Recent results in flavour physics

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Lyon University & CERN

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Why flavour physics?

CP violation:

The only CP violating parameter in the SM is the CKM phase. However, we know from baryogenesis that new sources of CP violation are needed.

The Standard Model flavour puzzle:

Why are the flavour parameters small and hierarchical?

The New Physics flavour puzzle:

If there is NP at the TeV scale, why are flavour changing neutral current (FCNC) so small? If NP has a generic flavour structure, it should contribute to FCNC processes

Flavour physics is sensitive to new physics at $\Lambda_{\rm NP} \gg E_{\rm experiments}$

Flavour physics can discover new physics or probe it before it is directly observed in experiments

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Indirect search for new physics

For a long time, flavour physics objectives were focused on the tests of the unitarity triangle, but this is now well established!

Focus is now towards the New Physics!

And search for the indirect signs of New Physics!

→ Many flavour observables under investigation!

→ Interesting interplay between flavour, collider and dark matter searches

Prime example: $B_s \to \mu^+\mu^-$, $A/H \to \tau^+\tau^-$ and direct dark matter detection (not covered in this talk)

→ Indirect hints for new physics: Flavour "anomalies"

Deviations from the Standard Model predictions in $b \to s\ell\ell$ transitions

Focus of the talk, since there are so few these days and they are still among our best bets!

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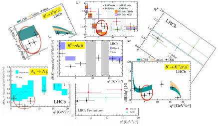

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- **Introduction**
  - → Theoretical framework
- **Observables**
  - → Definitions
  - → Recent anomalies
- **Theoretical uncertainties**
  - → Hadronic effects
  - → Statistical comparison of NP vs hadronic effects
- **NP global fits**
  - → Model independent implications
- **Specific NP models**
- **Future prospects**
- **Conclusions**

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Not covered in this talk:

- $b \to c$ charged currents
- Kaon anomalies

Theoretical framework

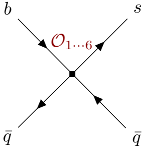
Effective field theory

$$\mathcal{H}_{\rm eff} = -\frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \Big(\sum_{i=1\cdots 10,S,P} \big(C_i(\mu)\mathcal{O}_i(\mu) + C'_i(\mu)\mathcal{O}'_i(\mu) \big) \Big)$$

Separation between short distance (Wilson coefficients) and long distance (local operators) effects

Operator set for $b \to s$ transitions:

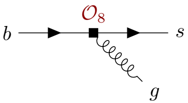
4-quark operators



$\mathcal{O}_{1,2} \propto (\bar{s}\Gamma_\mu c)(\bar{c}\Gamma^\mu b)$

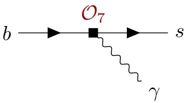
$\mathcal{O}_{3,4} \propto (\bar{s}\Gamma_\mu b)\sum_q(\bar{q}\Gamma^\mu q)$

chromomagnetic dipole operator



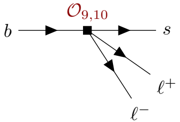
$\mathcal{O}_8 \propto (\bar{s}\sigma^{\mu\nu} T^a P_R) G^a_{\mu\nu}$

electromagnetic dipole operator



$\mathcal{O}_7 \propto (\bar{s}\sigma^{\mu\nu} P_R) F^a_{\mu\nu}$

semileptonic operators



$\mathcal{O}_9^\ell \propto (\bar{s}\gamma^\mu b_L)(\bar{\ell}\gamma_\mu \ell)$

$\mathcal{O}_{10}^\ell \propto (\bar{s}\gamma^\mu b_L)(\bar{\ell}\gamma_\mu\gamma_5 \ell)$

+ the chirality flipped counter-parts of the above operators, $\mathcal{O}'_i$

Wilson coefficients

The Wilson coefficients are calculated perturbatively and are process independent

Two main steps:

- matching between the effective and full theories → extraction of the $C_i^{eff}(\mu)$ at scale $\mu \sim M_W$

$$C_i^{eff}(\mu) = C_i^{(0)eff}(\mu) + \frac{\alpha_s(\mu)}{4\pi} C_i^{(1)eff}(\mu) + \cdots$$

- Evolving the $C_i^{eff}(\mu)$ to the scale relevant for B decays, $\mu \sim m_b$ using the RGE runnings.

SM contributions known to NNLL **(Bobeth, Misiak, Urban '99; Misiak, Steinhauser '04, Gorbahn, Haisch '04; Gorbahn, Haisch, Misiak '05; Czakon, Haisch, Misiak '06,...)**

$$C_7 = -0.294 \qquad C_9 = 4.20 \qquad C_{10} = -4.01$$

Hadronic quantities

To compute the amplitudes:

$$\mathcal{A}(A \to B) = \langle B|\mathcal{H}_{\rm eff}|A\rangle = \frac{G_F}{\sqrt{2}} \sum_i \lambda_i C_i(\mu) \langle B|\mathcal{O}_i|A\rangle(\mu)$$

$\langle B|\mathcal{O}_i|A\rangle$: hadronic matrix element

How to compute matrix elements?

$\to$ Model building, Lattice simulations, Light flavour symmetries, Heavy flavour symmetries, ...

$\to$ Describe hadronic matrix elements in terms of **hadronic quantities**

Two types of hadronic quantities:

- **Decay constants**: Probability amplitude of hadronising quark pair into a given hadron
- **Form factors**: Transition from a meson to another through flavour change

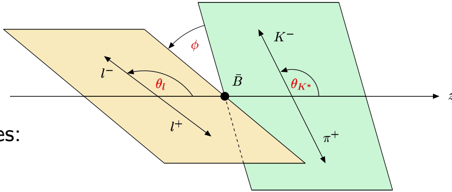
Once the Wilson coefficients and hadronic quantities calculated, the physical observables (branching fractions,...) can be calculated.

Observables and Anomalies

$b \to s\ell^+\ell^-$ transitions: $B \to K^*\mu^+\mu^-$

Angular distributions

The full angular distribution of the decay $\bar{B}^0 \to \bar{K}^{*0}\ell^+\ell^-$ ($\bar{K}^{*0} \to K^-\pi^+$) is completely described by four independent kinematic variables: q^2 (dilepton invariant mass squared), θ_ℓ, θ_{K^*}, ϕ



Differential decay distribution:

$$\frac{d^4\Gamma}{dq^2\, d\cos\theta_\ell\, d\cos\theta_{K^*}\, d\phi} = \frac{9}{32\pi} J(q^2, \theta_\ell, \theta_{K^*}, \phi)$$

$J(q^2, \theta_\ell, \theta_{K^*}, \phi) = \sum_i J_i(q^2)\, f_i(\theta_\ell, \theta_{K^*}, \phi)$

↘ angular coefficients J_{1-9}

↘ functions of the spin amplitudes A_0, $A_\parallel$, $A_\perp$, A_t, and A_S

Spin amplitudes: functions of Wilson coefficients and form factors

Main operators:

$$\mathcal{O}_9 = \frac{e^2}{(4\pi)^2}(\bar{s}\gamma^\mu b_L)(\bar{\ell}\gamma_\mu\ell), \quad \mathcal{O}_{10} = \frac{e^2}{(4\pi)^2}(\bar{s}\gamma^\mu b_L)(\bar{\ell}\gamma_\mu\gamma_5\ell)$$
$$\mathcal{O}_S = \frac{e^2}{16\pi^2}(\bar{s}^\alpha_L b^\alpha_R)(\bar{\ell}\,\ell), \quad \mathcal{O}_P = \frac{e^2}{16\pi^2}(\bar{s}^\alpha_L b^\alpha_R)(\bar{\ell}\gamma_5\ell)$$

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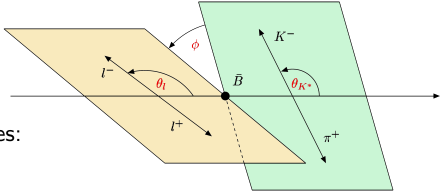

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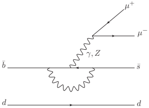
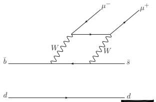




$B \to K^* \mu^+ \mu^-$ observables

Optimised observables: form factor uncertainties cancel at leading order

$$\langle P_1 \rangle_{\rm bin} = \frac{1}{2} \frac{\int_{\rm bin} dq^2 [J_3 + \bar{J}_3]}{\int_{\rm bin} dq^2 [J_{2s} + \bar{J}_{2s}]} \qquad \langle P_2 \rangle_{\rm bin} = \frac{1}{8} \frac{\int_{\rm bin} dq^2 [J_{6s} + \bar{J}_{6s}]}{\int_{\rm bin} dq^2 [J_{2s} + \bar{J}_{2s}]}$$

$$\langle P'_4 \rangle_{\rm bin} = \frac{1}{\mathcal{N}'_{\rm bin}} \int_{\rm bin} dq^2 [J_4 + \bar{J}_4] \qquad \langle P'_5 \rangle_{\rm bin} = \frac{1}{2\mathcal{N}'_{\rm bin}} \int_{\rm bin} dq^2 [J_5 + \bar{J}_5]$$

$$\langle P'_6 \rangle_{\rm bin} = \frac{-1}{2\mathcal{N}'_{\rm bin}} \int_{\rm bin} dq^2 [J_7 + \bar{J}_7] \qquad \langle P'_8 \rangle_{\rm bin} = \frac{-1}{\mathcal{N}'_{\rm bin}} \int_{\rm bin} dq^2 [J_8 + \bar{J}_8]$$

with

$$\mathcal{N}'_{\rm bin} = \sqrt{- \int_{\rm bin} dq^2 [J_{2s} + \bar{J}_{2s}] \int_{\rm bin} dq^2 [J_{2c} + \bar{J}_{2c}]}$$

+ CP violating clean observables and other combinations

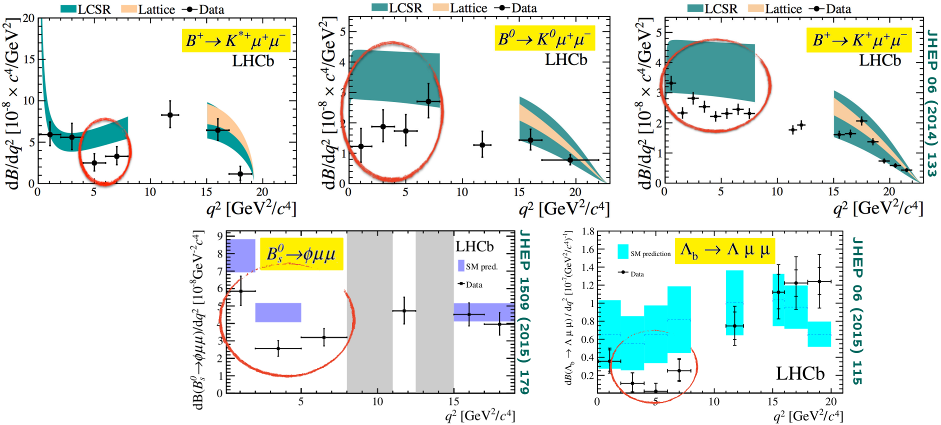
U. Egede et al., JHEP 0811 (2008) 032, JHEP 1010 (2010) 056
J. Matias et al., JHEP 1204 (2012) 104
S. Descotes-Genon et al., JHEP 1305 (2013) 137

Or alternatively:

$$S_i = \frac{J_{i(s,c)} + \bar{J}_{i(s,c)}}{\frac{d\Gamma}{dq^2} + \frac{d\bar{\Gamma}}{dq^2}} \,, \qquad P'_{4,5,8} = \frac{S_{4,5,8}}{\sqrt{F_L(1-F_L)}}$$

LHCb anomalies

A consistent deviation pattern with the SM predictions in $b \to s$ measurements with muons in the final state:



- deviations with the SM predictions between 1 and 3.5 σ
- general trend: EXP < SM in low q^2
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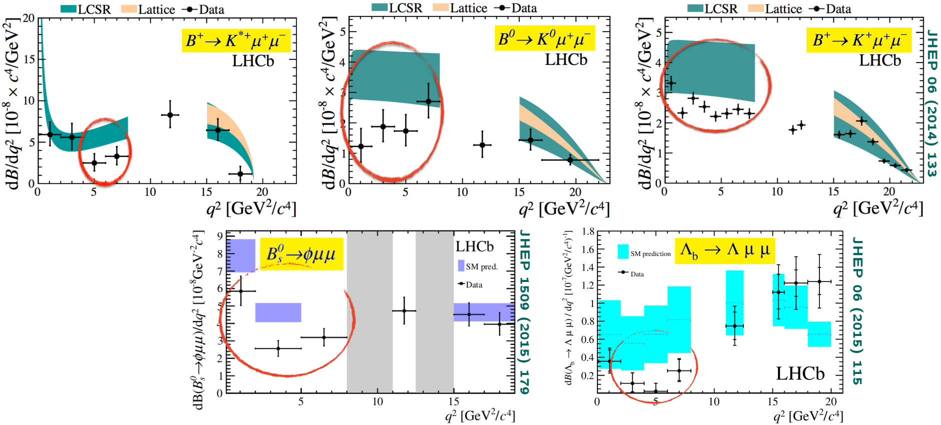

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$B \to K^* \mu^+ \mu^-$ angular observables, in particular P_5' / S_5

- 2013 (1 fb^{-1}): disagreement with the SM for P_2 and P_5' (PRL 111, 191801 (2013))
- March 2015 (3 fb^{-1}): confirmation of the deviations (LHCb-CONF-2015-002)
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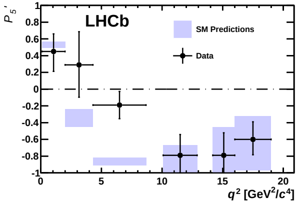

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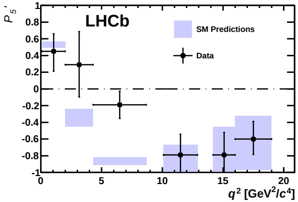

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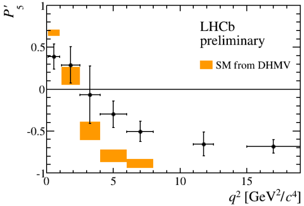

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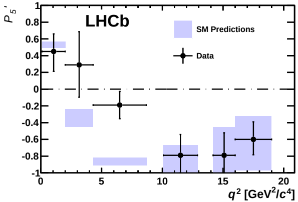

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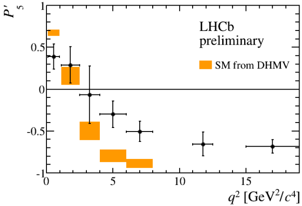

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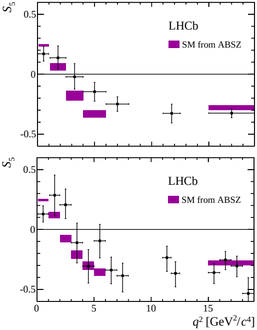

3.4σ combined fit (likelihood)

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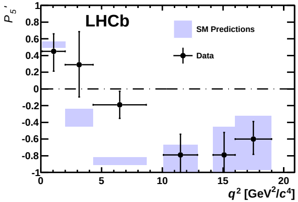

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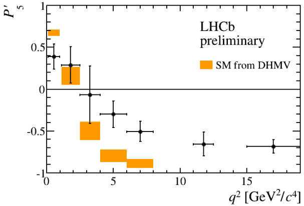

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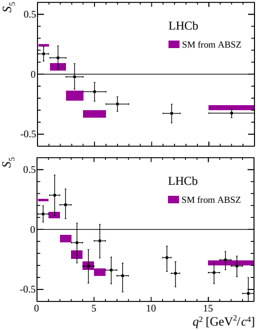
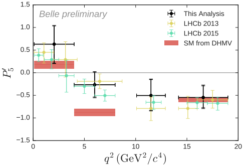


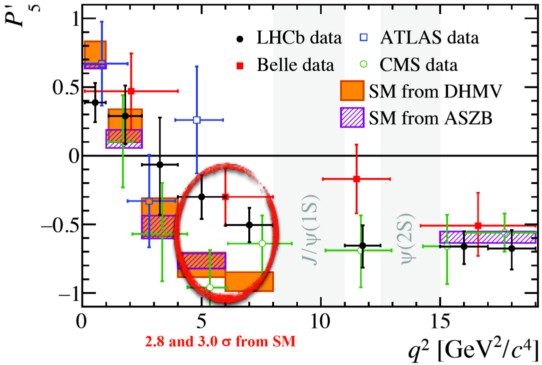
3.4σ combined fit (likelihood)

Belle supports LHCb (arXiv:1604.04042) tension at 2.1σ



The LHCb anomalies (1)

Current picture



LHCb, JHEP 02 (2016) 104; Belle, PRL 118 (2017); ATLAS, ATLAS-CONF-2017-023; CMS, CMS-PAS-BPH-15-008

The deviations are still there!

Difficult to think of statistical fluctuations...

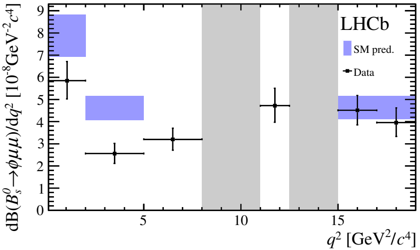
The LHCb anomalies (2)

$B_s \to \phi\mu^+\mu^-$ branching fraction

- Same theoretical description as $B \to K^*\mu^+\mu^-$
  - Replacement of $B \to K^*$ form factors with the $B_s \to \phi$ ones
  - Also consider the $B_s - \bar{B}_s$ oscillations

- June 2015 (3 fb^{-1}): the differential branching fraction is found to be 3.2σ below the SM predictions in the [1-6] GeV^2 bin

JHEP 1509 (2015) 179



The LHCb anomalies (3)

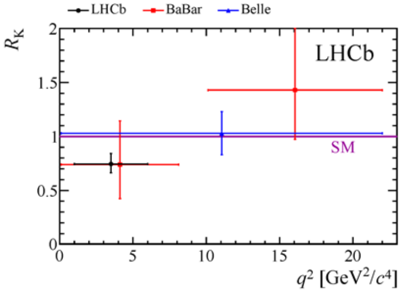
Lepton flavour universality in $B^+ \to K^+\ell^+\ell^-$

- Theoretical description similar to $B \to K^*\mu^+\mu^-$, but different since K is scalar
- June 2014 (3 fb^{-1}): measurement of R_K in the [1-6] GeV^2 bin

$$R_K = BR(B^+ \to K^+\mu^+\mu^-)/BR(B^+ \to K^+e^+e^-)$$

$$R_K^{SM} = 1.0006 \pm 0.0004$$

2.6σ tension in [1-6] GeV^2 bin



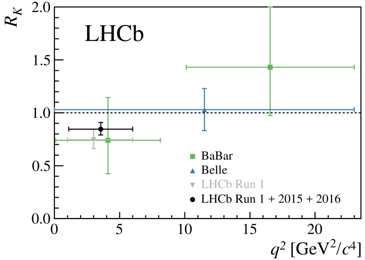
$$R_K^{exp} = 0.745^{+0.090}_{-0.074}(\text{stat}) \pm 0.036(\text{syst})$$

PRL 113, 151601 (2014)

BaBar, PRD 86 (2012) 032012; Belle, PRL 103 (2009) 171801

Recent results

$$R_K = BR(B^+ \to K^+\mu^+\mu^-)/BR(B^+ \to K^+e^+e^-)$$



Run 1 (PRL 113, 151601 (2014)):
$R_K([1.1, 6.0]\,\mathrm{GeV}^2) = 0.717^{+0.083+0.017}_{-0.071-0.016}$

Run 2 (arXiv:1903.09252):
$R_K([1.1, 6.0]\,\mathrm{GeV}^2) = 0.928^{+0.089+0.020}_{-0.076-0.017}$

$R_K^{\mathrm{SM}} = 1.0006 \pm 0.0004$

Bordone, Isidori, Pattori, Eur.Phys.J. C76 (2016) 8, 440

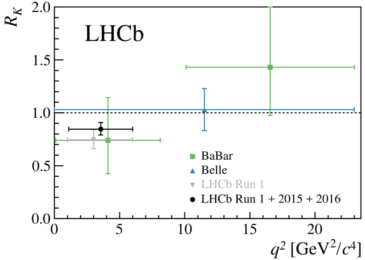
Combined result (arXiv:1903.09252):

$$R_K([1.1, 6.0]\,\mathrm{GeV}^2) = 0.846^{+0.060+0.016}_{-0.054-0.014}$$

Central value is now closer to the SM prediction, but the tension is still 2.5σ due to the smaller uncertainty of the new measurement.

Recent results

$$R_K = BR(B^+ \to K^+\mu^+\mu^-)/BR(B^+ \to K^+e^+e^-)$$



Run 1 (PRL 113, 151601 (2014)):
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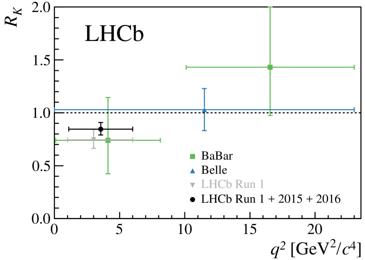

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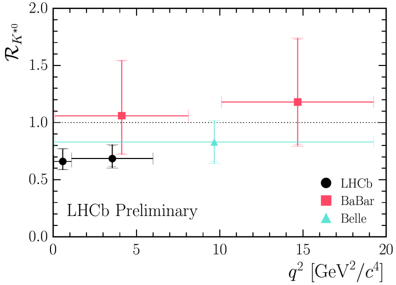
The LHCb anomalies (4)

Lepton flavour universality in $B \to K^* \ell^+ \ell^-$

- LHCb measurement (April 2017):

$$R_{K^*} = BR(B^0 \to K^{*0} \mu^+ \mu^-)/BR(B^0 \to K^{*0} e^+ e^-)$$

- Two q^2 regions: [0.045-1.1] and [1.1-6.0] GeV^2



$$R_{K^*}^{\text{exp,bin1}} = 0.660^{+0.110}_{-0.070}(\text{stat}) \pm 0.024(\text{syst})$$

$$R_{K^*}^{\text{exp,bin2}} = 0.685^{+0.113}_{-0.069}(\text{stat}) \pm 0.047(\text{syst})$$

JHEP 08 (2017) 055

$$R_{K^*}^{\text{SM,bin1}} = 0.906 \pm 0.028$$

$$R_{K^*}^{\text{SM,bin2}} = 1.000 \pm 0.010$$

BaBar, PRD 86 (2012) 032012; Belle, PRL 103 (2009) 171801

2.2-2.5σ tension with the SM predictions in each bin

Transversity amplitudes

Effective Hamiltonian for $b \to s\ell\ell$ transitions

$$\mathcal{H}_{\rm eff} = \mathcal{H}_{\rm eff}^{\rm had} + \mathcal{H}_{\rm eff}^{\rm sl}$$

$$\mathcal{H}_{\rm eff}^{\rm sl} = -\frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \Big[\sum_{i=7,9,10} C_i^{(\prime)} O_i^{(\prime)} \Big]$$

$\langle \bar{K}^* | \mathcal{H}_{\rm eff}^{\rm sl} | \bar{B} \rangle$: $B \to K^*$ form factors $V, A_{0,1,2}, T_{1,2,3}$

Transversity amplitudes:

$$A_\perp^{L,R} \simeq N_\perp \left\{ (C_9^+ \mp C_{10}^+) \frac{V(q^2)}{m_B + m_{K^*}} + \frac{2m_b}{q^2} C_7^+ T_1(q^2) \right\}$$

$$A_\parallel^{L,R} \simeq N_\parallel \left\{ (C_9^- \mp C_{10}^-) \frac{A_1(q^2)}{m_B - m_{K^*}} + \frac{2m_b}{q^2} C_7^- T_2(q^2) \right\}$$

$$A_0^{L,R} \simeq N_0 \Big\{ (C_9^- \mp C_{10}^-) \left[(\ldots) A_1(q^2) + (\ldots) A_2(q^2) \right] + 2m_b C_7^- \left[(\ldots) T_2(q^2) + (\ldots) T_3(q^2) \right] \Big\}$$

$$A_S = N_S (C_S - C_S^\prime) A_0(q^2)$$

$(C_i^\pm \equiv C_i \pm C_i^\prime)$

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$$\begin{aligned}\mathcal{A}_\lambda^{\rm (had)} &= -\, i\frac{e^2}{q^2} \int d^4x e^{-iq\cdot x} \langle \ell^+\ell^- | j_\mu^{\rm em,lept}(x)|0\rangle \\ &\times \int d^4y\, e^{iq\cdot y} \langle \bar{K}_\lambda^* | T\{ j^{{\rm em,had},\mu}(y) \mathcal{H}_{\rm eff}^{\rm had}(0)\} | \bar{B}\rangle \\ &\equiv \frac{e^2}{q^2} \epsilon_\mu L_V^\mu \Big[\underbrace{{\rm LO\ in\ } \mathcal{O}(\frac{\Lambda}{m_b}, \frac{\Lambda}{E_{K^*}})}_{\text{Non-Fact., QCDf}} + \underbrace{h_\lambda(q^2)}_{\text{power corrections}} \Big]\end{aligned}$$

Beneke et al.:
106067; 0412400

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$\to$ **unknown**

partial calculation: Khodjamirian et al., ... 1006.4945

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Non-Fact., QCDf

Beneke et al.: 106067; 0412400

power corrections

→ **unknown**

partial calculation: Khodjamirian et al., ... 1006.4945

The significance of the anomalies depends on the assumptions made for the unknown power corrections!

This does not affect R_K and R_{K^*}

Hadronic effects

Description also possible in terms of helicity amplitudes:

$$H_V(\lambda) = -\, i\, N' \Big\{ C_9\, \tilde{V}_{L\lambda}(q^2) + C_9'\, \tilde{V}_{R\lambda}(q^2) + \frac{m_B^2}{q^2} \Big[\frac{2\, \hat{m}_b}{m_B} (C_7\, \tilde{T}_{L\lambda}(q^2) + C_7'\, \tilde{T}_{R\lambda}(q^2)) - 16\pi^2 \mathcal{N}_\lambda(q^2) \Big] \Big\}$$

$$H_A(\lambda) = -i\, N' (C_{10}\, \tilde{V}_{L\lambda}(q^2) + C_{10}'\, \tilde{V}_{R\lambda}(q^2)), \qquad \mathcal{N}_\lambda(q^2) = \text{leading nonfact.} + h_\lambda$$

$$H_S = i\, N' \frac{\hat{m}_b}{m_W} (C_S - C_S') \tilde{S}(q^2) \qquad \left(N' = -\frac{4 G_F\, m_B}{\sqrt{2}} \frac{e^2}{16\pi^2} V_{tb} V_{ts}^* \right)$$

Helicity FFs $\tilde{V}_{L/R}, \tilde{T}_{L/R}, \tilde{S}$ are combinations of the standard FFs $V, A_{0,1,2}, T_{1,2,3}$

A possible parametrisation of the non-factorisable power corrections $h_{\lambda(=+,-,0)}(q^2)$:

$$h_\lambda(q^2) = h_\lambda^{(0)} + \frac{q^2}{1\mathrm{GeV}^2} h_\lambda^{(1)} + \frac{q^4}{1\mathrm{GeV}^4} h_\lambda^{(2)}$$

S. Jäger and J. Camalich, Phys.Rev. D93 (2016) 014028

M. Ciuchini et al., JHEP 1606 (2016) 116

It seems

$$h_\lambda^{(0)} \longrightarrow C_7^{NP}, \qquad h_\lambda^{(1)} \longrightarrow C_9^{NP}$$

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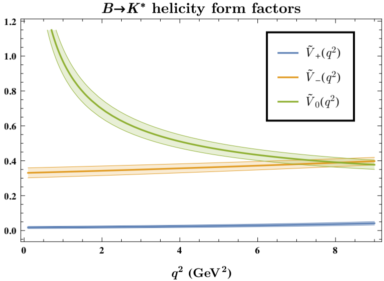

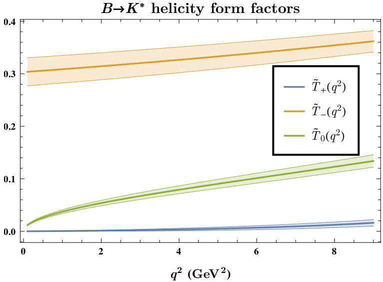

$\Longrightarrow$ q^4 terms can rise due to terms which multiply Wilson coefficients

$\Longrightarrow$ $C_7^{\rm NP}$ and $C_9^{\rm NP}$ can each cause effects similar to $h_\lambda^{(0,1,2)}$

Hadronic effects

Hadronic power correction effect:

$$\delta H_V^{\text{p.c.}}(\lambda) = iN' m_B^2 \frac{16\pi^2}{q^2} h_\lambda(q^2) = iN' m_B^2 \frac{16\pi^2}{q^2} \left(h_\lambda^{(0)} + q^2 h_\lambda^{(1)} + q^4 h_\lambda^{(2)} \right)$$

New Physics effect:

$$\delta H_V^{C_9^{\text{NP}}}(\lambda) = -iN' \tilde{V}_L(q^2) C_9^{\text{NP}} = iN' m_B^2 \frac{16\pi^2}{q^2} \left(a_\lambda C_9^{\text{NP}} + q^2 b_\lambda C_9^{\text{NP}} + q^4 c_\lambda C_9^{\text{NP}} \right)$$

and similarly for C_7

$\Rightarrow$ NP effects can be embedded in the hadronic effects.

We can do a fit for both (hadronic quantities $h_{+,-,0}^{(0,1,2)}$ (18 parameters) and Wilson coefficients C_i^{NP}(2 or 4 parameters))

Due to this embedding the two fits can be compared with the Wilk's test

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Wilk's test

SM vs 2 parameters and 4 parameters p-values were independently computed through 2D profile likelihood integration, and they give similar results

For low q^2 (up to 8 GeV^2):

	2 (δC_9)	4 ($\delta C_7, \delta C_9$)	18 ($h_{+,-,0}^{(0,1,2)}$)
0	3.7×10^{-5} (4.1σ)	6.3×10^{-5} (4.0σ)	6.1×10^{-3} (2.7σ)
2	–	0.13 (1.5σ)	0.45 (0.76σ)
4	–	–	0.61 (0.52σ)

$\rightarrow$ Adding δC_9 improves over the SM hypothesis by 4.1σ
$\rightarrow$ Including in addition δC_7 or hadronic parameters improves the situation only mildly
$\rightarrow$ One cannot rule out the hadronic option

Adding 16 more parameters does not really improve the fits

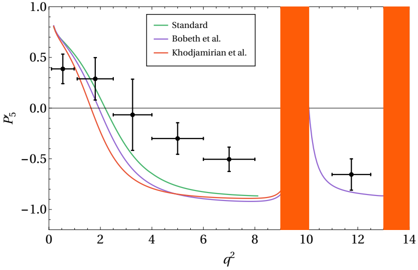
The situation is still inconclusive

Estimates of hadronic effects

Various methods for hadronic effects

$$\frac{e^2}{q^2}\epsilon_\mu L_V^\mu \left[Y(q^2)\tilde{V_\lambda} + \text{LO in } \mathcal{O}\left(\frac{\Lambda}{m_b}, \frac{\Lambda}{E_{K^*}}\right) + h_\lambda(q^2) \right]$$

	factorisable	non-factorisable	power corrections (soft gluon)	region of calculation	physical region of interest
Standard	✓	✓	✗	$q^2 \lesssim 7$ GeV^2	directly
Khodjamirian et al. [1006.4945]	✓	✗	✓	$q^2 < 1$ GeV^2	extrapolation by dispersion relation
Bobeth et al. [1707.07305]	✓	✓	✓	$q^2 < 0$ GeV^2	extrapolation by analyticity



Global fits

New Physics interpretation?

Many observables $\rightarrow$ **Global fits** of the LHCb data

Relevant $\mathcal{O}$perators:

$$\mathcal{O}_7,\ \mathcal{O}_8,\ \mathcal{O}_{9\mu,e}^{(\prime)},\ \mathcal{O}_{10\mu,e}^{(\prime)} \quad \text{and} \quad \mathcal{O}_{(S,P)} \propto (\bar{s}_L b_R)(\bar{\ell}(1,\gamma_5)\ell)$$

NP manifests itself in the shifts of the individual coefficients with respect to the SM values:

$$C_i(\mu) = C_i^{\rm SM}(\mu) + \delta C_i$$

$\rightarrow$ Scans over the values of δC_i

$\rightarrow$ Calculation of flavour observables

$\rightarrow$ Comparison with experimental results

$\rightarrow$ Constraints on the Wilson coefficients C_i

Global fits

Theoretical uncertainties and correlations

- Monte Carlo analysis
- variation of the "standard" input parameters: masses, scales, CKM, ...
- decay constants taken from the latest lattice results
- $B \to K^{(*)}$ and $B_s \to \phi$ form factors are obtained from the lattice+LCSR combinations (1411.3161, 1503.05534), including all the correlations
- Parameterisation of uncertainties from power corrections:

$$A_k \to A_k \left(1 + a_k \exp(i\phi_k) + \frac{q^2}{6\ \mathrm{GeV}^2} b_k \exp(i\theta_k)\right)$$

$|a_k|$ between 10 to 60%, $b_k \sim 2.5 a_k$
Low recoil: $b_k = 0$

$\Rightarrow$ Computation of a (theory + exp) correlation matrix

Global fits

Global fits of the observables obtained by minimisation of

$$\chi^2 = (\vec{O}^{\rm th} - \vec{O}^{\rm exp}) \cdot (\Sigma_{\rm th} + \Sigma_{\rm exp})^{-1} \cdot (\vec{O}^{\rm th} - \vec{O}^{\rm exp})$$

$(\Sigma_{\rm th} + \Sigma_{\rm exp})^{-1}$ is the inverse covariance matrix.

More than 100 observables relevant for leptonic and semileptonic decays:

- $\text{BR}(B \to X_s \gamma)$
- $\text{BR}(B \to X_d \gamma)$
- $\Delta_0(B \to K^* \gamma)$
- $\text{BR}^{\text{low}}(B \to X_s \mu^+ \mu^-)$
- $\text{BR}^{\text{high}}(B \to X_s \mu^+ \mu^-)$
- $\text{BR}^{\text{low}}(B \to X_s e^+ e^-)$
- $\text{BR}^{\text{high}}(B \to X_s e^+ e^-)$
- $\text{BR}(B_s \to \mu^+ \mu^-)$
- $\text{BR}(B_d \to \mu^+ \mu^-)$
- $\text{BR}(B \to K^0 \mu^+ \mu^-)$
- $\text{BR}(B \to K^{*+} \mu^+ \mu^-)$
- $\text{BR}(B \to K^+ \mu^+ \mu^-)$
- $\text{BR}(B \to K^* e^+ e^-)$
- R_K
- $B \to K^{*0} \mu^+ \mu^-$: BR, F_L, A_{FB}, S_3, S_4, S_5, S_7, S_8, S_9 in 8 low q^2 and 4 high q^2bins
- $B_s \to \phi \mu^+ \mu^-$: BR, F_L, , S_3, S_4, S_7 in 3 low q^2 and 2 high q^2bins

Single operator fits

Comparison of one-operator NP fits:

(under the assumption of 10% non-factorisable power corrections)

All observables except R_K, R_{K^*} ($\chi^2_{\rm SM} = 100.2$)			
	b.f. value	$\chi^2_{\rm min}$	${\rm Pull}_{\rm SM}$
δC_9	-1.00 ± 0.20	82.5	4.2σ
δC_9^μ	-1.03 ± 0.20	80.3	4.5σ
δC_9^e	0.72 ± 0.58	98.9	1.1σ
δC_{10}	0.25 ± 0.23	98.9	1.1σ
δC_{10}^μ	0.32 ± 0.22	98.0	1.5σ
δC_{10}^e	-0.56 ± 0.50	99.1	1.0σ
$\delta C_{\rm LL}^\mu$	-0.48 ± 0.15	89.1	3.3σ
$\delta C_{\rm LL}^e$	0.33 ± 0.29	99.0	1.1σ

Only R_K, R_{K^*} ($\chi^2_{\rm SM} = 16.9$)			
	b.f. value	$\chi^2_{\rm min}$	${\rm Pull}_{\rm SM}$
δC_9	-2.04 ± 5.93	16.8	0.3σ
δC_9^μ	-0.74 ± 0.28	8.4	2.9σ
δC_9^e	0.79 ± 0.29	7.7	3.0σ
δC_{10}	4.10 ± 11.87	16.7	0.5σ
δC_{10}^μ	0.77 ± 0.26	6.1	3.3σ
δC_{10}^e	-0.78 ± 0.27	6.0	3.3σ
$\delta C_{\rm LL}^\mu$	-0.37 ± 0.12	7.0	3.1σ
$\delta C_{\rm LL}^e$	0.41 ± 0.15	6.8	3.2σ

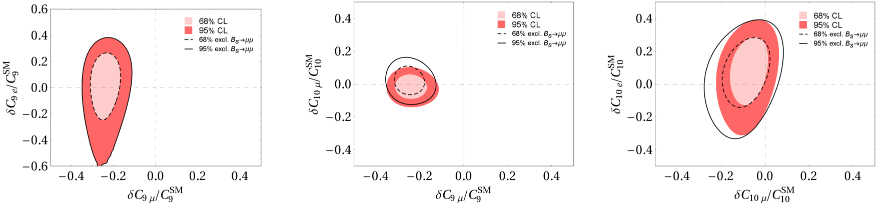
$\delta C_{\rm LL}^\ell$ basis corresponds to $\delta C_9^\ell = -\delta C_{10}^\ell$.

$\rightarrow$ C_9 and C_9^μ solutions are favoured with SM pulls of 4.2 and 4.5σ

$\rightarrow$ Good fits possible for $R_{K^{(*)}}$ ratios with NP in $C_9^{e/\mu}$, $C_{10}^{e/\mu}$ or $C_{\rm LL}^{e/\mu}$

Two operator fits

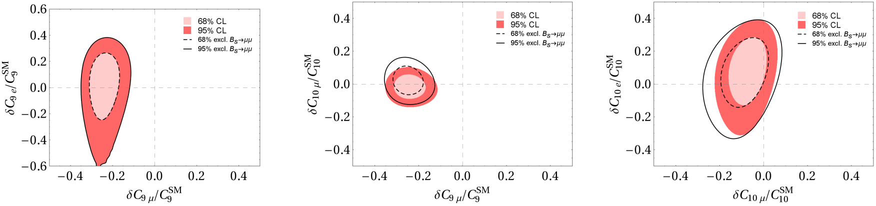
all observables except R_K and R_{K^*} (with the assumption of 10% power corrections)



Pull: 4.1σ 4.1σ 1.1σ

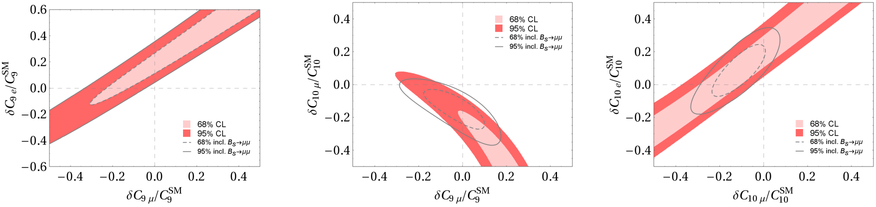
Two operator fits

all observables except R_K and R_{K^*} (with the assumption of 10% power corrections)



Pull: 4.1σ 4.1σ 1.1σ

Using only the data on R_K and R_{K^*}



Pull: 3.1σ 3.2σ 3.1σ

Updated fits - single operators

Using all the relevant data on $b \to s$ transitions:

assuming 10% error for the power corrections

All observables ($\chi^2_{\rm SM} = 117.03$)			
	b.f. value	$\chi^2_{\rm min}$	${\rm Pull}_{\rm SM}$
δC_9	-1.01 ± 0.20	99.2	4.2σ
δC_9^μ	-0.93 ± 0.17	89.4	5.3σ
δC_9^e	0.78 ± 0.26	106.6	3.2σ
δC_{10}	0.25 ± 0.23	115.7	1.1σ
δC_{10}^μ	0.53 ± 0.17	105.8	3.3σ
δC_{10}^e	-0.73 ± 0.23	105.2	3.4σ
$\delta C_{\rm LL}^\mu$	-0.41 ± 0.10	96.6	4.5σ
$\delta C_{\rm LL}^e$	0.40 ± 0.13	105.8	3.3σ

The NP significance is reduced by at least 0.5σ compared to before.

In cases of flavour-symmetric C_9 and C_{10}, which are independent from the changes in the ratios, one finds the same NP significance as expected.

More complete analyses

In a New Physics model:

- new vector bosons: C_7, C_9, C_{10}
- new fermions: C_7, C_8, C_9, C_{10}
- extended Higgs sector/new scalars: C_S, C_P

e.g. in the MSSM, 2HDM, ...: $C_7, C_8, C_9, C_{10}, C_S, C_P$

Considering only one or two Wilson coefficients may not give the full picture!

A generic set of Wilson coefficients:

$$\text{complex } C_7, C_8, C_9^\ell, C_{10}^\ell, C_S^\ell, C_P^\ell + \text{primed coefficients}$$

The available observables are mainly insensitive to the imaginary parts, one can limit the set to

$$\text{real } C_7, C_8, C_9^\ell, C_{10}^\ell, C_S^\ell, C_P^\ell + \text{primed coefficients}$$

corresponding to 20 degrees of freedom.

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corresponding to 20 degrees of freedom.

Full fit - results

Set: real C_7, C_8, C_9^ℓ, C_{10}^ℓ, C_S^ℓ, C_P^ℓ + primed coefficients (20 (16) degrees of freedom)

All observables with $\chi^2_{\rm SM} = 117.03$
$(\chi^2_{\rm min} = 71.96;\ {\rm Pull}_{\rm SM} = 3.3\ (3.8)\sigma)$

δC_7	δC_8
-0.01 ± 0.04	0.82 ± 0.72
$\delta C_7'$	$\delta C_8'$
0.01 ± 0.03	-1.65 ± 0.47

δC_9^μ	δC_9^e	δC_{10}^μ	δC_{10}^e
-1.37 ± 0.25	-6.55 ± 2.37	-0.11 ± 0.27	2.34 ± 3.11
$\delta C_9'^\mu$	$\delta C_9'^e$	$\delta C_{10}'^\mu$	$\delta C_{10}'^e$
0.23 ± 0.62	0.75 ± 2.82	-0.16 ± 0.36	1.67 ± 3.05

$C_{Q_1}^\mu$	$C_{Q_1}^e$	$C_{Q_2}^\mu$	$C_{Q_2}^e$
-0.01 ± 0.09	undetermined	-0.05 ± 0.19	undetermined
$C_{Q_1}'^\mu$	$C_{Q_1}'^e$	$C_{Q_2}'^\mu$	$C_{Q_2}'^e$
0.13 ± 0.09	undetermined	-0.18 ± 0.20	undetermined

Wilks' test:

- No real improvement in the fits when going beyond the C_9^μ case
- Pull with the SM decreases when all Wilson coefficients are varied
- Many parameters are very weakly constrained

NP scenarios

New physics scenarios

Global fits: New physics is likely to appear in C_9:

$$O_9 = \frac{e^2}{(4\pi)^2}(\bar{s}\gamma^\mu b_L)(\bar{\ell}\gamma_\mu \ell)$$

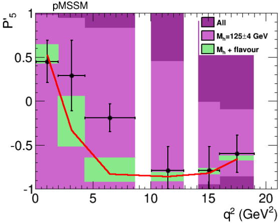
It can also affect other Wilson coefficients in a lesser extent.

However, difficult to generate $\delta C_9 \sim -1$ at loop level...

Very difficult in the MSSM!

MSSM

Fit results in the pMSSM



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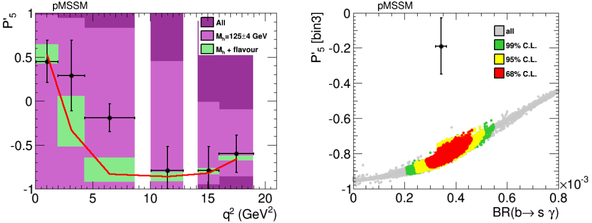

MSSM

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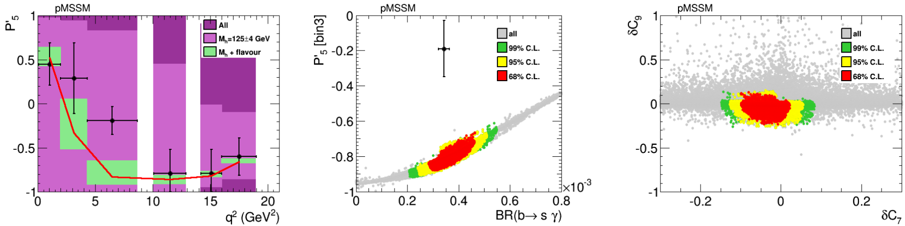
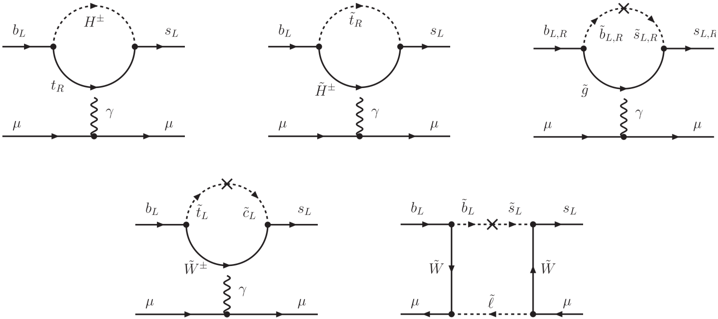


MSSM and C_9

Contributions to C_9 and C_9' can come from Z and photon penguins, and box diagrams



- Z-penguins suppressed by small vector coupling
- charged Higgs contributions proportional to $1/\tan^2\beta$
- other penguin diagrams suppressed by the LHC squark and gluino mass limits
- in any case, only box diagrams can lead to lepton flavour non-universality...
- ... but box diagrams suppressed by the LEP slepton and chargino mass bounds

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It can also affect other Wilson coefficients in a lesser extent.

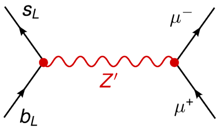
However, difficult to generate $\delta C_9 \sim -1$ at loop level...

$\rightarrow$ Need for tree level diagrams...

Mainstream scenarios:

- Z' bosons
- leptoquarks
- composite models

Z' bosons



Z' obvious candidate to generate the O_9 operator

Needs:

- Flavour-changing couplings to left-handed quarks
- Vector-like couplings to leptons
- Flavour violation or non-universality in the lepton sector

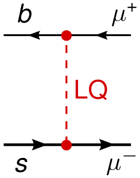
Strong constraints from $B_s - \bar{B}_s$ mixing and LEP contact interactions.

Anomalies consistent with a Z' of 1 to 10 TeV

Can appear in many models, like 331 models, gauge $L_\mu - L_\tau$ models, ...

See e.g. Altmannshofer et al. 1308.1501, Gauld et al. 1308.1959, Buras et al. 1309.2466, Gauld et al. 1310.1082, Buras et al. 1311.6729, Altmannshofer et al. 1403.1269, Buras et al. 1409.4557, Glashow et al. 1411.0565, Crivellin et al. 1501.00993, Altmannshofer et al. 1411.3161, Crivellin et al. 1503.03477, Niehoff et al. 1503.03865, Crivellin et al. 1505.02026, Celis et al. 1505.03079, ...

Leptoquarks



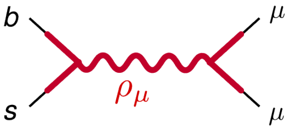
- t-channel diagrams
- Different possible representations, can be scalar or vector
- Cannot alter only C_9, but both C_9 and C_{10} $(= -C_9)$
- Cannot be lepton flavour non-universal and conserve lepton number simultaneously

Model can be tested with $R_{K^{(*)}}$ measurements and searches for $b \to s\mu^{\pm}e^{\mp}$ and $\mu \to e\gamma$

Possible scenario: two leptoquarks coupling to one lepton type only.

See e.g. Hiller et al. 1408.1627, Biswas et al. 1409.0882, Buras et al. 1409.4557, Sahoo et al. 1501.05193, Hiller et al. 1411.4773, Becirevic et al. 1503.09024, Alonso et al. 1505.05164, ...

Composite models



- Neutral resonance ρ_μ coupling to the muons via composite elementary mixing
- requires some compositeness for the muons
- can allow for lepton flavour violating couplings
- constrained by the LEP Z-width measurements and $B_s - \bar{B}_s$ mixing

Nonperturbative physics, making predictions more difficult...

See e.g. Gripaios et al. 1412.1791, Niehoff, et al. 1503.03865, Niehoff et al. 1508.00569, Carmona et al. 1510.07658, ...

Future prospects

How to resolve the issue?

1) Improving the precision of the theoretical calculations

- still some QCD ingrediants unknown, or only partially known

  → New methods and alternative approaches are required
  → Several attempts already in the literature

2) Cross-check with other $R_{\mu/e}$ ratios

- R_K and R_{K^*} ratios are theoretically very clean
- The tensions cannot be explained by hadronic uncertainties

Cross-checks needed with other ratios:

	Predictions assuming 12 fb^{-1} luminosity			
Obs.	C_9^μ	C_9^e	C_{10}^μ	C_{10}^e
$R_{F_L}^{[1.1,6.0]}$	$[0.785, 0.913]$	$[0.909, 0.933]$	$[1.005, 1.042]$	$[1.001, 1.018]$
$R_{S_5}^{[1.1,6.0]}$	$[-0.787, 0.394]$	$[0.603, 0.697]$	$[0.881, 1.002]$	$[1.053, 1.146]$
$R_{K^*}^{[15,19]}$	$[0.621, 0.803]$	$[0.577, 0.771]$	$[0.589, 0.778]$	$[0.586, 0.770]$
$R_K^{[15,19]}$	$[0.597, 0.802]$	$[0.590, 0.778]$	$[0.659, 0.818]$	$[0.632, 0.805]$
$R_\phi^{[1.1,6.0]}$	$[0.748, 0.852]$	$[0.620, 0.805]$	$[0.578, 0.770]$	$[0.578, 0.764]$
...				

A confirmation of the deviations in the ratios would indirectly confirm the NP interpretation of the anomalies in the angular observables!

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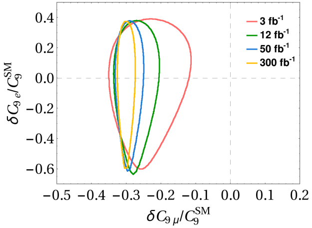
How to resolve the issue?

3) Future LHCb prospects

Global fits using the angular observables only (NO theoretically clean R ratios)

Considering several luminosities, assuming the current central values



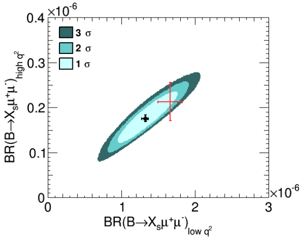
LHCb will be able to establish new physics within the angular observables even in the pessimistic case that there will be no theoretical improvements!

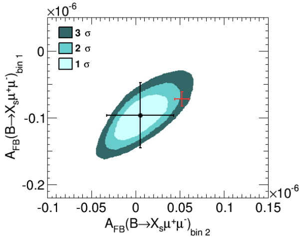
How to resolve the issue?

4) Cross-check with inclusive modes

Inclusive decays are theoretically cleaner (see e.g. T. Huber, T. Hurth, E. Lunghi, JHEP 1506 (2015) 176)

At Belle-II, for inclusive $b \to s\ell\ell$:





T. Hurth, FM, JHEP 1404 (2014) 097

T. Hurth, FM, S. Neshatpour, JHEP 1412 (2014) 053

Predictions based on our model-independent analysis

black cross: future measurements at Belle-II assuming the best fit solution

red cross: SM predictions

→ Belle-II will check the NP interpretation with theoretically clean modes

Cooking scenarios

Cooking a New Physics scenario

Cooking scenarios

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Model-independent approach

↓

gives us the ingredients
C_9, a bit of C_{10},...

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UV-complete theory

↓

The real model

Final remarks

Could the anomalies be explained by:

- Statistical fluctuations alone?
- Experimental issues alone?
- Underestimated theoretical uncertainties alone?
- Unknown pieces in the theoretical calculations alone?

  - Combination of above?

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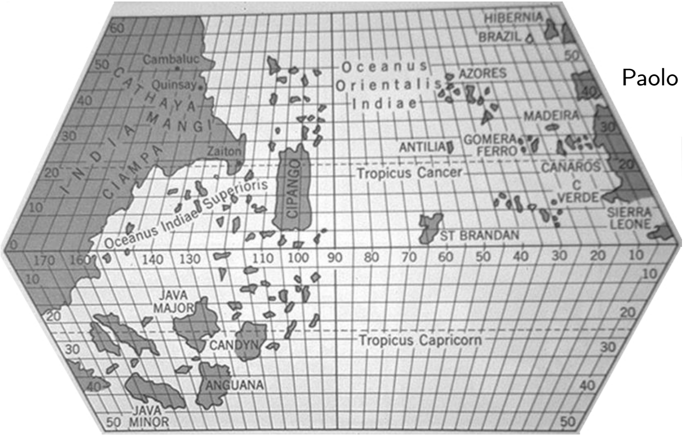
The next round of LHCb results will give us the verdict!

Path to New Physics

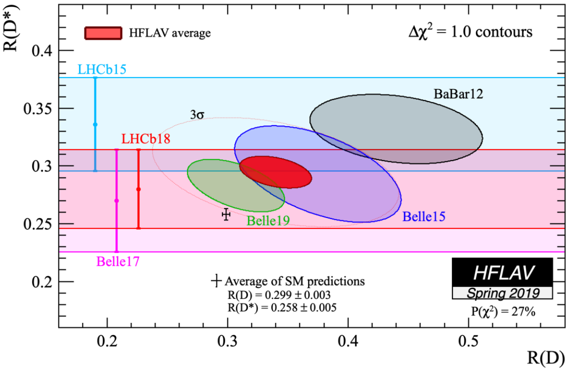
We may be in such a situation:



Paolo Toscanelli
1474

Columbus had Toscanelli's map.
It was terribly wrong, but served the purpose!

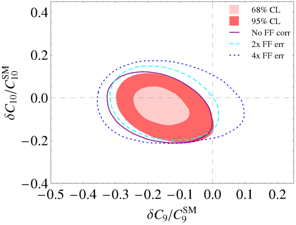
Backup


HFLAV average
Δχ² = 1.0 contours
LHCb15
BaBar12
3σ
LHCb18
Belle19
Belle15
Belle17
Average of SM predictions
R(D) = 0.299 ± 0.003
R(D*) = 0.258 ± 0.005
HFLAV
Spring 2019
P(χ²) = 27%
R(D*)
R(D)
0.4
0.35
0.3
0.25
0.2
0.2
0.3
0.4
0.5

Fits with different assumptions for the form factor uncertainties:

- correlations ignored (solid line)
- normal form factor errors (filled areas)
- 2 $\times$ form factor errors (dashed line)
- 4 $\times$ form factor errors (dotted line)

$(C_9 - C_{10})$ $(C_9 - C_9')$ $(C_9^e - C_9^\mu)$



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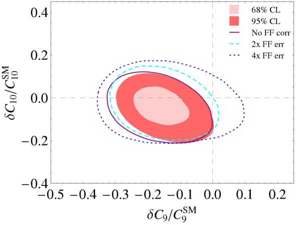

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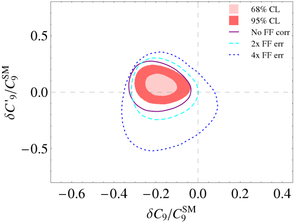

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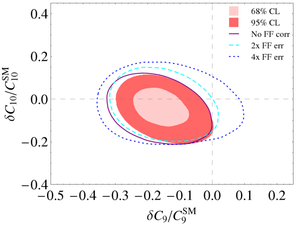

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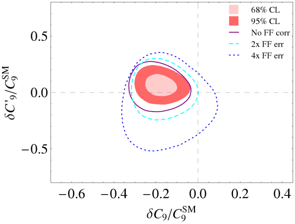
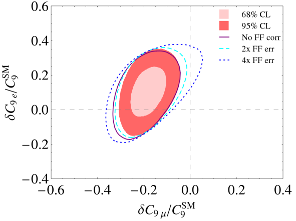


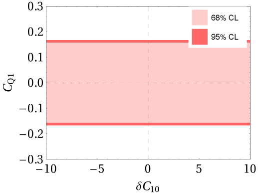
$(C_9^e - C_9^\mu)$

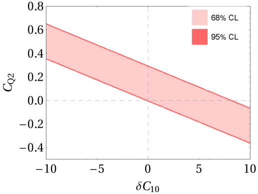


The size of the form factor errors has a crucial role in constraining the allowed region!

The role of (pseudo)scalar operators

Imposing BR($B_s \to \mu^+\mu^-$), if C_S and C_P independent, there exists a degeneracy between C_{10} and C_P so that large values for C_P are possible





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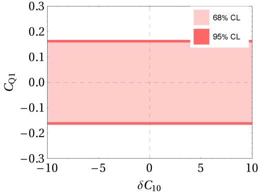
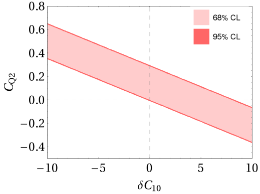
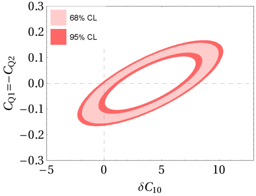




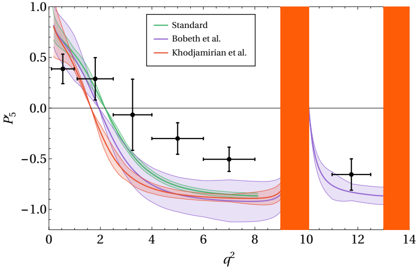
Even if $C_S = -C_P$, allowing for small variations of $C_{S,P}$ alleviates the constraints from $B_s \to \mu^+\mu^-$ on C_{10}



Various methods for hadronic effects

$$\frac{e^2}{q^2}\epsilon_\mu L_V^\mu\left[Y(q^2)\tilde{V}_\lambda + \mathrm{LO\ in}\ \mathcal{O}\left(\frac{\Lambda}{m_b},\frac{\Lambda}{E_{K^*}}\right) + h_\lambda(q^2)\right]$$

	factorisable	non-factorisable	power corrections (soft gluon)	region of calculation	physical region of interest
Standard	✓	✓	✗	$q^2 \lesssim 7$ GeV^2	directly
Khodjamirian et al. [1006.4945]	✓	✗	✓	$q^2 < 1$ GeV^2	extrapolation by dispersion relation
Bobeth et al. [1707.07305]	✓	✓	✓	$q^2 < 0$ GeV^2	extrapolation by analyticity



Full fit

A generic set of Wilson coefficients:

$$\text{complex } C_7, C_8, C_9^\ell, C_{10}^\ell, C_S^\ell, C_P^\ell + \text{primed coefficients}$$

The available observables are mainly insensitive to the imaginary parts, one can limit the set to

$$\text{real } C_7, C_8, C_9^\ell, C_{10}^\ell, C_S^\ell, C_P^\ell + \text{primed coefficients}$$

corresponding to 20 degrees of freedom.

Some of the coefficients may have only weak effects on the observables, and affect the number of dof without affecting the χ^2, acting as *spurious* degrees of freedom.

effective degrees of freedom (e-dof): degrees of freedom minus the parameters δC_i only weakly affecting the χ^2, defined such as

$$|\chi^2(\delta C_i = 1) - \chi^2(\delta C_i = 0)| < 1$$

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