HEP Summer School SM Problems 2015

(1) Show that under a local gauge transformation the non-abelian $F_{\mu\nu}$ field strength tensor transforms as $F_{\mu\nu} \to F'_{\mu\nu} = UF_{\mu\nu}U^{-1}$.
[Hint: Show first that $D_\mu \to D'_\mu = UD_\mu U^{-1}$, and then gauge transform the $[D_\mu, D_\nu]$ commutator definition for $F_{\mu\nu}$ (Eq.(1.20). Do **not**, unless you are a masochist, gauge transform $A_\mu \to A'_\mu$ and use the explicit expression for $F_{\mu\nu}$!]

(2) Defining $\gamma_5 = i\gamma^0\gamma^1\gamma^2\gamma^3$ show that $\gamma_5^2 = 1$, $\gamma_5^\dagger = \gamma_5$, and $\{\gamma_5, \gamma_\mu\} = 0$.

Consider a massless fermion with momentum p along the z direction, $p_\mu = (E, 0, 0, E)$. Show that $P_R u(p)$ and $P_L u(p)$ are eigenstates of helicity

$$h = -\frac{\gamma_0\gamma_5\vec{\gamma}\cdot p}{E} \ ,$$

with eigenvalues ± 1.

(3) Show that, as claimed, $\mathcal{L}(e)$ is $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ invariant, by checking explicitly that the $\chi_L, e_R, \overrightarrow{W}_\mu, B_\mu$ infinitessimal transformations given in the lecture leave it invariant.

(4) There is one Feynman diagram in lowest order electroweak theory for μ^- decay,

$$\mu^-(p) \to \nu_\mu(k) + e^-(p') + \bar{\nu}_e(k') \ .$$

Draw this diagram and use the electroweak Feynman rules to calculate the spin averaged $|\overline{\mathcal{M}}|^2$ for this decay. To simplify the calculation retain m_μ but set $m_e = 0$. Also, evaluate in the effective "Fermi theory" where you leave out the W propagator (set it to $g_{\mu\nu}$) and replace g at the vertices by g/M_W. Why is this a very good approximation for μ^- decay? Does setting $m_\mu = 0$ make any difference?
[You are given (I'm very merciful!!)

$$Tr[\gamma^\mu(1-\gamma_5)\not{p}_1\gamma^\nu(1-\gamma_5)\not{p}_2]Tr[\gamma_\mu(1-\gamma_5)\not{p}_3\gamma_\nu(1-\gamma_5)\not{p}_4] = 256(p_1\cdot p_3)(p_2\cdot p_4) \ .$$

]

Write out an expression for $d\Gamma$ (the differential decay rate) in terms of $|\overline{\mathcal{M}}|^2$ and phase space. A tedious phase space integration which you need not attempt then leads to the total μ^- decay rate

$$\Gamma(\mu^-) = \frac{g^4 m_\mu^5}{6144\pi^3 M_W^4} \ .$$

Given $m_\mu = 105.66$ MeV, and the μ^- lifetime

$$\tau(\mu^-)^{exp} = \frac{1}{\Gamma(\mu^-)} = (2.197138 \pm .000065) \times 10^{-6} \text{ sec}$$

estimate v the Higgs vev in the minimal Standard Model.
[In natural units 1 sec $= 1.52 \times 10^{24}$ GeV^{-1} .]

(5) Use the electroweak Feynman rules to calculate the polarization averaged decay width for Z^0 decay, $\Gamma(Z^0 \to f\bar{f})$, $f = e, \nu, q, \ldots$. Take f massless.
[For an external massive spin 1 vector boson with mass M_V you need the Feynman rule $\epsilon_\mu^{(\lambda)}$, where the $\epsilon_\mu^{(\lambda)}$ is the polarization vector of the vector boson, and the completeness sum over polarizations is

$$\sum_\lambda \epsilon_\mu^{(\lambda)*} \epsilon_\nu^{(\lambda)} = -g_{\mu\nu} + \frac{p_\mu p_\nu}{M_V^2} \ .$$

Suitable choices are ($\vec{p}$ along the z-axis)

$$\epsilon^{(\lambda=\pm 1)} = \mp(0, 1, \pm i, 0)/\sqrt{2}$$

and

$$\epsilon^{(\lambda=0)} = (|\vec{p}|, 0, 0, E)/M_V \ .$$

One then has

$$\Gamma(Z^0 \to f\bar{f}) = \frac{1}{64\pi^2 M_Z} \int |\overline{\mathcal{M}}|^2 \, d\Omega \ .$$

]
Estimate the total Z^0 decay width (take $M_Z = 91$ GeV, $g = 0.65$, $\sin^2(\theta_W) = 0.23$) which should have been observed at LEP. Don't forget three colours for each quark flavour!

(6) Show that given values for the seven SM parameters e, M_W, M_Z, M_H, m_e, m_μ, and m_τ one can determine the fifteen lepton sector parameters using relations between parameters in the notes. How many of these fifteen parameters could be determined if M_H were unknown?