# Applications of the Double Copy to Gravity

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UK Annual Theory Meeting

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# Outline

1. Complications with gravity perturbation theory.
2. Antidotes:
   - — Unitarity method.
   - — Double copy and color-kinematics duality.
3. Applications:
   - Web of theories.
   - Nonrenormalizability properties of gravity.
   - Gravitational wave physics.
4. Outlook.

# Complications with Gravity

# Perturbative Gravity

**Consider the Einstein gravity Lagrangian**

curvature   Flat-space metric

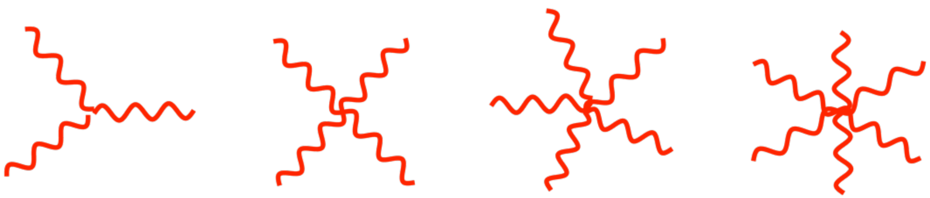
$$L_{\text{gravity}} = \frac{2}{\kappa^2}\sqrt{-g}\,R \qquad g_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu}$$

graviton field

metric

$$\kappa^2 = 32\pi\, G_{\text{Newton}}$$

**Infinite number of complicated interactions**

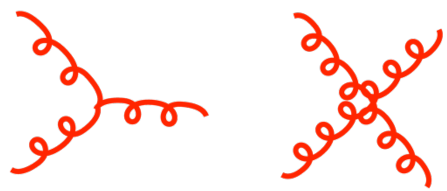


$+ \cdots$

**Very nonlinear**

**Compare to gauge-theory Lagrangian on which QCD is based**

$$L_{\text{YM}} = \frac{1}{g^2}F^2$$



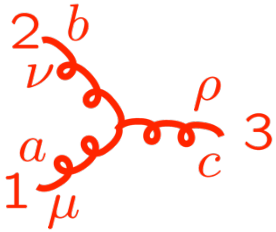
**Only three and four point interactions**

**Gravity seems so much more complicated than gauge theory.**

**Gauge and gravity theories seem rather different.**

# Three-Point Interactions

## Standard perturbative approach:




**Three-gluon vertex from strong interactions**:

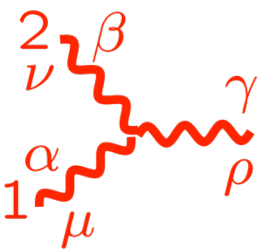
$$V_{3\,\mu\nu\sigma}^{abc} = -g f^{abc}(\eta_{\mu\nu}(k_1-k_2)_\rho+\eta_{\nu\rho}(k_1-k_2)_\mu+\eta_{\rho\mu}(k_1-k_2)_\nu)$$

**Three-graviton vertex**:

$$k_i^2 = E_i^2 - \vec{k}_i^2 \neq 0$$

$$G_{3\mu\alpha,\nu\beta,\sigma\gamma}(k_1,k_2,k_3) =$$

$$\begin{aligned}
&\mathrm{sym}[-\tfrac{1}{2}P_3(k_1\cdot k_2\eta_{\mu\alpha}\eta_{\nu\beta}\eta_{\sigma\gamma}) - \tfrac{1}{2}P_6(k_{1\nu}k_{1\beta}\eta_{\mu\alpha}\eta_{\sigma\gamma}) + \tfrac{1}{2}P_3(k_1\cdot k_2\eta_{\mu\nu}\eta_{\alpha\beta}\eta_{\sigma\gamma})\\
&+P_6(k_1\cdot k_2\eta_{\mu\alpha}\eta_{\nu\sigma}\eta_{\beta\gamma}) + 2P_3(k_{1\nu}k_{1\gamma}\eta_{\mu\alpha}\eta_{\beta\sigma}) - P_3(k_{1\beta}k_{2\mu}\eta_{\alpha\nu}\eta_{\sigma\gamma})\\
&+P_3(k_{1\sigma}k_{2\gamma}\eta_{\mu\nu}\eta_{\alpha\beta}) + P_6(k_{1\sigma}k_{1\gamma}\eta_{\mu\nu}\eta_{\alpha\beta}) + 2P_6(k_{1\nu}k_{2\gamma}\eta_{\beta\mu}\eta_{\alpha\sigma})\\
&+2P_3(k_{1\nu}k_{2\mu}\eta_{\beta\sigma}\eta_{\gamma\alpha}) - 2P_3(k_1\cdot k_2\eta_{\alpha\nu}\eta_{\beta\sigma}\eta_{\gamma\mu})]
\end{aligned}$$



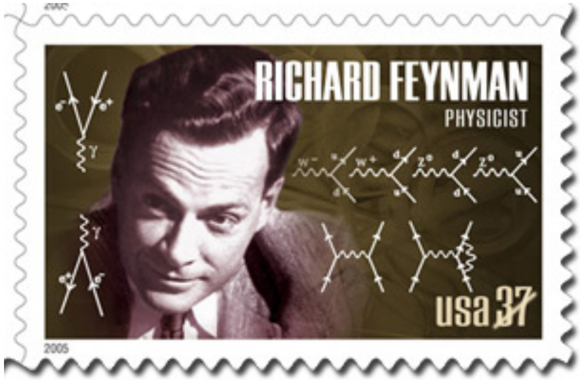

**About 100 terms in three vertex**

**Naïve conclusion: Gravity is a nasty mess.**

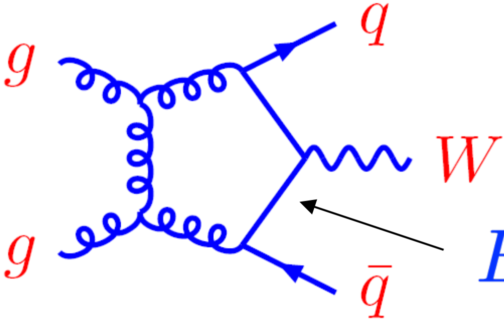
# Antidotes to Complexity

# Why are Feynman diagrams difficult for high-loop or high-multiplicity processes?

**Vertices and propagators involve unphysical gauge-dependent off-shell states. An important origin of the complexity.**




$$\int \frac{d^3\vec{p}\, dE}{(2\pi)^4}$$




$$E^2 - \vec{p}^2 \neq m^2$$

**Individual Feynman diagrams unphysical**

**Einstein's relation between momentum and energy violated in the loops. Unphysical states! Not gauge invariant.**

- **Use gauge invariant on-shell physical states.**
- **On-shell formalism.**
- **Don't violate Einstein's relation!** ZB, Dixon, Dunbar, Kosower (1994)

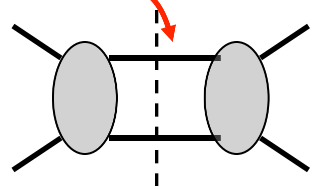
# From Tree to Loops: Generalized Unitarity Method

**No Feynman rules; no need for virtual particles.**

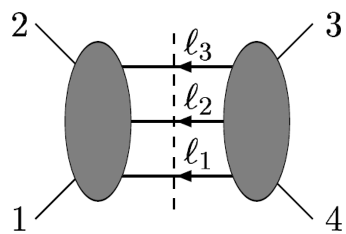
$$E^2 = \vec{p}^{\,2} + m^2$$ ← **on-shell**

ZB, Dixon, Dunbar and Kosower (1994)

**Two-particle cut:**



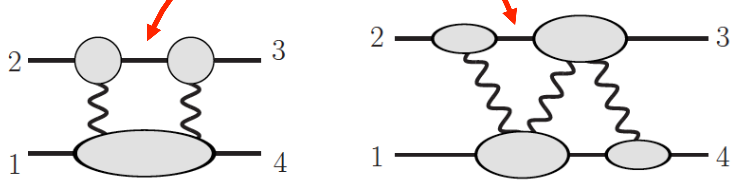
**Three-particle cut:**




- **Systematic assembly of complete loop amplitudes from tree amplitudes.**
- **Works for any number of particles or loops.**

**Generalized unitarity as a practical tool for loops.**

**on-shell**




ZB, Dixon and Kosower;
ZB, Morgan;
Britto, Cachazo, Feng;
Ossala,Pittau,Papadopoulos;
Ellis, Kunszt, Melnikov;
Forde; Badger;
ZB, Carrasco, Johansson, Kosower
and many others

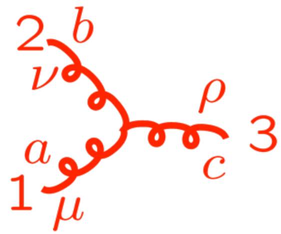
**Idea used in the "NLO revolution" in QCD collider physics.**
**Want to apply it to gravitational wave problem.**

# Simplicity of Gravity Amplitudes

**People were looking at gravity amplitudes the wrong way.**
**On-shell viewpoint much more powerful.**

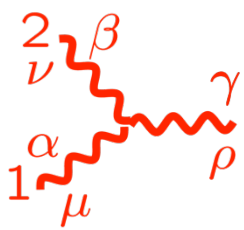
***On-shell* three vertices contains all information:** $E_i^2 - \vec{k}_i^2 = 0$

**Yang-Mills gauge theory:**




$$-gf^{abc}(\eta_{\mu\nu}(k_1 - k_2)_\rho + \text{cyclic})$$

**Einstein gravity:**




$$i\kappa(\eta_{\mu\nu}(k_1 - k_2)_\rho + \text{cyclic})$$
$$\times\,(\eta_{\alpha\beta}(k_1 - k_2)_\gamma + \text{cyclic})$$

**"square" of Yang-Mills vertex.**

**Very simple interactions.**

# KLT Relation Between Gravity and Gauge Theory

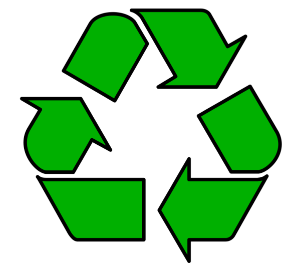
KLT (1985)

**Kawai-Lewellen-Tye string relations in low-energy limit:**

**gravity** **gauge-theory color ordered**

$$M_4^{\rm tree}(1,2,3,4) = -is_{12} A_4^{\rm tree}(1,2,3,4)\, A_4^{\rm tree}(1,2,4,3)\,,$$

$$\begin{aligned} M_5^{\rm tree}(1,2,3,4,5) &= i s_{12} s_{34} A_5^{\rm tree}(1,2,3,4,5)\, A_5^{\rm tree}(2,1,4,3,5) \\ &\quad + i s_{13} s_{24} A_5^{\rm tree}(1,3,2,4,5)\, A_5^{\rm tree}(3,1,4,2,5) \end{aligned}$$

**Generalizes to explicit all-leg form.** ZB, Dixon, Perelstein, Rozowsky

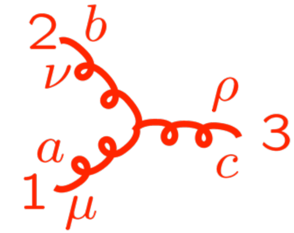
1. **Gravity is derivable from gauge theory. Standard Lagrangian methods offers no hint why this is possible.**
2. **It is very generally applicable.**

# Duality Between Color and Kinematics

ZB, Carrasco, Johansson (2007)

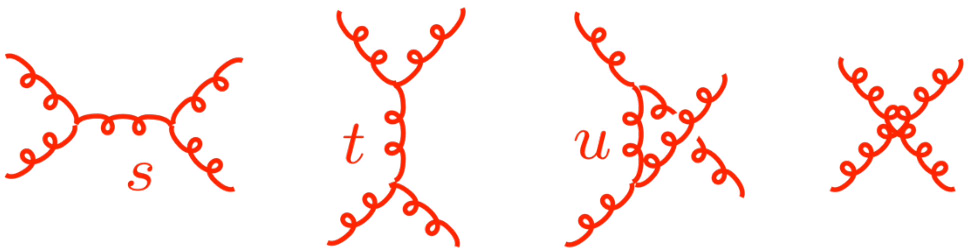
**coupling constant** → **color factor** ← **momentum dependent kinematic factor**

$$-g f^{abc} (\eta_{\mu\nu}(k_1 - k_2)_\rho + \text{cyclic})$$




**Color factors based on a Lie algebra:** $[T^a, T^b] = i f^{abc} T^c$

**Jacobi Identity** $f^{a_1 a_2 b} f^{b a_4 a_3} + f^{a_4 a_2 b} f^{b a_3 a_1} + f^{a_4 a_1 b} f^{b a_2 a_3} = 0$




**Use 1 = *s/s* = *t/t* = *u/u* to assign 4-point diagram to others.**

$s = (k_1 + k_2)^2 \qquad t = (k_1 + k_4)^2$

$u = (k_1 + k_3)^2$

$$\mathcal{A}_4^{\text{tree}} = g^2 \left( \frac{n_s c_s}{s} + \frac{n_t c_t}{t} + \frac{n_u c_u}{u} \right)$$

**Color factors satisfy Jacobi identity:**

**Numerator factors satisfy similar identity:**

$$\boxed{\begin{aligned} c_u &= c_s - c_t \\ n_u &= n_s - n_t \end{aligned}}$$

**Proven at tree level**

ZB, Carrasco, Johansson; Kiermaier; Bjerrum-Bohr, Damgaard, Sondergaard, Vanhove; Cachazo, etc

# Duality Between Color and Kinematics

## Consider five-point tree amplitude:

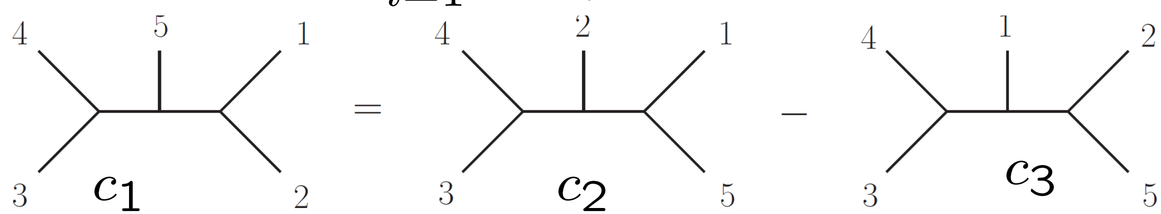
ZB, Carrasco, Johansson (BCJ)

color factor

kinematic numerator factor

$$\mathcal{A}_5^{\text{tree}} = \sum_{i=1}^{15} \frac{c_i n_i}{\prod_{\alpha_i} p^2_{\alpha_i}}$$

Feynman propagators




$$c_1 = f^{a_3 a_4 b} f^{b a_5 c} f^{c a_1 a_2} \qquad c_2 = f^{a_3 a_4 b} f^{b a_2 c} f^{c a_1 a_5} \qquad c_3 = f^{a_3 a_4 b} f^{b a_1 c} f^{c a_2 a_5}$$

$$n_i \sim k_4 \cdot k_5 \, k_2 \cdot \varepsilon_1 \, \varepsilon_2 \cdot \varepsilon_3 \, \varepsilon_4 \cdot \varepsilon_5 + \cdots$$

$$\boxed{c_1 + c_2 + c_3 = 0 \;\Leftrightarrow\; n_1 + n_2 + n_3 = 0}$$

**Claim: We can always find a rearrangement so color and kinematics satisfy the *same* algebraic constraint equations.**

**Progress on unraveling relations.**

BCJ, Bjerrum-Bohr, Feng, Damgaard, Vanhove, ; Mafra, Stieberger, Schlotterer;
Tye and Zhang; Feng, Huang, Jia; Chen, Du, Feng; Du, Feng, Fu; Naculich, Nastase, Schnitzer
O'Connell and Montiero; Bjerrum-Bohr, Damgaard, O'Connell and Montiero; O'Connell, Montiero, White;
Du, Feng and Teng, Song and Schlotterer, etc.

# Higher-Point Gravity and Gauge Theory

ZB, Carrasco, Johansson

color factor

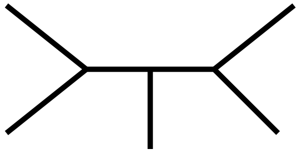
kinematic numerator factor

Feynman propagators

**gauge theory (QCD):** $$\mathcal{A}_n^{\text{tree}} = i g^{n-2} \sum_i \frac{c_i\, n_i}{D_i}$$

$$c_k = c_i - c_j$$
$$n_k = n_i - n_j$$

$$\boxed{c_i \to n_i}$$



**Einstein gravity:** $$\mathcal{M}_n^{\text{tree}} = i \kappa^{n-2} \sum_i \frac{n_i^2}{D_i}$$

$$n_i \sim k_4 \cdot k_5\, k_2 \cdot \varepsilon_1\, \varepsilon_2 \cdot \varepsilon_3\, \varepsilon_4 \cdot \varepsilon_5 + \cdots$$

**sum over diagrams with only 3 vertices**

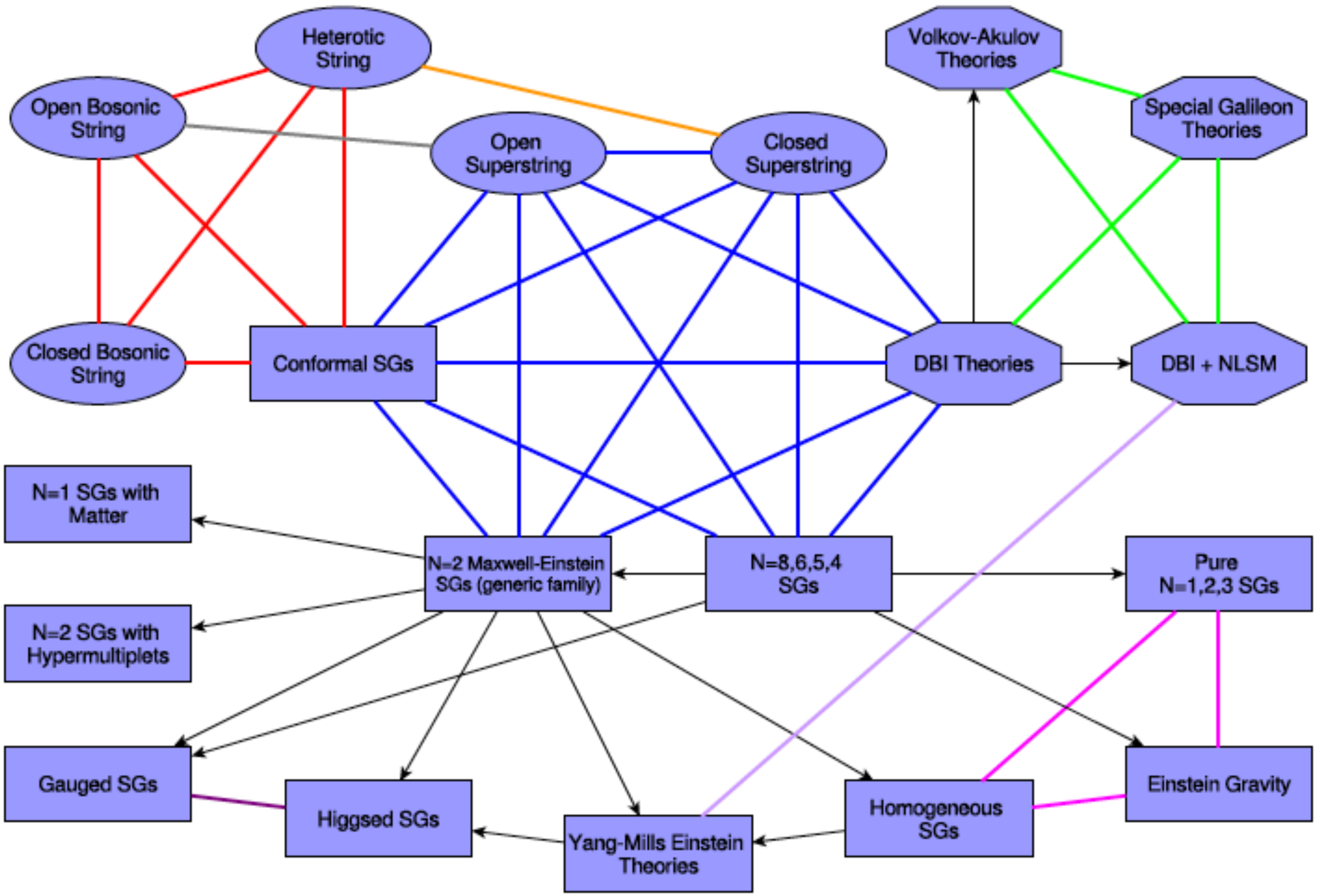
**Gravity and gauge theory kinematic numerators are the same!**

**Same ideas conjectured to hold at loop level.**

**Cries out for a unified description of gravity with gauge theory, presumably along the lines of string theory.**

# Web of Theories

ZB, Carrasco, Chiodaroli, Johansson, Roiban arXiv:1909.01358, Section 5.

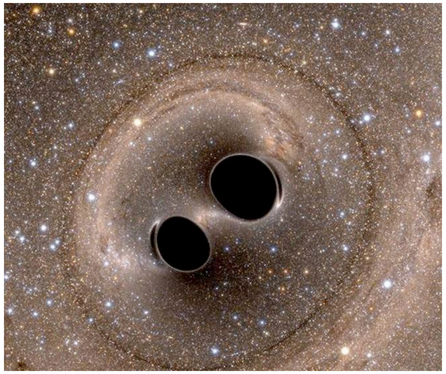



**Double copy links various theories through their component theories.**

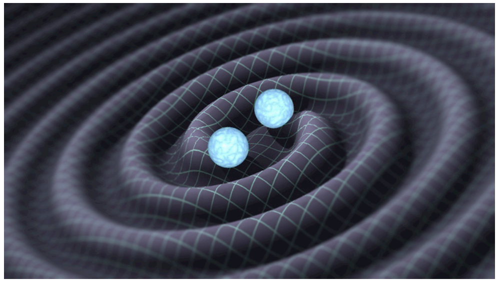
# Double Copy for Classical Solutions

**Goal is to formulate gravity solutions directly in terms of gauge theory**

## Variety of special cases:



- **Schwarzschild and Kerr black holes.**
- **Solutions with cosmological constant.**
- **Radiation from accelerating black hole.**
- **Maximally symmetric space times.**
- **Plane wave background.**
- **Gravitational radiation.**



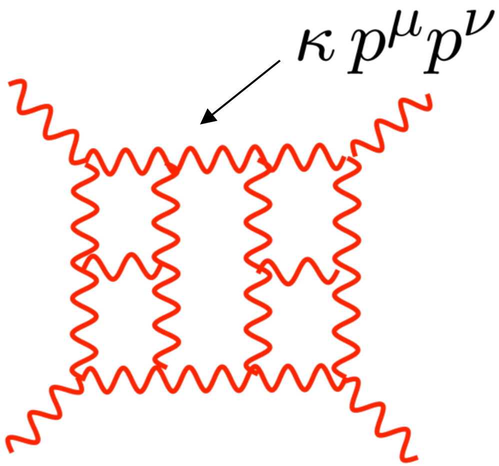
**Monteiro, O'Connell and White;**
Luna, Monteiro, O'Connell and White;
Luna, Monteiro, Nicholsen, O'Connell and White;
Ridgway and Wise; Carrillo González, Penco, Trodden;
Adamo, Casali, Mason, Nekovar;
Goldberger and Ridgway; Chen;
Luna, Monteiro, Nicholson, Ochirov;
Bjerrum-Bohr, Donoghue, Vanhove;
O'Connell, Westerberg, White; Kosower, Maybee, O'Connell; Adamo, Casali, Mason, Nekovar
etc

**Still no general understanding.**
**But plenty of examples.**

# Understanding UV of Gravity

# UV Behavior of Gravity?

$\kappa = \sqrt{32\pi G_N}$ ⟵ Dimensionful coupling




**Gravity:** $$\int \prod_{i=1}^{L} \frac{d^D p_i}{(2\pi)^D} \frac{\cdots \kappa p_j^\mu p_j^\nu \cdots}{\text{propagators}}$$

**Gauge theory:** $$\int \prod_{i=1}^{L} \frac{d^D p_i}{(2\pi)^D} \frac{\cdots g p_j^\nu \cdots}{\text{propagators}}$$

- **Extra powers of loop momenta in numerator means integrals are badly behaved in the UV and must diverge at some loop order.**
- **Much more sophisticated power counting in supersymmetric theories but this is basic idea.**

- **With more supersymmetry expect better UV properties.**
- **Need to worry about "hidden cancellations".**
- ***N*** **= 8 supergravity *best* theory to study.**

# $N = 8$ supergravity: Where is First $D = 4$ UV Divergence?

| | | |
|---|---|---|
| 3 loops $N = 8$ | Green, Schwarz, Brink (1982); Howe and Stelle (1989); Marcus and Sagnotti (1985) | ✗ |
| 5 loops $N = 8$ | Bern, Dixon, Dunbar, Perelstein, Rozowsky (1998); Howe and Stelle (2003,2009) | ✗ |
| 6 loops $N = 8$ | Howe and Stelle (2003) | ✗ |
| 7 loops $N = 8$ | Grisaru and Siegel (1982); Bossard, Howe, Stelle (2009);Vanhove; Björnsson, Green (2010); Kiermaier, Elvang, Freedman(2010); Ramond, Kallosh (2010); Biesert et al (2010); Bossard, Howe, Stelle, Vanhove (2011) | ? |
| 3 loops $N = 4$ | Bossard, Howe, Stelle, Vanhove (2011) | ✗ |
| 4 loops $N = 5$ | Bossard, Howe, Stelle, Vanhove (2011) | ✗ |
| 4 loops $N = 4$ | Vanhove and Tourkine (2012) | ✓ |
| 9 loops $N = 8$ | Berkovits, Green, Russo, Vanhove (2009) | ✗ |

**"shut up and calculate"**

ZB, Kosower, Carrasco, Dixon, Johansson, Roiban; ZB, Davies, Dennen, A. Smirnov, V. Smirnov; series of calculations.

**This is what we are most interested in.**

**Weird structure. Anomaly-like behavior of divergence.**

**Retracted, but perhaps to be unretracted.**

- **Track record of predictions from standard symmetries not great.**
- **Conventional wisdom holds that it will diverge sooner or later.**

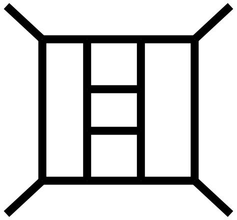
# Supersymmetry and Ultraviolet Divergences

Bossard, Howe, Stelle; Elvang, Freedman, Kiermaier; Green, Russo, Vanhove ; Green and Björnsson ; Bossard , Hillmann and Nicolai; Ramond and Kallosh; Broedel and Dixon; Elvang and Kiermaier; Beisert, Elvang, Freedman, Kiermaier, Morales, Stieberger; Bossard, Howe, Stelle, Vanhove, etc

**Poor UV behavior, unless new types of cancellations between diagrams exist that are "not consequences of supersymmetry in any conventional sense"**

Bjornsson and Green

- **$N = 4$ sugra should diverge at 3 loops in $D = 4$.**
- **$N = 5$ sugra should diverge at 4 loops in $D = 4$.**
- **Half maximal sugra diverges at 2 loops in $D = 5$.**
- **$N = 8$ sugra should diverge at 5 loops in $D = 24/5$.**
- **$N = 8$ sugra should diverge at 7 loops in $D = 4$.**



**Consensus agreement from all power-counting methods.**

# Scorecard on Symmetry Predictions

- **$N = 4$ sugra should diverge at 3 loops in $D = 4$.** ✗
- **$N = 5$ sugra should diverge at 4 loops in $D = 4$.** ✗
- **Half maximal sugra diverges at 2 loops in $D = 5$.** ✗
- **$N = 8$ sugra should diverge at 5 loops in $D = 24/5$.** ✓
- **$N = 8$ sugra should diverge at 7 loops in $D = 4$.** ? **key question**

ZB, Davies, Dennen (2012, 2014); ZB, Davies, Dennen, Huang(2012)
ZB, Carrasco, Chen, Edison, Johansson, Roiban, Parra-Martinez, Zeng (2018)

- **UV cancellation of $N = 5$ supergravity at 4 loops in $D = 4$ definite mystery. Problem with standard symmetry arguments.**

Freedman, Kallosh and Yamada (2018)

**What is the difference between $N = 5$ and $N = 8$?**
**$D = 4$ has extra cancellations.**

Edison, Herrmann, Parra-Martinez, Trnka (2019)

**Goal is to provide definitive answers. Need to go to 7 loops!**

# $\mathcal{N} = 8$ UV at Five loops

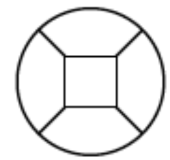
ZB, Carrasco, Chen, Edison, Johansson, Roiban, Parra-Martinez, Zeng (2018)

**Using the above ideas and some generalization even 5 loops possible.**

**In $D = 24/5$ we obtain a divergence:**

**Basis of UV divergent integrals**

$$\mathcal{M}_4^{(5)}\Big|_{\text{leading}} = -\frac{16 \times 629}{25}\left(\frac{\kappa}{2}\right)^{12}(s^2+t^2+u^2)^2 stu M_4^{\text{tree}} \left(\frac{1}{48}\right.$$



$$+ \frac{1}{16}$$

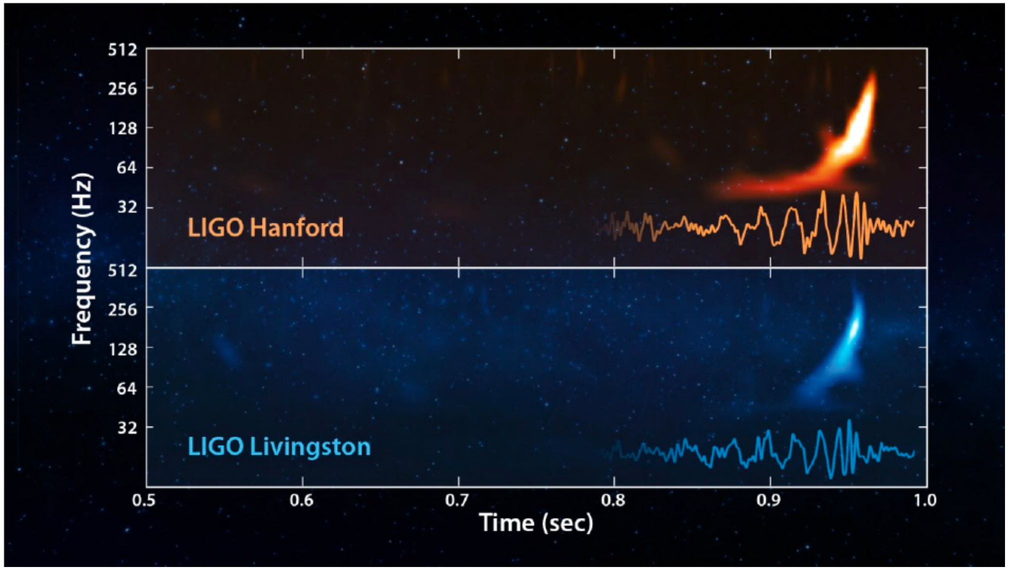
$$\left.\right)$$

- **Mysterious "enhanced UV cancellations" exist in supergravity. Still need to be understood. Role of four dimensions?**
- **"Impossible" multiloop gravity calculations are pretty standard by now using modern amplitude methods.**

# Applications to Gravitational Wave Physics

# Outline

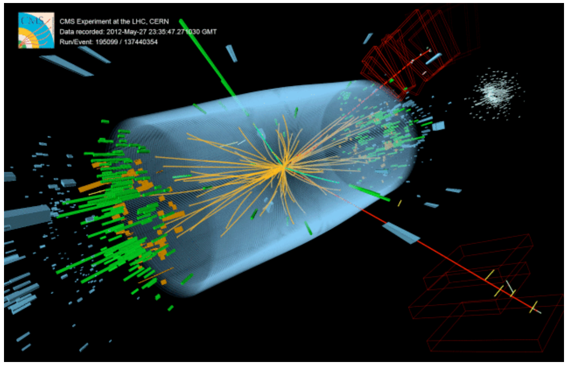
**Era of gravitational wave astronomy has begun.**

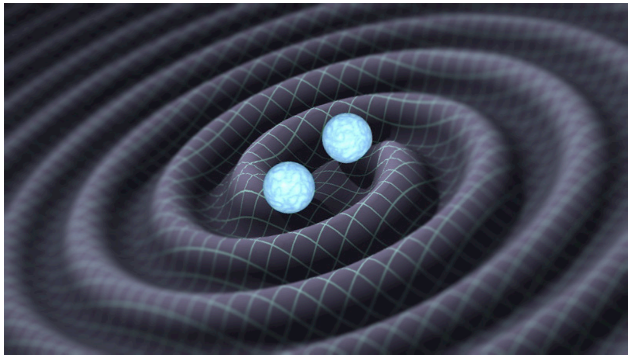



**For an instant, brighter in gravitational radiation than all the stars in the visible universe are in EM radiation!**

# Can Particle Theory Help with Gravitational Waves?

**What does particle physics have to do with classical dynamics of astrophysical objects?**



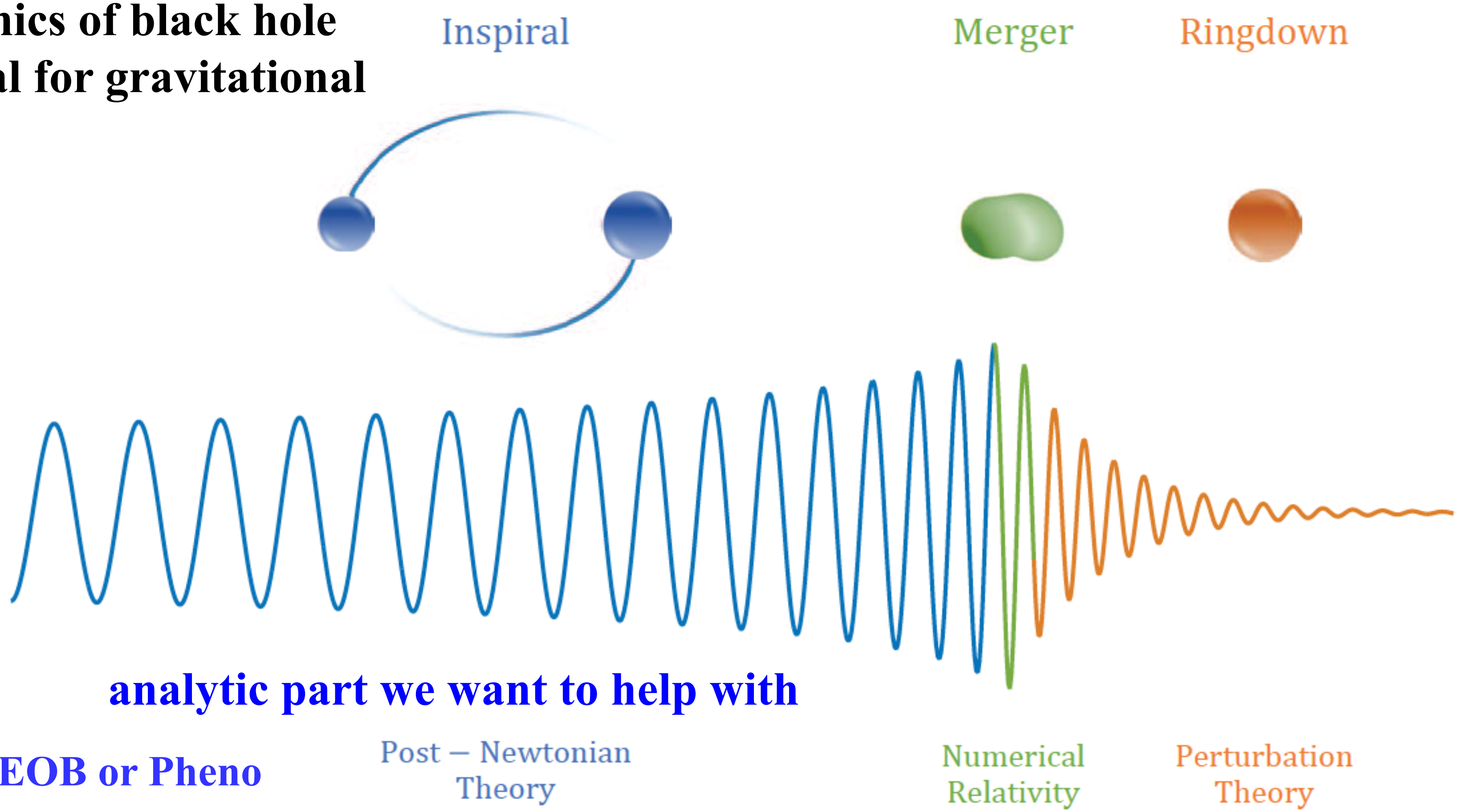
**Black-holes and neutron stars are point particles as far as long wavelength radiation is concerned.**

EFT approach: Goldberger, Rothstein, Porto; Vaydia, Foffa , Porto, Rothstein, Sturani; etc

**Will explain that the tools that we use in particle physics are ideally suited to push the state of the art in gravitational-wave physics.**

# Goal: Improve on post-Newtonian Theory

**Dynamics of black hole inspiral for gravitational waves.**




**analytic part we want to help with**

**PN + EOB or Pheno**

**Small errors accumulate. Need for high precision.**

From Antelis and Moreno, arXiv:1610.03567

# Post Newtonian Approximation

**For orbital mechanics:**

$$v^2 \sim \frac{GM}{R} \ll 1$$

**Expand in *G* and $v^2$**

**virial theorem**

**In center of mass frame:**

$$m = m_A + m_B, \quad \nu = \mu/M,$$

$$\mu = m_A m_B/m, \quad P_R = P \cdot \hat{R}$$

$$\frac{H}{\mu} = \frac{P^2}{2} - \frac{Gm}{R}$$

**Newton**

$$+ \frac{1}{c^2}\left\{ -\frac{P^4}{8} + \frac{3\,\nu\,P^4}{8} + \frac{Gm}{R}\left( -\frac{{P_R}^2\,\nu}{2} - \frac{3\,P^2}{2} - \frac{\nu\,P^2}{2} \right) + \frac{G^2 m^2}{2R^2} \right\}$$

$$+ \ldots$$

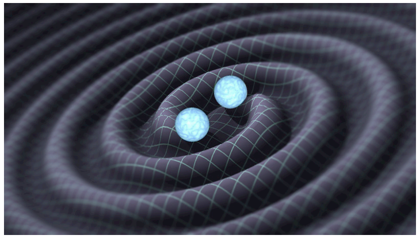
**1PN: Einstein, Infeld, Hoffmann**

**Hamiltonian known to 4PN order.**

**2PN: Ohta, Okamura, Kimura and Hiida.**
**3PN: Damour, Jaranowski and Schaefer; L. Blanchet and G. Faye.**
**4PN: Damour, Jaranowski and Schaefer; Foffa, Porto, Rothstein, Sturani.**

# Scattering Amplitudes and Gravitational Radiation

**Can we simplify the types of calculations needed for LIGO?**



**A small industry has developed to study this.**

- **Connection to scattering amplitudes.**

  Bjerrum-Bohr, Donoghue, Holstein, Plante, Pierre Vanhove; Luna, Nicholson, O'Connell, White; Guevara; Bjerrum-Bohr, Damgaard, Festuccia, Planté, Vanhove; Cheung, Rothstein, Solon; Damour; Bautista, Guevara; Kosower, Maybee, O'Connell; Plefka, Steinhoff, Wormsbecher; Foffa, Mastrolia, Sturani, Sturm; Guevara, Ochirov, Vines; Chung, Huang, Kim, Lee; etc.
- **Worldline approach for radiation and double copy.**

  Goldberger and Ridgway; Goldberger, Li, Prabhu, Thompson; Chester; Shen
- **Removing the dilaton contamination.**

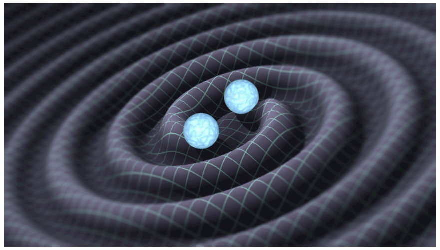
  Luna, Nicholson, O'Connell, White; Johansson, Ochirov; Johansson, Kalin; Henrik Johansson, Gregor Kälin, Mogull.

**Key Question: Can we calculate something of direct interest to LIGO/Virgo, decisively *beyond* previous state of the art?**

# Which problem to solve?

ZB, Cheung, Roiban, Shen, Solon, Zeng

**Some problems for (analytic) theorists:**



1. **Spin.**
2. **Finite size effects.**
3. **New physics effects.**
4. **Radiation.**
5. **High orders in perturbation theory.**

**Which problem should we solve?**

- **Needs to be extremely difficult using standard methods.**
- **Needs to be of direct importance to LIGO theorists.**
- **Needs to be in a form that can in principle enter LIGO analysis pipeline.**

**2-body Hamiltonian at 3$^{rd}$ post-Minkowskian order**

# PN versus PM expansion for conservative two-body dynamics

**From Buonanno Amplitudes 2018**

$$\mathcal{L} = -Mc^2 + \underbrace{\frac{\mu v^2}{2} + \frac{GM\mu}{r}}_{\downarrow} + \frac{1}{c^2}\left[\ldots\right] + \frac{1}{c^4}\left[\ldots\right] + \ldots$$

$$E(v) = -\frac{\mu}{2}v^2 + \cdots$$

non-spinning compact objects

| | | 0PN | 1PN | 2PN | 3PN | 4PN | 5PN | $\ldots$ |
|---|---|---|---|---|---|---|---|---|
| $\boxed{\text{0PM:}}$ | 1 | $v^2$ | $v^4$ | $v^6$ | $v^8$ | $v^{10}$ | $\boxed{v^{12}}$ | $\ldots$ |
| $\boxed{\text{1PM:}}$ | | $1/r$ | $v^2/r$ | $v^4/r$ | $v^6/r$ | $v^8/r$ | $\boxed{v^{10}/r}$ | $\ldots$ |
| $\boxed{\text{2PM:}}$ | | | $1/r^2$ | $v^2/r^2$ | $v^4/r^2$ | $v^6/r^2$ | $\boxed{v^8/r^2}$ | $\ldots$ |
| 3PM: | | | | $1/r^3$ | $v^2/r^3$ | $v^4/r^3$ | $v^6/r^3$ | $\ldots$ |
| 4PM: | | | | | $1/r^4$ | $v^2/r^4$ | $v^4/r^4$ | $\ldots$ |
| $\ldots$ | | | | | | $\ldots$ | $\ldots$ | |

*(credit: Justin Vines)*

$$1 \to Mc^2, \qquad v^2 \to \frac{v^2}{c^2}, \qquad \frac{1}{r} \to \frac{GM}{rc^2}.$$

current known PN results — current known PM results — overlap between PN & PM results — unknown

- **PM results** *(Westfahl 79, Westfahl & Goller 80, Portilla 79-80, Bel et al. 81, Ledvinka et al. 10, Damour 16-17, Guevara 17, Vines 17, Bini & Damour 17-18, Vines in prep)*

# Our Approach

ZB, Cheung, Roiban, Shen, Solon, Zeng

**Cheung, Rothstein, Solon (2018)**

**Amplitudes community**

**Gravitational Scattering Amplitudes**

**Effective Field Theory Methods**

**EFT community**

Kawai, Lewellen, Tye
ZB, Dixon, Dunbar and Kosower
ZB, Dixon, Dunbar, Perelstein, Rozowsky
ZB, Carrasco, Johansson; Etc

Goldberger, Rothstein;
Porto; Neill, Rothstein;
Vaydia, Foffa , Porto, Rothstein, Sturani; etc

**Post Minkowskian Potentials**

Beautiful paper from Kosower, Maybee and O'Connell gives alternate approach

**Inefficient: Start with quantum theory and take $\hbar \to 0$**

**Efficient: Almost magical simplifications for gravity amplitudes.**
**EFT methods efficiently target pieces we want.**

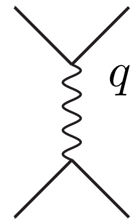
**Efficiency wins**

# 2 Body Potentials and Amplitudes

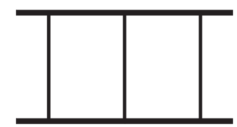
Iwasaki; Gupta, Radford; Donoghue; Holstein, Donoghue; Holstein and A. Ross; Bjerrum-Bohr, Donoghue, Vanhove; Neill, Rothstein; Bjerrum-Bohr, Damgaard, Festuccia, Planté. Vanhove; Chueng, Rothstein, Solon; Chung, Huang, Kim, Lee; etc.

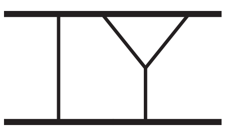
**Tree-level: Fourier transform gives classical potential.**

$$V(r) = \int \frac{d^3q}{(2\pi)^3} e^{-i\mathbf{q}\cdot\mathbf{r}} A^{\text{tree}}(\mathbf{q})$$




**Beyond 1 loop things quickly become less obvious:**





- **What you learned in grad school on $\hbar$ and classical limits is wrong! Loops have classical pieces.**
- $1/\hbar^L$ **scaling of at $L$ loop.**
- **Double counting and iteration, IR singularities.**
- **Cross terms between $1/\hbar$ and $\hbar$.**
- **Which corners of multiloop integrals are classical? How to extract?**

$$e^{iS_{\text{classical}}/\hbar}$$

**Piece of loops are classical: Our task is to efficiently extract these pieces.**

**We harness EFT to clean up confusion**

# EFT is a Clean Approach

**Build EFT from which we can read off potential. Avoids a variety of confusions related to taking classical limit.**

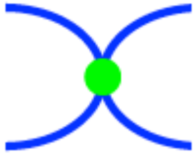
Goldberger and Rothstein
Neill, Rothstein
**Cheung, Rothstein, Solon (2018)**

$$L_{\rm kin} = \int_{\boldsymbol{k}} A^\dagger(-\boldsymbol{k}) \left( i\partial_t + \sqrt{\boldsymbol{k}^2 + m_A^2}\right) A(\boldsymbol{k}) + \int_{\boldsymbol{k}} B^\dagger(-\boldsymbol{k}) \left( i\partial_t + \sqrt{\boldsymbol{k}^2 + m_B^2}\right) B(\boldsymbol{k})$$

***A*, *B* scalars represents spinless black holes**

$$L_{\rm int} = -\int_{\boldsymbol{k},\boldsymbol{k}'} V(\boldsymbol{k},\boldsymbol{k}')\, A^\dagger(\boldsymbol{k}')A(\boldsymbol{k})B^\dagger(-\boldsymbol{k}')B(-\boldsymbol{k})$$

**potential**



$$H = \sqrt{k^2+m_1^2} + \sqrt{k^2+m_2^2} + V$$

**2 body Hamiltonian in c.o.m. frame.**

**Match amplitudes of this theory to the full theory in classical limit to extract a potential.**

**Our friends at LIGO/Virgo want Hamiltonians.**

# EFT Matching

Cheung, Rothstein, Solon

**full general relativity**
(complicated)

Amplitude methods
double copy

**tree amplitude**

generalized
unitarity
$\hbar \to 0$

**loop integrand**

loop
integration

**GR loop amplitude**

**effective theory**
(simpler)

build
ansatz

**potential**

Feynman
diagrams

**loop integrand**

loop
integration

**EFT loop amplitude**

**identical physics**

**GR loop amplitude** $=$ **EFT loop amplitude**

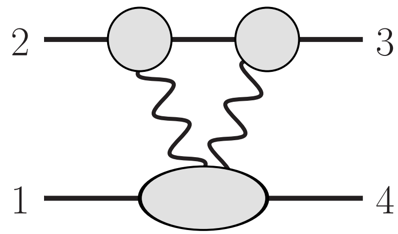
**Roundabout, but efficiently determines potential.**
**IR singularities cancel trivially.**

# Full Theory: Unitarity + Double Copy

- **Long-range force: Two matter lines must be separated by cut propagators.**
- **Classical potential: 1 matter line per loop is cut (on-shell).**

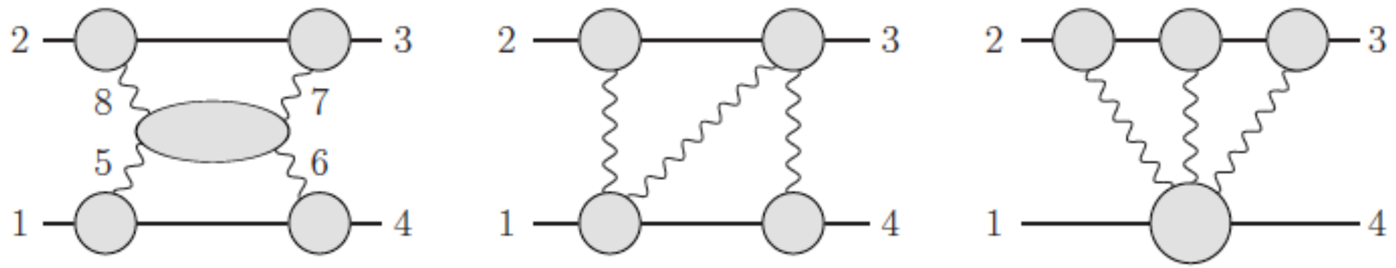
Neill and Rothstein ; Bjerrum-Bohr, Damgaard, Festuccia, Planté, Vanhove; Cheung, Rothstein, Solon

**Only independent unitarity cut for 2 PM.**




**exposed lines on-shell (long range).**
**Classical pieces simple!**

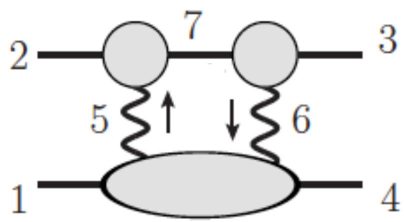
**Independent generalized unitarity cuts for 3 PM.**




**Use our amplitudes tools for this part.**

# Generalized Unitarity Cuts

**Primary means of construction uses BCJ in *D* dimensions, but KLT with helicity should have better scaling at higher loops and gives compact expressions.**




**$2^{\mathrm{nd}}$ post-Minkowkian order**

$$
\begin{aligned}
C_{\mathrm{GR}} &= \sum_{h_5,h_6=\pm} M_3^{\mathrm{tree}}(3^s,6^{h_6},-7^s)\, M_3^{\mathrm{tree}}(7^s,-5^{h_5},2^s)\, M_4^{\mathrm{tree}}(1^s,5^{-h_5},-6^{-h_6},4^s) \\
&= \sum_{h_5,h_6=\pm} it[A_3^{\mathrm{tree}}(3^s,6^{h_6},-7^s)\, A_3^{\mathrm{tree}}(7^s,-5^{h_5},2^s)\, A_4^{\mathrm{tree}}(1^s,5^{-h_5},-6^{-h_6},4^s)] \\
&\qquad \times [A_3^{\mathrm{tree}}(3^s,6^{h_6},-7^s)\, A_3^{\mathrm{tree}}(7^s,-5^{h_5},2^s)\, A_4^{\mathrm{tree}}(4^s,5^{-h_5},-6^{-h_6},1^s)]
\end{aligned}
$$

**By correlating gluon helicities, removing dilaton is trivial.**

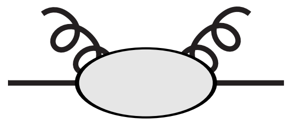
$h^-_{\mu\nu} \to A^-_\mu A^-_\nu$ $\qquad$ $h^+_{\mu\nu} \to A^+_\mu A^+_\nu$ $\qquad$ **Forbid:** $A^+_\mu A^-_\nu$

**Problem of computing the generalized cuts in gravity is reduced to multiplying and summing gauge-theory tree amplitudes.**

# Gauge-Theory Warm-up

$$A^{\text{tree}}(1^s, 2^+, 3^+, 4^s) = i\frac{m_1^2[23]}{\langle 23\rangle t_{12}}$$

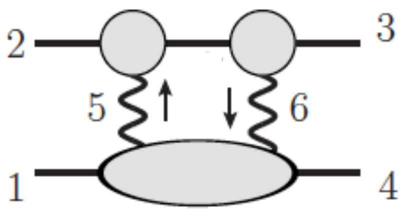
$$A^{\text{tree}}(1^s, 2^+, 3^-, 4^s) = -i\frac{\langle 3|1|2]^2}{\langle 23\rangle [23] t_{12}}$$



color-ordered gauge-theory tree amplitudes

- This is all you need for 2 PM.
- Scaling with number of loops is very good.

$$s_{23} = (p_2 + p_3)^2$$
$$t_{ij} = 2p_i \cdot p_j$$




$$\boxed{C_{\text{YM}} = 2\left(\frac{\mathcal{E}^2 + \mathcal{O}^2}{s_{23}^2} + m_1^2 m_2^2\right)\frac{1}{t_{15}}}$$

sum over helicities

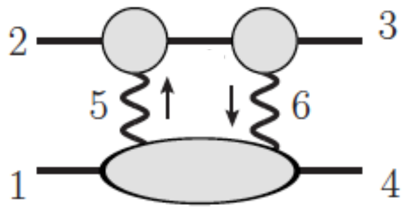
gauge theory

$$\mathcal{E}^2 = \frac{1}{4}s_{23}^2(t_{15} - t_{12})^2$$
$$\mathcal{O}^2 = \mathcal{E}^2 - s_{23}m_2^2(s_{23}m_1^2 + s_{23}t_{15} + t_{15}^2)$$

Simple cut contains all information we need for classical potential

# One-Loop Gravity Warmup

$$s_{23} = (p_2 + p_3)^2$$
$$t_{ij} = 2p_i \cdot p_j$$




**Apply unitarity and KLT relations.**
**Import gauge-theory results.**

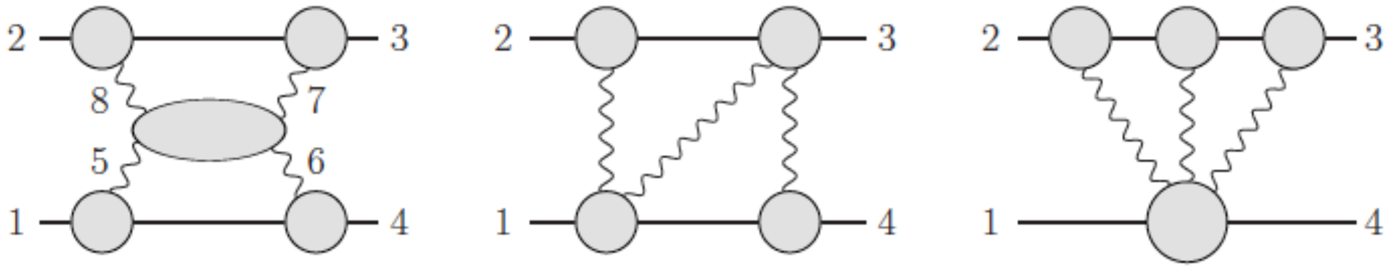
$$\boxed{C_{\rm GR} = 2\left[\frac{1}{t^4}\left(\mathcal{E}^4 + \mathcal{O}^4 + 6\mathcal{E}^2\mathcal{O}^2\right) + m_1^4 m_2^4\right]\left[\frac{1}{t_{15}} + \frac{1}{t_{45}}\right]}$$

- **Same building blocks as gauge theory.**
- **Double copy is visible even though we removed dilaton and axion.**

**We can extract classical scattering angles or potentials following literature.**

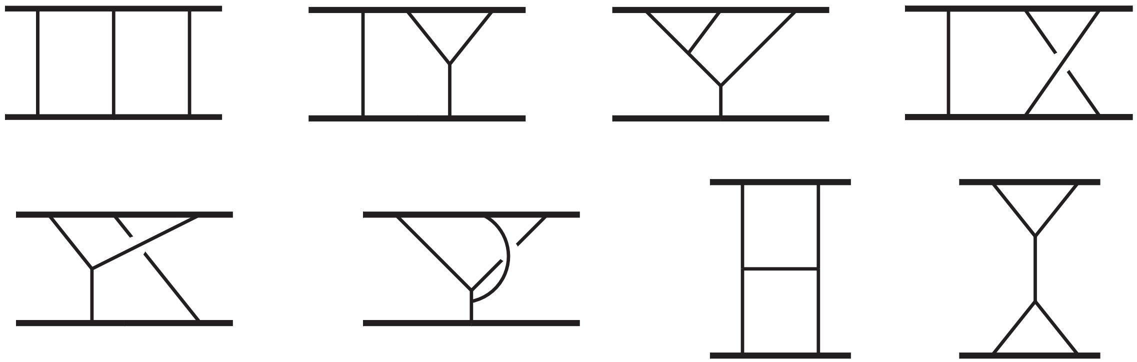
Damour; Bjerrum-Bohr, Damgaard, Festuccia, Planté, Vanhove; Cheung, Rothstein, Solon; Koemans Collado, Di Vecchia, Russo; etc.

# Two Loops and 3 PM




- Somewhat more complicated than previous one loop, but no problem.
- Evaluated using both BCJ and KLT double copies.
- To interface easily with EFT approach, merged unitarity cuts into Feynman-like diagrams to get integrand.

**Integrand organized into 8 independent diagrams that may contribute in classical limit:**



# Integration + Extraction of Potential

**To integrate, follow path of Cheung, Rothstein and Solon.**

- **Efficiently targets the classical pieces of integrals we want.**
- **Integrals reduce via energy residues to 3 dimensional integrals.**

$$\ell_0 \ll |\vec{\ell}| \ll |\vec{p}| \ll m_i$$

potential classical nonrelativistic

**Integrals much simpler than for full quantum theory**

**Detail are found in our paper.**

**Various checks using standard machinery used in QCD: IBP, Mellin-Barnes, differential equations, sector decomposition.**

# Amplitude in Classical Potential Limit

ZB, Cheung, Roiban, Shen, Solon, Zeng (2019)

**Classical limit. The O($G^3$) or 3PM terms are:**

rapidity

$$\mathcal{M}_3 = \frac{\pi G^3 \nu^2 m^4 \log \boldsymbol{q}^2}{6\gamma^2\xi}\left[3 - 6\nu + 206\nu\sigma - 54\sigma^2 + 108\nu\sigma^2 + 4\nu\sigma^3 - \frac{48\nu\left(3+12\sigma^2-4\sigma^4\right)\operatorname{arcsinh}\sqrt{\frac{\sigma-1}{2}}}{\sqrt{\sigma^2-1}}\right.$$
$$\left. - \frac{18\nu\gamma\left(1-2\sigma^2\right)\left(1-5\sigma^2\right)}{(1+\gamma)(1+\sigma)}\right] + \frac{8\pi^3 G^3 \nu^4 m^6}{\gamma^4 \xi}\left[3\gamma\left(1-2\sigma^2\right)\left(1-5\sigma^2\right)F_1 - 32 m^2\nu^2\left(1-2\sigma^2\right)^3 F_2\right]$$

$$m = m_A + m_B, \qquad \mu = m_A m_B/m, \qquad \nu = \mu/m, \qquad \gamma = E/m,$$
$$\xi = E_1 E_2/E^2, \qquad E = E_1 + E_2, \qquad \sigma = p_1 \cdot p_2 / m_1 m_2,$$

$F_1$ and $F_2$ IR divergent iteration terms that don't affect potential.

**Amplitude containing classical potential is remarkably simple!**

# Conservative 3PM Hamiltonian

ZB, Cheung, Roiban, Shen, Solon, Zeng (2019)

## The 3PM Hamiltonian:

$$H(\boldsymbol{p},\boldsymbol{r}) = \sqrt{\boldsymbol{p}^2+m_1^2} + \sqrt{\boldsymbol{p}^2+m_2^2} + V(\boldsymbol{p},\boldsymbol{r})$$

$$V(\boldsymbol{p},\boldsymbol{r}) = \sum_{i=1}^{3} c_i(\boldsymbol{p}^2)\left(\frac{G}{|\boldsymbol{r}|}\right)^i ,$$

$$c_1 = \frac{\nu^2 m^2}{\gamma^2\xi}\left(1-2\sigma^2\right), \qquad c_2 = \frac{\nu^2 m^3}{\gamma^2\xi}\left[\frac{3}{4}\left(1-5\sigma^2\right) - \frac{4\nu\sigma\left(1-2\sigma^2\right)}{\gamma\xi} - \frac{\nu^2(1-\xi)\left(1-2\sigma^2\right)^2}{2\gamma^3\xi^2}\right],$$

$$\begin{aligned} c_3 = \frac{\nu^2 m^4}{\gamma^2\xi}\Bigg[ & \frac{1}{12}\left(3-6\nu+206\nu\sigma-54\sigma^2+108\nu\sigma^2+4\nu\sigma^3\right) - \frac{4\nu\left(3+12\sigma^2-4\sigma^4\right)\operatorname{arcsinh}\sqrt{\frac{\sigma-1}{2}}}{\sqrt{\sigma^2-1}} \\ & - \frac{3\nu\gamma\left(1-2\sigma^2\right)\left(1-5\sigma^2\right)}{2(1+\gamma)(1+\sigma)} - \frac{3\nu\sigma\left(7-20\sigma^2\right)}{2\gamma\xi} - \frac{\nu^2\left(3+8\gamma-3\xi-15\sigma^2-80\gamma\sigma^2+15\xi\sigma^2\right)\left(1-2\sigma^2\right)}{4\gamma^3\xi^2} \\ & + \frac{2\nu^3(3-4\xi)\sigma\left(1-2\sigma^2\right)^2}{\gamma^4\xi^3} + \frac{\nu^4(1-2\xi)\left(1-2\sigma^2\right)^3}{2\gamma^6\xi^4}\Bigg], \end{aligned}$$

$$m = m_A + m_B, \qquad \mu = m_A m_B/m, \qquad \nu = \mu/m, \qquad \gamma = E/m,$$

$$\xi = E_1 E_2/E^2, \qquad E = E_1 + E_2, \qquad \sigma = p_1\cdot p_2/m_1 m_2,$$

# Key Checks

**Primary check:**

ZB, Cheung, Roiban, Shen, Solon, Zeng

**Compare to 4PN Hamiltonian of Damour, Jaranowski, Schäfer, used in high precision template construction.**

**Need canonical transformation:**

$$(\boldsymbol{r}, \boldsymbol{p}) \to (\boldsymbol{R}, \boldsymbol{P}) = (A\,\boldsymbol{r} + B\,\boldsymbol{p}, C\,\boldsymbol{p} + D\,\boldsymbol{r})$$

$$A = 1 - \frac{Gm\nu}{2|\boldsymbol{r}|} + \cdots\,, \quad B = \frac{G(1-2/\nu)}{4m|\boldsymbol{r}|}\boldsymbol{p}\cdot\boldsymbol{r} + \cdots$$

$$C = 1 + \frac{Gm\nu}{2|\boldsymbol{r}|} + \cdots\,, \quad D = -\frac{Gm\nu}{2|\boldsymbol{r}|^3}\boldsymbol{p}\cdot\boldsymbol{r} + \cdots\,,$$

**Our Hamiltonian equivalent to 4PN Hamiltonian on overlap.**

**Bini and Damour recently verified we correctly overlap the 5 PN result.**

# 4 PN Hamiltonian

Damour, Jaranowski, Schaefer

$\mathbf{n} = \hat{\mathbf{r}}$

$$\widehat{H}_{\mathrm{N}}(\mathbf{r},\mathbf{p}) = \frac{\mathbf{p}^2}{2} - \frac{1}{r},$$

$$c^2\,\widehat{H}_{1\mathrm{PN}}(\mathbf{r},\mathbf{p}) = \frac{1}{8}(3\nu-1)(\mathbf{p}^2)^2 - \frac{1}{2}\Big\{(3+\nu)\mathbf{p}^2 + \nu(\mathbf{n}\cdot\mathbf{p})^2\Big\}\frac{1}{r} + \frac{1}{2r^2},$$

$$c^4\,\widehat{H}_{2\mathrm{PN}}(\mathbf{r},\mathbf{p}) = \frac{1}{16}\left(1-5\nu+5\nu^2\right)(\mathbf{p}^2)^3 + \frac{1}{8}\Big\{\left(5-20\nu-3\nu^2\right)(\mathbf{p}^2)^2 - 2\nu^2(\mathbf{n}\cdot\mathbf{p})^2\mathbf{p}^2 - 3\nu^2(\mathbf{n}\cdot\mathbf{p})^4\Big\}\frac{1}{r}$$
$$+ \frac{1}{2}\Big\{(5+8\nu)\mathbf{p}^2 + 3\nu(\mathbf{n}\cdot\mathbf{p})^2\Big\}\frac{1}{r^2} - \frac{1}{4}(1+3\nu)\frac{1}{r^3},$$

$$c^6\,\widehat{H}_{3\mathrm{PN}}(\mathbf{r},\mathbf{p}) = \frac{1}{128}\left(-5+35\nu-70\nu^2+35\nu^3\right)(\mathbf{p}^2)^4 + \frac{1}{16}\Big\{\left(-7+42\nu-53\nu^2-5\nu^3\right)(\mathbf{p}^2)^3$$
$$+ (2-3\nu)\nu^2(\mathbf{n}\cdot\mathbf{p})^2(\mathbf{p}^2)^2 + 3(1-\nu)\nu^2(\mathbf{n}\cdot\mathbf{p})^4\mathbf{p}^2 - 5\nu^3(\mathbf{n}\cdot\mathbf{p})^6\Big\}\frac{1}{r}$$
$$+ \Big\{\frac{1}{16}\left(-27+136\nu+109\nu^2\right)(\mathbf{p}^2)^2 + \frac{1}{16}(17+30\nu)\nu(\mathbf{n}\cdot\mathbf{p})^2\mathbf{p}^2 + \frac{1}{12}(5+43\nu)\nu(\mathbf{n}\cdot\mathbf{p})^4\Big\}\frac{1}{r^2}$$
$$+ \Big\{\Big(-\frac{25}{8} + \Big(\frac{\pi^2}{64} - \frac{335}{48}\Big)\nu - \frac{23\nu^2}{8}\Big)\mathbf{p}^2 + \Big(-\frac{85}{16} - \frac{3\pi^2}{64} - \frac{7\nu}{4}\Big)\nu(\mathbf{n}\cdot\mathbf{p})^2\Big\}\frac{1}{r^3} + \Big\{\frac{1}{8} + \Big(\frac{109}{12} - \frac{21}{32}\pi^2\Big)\nu\Big\}\frac{1}{r^4},$$

$G^4$

# 4 PN Hamiltonian

Damour, Jaranowski, Schaefer

$$\mathbf{n} = \hat{\mathbf{r}}$$

$$
\begin{aligned}
c^8\,\widehat{H}^{\text{local}}_{4\text{PN}}(\mathbf{r},\mathbf{p}) &= \left(\frac{7}{256} - \frac{63}{256}\nu + \frac{189}{256}\nu^2 - \frac{105}{128}\nu^3 + \frac{63}{256}\nu^4\right)(\mathbf{p}^2)^5 \\
&+ \left\{\frac{45}{128}(\mathbf{p}^2)^4 - \frac{45}{16}(\mathbf{p}^2)^4\,\nu + \left(\frac{423}{64}(\mathbf{p}^2)^4 - \frac{3}{32}(\mathbf{n}\cdot\mathbf{p})^2(\mathbf{p}^2)^3 - \frac{9}{64}(\mathbf{n}\cdot\mathbf{p})^4(\mathbf{p}^2)^2\right)\nu^2\right. \\
&+ \left(-\frac{1013}{256}(\mathbf{p}^2)^4 + \frac{23}{64}(\mathbf{n}\cdot\mathbf{p})^2(\mathbf{p}^2)^3 + \frac{69}{128}(\mathbf{n}\cdot\mathbf{p})^4(\mathbf{p}^2)^2 - \frac{5}{64}(\mathbf{n}\cdot\mathbf{p})^6\mathbf{p}^2 + \frac{35}{256}(\mathbf{n}\cdot\mathbf{p})^8\right)\nu^3 \\
&+ \left.\left(-\frac{35}{128}(\mathbf{p}^2)^4 - \frac{5}{32}(\mathbf{n}\cdot\mathbf{p})^2(\mathbf{p}^2)^3 - \frac{9}{64}(\mathbf{n}\cdot\mathbf{p})^4(\mathbf{p}^2)^2 - \frac{5}{32}(\mathbf{n}\cdot\mathbf{p})^6\mathbf{p}^2 - \frac{35}{128}(\mathbf{n}\cdot\mathbf{p})^8\right)\nu^4\right\}\frac{1}{r} \\
&+ \left\{\frac{13}{8}(\mathbf{p}^2)^3 + \left(-\frac{791}{64}(\mathbf{p}^2)^3 + \frac{49}{16}(\mathbf{n}\cdot\mathbf{p})^2(\mathbf{p}^2)^2 - \frac{889}{192}(\mathbf{n}\cdot\mathbf{p})^4\mathbf{p}^2 + \frac{369}{160}(\mathbf{n}\cdot\mathbf{p})^6\right)\nu\right. \\
&+ \left(\frac{4857}{256}(\mathbf{p}^2)^3 - \frac{545}{64}(\mathbf{n}\cdot\mathbf{p})^2(\mathbf{p}^2)^2 + \frac{9475}{768}(\mathbf{n}\cdot\mathbf{p})^4\mathbf{p}^2 - \frac{1151}{128}(\mathbf{n}\cdot\mathbf{p})^6\right)\nu^2 \\
&+ \left.\left(\frac{2335}{256}(\mathbf{p}^2)^3 + \frac{1135}{256}(\mathbf{n}\cdot\mathbf{p})^2(\mathbf{p}^2)^2 - \frac{1649}{768}(\mathbf{n}\cdot\mathbf{p})^4\mathbf{p}^2 + \frac{10353}{1280}(\mathbf{n}\cdot\mathbf{p})^6\right)\nu^3\right\}\frac{1}{r^2} \\
&+ \left\{\frac{105}{32}(\mathbf{p}^2)^2 + \left(\left(\frac{2749\pi^2}{8192} - \frac{589189}{19200}\right)(\mathbf{p}^2)^2 + \left(\frac{63347}{1600} - \frac{1059\pi^2}{1024}\right)(\mathbf{n}\cdot\mathbf{p})^2\mathbf{p}^2 + \left(\frac{375\pi^2}{8192} - \frac{23533}{1280}\right)(\mathbf{n}\cdot\mathbf{p})^4\right)\nu\right. \\
&+ \left(\left(\frac{18491\pi^2}{16384} - \frac{1189789}{28800}\right)(\mathbf{p}^2)^2 + \left(-\frac{127}{3} - \frac{4035\pi^2}{2048}\right)(\mathbf{n}\cdot\mathbf{p})^2\mathbf{p}^2 + \left(\frac{57563}{1920} - \frac{38655\pi^2}{16384}\right)(\mathbf{n}\cdot\mathbf{p})^4\right)\nu^2 \\
&+ \left.\left(-\frac{553}{128}(\mathbf{p}^2)^2 - \frac{225}{64}(\mathbf{n}\cdot\mathbf{p})^2\mathbf{p}^2 - \frac{381}{128}(\mathbf{n}\cdot\mathbf{p})^4\right)\nu^3\right\}\frac{1}{r^3} \\
&+ \left\{\frac{105}{32}\mathbf{p}^2 + \left(\left(\frac{185761}{19200} - \frac{21837\pi^2}{8192}\right)\mathbf{p}^2 + \left(\frac{3401779}{57600} - \frac{28691\pi^2}{24576}\right)(\mathbf{n}\cdot\mathbf{p})^2\right)\nu\right. \\
&+ \left.\left(\left(\frac{672811}{19200} - \frac{158177\pi^2}{49152}\right)\mathbf{p}^2 + \left(\frac{110099\pi^2}{49152} - \frac{21827}{3840}\right)(\mathbf{n}\cdot\mathbf{p})^2\right)\nu^2\right\}\frac{1}{r^4} \qquad \longleftarrow G^4 \\
&+ \left\{-\frac{1}{16} + \left(\frac{6237\pi^2}{1024} - \frac{169199}{2400}\right)\nu + \left(\frac{7403\pi^2}{3072} - \frac{1256}{45}\right)\nu^2\right\}\frac{1}{r^5}. \qquad \longleftarrow G^5
\end{aligned}
$$

**After canonical transformation we match all but $G^4$ and $G^5$ terms**

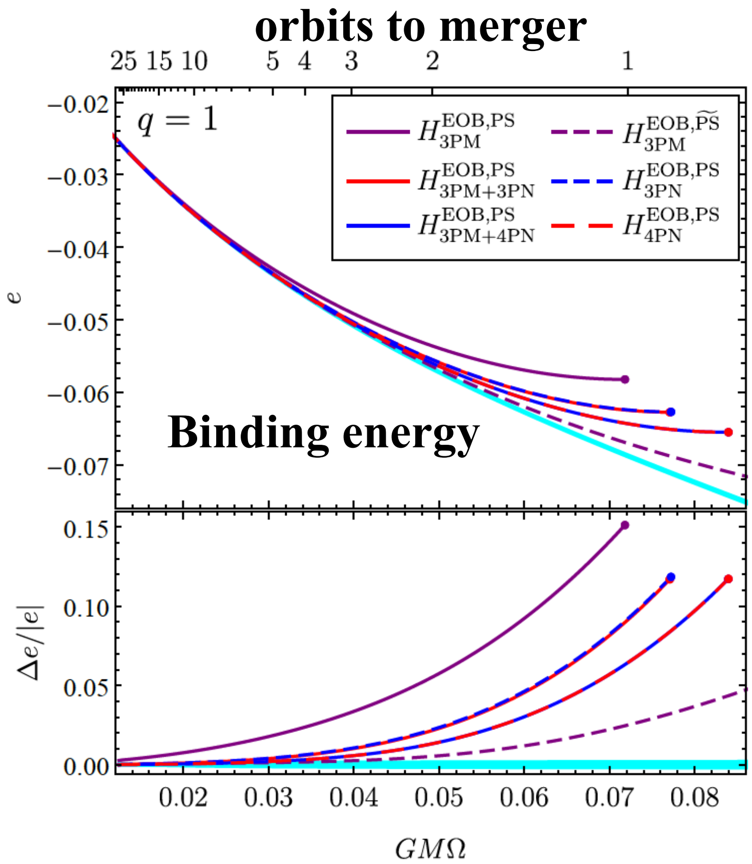
**Mess is partly due to their gauge choice.**

**Ours is all orders in $p$ at $G^3$**

# Tests of Our 3PM Hamiltonian for LIGO

Antonelli, Buonanno, Steinhoff, van de Meent, and Vines, arXiv:1901.07102

(8 days after our paper!)




**Test against numerical relativity.**

**Fed into EOB models, which are needed for good agreement.**

**Note: Not conclusive, e. g. radiation not taken into accounted**

**numerical relativity taken as truth**

**"This rather encouraging result motivates a more comprehensive study…"**

**Our GR friends are definitely interested!**

# Outlook

- **Most exciting part is that methods are far from exhausted.**
- **Started working on 4$^{th}$ PM order. Methods certainly look up to the task.**
- **Greater improvements on horizon.**


**Obvious topics to investigate:**

- **Higher orders. Resummation in *G*.**
- **Radiation.**
- **Spin (hot topic).**
- **Finite size effects.**

**Expect many advances in coming years.**

# Summary

- **Particle physics give us new ways to think about problems of current interest in general relativity.**
- **Double-copy idea gives a unified framework for gravity and gauge theory.**
- **Sample applications:**
  - — **Web of theories.**
  - — **High loop order explorations of supergravity, especially UV.**
  - — **2 body Hamiltonians for gravitional radiation.**

**Expect many more advances in coming years, not only for gravitational-wave physics, but more generally for understanding gravity and its relation to the other forces via double copy.**

# Further Reading

## Double Copy:

**Z. Bern, J. J. Carrasco, M. Chiodaroli, H. Johansson, R. Roiban.**
**"The Duality Between Color and Kinematics and its Applications"**
**arXiv:1909.01358**

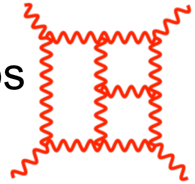
## Binary Black Hole Physics from Amplitudes:

**Z. Bern, C. Cheung, R. Roiban, Chia-Hsien Shen, M. P. Solon, M. Zeng**
**"Black Hole Binary Dynamics from the Double Copy and Effective Theory"**
**arXiv:1908.01493**

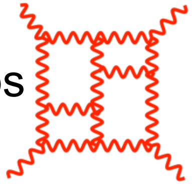
# Extra Slides

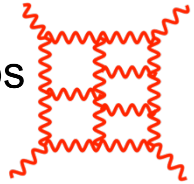
# Feynman Diagrams for Gravity

**We will be talking about high order processes**

3 loops  $\sim 10^{20}$ TERMS

No surprise it has never been calculated via Feynman diagrams.

4 loops  $\sim 10^{26}$ TERMS

5 loops  $\sim 10^{31}$ TERMS

More terms than atoms in your brain!

- Calculations to settle this seemed utterly hopeless!
- Seemed destined for dustbin of undecidable questions.

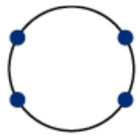
**At present there is only one basic approach to carry out such computations, which I will outline.**

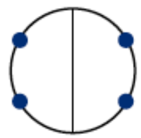
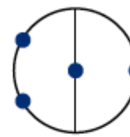
# Higher-loop Structure.

Green Schwarz, Brink; ZB, Dixon, Dunbar, Perelstein, Rozowsky;Carrasco, Dixon, Johansson, Roiban; ZB, Carrasco, Chen, Edison, Johansson, Roiban, Parra-Martinez, Zeng

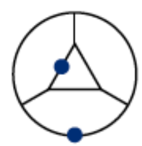
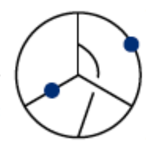
**Over the years we've obtained results for $N = 8$ sugra through five loops. Vacuum diagrams capture UV divergences.**

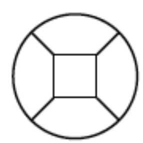
**dots represent extra propagators**

$\mathcal{M}_4^{(1)}\Big|_{\text{leading}} = -3\,\mathcal{K}_{\text{G}}\left(\frac{\kappa}{2}\right)^4$  , $\qquad D_c = 8$

$\mathcal{M}_4^{(2)}\Big|_{\text{leading}} = -8\,\mathcal{K}_{\text{G}}\left(\frac{\kappa}{2}\right)^6 (s^2+t^2+u^2)\Big(\frac{1}{4}$  $+\frac{1}{4}$  $\Big)$ , $\qquad D_c = 7$

$\mathcal{M}_4^{(3)}\Big|_{\text{leading}} = -60\,\mathcal{K}_{\text{G}}\left(\frac{\kappa}{2}\right)^8 stu\Big(\frac{1}{6}$  $+\frac{1}{2}$  $\Big)$ , $\qquad D_c = 6$

$\mathcal{M}_4^{(4)}\Big|_{\text{leading}} = -\frac{23}{2}\,\mathcal{K}_{\text{G}}\left(\frac{\kappa}{2}\right)^{10} (s^2+t^2+u^2)^2\Big(\frac{1}{4}$  $+\frac{1}{2}$  $+\frac{1}{4}$  $\Big)$ , $\qquad D_c = \frac{11}{2}$

$\mathcal{M}_4^{(5)}\Big|_{\text{leading}} = -\frac{16\times 629}{25}\,\mathcal{K}_{\text{G}}\left(\frac{\kappa}{2}\right)^{12} (s^2+t^2+u^2)^2\Big(\frac{1}{48}$  $+\frac{1}{16}$  $\Big)$ , $\qquad D_c = \frac{24}{5}$

**We now have a lot of theoretical "data" to guide us to 6,7 loops.**