STFC HEP summer school 2019

Phenomenology Problems

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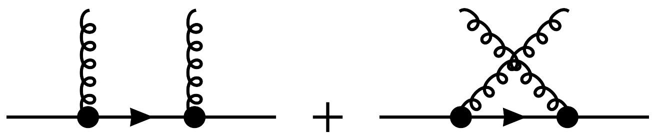
1 Leading colour approximation



Consider a quark line emitting two gluons as shown in the above figure. Compute the colour factors for the squared amplitude using the pictorial method (or otherwise) to show that the interference term is sub-leading in N_c.

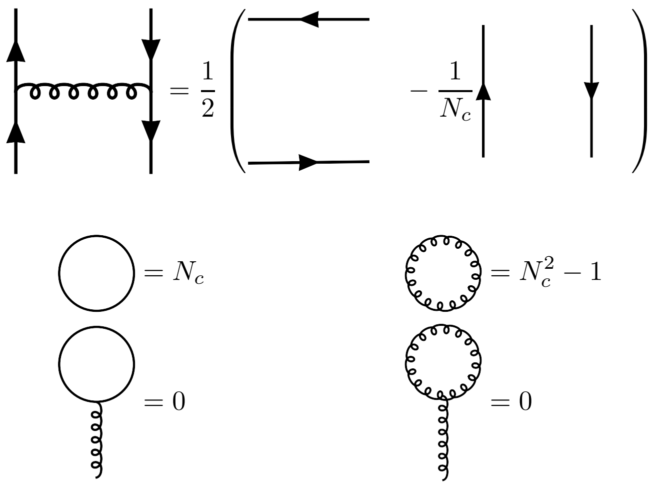


Figure 1: Diagrammatic rules for the $SU(N_c)$ colour algebra.

2 $e^+e^- \to$ hadrons

By studying the shape of the Z-resonance in the R-ratio we can try to see the effect of some parameters in the electro-weak interactions. You can answer the questions qualitatively but you could also plot the function if you have time.

Recall from the lectures that:

$$
\begin{aligned}
d\sigma\left(f\bar{f} \to f'\bar{f}'\right) = \alpha^2 \frac{\pi}{2s} d(\cos\theta) \Bigg\{ \\
\left(1+\cos^2\theta\right)\left(q_f^2 q_{f'}^2 + \frac{g_z^2}{4g_e^2} q_f q_{f'} v_f v_{f'} \chi_1 + \frac{g_z^4}{16 g_e^4}(a_f^2 + v_f^2)(a_{f'}^2 + v_{f'}^2)\chi_2\right) \\
+ \cos\theta \left(\frac{g_z^2}{2g_e^2} q_f q_{f'} v_f v_{f'} \chi_1 + \frac{g_z^4}{2g_e^4} a_f a_{f'} v_f v_{f'} \chi_2\right) \Bigg\}
\end{aligned}
$$

where

$$
\frac{g_Z}{g_e} = \frac{1}{\cos\theta_w \sin\theta_w} \qquad \chi_1 = \frac{s(s-m_Z^2)}{(s-m_Z^2)^2 + m_Z^2\Gamma_Z^2} \qquad \chi_2 = \frac{s^2}{(s-m_Z^2)^2 + m_Z^2\Gamma_Z^2}
$$

and

$$g_z^2 = \frac{4\pi\alpha}{\cos^2\theta_w \sin^2\theta_w} \qquad \Gamma_Z \approx \sum_l \Gamma_{Z\to l\bar{l}} + \sum_q N_c \Gamma_{Z\to q\bar{q}} \qquad \Gamma_{Z\to f\bar{f}} = \frac{m_Z\alpha}{12\cos^2\theta_w \sin^2\theta_w}\left(a_f^2 + v_f^2\right)$$

The axial and vector couplings in the Standard Model are given in Table 1.

a) Try to plot $R(\frac{e^+e^-\to\text{hadrons}}{e^+e^-\to\mu^+\mu^-})$ both in the case where the full corrections are kept in both numerator and denominator and in the case where the denominator includes only the photon exchange.

b) What happens to the shape of the resonance as you change θ_w?

c) Does the shape of the resonance change if the Z propagator had the opposite sign?

3 Event shapes

The thrust is defined as

$$T = \max_{\vec{n}} \frac{\sum_i |\vec{p}_i \cdot \vec{n}|}{\sum_i |\vec{p}_i|},$$

where the sum is over all the particles, the ith particle has 3-momentum $\vec{p}_i$, and $\vec{n}$ is a unit-vector.

Explain why the value of the thrust is given by

a) 1 for back-to-back configurations,

b) $\frac{1}{2}$ for a perfectly spherical event (i.e. uniform ditribution of momenta).

c) Calculate the minimum possible value of the thrust for a $q\bar{q}g$ state. [Hint: This occurs for the 'Mercedes' configuratio where all the particles have the same energy and the angle between any two particles is 120^0.]

Consider the two event shape variables

$$S_{\text{lin}} = \left(\frac{4}{\pi}\right)^2 \min_{\vec{n}} \left(\frac{\sum_i |\vec{p}_i \times \vec{n}|}{\sum_i |\vec{p}_i|}\right)^2, \qquad S_{\text{quad}} = \frac{3}{2}\min_{\vec{n}} \frac{\sum_i |\vec{p}_i \times \vec{n}|^2}{\sum_i |\vec{p}_i|^2}.$$

d) Determine whether these observables are infrared safe or not.

e) What are the limiting values of this event shape for pencil-like (back-to-back) and spherical events?

f) What is the value for the Mercedes configuration?

4 DGLAP splitting kernels

Show that in the collinear limit $p_3 \to z p_{\tilde{13}},\ p_1 \to (1-z)p_{\tilde{13}}$ the matrix element

$$\left\langle |\mathcal{M}(a_{e^+}, b_{e^-}, 1_q, 2_{\bar{q}}, 3_g)|^2 \right\rangle = \frac{4e^2 e_q^2 g_s^2 N_c}{s} C_F \frac{s_{a1}^2 + s_{a2}^2 + s_{b1}^2 + s_{b2}^2}{s_{ab}s_{13}s_{23}}$$

factorizes to

$$|\mathcal{M}_{q\bar{q}g}|^2 \to |\mathcal{M}_{q\bar{q}}|^2 \times \frac{2g_s^2}{s_{13}} \times C_F \frac{1+(1-z)^2}{z}$$

$$\left\langle |\mathcal{M}(a_{e^+}, b_{e^-}, 1_q, 2_{\bar{q}}, 3_g)|^2 \right\rangle \overset{3||1}{\to} \left\langle |\mathcal{M}(a_{e^+}, b_{e^-}, (\tilde{13})_q, 2_{\bar{q}})|^2 \right\rangle \frac{2g_s^2 C_F}{s_{13}} \frac{1+(1-z)^2}{z}$$

with

$$\left\langle |\mathcal{M}(a_{e^+}, b_{e^-}, 1_q, 2_{\bar{q}})|^2 \right\rangle = 2e^2 e_q^2 N_c \frac{s_{a1}^2 + s_{a2}^2}{s_{ab}}$$

	q_f	a_f	v_f
u, c, t	$2/3$	$1/2$	$1/2 - 4/3\sin^2\theta_w$
d, s, b	$-1/3$	$-1/2$	$-1/2 + 2/3\sin^2\theta_w$
e, μ, τ	-1	$-1/2$	$-1/2 + 2\sin^2\theta_w$
ν_e, μ, τ	0	$1/2$	$1/2$

Table 1: EW couplings in the Standard Model.

5 Infrared safety

Are these observables infrared safe at a hadron collider? If not, how would you modify them to make them infrared safe?

a) Partonic center of mass energy (defined as a the invariant mass of the sum of all final state particles in the event).

b) The sum of the energies of all jets with transverse momentum above a given p_T threshold.

c) The invariant mass of all jets in the event.

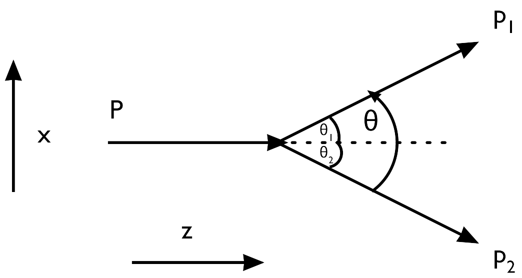
d) The number of partons.

6 Jet mass

In the lectures we looked at the change in the transverse momentum of a jet due to perturbative radiation, non-perturbative effects and the underlying event. Another interesting quantity which can be calculated in the same way is the invariant mass of a jet.

a) Show that at leading order the invariant mass of a jet is zero. (At leading order jet contains a single parton)

b) Consider a jet containing a pair of collinear partons,



If the partons p_1 and p_2 carry a fraction z and $1-z$ of the energy, E, of the original parton P, show that the invariant mass of the jet $m^2 = (p_1 + p_2)^2$ in the small angle approximation is,

$$m^2 = E^2 z(1-z)\theta^2,$$

where θ is the angle between the two out-going partons. You may like choose the partons 1 and 2 to lie in the (x, z) plane.

c) Use this result to calculate the average invariant mass squared of a quark and gluon jet due to perturbative radiation.

d) Using the formulae for the hadronic and underlying event corrections

$$\langle m^2 \rangle_{q,had} = 2ERC_F\mathcal{A}\ , \quad \langle m^2 \rangle_{g,had} = 2ERC_A\mathcal{A}$$

and

$$\langle m^2 \rangle_{\mathrm{UE}} = \frac{E\Lambda_{\mathrm{UE}}R^4}{4}$$

sketch the different contributions to the jet mass as a function of R for a quark jet with energy 50 GeV. Take $\mathcal{A} = 0.20$ GeV, $\alpha_S = 0.125$ and $\Lambda_{\mathrm{UE}} = 3.3$ GeV.

7 Jet algorithms

Recall the distance measure used by the anti-k_T jet algorithm is given by:

$$d_{i,j} = \min(p_{i,\perp}^{-2}, p_{j,\perp}^{-2})\frac{\Delta R_{ij}^2}{R^2}\ , \quad d_{iB} = p_{i,\perp}^{-2}\ .$$

where $\Delta R_{ij} = \sqrt{(\eta_i - \eta_j)^2 + (\phi_i - \phi_j)^2}$. Consider the clustering of a three particle system with one hard jet $j_H = (p_{H,\perp}, \eta, \phi)$ and two softer jets $j_1 = (p_{1,\perp}, 0, 0)$ and $j_2 = (p_{2,\perp}, \eta_0, 0)$ where $p_{H,\perp} \gg p_{i,\perp}$. You may also take $R = 1$.

a) Find the conditions for the soft jets to be clustered with the hard jet.

b) Show that anti-k_T jets are circular in the $\eta - \phi$ plane.

c) What happens to the clustering when cones of jets 1 and 2 overlap?

d) How does it depend on the relative size of the transverse momentum, $r = \frac{p_{1,\perp}}{p_{2,\perp}}$?