserves same symmetries as the thermal circle.
- There is another background for A_μ that preserves all the same symmetries. This is a uniform spatial magnetic field. Quite remarkably turns out to be possible to do some exact computations with mangetic field and chemical potential.

What we compute

- We would like to compute something like the colour averaged fermion two point function

$$\alpha(x,y) = \sum_m < \psi(x)_m \bar{\psi}(y)^m b >$$

  However this quantity has two issues. First it is not invariant under background gauge transformations. Second it is not invariant under $SU(N)$ gauge transformation.

- The first issue is easily dealt with. We make the propagator gauge invariant by dressing it with a background open Wilson line. In the background rotationally invariant gauge this amounts to moving to the variable $\alpha_R(x,y)$ defined by

$$\alpha(x,y) = e^{-i\frac{b}{2}(x^1y^2 - x^2y^1)}\alpha_R(x-y) \tag{5}$$

What we compute

- While we could, in principle, deal with the second issue in the same way, the introduction of a dynamical Wilson line makes all computations prohibitively difficult. Instead we just live with the fact that the quantity we study is (dynamical) gauge non invariant. Consequently much of what we compute will not be physical. As in S matrix theory, however, the locations of the poles of our two point functions are presumably physical. After solving for α_R we focus on this physical information.
- It is not too difficult to set up the Schwinger Dyson equations for $\alpha_R(x, y)$

Gap equation in terms of the star product

- The gap equation may be rewritten as

$$\left(\gamma^\mu D_\mu^{(x-y)} + m_F\right)\alpha_R(x-y) + (\Sigma_R *_b \alpha_R)(x-y) = -\delta^3(x-y)$$
$$\Sigma_R(x-y) = -\frac{N}{2}\gamma^\mu \alpha_R(x-y)\gamma^\nu G^A_{\mu\nu}(x-y) \tag{6}$$

  where the twisted convolution - or star product - $*_b$ is defined as

-

$$(A *_b B)(x-y) = \int d^2w A(x-w) e^{-i\frac{b}{2}\epsilon_{ij}(x^i-w^i)(w^j-y^j)} B(w-y) \tag{7}$$

  $*_b$ is simply the (Fourier Transformed) formula for the famous noncommuative but associative Moyal Star product. First line in (6) simply definition of propagator in terms of self energy. Second line in (6) is the self consistency equation for self energy.

Solvability of the gap equation

- Quite incredibly it turns out to be possible to analytically solve these gap equations completely - in terms of one constant (roughly the renormalized mass of the fermion) that is given as the solution to a single explicit but complicated equation.
- Turns out that this dressed propagator has only pole type singularities in the time frequency ω. At zero chemical potential the poles in this dressed propagator turn out to occur at $\omega = \chi^+(s, \nu)$ and $\omega = -\chi^-(s, \nu)$
- $$(\chi^+_{s,\nu})^2 = m^2 + 2b\left(\nu + \frac{1}{2} - \frac{\mathrm{sgn}(gs)}{2}\right) \quad (8)$$
  $$(\chi^-_{s,\nu})^2 = m^2 + 2b\left(\nu + \frac{1}{2} + \frac{\mathrm{sgn}(gs)}{2}\right) \quad (9)$$
  Here ν is an integer and
  $$g = 2, \quad s = \frac{\mathrm{sgn}(m_F) - \lambda_F}{2}$$

Structure of Poles

- s above is the effective spin of a fermion. The Chern Simons coupling renormalizes the free spin $\frac{\mathrm{sgn}(m)}{2}$ by $-\frac{\lambda_F}{2}$.
- The energy spectrum above is simply the naive generalization of the Landau Level spectrum to a 'free relativistic field of spin s' (the shfits in energy proportional to s can be thought of as a consequence of a $B.S$ coupling). The effective mass m is determined in terms of the fermion pole mass by a self consistency equation.
- Result more interesting when we turn on a chemical potential. Answer similar in form. However the effective value of m depends on the value of the chemical potential, more precisely on which 'gap' it lies in, or equivalently how many Landau Levels are filled.

Turning on a chemical potential

- μ_{up}^{M} is the lowest potential upto which it is consistent to assume that the M^{th} Landau Level is totally filled. On the other hand μ_{down}^{M-1} is the highest potential upto which it is consistent to assume that the M^{th} Landau Level is completely empty.
- It follows that

$$\delta\mu^{M} = \mu_{up}^{M} - \mu_{down}^{M}$$

  is of great physical significance. When $\delta\mu^{M}$ is positive it follows that the M^{th} Landau Level - which is perfectly degenerate in the free theory - broadens out as you start filling the level.
- When $\delta\mu^{M}$ is negative, on the other hand we have a 'first order phase transition' type situation. In this case there is a range of μ over which both solutions (completely filled and completely empty) are consistent. The system will pick out the state with the lower free energy. Physical interpretation more mysterious.

Gaps in Landau Levels

- Actually determining $\delta\mu^M$ requries explicit solutions for the quantities c_F^M. In general we have only been able to solve these equations numerically. Our paper contains lots of plots.
- Easy to find $\delta\mu^M$ perturbatively in λ_F. Our result

$$\delta\mu^M = -2\lambda_F b \left(1 - \frac{m_F}{\sqrt{m_F^2 + 2b(M+1)}}\right) + \mathcal{O}(\lambda_F^2) \quad (10)$$

- Note the sign of this quantity is opposite to that of λb. Note also that the bracket behaves very differently at small b depending on the sign of m_F.
- Many generalizations of this computation: e.g. finite temperature free energy - may well be possible to compute.

Within the Band

- Let us focus on a situation in which $\delta\mu^M$ is positive. What is the solution to the fermion propagator when μ lies in this band? More delicate question. We are thinking about it. Unlikely that the analytic singularities in ω will be only poles.
- The expectation from condensed matter physics is that this interacting fermion problem should display the fractional quantum hall effect. Should, for instance, expect to see a mass gap at $1/3$ filling. Would be fantastic to be able to see anything like this in this exactly solvable problem. Not clear it will be easy (perhaps the mass gap is of order e^{-N} in this problem?). At any rate it sounds like a very interesting question that one can atleast begin to address within a cleanly posed relativistic field theory that appears both nontrivial and remarkably easy to just solve.

Summary

- Large N Chern Simons theories with fundamental matter are remarkable theories. They are simple enough that they can be exactly solved. But they also exhibit a fair degree of richness in their dynamics.
- The study of these theories has taught us many lessons that go beyond these theories. Some of these lessons - like duality at finite N - are well understood
- Other hints of interesting structure- like the modification of crossing transformation rules and the WZW singlet condition - have not yet been completely worked out.
- Every qualitatively new computation in these theories has uncovered new surprises and new general lessons. One can hope that this trend will continue - for instance with the new computations in background magnetic fields.
- I have unfortunately not had the time to touch on my original motivation for looking at these theories - namely of the study of bulk dualdescriptions to these theories. For