Double parton scattering in QCD

Jonathan Gaunt (CERN)

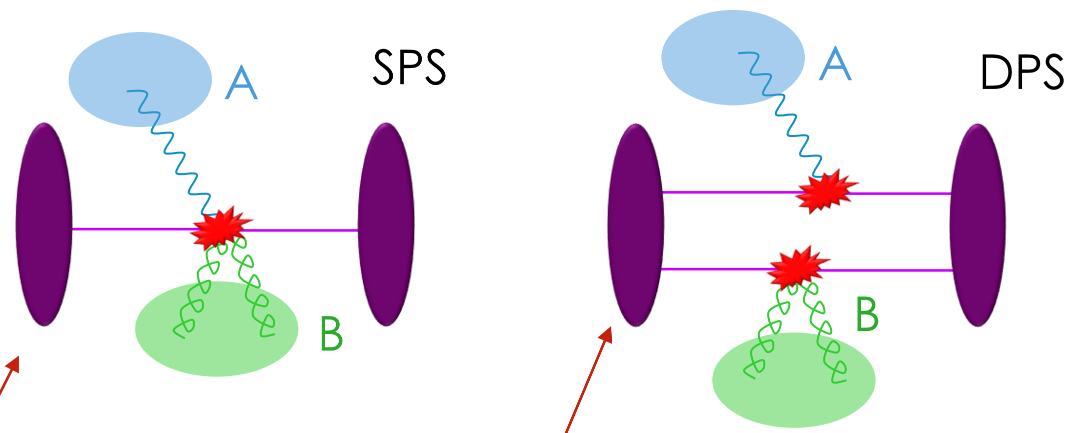
COFUND. A project supported by the European Union

James Stirling Memorial Conference

17/09/19

DOUBLE PARTON SCATTERING: BASICS

Certain sets of scattering products can be formed either from one hard collision (SPS), or two separate hard collisions (double parton scattering, DPS):



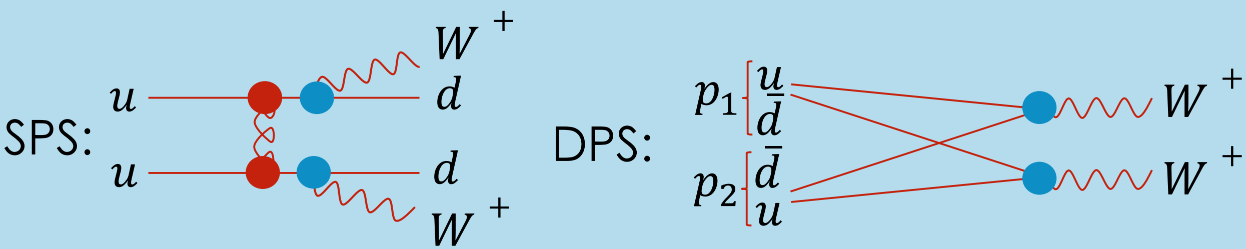
Parton density functions (PDFs)

Double parton densities (DPDs)

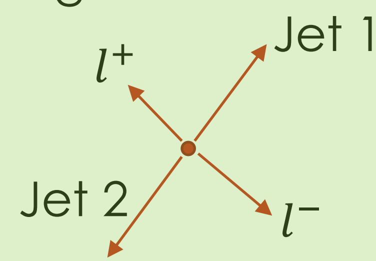
In terms of the total cross section for the production of AB, the DPS mechanism is power suppressed: $\sigma_{DPS}/\sigma_{SPS} \sim \Lambda_{QCD}^2/Q^2$

WHY STUDY DPS?

DPS can be a significant background to processes suppressed by small/multiple coupling constants…

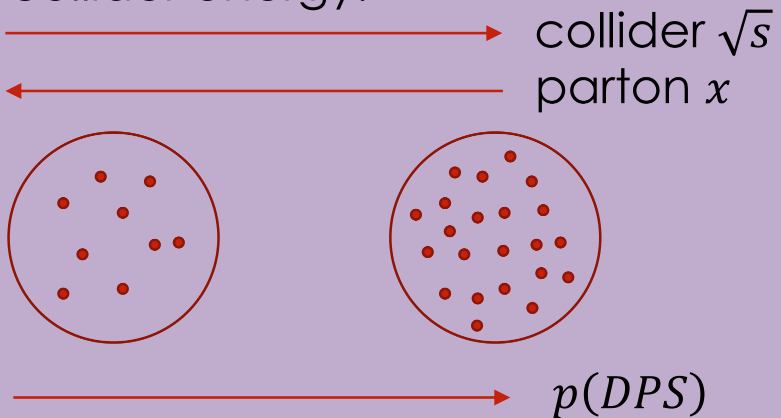


…or in certain phase space regions

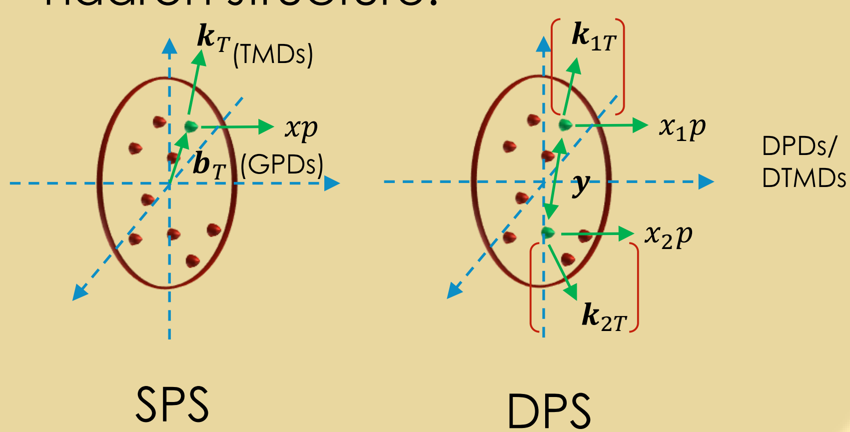


$p_T(l^+l^-), p_T(jj) \ll Q$

DPS importance increases with collider energy:



DPS tells us new information on hadron structure:



JAMES AND DPS

DPS a long-standing interest for James!

EVIDENCE FOR MULTIPLE PARTON INTERACTIONS FROM THE OBSERVATION OF MULTI-MUON EVENTS IN DRELL–YAN EXPERIMENTS

F. HALZEN [1], P. HOYER [2]
CERN, CH-1211 Geneva 23, Switzerland

and

W.Y. STIRLING
Department of Physics and Mathematical Sciences, University of Durham, Durham DH1 3LE, UK

Received 15 January 1987

Like-sign W boson production at the LHC as a probe of double parton scattering

Anna Kulesza [a], W. James Stirling [a,b]

[a] *Department of Physics, University of Durham, Durham, DH1 3LE, UK*
[b] *Department of Mathematical Sciences, University of Durham, Durham, DH1 3LE, UK*

Received 7 December 1999; accepted 28 December 1999
Editor: P.V. Landshoff

PHD GOALS

Goals of PhD (for STFC):

1) Build a numerical DPD set based on proper QCD description of DPS.

2) Use DPD set for phenomenological analysis of key processes (e.g. W^+W^+).

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4) Perform phenomenological analyses at NLO.

HOW TO DESCRIBE DPS IN QCD?

QCD framework:

[Snigirev, Phys.Rev. D68 (2003) 114012]

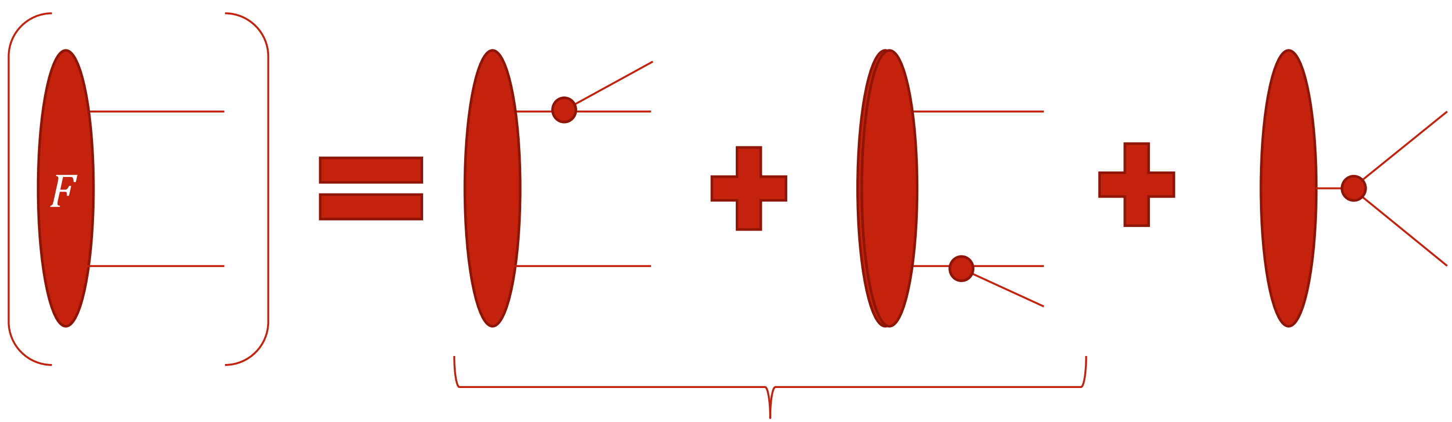
DPDs

$$\sigma_{DPS}^{(A,B)} = \frac{1}{\sigma_{eff}} \int \mathrm{d}x_i F_{ik}(x_1, x_2; Q) F_{jl}(x_3, x_4; Q) \hat{\sigma}_{ij\to A}(x_1, x_3) \hat{\sigma}_{kl\to B}(x_2, x_4)$$

Geometrical factor $\sim R_p^2$

'Inhomogeneous double DGLAP equation'

$$\frac{d}{d(\ln Q^2)}$$



Normal DGLAP splitting

'$1 \to 2$' splitting

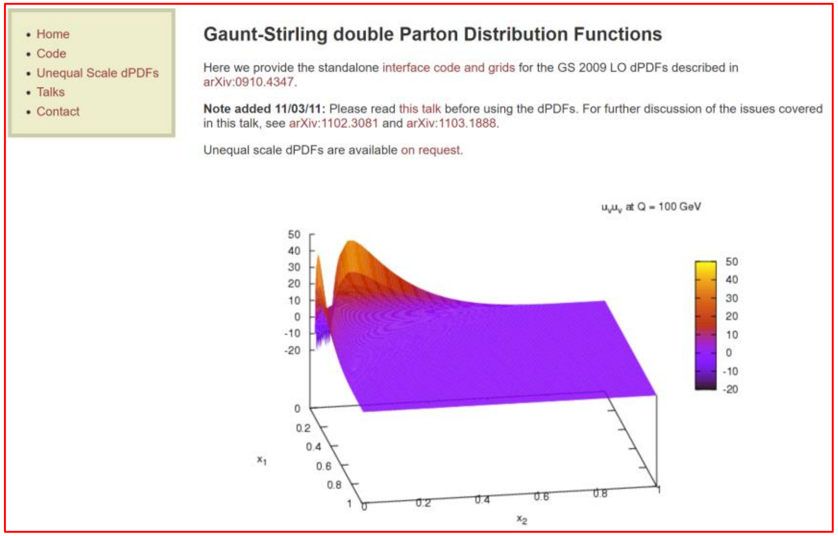
GS SUM RULES AND DPDS

DPD sum rules: preserved by inhomogeneous double DGLAP

e.g. momentum sum rule: $$\sum_j \int_0^{1-x_1} dx_2 x_2 F_{ij}(x_1, x_2; Q) = (1 - x_1) f_i(x_1; Q)$$

Useful for nonperturbative DPD modelling! Used in creation of GS2009 DPD set.

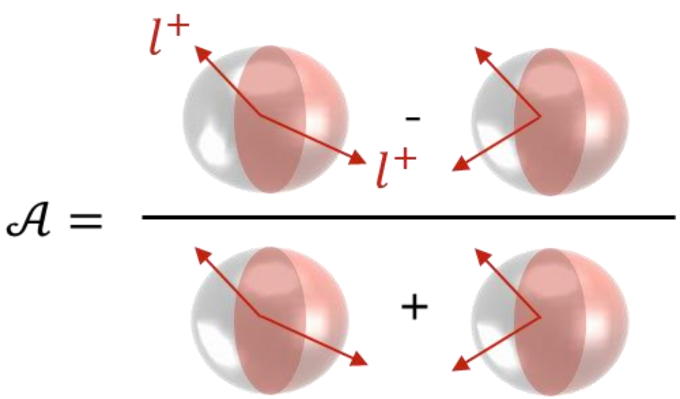
Gaunt, Stirling, JHEP 1003 (2010) 005



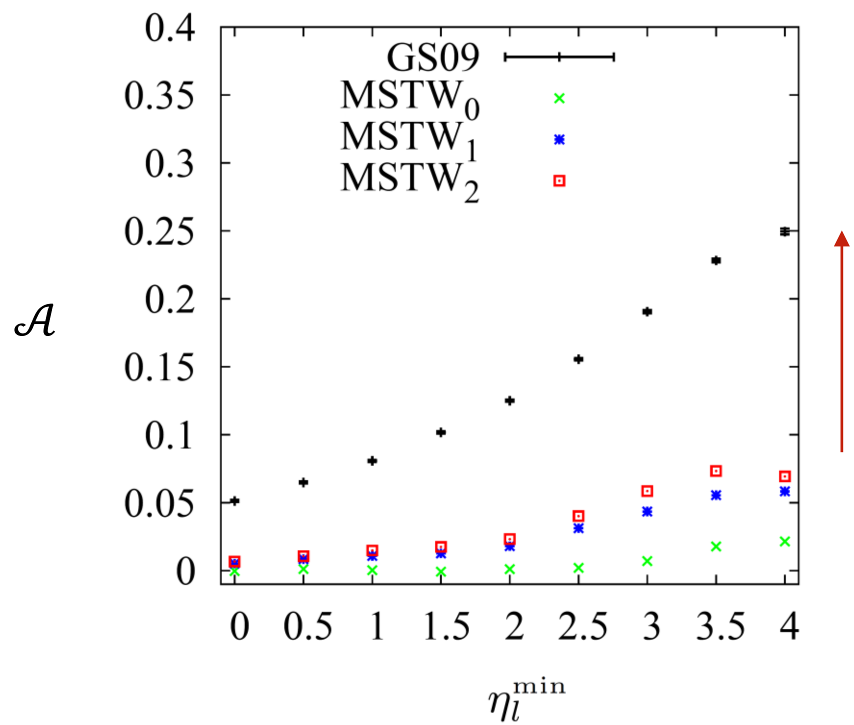
DPS PHENOMENOLOGY: WW

Phenomenological analysis of same-sign WW using GS09 DPDs, together with Anna Kulesza and Steve Kom:

$$pp \to W^+W^+ \to e^+\nu_e\mu^+\nu_\mu$$



JG, Kom, Kulesza, Stirling, Eur.Phys.J. C69 (2010) 53-65



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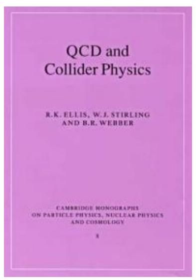
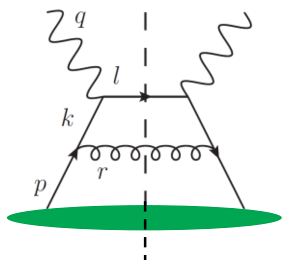
TOWARDS NLO?

First exercise: rederive '1→2' splitting functions at LO. How to do this?

DIS:



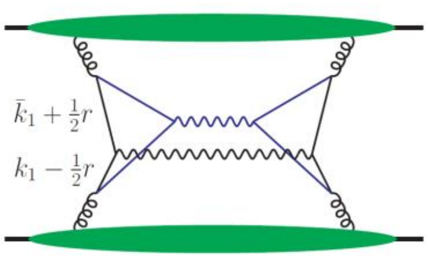


$P_{qq}(z)$

$$\frac{d^2\sigma}{dx\,dQ^2}\Big|_{qe\to egq,a} = \int_0^1 dz \int^{Q^2} \frac{dk_\perp^2}{k_\perp^2}\frac{\alpha_s}{2\pi} C_F \left[\frac{1+z^2}{1-z}\right] \frac{d^2\sigma}{dx\,dQ^2}\Big|_{qe\to eq}(z)$$ + non divergent

Logarithmic divergence.

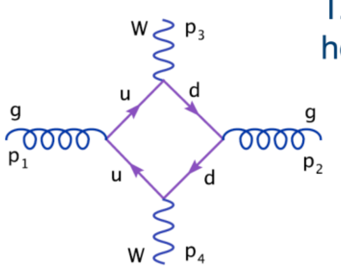
'Absorb' into PDF, up to scale μ_F



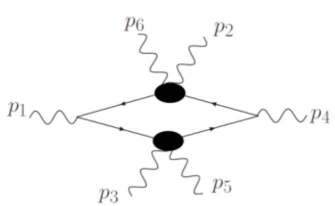
'1v1' graph with two 1→2' splittings. Find coefficient of double log divergence → splitting function?

PROBLEMS….

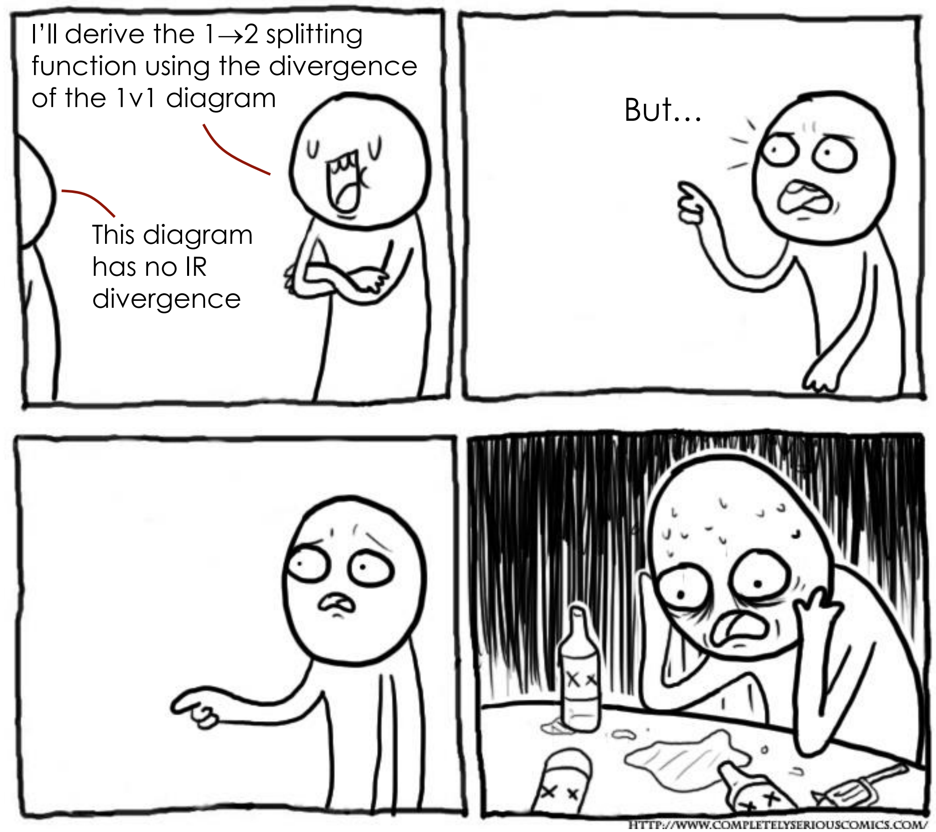
- Why assign geometrical factor $\backsim 1/R_p^2$ to 1v1 graphs?
- Graphs computed as loop corrections to SPS – no divergences!



T. Binoth et al.
hep-ph/0611170



Z. Bern et al.
0803.0494



(NEW) GOALS OF PHD

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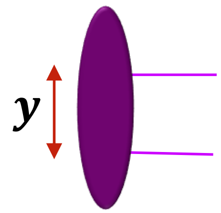
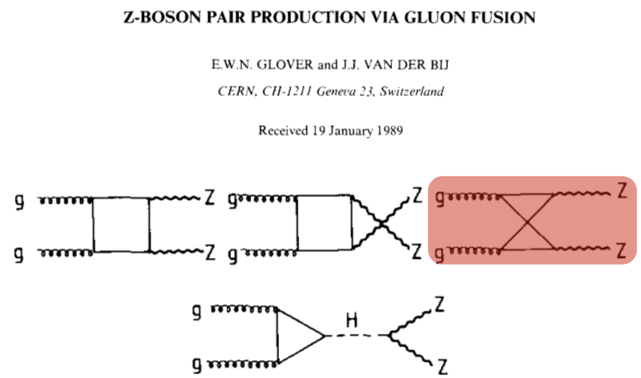
1V1 GRAPHS

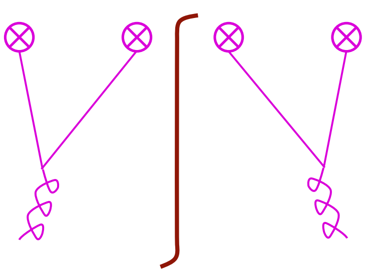
Study papers with 1v1 graphs, look at structure of results & graphs.

Close-to-collinear part of loops can be described by a DPS-like formula, but now with y dependence!





$$\sigma^{(A,B)}_{\text{1v1,DPS sing.}} = \int d^2\boldsymbol{y}\, \mathrm{d}x_i F_{ik}(x_1, x_2, \boldsymbol{y}; Q) F_{jl}(x_3, x_4, \boldsymbol{y}; Q) \hat{\sigma}_{ij\to A}(x_1, x_3) \hat{\sigma}_{kl\to B}(x_2, x_4)$$



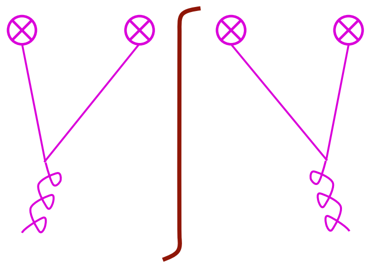
$$F_{ik}(x_1, x_2, \boldsymbol{y}; Q) = \frac{\alpha_s}{2\pi^2 y^2} \frac{P_{j\to ik}\left(\frac{x_1}{x_1 + x_2}\right)}{x_1 + x_2} f_j(x_1 + x_2; Q)$$

Perturbative y dependence!

JG, Stirling, JHEP 1106 (2011) 048
Diehl, Ostermeier, Schäfer, JHEP 1203 (2012) 089

1V1 GRAPHS

$$\sigma^{(A,B)}_{\mathrm{1v1,DPS\ sing.}} = \int d^2\boldsymbol{y}\, \mathrm{d}x_i F_{ik}(x_1,x_2,\boldsymbol{y};Q)F_{jl}(x_3,x_4,\boldsymbol{y};Q)\hat{\sigma}_{ij\to A}(x_1,x_3)\hat{\sigma}_{kl\to B}(x_2,x_4)$$



$$F_{ik}(x_1,x_2,\boldsymbol{y};Q) = \frac{\alpha_s}{2\pi^2 y^2}\frac{P_{j\to ik}\left(\frac{x_1}{x_1+x_2}\right)}{x_1+x_2} f_j(x_1+x_2;Q)$$

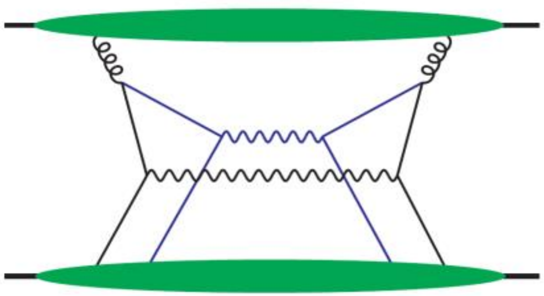
Proper QCD description of DPS must incorporate proper description of y dependence.

1v1 loops have no IR divergence, and already computed (at fixed order) as part of SPS: just leave as SPS?

JG, Stirling, JHEP 1106 (2011) 048
Also: Blok et al. Eur.Phys.J. C72 (2012) 1963
Manohar, Waalewijn Phys.Lett. B713 (2012) 196

2V1 GRAPHS

What about graphs with one 1→2 splitting ("2v1 graphs")?

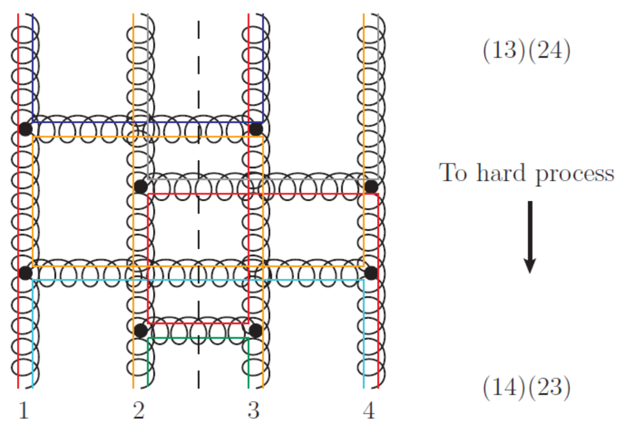


Technical complication: 3 → 2 process

Does have logarithmic IR divergence → include 2v1 in DPS.

Geometrical enhancement of these graphs by factor ~2

Other fun effects

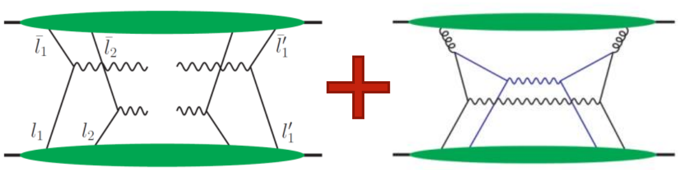


JG, JHEP 1301 (2013) 042
Also: Blok et al., Eur.Phys.J. C72 (2012) 1963
Ryskin, Snigirev, Phys.Rev.D83:114047,2011

PHD OVER!

Goals of PhD (for STFC):

2v2 2v1



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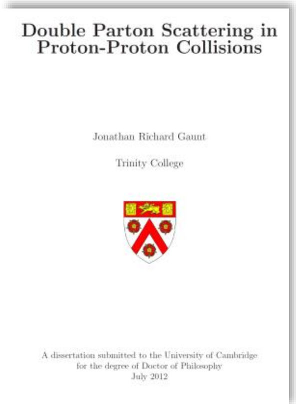

Double Parton Scattering in Proton-Proton Collisions

Jonathan Richard Gaunt

Trinity College

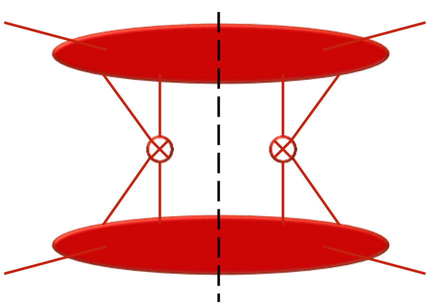
A dissertation submitted to the University of Cambridge
for the degree of Doctor of Philosophy
July 2012

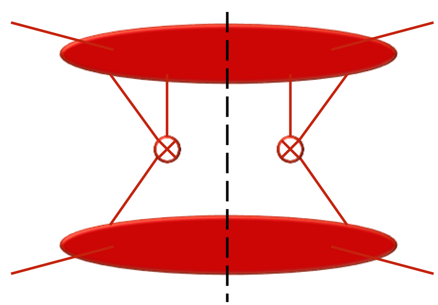
DRAWBACKS…

Drawback of this DPS framework:

$$\sigma_{DPS,2v2}^{(A,B)} + \sigma_{DPS,2v1}^{(A,B)} \quad \not\Longrightarrow \quad \sigma_{DPS}^{(A,B)} = \int d^2\boldsymbol{y} \mathrm{d}x_i F_{ik}(\boldsymbol{y}) F_{jl}(\boldsymbol{y}) \hat{\sigma}_{ij\to A} \hat{\sigma}_{kl\to B}$$

$$A^2 + 2AB \neq (A+B)\cdot(A+B)!$$





No well-defined DPD for single hadron.

NLO corrections?

A NEW SCHEME

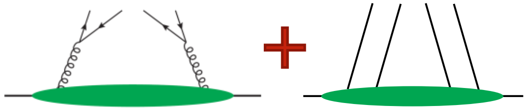
Diehl, Gaunt, Schönwald, JHEP 1706 (2017) 083

Alternative idea: *can* regard some of the 1v1 loop diagrams as DPS if we want.

Split diagrams up into DPS/SPS according to value of partonic separation y.

Step1: Insert cutoff into DPS cross section:

$$\sigma_{\mathrm{DPS}} = \int d^2y\, \Phi^2(\nu y)\, F(x_1, x_2; y) F(\bar{x}_1, \bar{x}_2; y)$$



If one just combined this with SPS naively, would have double counting up to scale ν!

A NEW SCHEME

<u>Step 2:</u> For total cross section for production of *AB*, include a subtraction term to remove double counting.

$$\sigma_{tot} = \sigma_{DPS} + \sigma_{SPS} - \sigma_{sub}$$

Subtraction term constructed by adapting techniques used by Collins in single scattering factorisation treatment.

By construction:

$$\sigma_{tot}(y \sim 1/Q) \approx \sigma_{SPS} \qquad \sigma_{tot}(y \sim 1/\Lambda) \approx \sigma_{DPS}$$

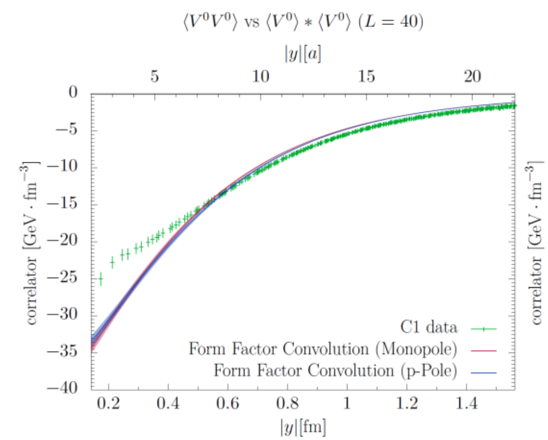
A NEW SCHEME

Some key advantages of this scheme:

- Retain concept of the DPD for an individual hadron with rigorous operator definition.

- All-order formulation, with corrections that are practicably computable.

- Resum logs in all diagrams where appropriate (2v2, 2v1 and 1v1).



Lattice computations of DPDs!

Preliminary, from talk by Christian Zimmermann at MPI@LHC 2018

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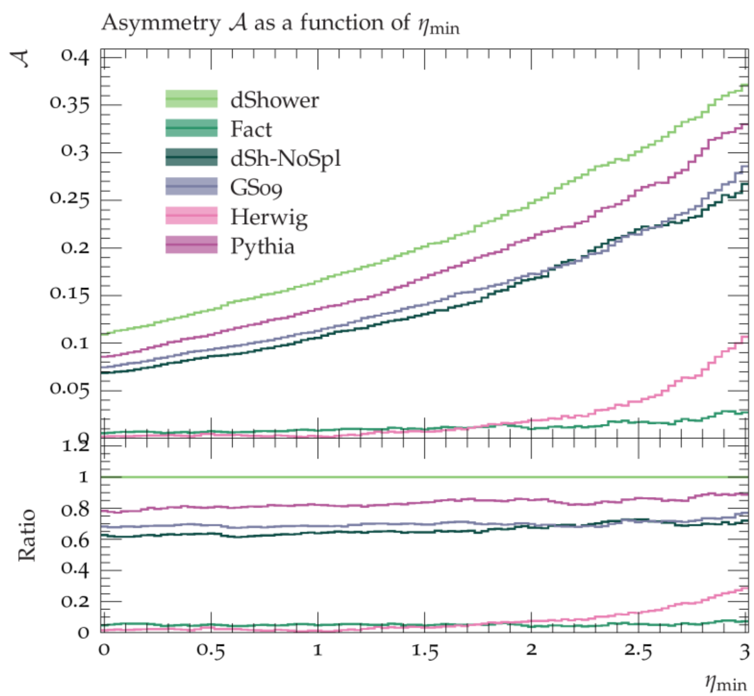
MONTE CARLO IMPLEMENTATION OF DPS

Construction of parton shower implementation of DGS framework in progress, together with B. Cabouat, K. Ostrolenk: dShower.



Effect of QCD splittings

Cabouat, JG, Ostrolenk, arXiv:1906.04669

N.B. DPD sum rules still hold, and retain their utility for DPD modelling!

JG, PhD thesis
Diehl, Plößl, Schäfer, Eur.Phys.J. C79 (2019) no.3, 253

RESEARCH GOALS

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0) Build proper QCD description of DPS. ✓ $\boxed{\sigma_{tot} = \sigma_{DPS} + \sigma_{SPS} - \sigma_{sub}}$

1) Build a numerical DPD set based on proper QCD description of DPS. ✓ + WIP

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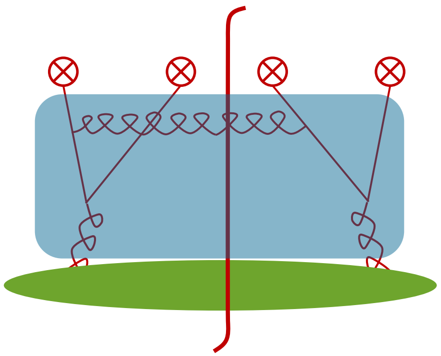
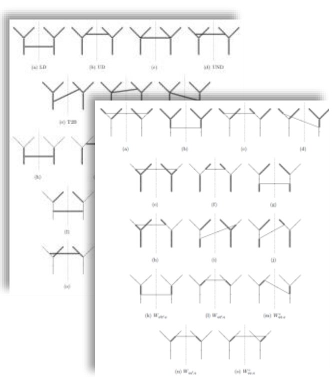
DPS AT NLO

Missing ingredient required for NLO computations: NLO corrections to 1→2 splitting

$$F_{a_1 a_2}(x_1, x_2, y, \mu) = \frac{1}{\pi y^2} \sum_{a_0} \int_{x_1+x_2}^{1} \frac{dz}{z^2} \boxed{V_{a_1 a_2, a_0}\left(\frac{x_1}{z}, \frac{x_2}{z}, a_s(\mu), \log \frac{\mu^2 y^2}{b_0^2}\right)} \boxed{f_{a_0}(z, \mu)}$$



Recently we computed the NLO corrections to V for all flavour channels (Diehl, JG, Plößl, Schäfer, SciPost Phys. 7 (2019) 2, 017).



Graphs

Integration-by-parts identities

$$I_1(a_1, a_2, a_3, a_4) = \int \frac{d^{d-2}\boldsymbol{k}_1 d^{d-2}\boldsymbol{k}_2}{\prod_{i=1..4} D_i^{a_i}}$$

'Master integrals'

Differential equations, boundary conditions from method of regions

Results for bare graphs!

Opens the way for the first full NLO computations of DPS!

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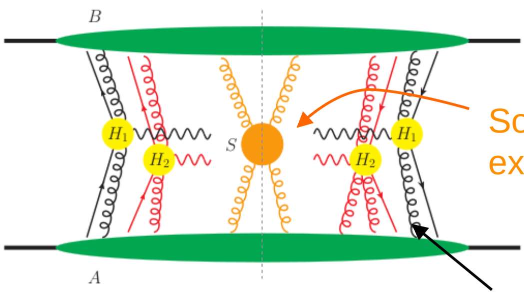
4) Perform phenomenological analyses at NLO. Next!

FACTORISATION IN DPS

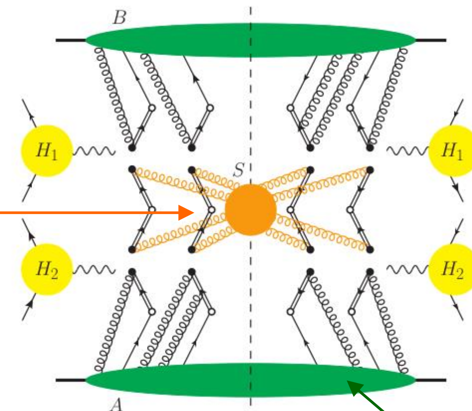


Soft and Glauber exchanges

Initial picture

Extra (unphysically polarised) gluon connections to hard

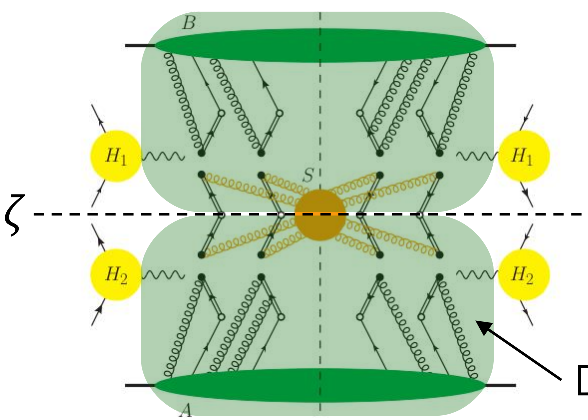
Diehl, JG, Ostermeier, Plößl, Schafer, JHEP 1601 (2016) 076, Diehl, Ostermeier, Schafer, JHEP 1203 (2012) 089, Diehl, Nagar, JHEP 1904 (2019) 124.



Soft factor

Collinear factor

(Vladimirov, JHEP 1804 (2018) 045)



DPDs

$$\sigma \sim F \otimes F \otimes \hat{\sigma} \otimes \hat{\sigma}$$

Proven, at least for double Drell-Yan production!

CONCLUSION

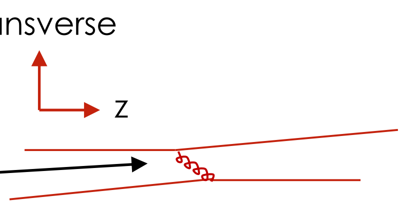
Theory of DPS is now in great shape!

Thanks to James, for starting me on this pathway through DPS theory, and for his guidance and constant encouragement during my PhD.

Backup

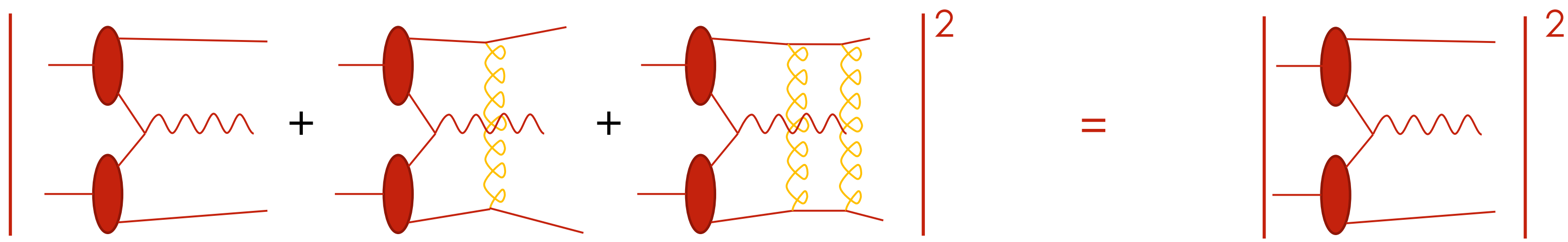
FACTORISATION: SOFT EXCHANGES

However, there is a particular type of soft exchange for which this doesn't work: Glauber exchanges. Soft particles mediating forward scattering.



Treatment of Glauber exchanges is the trickiest part of a factorisation proof!

Single scattering production of colour singlet V: Collins, Soper, Sterman showed that effect of Glauber exchanges cancels if we measure only properties of V, and sum over everything else!



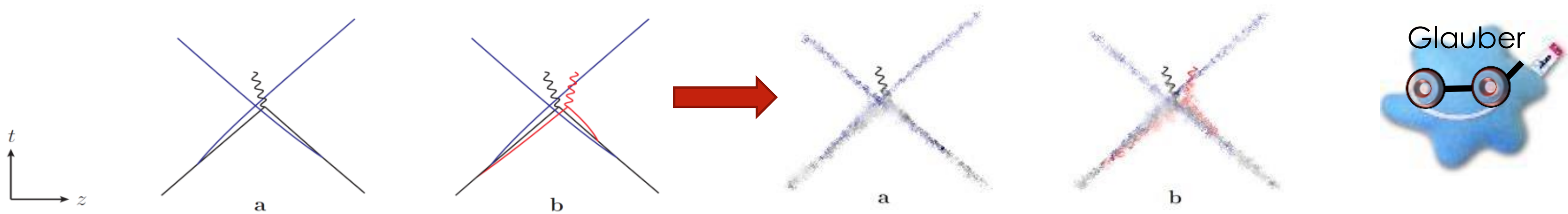
If one starts measuring properties of radiation accompanying V (e.g. global event shape variables), this argument breaks down!

JG, JHEP 1407 (2014) 110
Zeng, JHEP 1510 (2015) 189
Schwartz, Yan, Zhu, Phys.Rev. D97 (2018) no.9, 096017

GLAUBER CANCELLATION IN DPS

In JHEP 1601 (2016) 076 (Diehl, JG, Schäfer, Ostermeier, Plößl) we adapted the methodology of Collins, Soper, Sterman to show that Glauber exchanges also cancel for DPS production of two colourless systems.

Full proof is very technical, but can get some insight as to why it works by looking at spacetime pictures of single and double scattering:



Other important steps towards factorisation proof made in Diehl, Ostermeier, Schafer, JHEP 1203 (2012) 089 Vladimirov, JHEP 1804 (2018) 045, Diehl, Nagar, arXiv:1812.09509.