The Standard Model

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1 Introduction. From Fermi theory to Higgs mechanism

Currently Standard Model describes to an excellent precision the vast majority of the laboratory experiments. There are several notable exceptions, which indicate that our understanding of the fundamental details of everything is yet far from complete. However, the SM itself does not contain indications of any problems.

This was not always the case. Actually, SM itself emerged as a theory that solved the intrinsic problems of its predecessors. Let us do a small historic[1] review.

When we study a theory at low energy, up to several hundred MeV, we have lot's of weak and rare effects, like nuclear β decay, μ and π decays, neutrino scattering. The phenomenology of these processes is very rich, and all have stunning properties – parity is maximally violated, flavour is violated. All these effects are perfectly grasped by the Fermi theory of 4-fermion interactions. All these effects are due to the interaction between "charged currents"

$$\mathcal{L} = -4\frac{G_f}{\sqrt{2}} J^+_\mu J^-_\mu + \text{h.c.}, \tag{1}$$

where the *Fermi constant*

$$G_F = 1.116 \times 10^{-5}\ \text{GeV}^{-2} \sim \frac{1}{(300\ \text{GeV})^2}.$$

The currents are composed of leptonic and hadronic parts $J^\pm_\mu = J^\pm_{\mu,\text{lepton}} + J^\pm_{\mu,\text{hardon}}$. For the first two generations (not much sense to use Fermi theory for the third generation)

$$J^+_{\mu,\text{lepton}} = \bar{\nu}_L \gamma_\mu e_L + \bar{\nu_\mu}_L \gamma_\mu \mu_L. \tag{2}$$

Here the index 'L' means left fermion $\psi_L \equiv \frac{1-\gamma^5}{2}\psi$. The hadronic part is a bit more complicated, as far as it mixes first and second generation quarks

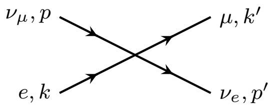
$$J^+_{\mu,\text{hadron}} = \bar{u}_L \gamma_\mu d'_L + \bar{c}_L \gamma_\mu s'_L, \tag{3}$$

[1]Not very historically precise.

where the down quarks are mixed with the Cabibo angle θ_c

$$d'_L = \cos\theta_c d_L + \sin\theta_c s_L,$$
$$s'_L = -\sin\theta_c d_L + \cos\theta_c s_L.$$

The interaction (1) looks rather innocent (though a careful reader with QFT knowledge may notice the suspicious negative dimension of the coupling constant). Let us see what is the problem. For this, we can calculate the cross-section $\nu_\mu e \to \nu_e \mu$ scattering.[2] This process has only one diagram to compute



$$= i\mathcal{M} = i\frac{4G_F}{\sqrt{2}}\bar{u}(p')\gamma^\lambda \frac{1-\gamma^5}{2}u(k)\cdot \bar{u}(k')\gamma^\lambda\frac{1-\gamma^5}{2}u(p), \qquad (4)$$

where $u(p)$ are fermionic wave functions for corresponding fermions (neutrinos, electrons, or muons). Squared modulus of the matrix element summed over all spin states, following standard rules, is

$$\sum_{\text{spins}}|\mathcal{M}|^2 \;=\; 8G_F^2 \,\mathrm{tr}\left(\gamma_\rho\frac{1-\gamma^5}{2}\not{p}\gamma_\lambda\frac{1-\gamma^5}{2}(\not{k}'+m_\mu)\right)\mathrm{tr}\left(\gamma_\rho\frac{1-\gamma^5}{2}\not{k}\gamma_\lambda\frac{1-\gamma^5}{2}\not{p}'\right), \quad (5)$$

where we have neglected masses of neutrinos and electron. Actually, the term with muon mass gives zero also, as far as it enters under trace multiplied by an odd number of gamma matrices. Using the standard trace rules, and contracting indexes for the epsilon symbols we get after some tedious algebra

$$\sum_{\text{spins}}|\mathcal{M}|^2 = 128G_F^2(pk)(p'k') = 128G_F^2\frac{s}{2}\frac{s-m_\mu^2}{2},$$

where $s \equiv (p+k)^2 = (p'+k')^2$ characterises the collision energy.

As far as the matrix element does not depend on directions of the final particles, we can immediately convert it to the cross-section

$$\sigma = \frac{1}{8\pi s}\left(\frac{1}{2}\sum_{\text{spins}}|\mathcal{M}|^2\right)\frac{|\mathbf{p}|}{s} = \frac{G_F^2}{\pi}\frac{(s-m_\mu^2)^2}{s},$$

where 1/2 is present because we have to average over the initial electron spin states (no averaging for neutrinos as far as the have only one helicity state, and nothing for muon, because we sum over all final states).

This result is rather peculiar. First, it iz zero at threshold, $s = m_\mu^2$. Then it *grows* with energy. Of course, while *sqrts* $\ll G_F^{-2} \sim 300$ GeV it remains small due to the Fermi constant, but for higher energies it becomes large, and grows $\propto s$. At the same time, scattering cross-sections can not be too large, the unitarity (basically, requirement

[2] Probably not the easiest process for experiment, but it requires only one diagram for us to study.

that probability should be smaller than one) states that all cross-sections[3] can not grow faster than $\propto (\ln s)^2$. Thus, the Fermi theory fails to produce physically sensible results at energies above $G_F^{-2} \sim 300$ GeV. Note, that this scale is encoded in the theory, and follows from the low-energy cross-section measurements.

This inconsistency asked for invention of better theories. First, thing that can be done is to replace the point-like 4-fermion interaction by an interaction with the exchange of a vector boson. Then, instead of the Fermi constant one would expect an expression of the form $g^2/(p^2 - m_W^2)$, where g is the coupling constant, and m_W is the boson mass. For momenta much smaller than the mass it would give approximately constant $\sim g^2/M_W^2$, while at high energies the amplitude will become suppressed. Unfortunately, this does not work immediately – the massive vector boson propagator is more complicated

$$\frac{i}{p^2 - m_W^2}\left(g_{\mu\nu} - \frac{p_\mu p_\nu}{m_W^2}\right).$$

The last term does not become small at large momenta. However, it is possible to get away from this problem, if this term, which has peculiar tensor structure, cancels in the final answer. This is similar to cancellation of gauge dependence in QED, and actually happens if the theory has gauge symmetry. A new problem appears here – normally, massive vector bosons are not gauge invariant. This problem itslef can be solved by introducing a new particle, the Higgs boson.

In the following, we will introduce the Higgs mechanism, and describe it specifically for the case of the electroweak interactions.

[3] See C. Itzykson, J.-B. Zuber, Quantum Field Theory, McGraw-Hill: 1980, vol. 1. Strictly speaking, this is true for theories without massless particles. Really, the “total” cross-section of electron-electron scattering is infinite, because massless photons lead to scattering at arbitrary distance, but for infinitely small scattering angle.