#### Lattice QCD for Nuclear and Particle Physics

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Image Credit: 2018 EIC User's Group Meeting

UK Annual Theory Meeting, IPPP Durham U, 2019

Massachusetts Institute of Technology

# Lattice QCD for HEP & NP

We are entering the

- Precision era of lattice QCD for simple systems
- Beginning of reliable lattice QCD results for nuclear matrix elements

I will highlight some new results in these areas in the last few years

#### Lattice QCD can provide input for

Decay constants, form factors, mixing parameters



Hadronic vacuum polarisation and lightby-light scattering



Neutrino-nucleus interactions



Dark matter-nucleon and DM-nucleus interactions



Muon-nucleus cross-sections

Parton distribution functions





#### Lattice QCD

Numerical first-principles approach to non-perturbative QCD

• Discretise QCD onto 4D space-time lattice

- Approximate QCD path integral using Monte-Carlo methods and importance sampling
- Run on supercomputers and dedicated clusters
- Take limit of vanishing discretisation, infinite volume, physical quark masses



#### Lattice QCD

#### Numerical first-principles approach to non-perturbative QCD

#### INPUT

Lattice QCD action has same free parameters as QCD: quark masses,  $\alpha_S$ 

- Fix quark masses by matching to measured hadron masses, e.g.,  $\pi, K, D_s, B_s$  for u, d, s, c, b
- One experimental input to fix lattice spacing in GeV (and also  $\alpha_S$ ), e.g., 2S-1S splitting in Y, or  $f_{\pi}$  or  $\Omega$  mass

#### OUTPUT

Calculations of all other quantities are QCD predictions



#### Lattice QCD

Numerical first-principles approach to non-perturbative QCD

#### Calculations use world's largest computers

- Many millions of CPU/ GPU/KNL hours
- Specifically designed processors for QCD (QCDOC precursor of BlueGene computers)



# Highlights in LQCD for HEP & NP

- •Quark masses and  $lpha_s$
- Flavour physics
- Parton distribution functions
- Pressure in the proton
- Neutrino physics
- Dark matter

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#### Quark masses and $\alpha_s$

The quark masses and  $\alpha_{\rm s}$  are the fundamental parameters of QCD Their precise values are important for precision tests of the Standard Model

e.g., Next-generation of high-luminosity colliders will measure **Higgs partial** widths to sub-percent precision to look for deviations from Standard-Model expectations



Need Standard Model calculations at same sub-percent precision; largest uncertainties are currently in  $m_c$ ,  $m_b$ , &  $\alpha_s$  [LHCHXSWG-DRAFT-INT-2016-008]



Lepage, Mackenzie, Peskin, [arXiv:1404.0319]

- Precision goals for  $m_c$ ,  $m_b$ , &  $\alpha_s$  needed by high-luminosity ILC
- Outlined timeline for lattice
   QCD progress

Continued progress towards precision goals in last year

Next goals:

- Correlated determinations of m<sub>c</sub>, m<sub>b</sub>, and  $\alpha_s$
- Dynamical QED



### $\alpha_s$ update

Best lattice QCD uncertainties ~0.6-0.7%, approaching ILC target: 0.6%. Twice as precise as non-lattice world average

- Several independent lattice QCD methods available to obtain  $\alpha_{\rm s}$
- Results consistent, despite significantly different sources of systematic uncertainty

#### **Recent Highlight:**

New lattice QCD determination based on finite size scaling (rather than Wilson Loops and quarkonia) consistent and precise:  $\alpha_s = 0.1185(8)(3)$  [PRL119, 102001]

#### Quark masses update



Recent Highlight: Significant improvement in heavy quark mass determinations using new method based on heavyquark effective theory [Phys. Rev. D97, 034503]

Precision	ILC goals
$\delta m_b \sim 0.3$	0.3
$\delta m_c \sim 0.8$	0.7

Will improve further with inclusion of finer lattice spacings

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# Flavour physics from lattice QCD

Lattice QCD can provide precise hadronic matrix elements for flavour physics:

[experiment] = [known] x [CKM] x [hadronic matrix element]

This leads to effort in two main directions:

1 Lattice QCD + experiment CKM quark mixing matrix elements Simple tree-level processes in lattice QCD allow determination of all elements and phases other than V<sub>tb</sub>

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ & K \rightarrow \pi \, l\nu & B \rightarrow \pi \, l\nu \\ \pi \rightarrow \mu\nu & K \rightarrow \mu\nu & & \\ V_{cd} & V_{cs} & V_{cb} \\ D \rightarrow \pi \, l\nu & D \rightarrow K \, l\nu & B \rightarrow D, D^* \, l\nu \\ D \rightarrow l\nu & D_s \rightarrow \, l\nu & & \\ V_{td} & V_{ts} & V_{tb} \\ B^0 - \overline{B^0} & \text{mixing} & B_s - \overline{B_s} & & \\ \end{pmatrix}$$

2 Lattice QCD + CKM
 Standard Model expectations for rare decays & mixing
 Target processes sensitive to beyond-SM physics

# Flavour Lattice Averaging Group R Flavour Lattice Averaging Group

Many lattice calculations of what are now "simple" flavour physics quantities

FLAG: Flavour Lattice Averaging Group Similar effort to the PDG, for Lattice QCD

- Members from most major lattice QCD collaborations
- Evaluates and grades different aspects of each calculation  $\prec$  O  $\blacksquare$
- Provides averages as the "Lattice QCD community consensus" value for a given quantity
- Includes lattice dictionary and summaries for non-experts
- Summary report every ~2 years: [arXiv:1304.5422] [1607.00299] [1902.08191]
   March 2019 update at http://flag.unibe.ch
- New version released in 2019: coverage expanded to include simple baryon quantities e.g., g<sub>A</sub>



- Decay constants, form factors, kaon mixing, LECs...
- Colour coded for quality of calculation (# lattice spacings, volumes,...)



http://flag.unibe.ch

#### $B \rightarrow D\ell\nu$ and $B \rightarrow D^*\ell\nu$

Lattice QCD results for  $B \rightarrow D\ell v$  reveal  $\sim 3\sigma$  tension with experimental measurements [HPQCD arXiv:1505.03925, MILC arXiv:1503.07237]

Latest Belle results reduce the tension

Standard Model predication for R(D\*) uses experimental data plus HQET: uncertainties may be underestimated [PRD 85, 094025]

Preliminary results for LQCD determinations of  $\overline{B} \rightarrow D^* \ell \overline{\nu}$  form factors at nonzero recoil: will provide independent constraints on R(D\*) [arXiv:1710.09817, 1906.01019]



 $K^+ \rightarrow \pi^+ \nu \overline{\nu}$ 

K+→π<sup>+</sup>ν decay is a rare process sensitive to many new physics scenarios e.g., Randall-Sundrum, MSSM, Z, Z' models, Littlest Higgs with T-parity, LFU violation models

 NA62 Experiment aims to test Standard Model at 10% precision with data to end 2018; first dataset 2016, one event observed



 Short distance contribution to decay amplitude: perturbation theory + semileptonic kaon decay form factors
 Long-distance contribution O(5%) from phenomenological estimates.

#### 2018 Lattice QCD highlight:

Exploratory calculation demonstrated feasibility of decay amplitude calculation, in particular long-distance component [Bai et al., Phys. Rev. D 98, 074509 (2018)] **Expectation that a fully controlled calculation will be possible within four years** 

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# Parton physics from Lattice QCD

Understanding the quark and gluon structure of matter



Three-dimensional partonic structure of the proton

How do quarks and gluons carry the proton's

- Mass
- Momentum
- Angular momentum, spin
- Pressure and shear

#### Parton distribution functions

Parton distribution functions  $f(x, \mu^2)$ 

Number densities of partons of type f with momentum fraction x at scale  $\mu^2$  in a given hadron

PDFs quantify fundamental aspects of hadron structure

Nucleon PDFs are needed for e.g., searches for new physics at the LHC through top-quark and Higgs-boson coupling measurements

#### Lattice QCD can provide

- Moments of PDFs with controlled uncertainties:  $\int_0^1 x^n f(x,\mu^2) = \langle x^n \rangle_f(\mu^2)$ Inclusion in global PDF fits can reduce uncertainties see workshop slides <u>http://www.physics.ox.ac.uk/confs/PDFlattice2017</u> and community white paper [Prog.Part.Nucl.Phys.100 (2018) 107]
- First calculations of x-dependence of nucleon PDFs

# Parton physics from Lattice QCD

#### **Precision Era**

Fully-controlled w/ few-percent errors within ~5y

- Static properties of nucleon incl. spin, flavour decomp.
- Mellin moments of PDFs, GPDs

#### **Early Era**

Fully-controlled w/ ~15-percent errors within ~7y

- Nuclear structure A<5
- Spin, flavour decomp. of EMC-type effects

#### **Exploratory Era**

First calculations, timeline for controlled calculations unclear

• x-dependence of PDFs

Phiala Shanahan, MIT

• TMDs

# Moments of PDFs

# Lattice QCD can cleanly access low moments of PDFs (n $\leq$ 3)

[work to move beyond: Chambers et al., arXiv:1703.01153,

Davoudi & Savage, arXiv:1204.4146]

$$\int_0^1 x^n f(x,\mu^2) = \langle x^n \rangle_f(\mu^2)$$

State-of-the-art calculations have:

- Fully-controlled systematic uncertainties competitive with or better than experiment for some quantities
- Separate contributions from
  - Strangeness and light flavours
  - Charge symmetry violation
  - Gluons

**Recent highlight:** All terms of nucleon momentum decomposition calculated with controlled uncertainties

MS-scheme at 2 GeV



# Constraints on global PDF fits

 Including lattice QCD results for moments in global PDF fits can yield significant improvements



Yellow: SIDIS data only: direct constraints in region indicated by dashes Blue/Red: SIDIS + lattice QCD for tensor charge (zeroth moment)

[H-W. Lin et al., PRL 120 (2018), 152502]

- First calculations of x-dependence of nucleon PDFs Quasi and pseudo-PDF calculations use non-local Euclidean correlators and perturbative QCD matching in high momentum limit [X. Ji, arXiv:1305.1539]
- Renormalisation and perturbative matching understood (~5 year effort)
- Low-x, high-x, regions particularly challenging (lattice systematics)
- Flavour separation is relatively straightforward



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# Energy Momentum Tensor

- Generalised form factors encode e.g., Energy-Momentum Tensor
- Matrix elements of traceless gluon EMT for spin-half nucleon:

$$\langle p', s' | G_{\{\mu\alpha}^{a\alpha} G_{\nu}^{a\alpha} | p, s \rangle = \bar{U}(p', s') \begin{pmatrix} A_g(t) \gamma_{\{\mu} P_{\nu\}} + B_g(t) \frac{i P_{\{\mu} \sigma_{\nu\}\rho} \Delta^{\rho}}{2M_N} + D_g(t) \frac{\Delta_{\{\mu} \Delta_{\nu\}}}{4M_N} \end{pmatrix} U(p, s)$$
Gluon field-
strength tensor
$$\Delta_{\mu} = p'_{\mu} - p_{\mu} \quad P_{\mu} = (p_{\mu} + p'_{\mu})/2, \quad t = \Delta^2$$

Sum rules of gluon and quark GFFs in forward limit

• Momentum fraction  $A_a(0) = \langle x \rangle_a$   $\longrightarrow$   $\sum A_a(0) =$ 

• Spin  $J_a(t) = \frac{1}{2}(A_a(t) + B_a(t))$ 

$$\sum_{a=q,g} A_a(0) = 1$$
$$\sum_{a=q,g} J_a(0) = \frac{1}{2}$$

• D-terms  $D_a(0)$  unknown but equally fundamental!

• D<sub>a</sub>(t) GFFs encodes pressure and shear distributions

### D-term from JLab DVCS

Experimental determination of DVCS D-term and extraction of proton pressure distribution [Burkert, Elouadrhiri, Girod, Nature 557, 396 (2018)]

$$s(r) = -\frac{r}{2}\frac{d}{dr}\frac{1}{r}\frac{d}{dr}\widetilde{D}(r), \quad p(r) = \frac{1}{3}\frac{1}{r^2}\frac{d}{dr}r^2\frac{d}{dr}\widetilde{D}(r)$$

- Peak pressure near centre ~10<sup>35</sup> Pascal, greater than pressure estimated for neutron stars
- Key assumptions: gluon D-term same as quark term, tripole form factor model,  $D_u(t,\mu) = D_d(t,\mu)$

#### EXP + LQCD

first complete pressure determination

[Shanahan, Detmold PRL 122 072003 (2019)]



#### Radial pressure distribution



#### Nucleon D-term GFFs from LQCD

#### EXP + LQCD

#### first complete pressure determination

[Shanahan, Detmold PRL 122 072003 (2019)]

#### Key assumptions in pressure extraction from DVCS

- Gluon D-term same as quark term in magnitude and shape
   Factor of ~2 difference in magnitude, somewhat different t-dependence
- Tripole form factor model LQCD results consistent with ansatz, but more general form is less well constrained
- Isovector quark D-term vanishes  $D_{u-d}(t) \sim 0$  from other LQCD studies



Gluon GFFs: Shanahan, Detmold, PRD 99, 014511 Quark GFFs: P. Hägler et al. (LHPC), PRD77, 094502 (2008) Expt quark GFFs (BEG): Burkert et al, Nature 557, 396 (2018)

#### Nucleon D-term GFFs from LQCD EXP + LQCDfirst complete pressure determination [Shanahan, Detmold PRL 122 072003 (2019)] 1.0total $r^2 p(r) \ (\times 10^{-2} \text{ GeV fm}^{-1})$ total BEG - gluon cont. - - quark cont. 0.5 $D_a(t)$ quark BEG quark LQCD gluon LQCD $\diamond$ 0.0 $\overline{\mathrm{MS}} \left( \mu = 2 \text{ GeV} \right)$ gluon contribution shifts peaks, extends region over 1.01.52.0which pressure is non-zero $-t \; (\text{GeV}^2)$ -0.5Gluon GFFs: Shanahan, Detmold, PRD 99, 014511 0.51.52.00.01.0 Quark GFFs: P. Hägler et al. (LHPC), PRD77, 094502 (2008) Expt quark GFFs (BEG): Burkert et al, Nature 557, 396 (2018) $r \, (\mathrm{fm})$

#### Next: pressure distribution in nuclei

Pressure in light nuclei c.f. pressure in the nucleon?



Pion & Nucleon quark and gluon momentum fractions consistent within uncertainties, but very different pressure distributions!

m) Gluon GFFs: Shanahan, Detmold, PRD 99, 014511 Quark GFFs: P. Hägler et al. (LHPC), PRD77, 094502 (2008)

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# Long-baseline neutrino experiments



Seek to determine neutrino mass hierarchy, mixing parameters, CP violating phase

To differentiate between mixing & CP parameter scenarios



Need neutrino energy reconstruction from final state to better than 100 MeV



Lattice QCD: direct non-perturbative QCD predictions for nucleon and nuclear matrix elements

e.g., axial and pseudo-scalar form factors important in quasi-elastic region

# Constraining v-nucleus interactions

- For LBNEs neutrino energy distributions peak at 1-10 GeV
- Challenging region: several processes contribute
  - Quasielastic lepton scattering
  - Inelastic continuum / shallowinelastic region
  - Resonances
- Lattice QCD can provide direct non-perturbative QCD predictions of nucleon and nuclear matrix elements



J.A. Formaggio, G.P. Zeller, Rev. Mod. Phys. 84 (2012) 1307

#### Quasi-elastic scattering

Cross-section for quasi-elastic neutrinonucleon scattering

$$\begin{bmatrix} \frac{d\sigma}{dQ^2} = \frac{G_f^2 M^2 \cos^2 \theta_C}{8\pi E_v^2} \begin{bmatrix} A \mp \frac{(s-u)}{M^2} B + \frac{(s-u)^2}{M^4} C \end{bmatrix}$$
$$A = \frac{(m^2 + Q^2)}{M^2} [(1+\tau) G_A^2 - (1-\tau) F_1^2 + \tau (1-\tau) F_2^2 + 4\tau F_1 F_2$$
$$-\frac{m^2}{4M^2} \Big( (F_1 + F_2)^2 + (G_A + 2G_P)^2 - \Big(\frac{Q^2}{M^2} + 4\Big) G_P^2 \Big) \Big]$$
$$B = \frac{Q^2}{M^2} G_A(F_1 + F_2)$$



u charged-current cross-section





 $C = \frac{1}{4}(G_A^2 + F_1^2 + \tau F_2^2)$ 

Well-determined from electron scattering expts

dominant contribution

Iargest uncertainty

 $G_P$  can be related to  $G_A$  by pion pole dominance

#### Nucleon axial form factors

- Nucleon properties are historically difficult calculations
- Recent calculations of nucleon form factors including axial in agreement with experiment with fully-controlled uncertainties
- Q<sup>2</sup>-dependence well-determined in LQCD: competitive with experiment



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Strange quark contributions are determined separately and can be isolated

[Also Gupta et al., EPJ Web Conf. 175 (2018) 06029, Alexandrou et al., PRD96 (2017), 054507]

#### Nucleon pseudo-scalar form factors

- First calculations with controlled uncertainties
- Clear deviations from pion-pole dominants ansatz at low  $Q^2$

$$\tilde{G}_P(Q^2) = G_A(Q^2) \left[ \frac{4M_N^2}{Q^2 + M_\pi^2} \right]$$



#### Resonance region

- Energies above ~200 MeV, inelastic excitations from pion production
- Dominant contribution from
   Δ resonance
- N\*'s also important at high  $E_v$
- <u>Very</u> difficult to access experimentally Constrained only from PCAC
- QCD calculations possible:
   Need to account for unstable nature of resonance, extract N→Nπ transition FFs



 ${\cal V}$  charged-current cross-section



#### **Transition form factors**

Lattice QCD calculation of axial N  $\Delta$  transition form factor:



CAVEAT: Complexities at physical point with unstable resonances, but formalism exists: [Lellouch-Lüscher hep-lat/0003023]

# Constraining $\nu$ -nucleus interactions

- Lattice efforts have potential to impact
   v energy determinations
- Precise determinations with controlled percent-level uncertainties within ~5 years



- Axial and pseudoscalar FFs determined with momenta less than a few GeV
- BUT: large momentum FFs (≈3 GeV) more difficult. Novel ideas exist, need testing
- Early results with promising applications
  - Transition FFs

Formalism exists but developments still necessary for higher states above  $N\pi\pi$  inelastic threshold

• Application of EFT using 2-, 3- body matrix elements to constrain nuclear effects

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Look for scattering of WIMP dark matter on nuclear target

Detection rate depends on

- Dark matter properties
- Probability of interaction with nucleus i.e., *nuclear* effects are important



Low-energy limit of a generic spin-independent interaction is scalar

Determine **nucleon and nuclear scalar matrix elements** from lattice QCD

Other e.g., spin-dependent couplings can also be constrained e.g., [Hoferichter et al., arXiv:1503.04811], [Hill et al., arXiv:1409.8290], [Fitzpatrick et al., arXiv:1203.3542]

Spin-independent scattering of WIMP candidates is governed by scalar matrix elements

Lattice QCD nucleon scalar matrix elements



Light quark: competitive with phenomenology

Note: tension with extraction using Roy–Steiner equations [Hoferichter arXiv:1506.04142]

Strange quark: much more precise than phenomenology

Direct detection experiments use nuclear targets e.g., Xenon

Determine interaction cross-section (with nucleus) for a given dark matter model

• Born approximation – interacts with a single nucleon  $\sigma \sim |A \ \langle N | DM | N \rangle|^2$ 

known from LQCD



Interacts non-trivially with multiple nucleons

 $\sigma \sim |A \langle N|DM|N \rangle + \alpha \langle NN|DM|NN \rangle + \dots |^2$ 

poorly known!

Second term may be significant!



Spin-independent scattering of WIMP candidates is governed by scalar matrix elements

- Lattice QCD calculation with  $m_{\pi}$ ~800 MeV shows 10% nuclear effects in <sup>3</sup>He potentially very significant effects in e.g., Xenon
- Same calculation gives axial and tensor nuclear effects around ~1%



#### Larger nuclei

- What about larger (phenomenologically-relevant) nuclei?
- Nuclear effective field theory:
  - 1-body currents are dominant
  - 2-body currents are sub-leading but non-negligible
- Determine one body contributions from single nucleon
- Determine few-body contributions from A=2,3,4...
- Match EFT and many body methods to LQCD to make predictions for larger nuclei



# NPLQCD effort in lattice QCD for nuclei

- Nuclei with A<5</li>
- QCD with unphysical quark masses

 $m_{\pi}$ ~800 MeV,  $m_{N}$ ~1,600 MeV

 $m_{\pi}$ ~450 MeV,  $m_{N}$ ~1,200 MeV

 First calculation of spectrum of light nuclei in 2013



#### **Recent highlights**

- Proton-proton fusion
   and tritium β-decay
   [PRL 119, 062002 (2017)]
- Double  $\beta$ -decay



[PRL 119, 062003 (2017), PRD 96, 054505 (2017)]

- Gluon structure of light
   nuclei
   [PRD 96 094512 (2017)]
- Scalar, axial, tensor MEs [PRL 120 152002 (2018)]

# Summary and outlook

Lattice QCD (+QED) is providing essential Standard Model input for high-energy and nuclear physics experiments



Precision lattice QCD results for simple systems including hadrons

- FLAG lattice averaging: first inclusion of hadron structure in 2019
- Many more quantities will have fully-controlled systematic uncertainties by 2021
- Beginning of reliable lattice QCD results for nuclear matrix elements

#### Parton distribution functions

Parton distribution functions  $f(x, \mu^2)$ 

- Non-local light-cone correlations
- Encode non-perturbative physics
- Correlations at light-like separation not directly accessible in Euclidean-space calculations

$$f(x) = \int \frac{d\xi^{-}}{2\pi} e^{-2i\xi^{-}(xP^{+})} \langle p | \overline{\psi}_{f}(\xi^{-}) \gamma^{+} W[\xi^{-}, -\xi^{-}] \psi_{f}(-\xi^{-}) | p \rangle$$

 Operator Product Expansion relates Mellin moments of PDFs to local operators

$$\langle h | \overline{\psi}_f D_{\mu_1} \dots D_{\mu_n} \psi_f | h \rangle \sim \langle x^n \rangle_f^h = \int_0^1 dx \, x^n f(x)$$



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  - Quasi-PDFs [Ji, PRL 110 (2013) 262002]
  - Pseudo-PDFs [Radyushkin, PRD 96 (2017) 034025]
  - Factorisable matrix elements [Ma & Qiu, PRL 120 (2018) 022003]
  - (Heavy quark) Compton tensor
     [Braun & Müller, EPJ C55 (2008) 349; Chambers et al., PRL 118 (2017) 242001, Detmold & Lin, PRD 73 (2006) 014501, Liu & Dong, PRL 72 (1994) 1790]





#### Also first results in 2017-2019 for

- Gluon quasi-PDFs [Z. Y. Fan, et al., PRL 121, no. 24 (2018) 242001]
- Pion quasi-PDFs [J. Chen, et al., arXiv:1804.01483]
- Quasi-GPDs of nucleon and pion, matching to GPDs available [Bhattacharya, Cocuzza and Metz, PLB 788 (2019) 453, Chen, Lin and Zhang, arXiv: 1904.12376, Y.-S. Liu et al., arXiv:1902.00307]
- Nucleon pseudo-PDFs [Orginos et al., PRD96 (2017)]



#### **Generalised Parton Distributions**

- Quark GPDs: constraints from JLab, HERA, COMPASS, by DVCS, DVMP, future improvements from JLab 12GeV, EIC
- Gluon GPDs: almost unknown from experiment, future constraints central goal of EIC

• Moments of GPDs: Generalised Form Factors (GFFs) e.g.,  $\int_0^1 dx \ H_g(x,\xi,t) = A_g(t) + \xi^2 D_g(t), \qquad \int_0^1 dx \ E_g(x,\xi,t) = B_g(t) - \xi^2 D_g(t)$ 



#### Transverse Momentum Dependent PDFs

- Most robust results for ratios of TMDPDFs: cancellation of renormalisation ambiguities and soft factors
  - e.g., Generalised Sivers shift (~ratio of Sivers TMD over unpolarised TMD)
  - Encouraging comparison with expt: global fit to HERMES, COMPASS, JLab Light cone:  $\hat{\zeta} \to \infty$



- First study of Generalized Transverse Momentum-Dependent Distributions (GTMDs) to obtain quark orbital angular momentum (OAM) in proton [Engelhardt, PRD 95 (2017), USQCD 1904.09512]
- First results for x-dependence of TMDs [Engelhardt, Lattice 2018]

#### TMD Evolution

**Collins-Soper Evolution Kernel**  $\gamma_{\zeta}^{q}(\mu, b_{T}) = \zeta \frac{d}{d\zeta} \ln f_{q}(x, \vec{b}_{T}, \mu, \zeta)$ 

- Governs TMD evolution
- Needed to match quasi-TMD to physical TMD
- Perturbative at short distances  $\mu, b_T^{-1} \gg \Lambda_{\rm QCD}$
- Non-perturbative for  $b_T^{-1} \lesssim \Lambda_{\text{QCD}}$ Can be accessed via ratio of non-local MEs in LQCD [Ebert, Stewart, Zhao, PRD99 (2019)]
- First calculation in progress [PES, Wagman, Zhao]
- CS-kernel independent of state: study unphysically-heavy pion with no systematic bias
- 5x statistics, 1.5x  $b_T$  range will constrain  $\gamma^q_\zeta(\mu, b_T)$  in non-perturbative region



#### Nucleon axial charge

Complete calculations with controlled uncertainties from multiple collaborations in 2018



#### Red star:

$$g_A = 1.2671$$

determined with high precision from nuclear beta decay

# Inelastic region

- In inelastic regime, quark PDFs of the nucleon control scattering cross-section
- In inelastic region, both resonances and DIS are important
- Multi-meson channels may become important
- Nuclear effects are different in vA vs. eA (MINERvA)
- DIS structure functions accessible in LQCD
  - low moments of structure functions controlled

$$M_n = \int_{-1}^1 x^n f(x) dx, \quad n \lessapprox 4$$

• x-dependence difficult but promising







# Gluon momentum fraction of nuclei

Matrix elements of the spin-independent gluon operator in nucleon + light nuclei [NPLOCD PRD96 094512 (2017)]

first determination of gluon momentum fraction of nuclei

• Present statistics: can't distinguish from no-EMC effect scenario



#### Non-nucleonic glue in deuteron

Contributions to nuclear structure from gluons not associated with individual nucleons in nucleus

- First moment of gluon transversity distribution in the deuteron [Jaffe, Manohar PLB223 (1989) 218]
- First evidence for non-nucleonic gluon contributions to nuclear structure: LQCD with m\_~800 MeV [NPLQCD PRD96 (2017)]
- Magnitude relative to momentum fraction as expected from large-N<sub>c</sub>

nucleon:  $\langle p|\mathcal{O}|p\rangle = 0$ nucleus:  $\langle N, Z|\mathcal{O}|N, Z\rangle \neq 0$ 



#### Nucleon electromagnetic form factors

State-of-the-art calculations have physical quark masses, large lattice volumes, and fine lattice spacings, but systematic uncertainties not yet controlled at a level comparable to flavour physics quantities



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• 
$$g - 2$$

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- Neutrino physics
- Dark matter

#### μ m. <u>g-2</u>m

### Muon g-2: status and challenges

Long standing discrepancy between measured value and SM estimate for muon anomalous magnetic moment (~ $3\sigma$ )



#### Sign of new physics?

New experiments aiming at 4-fold uncertainty reduction (E989 @ Fermilab, E34 @ JPARC) if no shift in central values, tension will be  $\sim 7\sigma$  with projected theory improvements by 2020

Commensurate control of theory needed: **Muon g-2 Theory Initiative formed** https://indico.fnal.gov/event/13795/

#### Standard Model muon g-2

Measured value

 $a_{\mu}^{\text{E821}} = (116\,592\,089\pm63) \times 10^{-11}$ 

Breakdown of SM contributions (2 evaluations of HVP)

	Value ( $\times 10^{-11}$ ) units
QED $(\gamma + \ell)$	$116584718.951\pm0.009\pm0.019\pm0.007\pm0.077_{\alpha}$
HVP(lo) [20]	$6923\pm42$
HVP(lo) [21]	$6949\pm43$
HVP(ho) [21]	$-98.4\pm0.7$
HLbL	$105 \pm 26$
${ m EW}$	$154 \pm 1$
Total SM $[20]$	$116591802 \pm 42_{\rm H-LO} \pm 26_{\rm H-HO} \pm 2_{\rm other}(\pm 49_{\rm tot})$
Total SM $[21]$	$116591828\pm43_{\rm H\text{-}LO}\pm26_{\rm H\text{-}HO}\pm2_{\rm other}(\pm50_{\rm tot})$

[T. Blum et al., arXiv:1311.2198]

Deviation

 $\Delta a_{\mu} (E821 - SM) = (287 \pm 80) \times 10^{-11} [20]$ =  $(261 \pm 78) \times 10^{-11} [21]$ 

Dominant uncertainties from hadronic corrections—calculable in lattice QCD

QED (5 loop) [Aoyama et al. 2012]





# Standard Model muon g-2

Dominant uncertainties in SM determination from hadronic corrections Both calculable (in principle) from lattice QCD



Dispersion relation plus experimental data on "R-ratio" **o**(e+e-→hadrons)

2018: First lattice QCD calculation with QED and isospin breaking [T. Blum et al., arXiv:1801.07224]

# Hadronic light-by-light $\star + \star + \star$

Estimated from models including large-N<sub>c</sub>, chiPT, vector meson dominance, etc.

Since ICHEP2016: disconnected terms and lattice volume better controlled [PRL118(2016)022005, PRD96(2017)034515]

#### Hadronic Vacuum Polarisation

Combining lattice QCD and dispersion relations yields best current determination of HVP contribution [T. Blum et al., arXiv:1801.07224]

Use "R-ratio" experimental data on  $\sigma(e+e-\rightarrow hadrons)$  at short and long distances *t*, lattice QCD in intermediate *t* region

$$a_{\mu}^{HVP} = \sum_{t} w_{t}C(t)$$
$$C(t) = \frac{1}{3} \sum_{\vec{x}} \sum_{j=0,1,2} \langle J_{j}(\vec{x},t) J_{j}(0) \rangle$$

Flavour breakdown: light~90%, strange~8% and charm~2%

