

PDFs with MHOUs

- MHOU via Scale Varn
- Validation
- PDFs and Predictions

Richard Ball (for NNPDF)

[arXiv: 1905.04311](#)

[arXiv: 1906.10698](#)

James Stirling Memorial
PDF4LHC Sep 2019
Durham

Uncertainties in PDF fits

- Experimental uncertainties
- Parametric uncertainties
(α_s , m_q , etc)
- Methodological uncertainties
(closure testing: small)
- Theoretical uncertainties
(nuclear, MHOU, HT, etc)

Ultimate aim: 1% uncertainties: need to have **all** these under control.

Theoretical Uncertainties in PDF fits

- Statistical (Monte Carlo, etc) [arXiv: 1704.00428](#)
- Nuclear Corrections [arXiv: 1812.09074](#)
- **Missing Higher orders (MHOU)** [arXiv: 1906.10698](#)
- Power corrections (HT)
- Final state corrections

General Strategy:

- compute a theory covariance matrix S_{ij} for each theoretical uncertainty
- add to experimental covariance matrix C_{ij} (exp and th unc independent)
- fit using combined covariance matrix $C_{ij} + S_{ij}$ [arXiv: 1801.04842](#)

Increases overall PDF uncertainty, but removes theoretical tensions or biases
(by reducing impact of data sets with significant theoretical uncertainties)

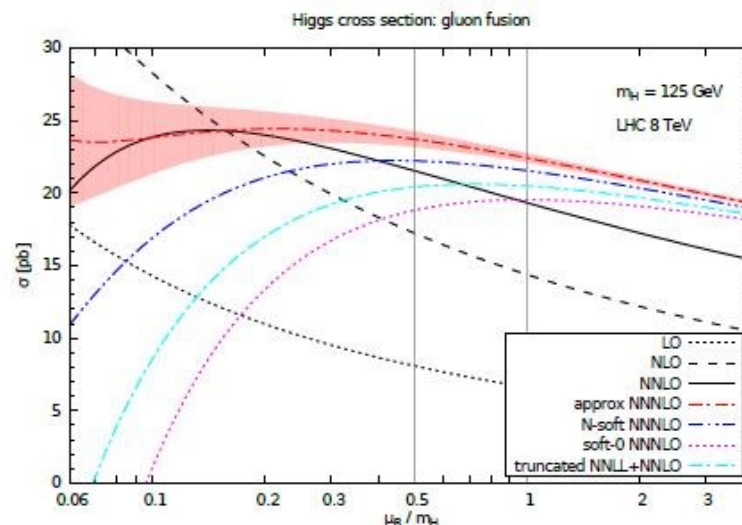
Estimating MHOU by Scale Variation

Ren grp invariance:

$$\mu^2 \frac{d}{d\mu^2} \bar{\sigma}(\alpha_s(\mu^2), \mu^2/Q^2) = 0.$$

- Q^2 is the physical scale
- μ^2 is the ren scale

Estimate MHOU by evaluating xsec at various scales:



Various prescriptions: ren scale, fact scale, used to give **envelope** estimates

Here want to instead estimate theory **covmat** by varying parameters and **averaging**

$$S_{ij} \simeq \langle (T_i(\mu) - T_i)(T_j(\mu) - T_j) \rangle_\mu$$

Scale Variation: Pros and Cons

Pros:

- naturally incorporates ren grp inv: MHOU goes to zero at high orders
- $\alpha_s(\mu^2)$ is universal: same procedure works for any process
- naturally incorporates sum rule constraints
- smooth: kinematic corrlns in e.g. Q^2 , x , etc, automatically included
- can also account for corrlns between different processes

Cons:

- how much should we vary the scale? Factor $2, \frac{1}{2}$ usual ...
- can fail for new channels, color structures, kinematic sings...

Validation:

- At NLO, can compare NLO MHOU estimate to NNLO

Scale variation for PDF determination

Factorization (eg DIS):

$$\overline{F}(Q^2, \mu_f^2, \mu_r^2) = \overline{C}(\alpha_s(\mu_r^2), \mu_r^2/Q^2) \otimes \overline{f}(\alpha_s(\mu_f^2), \mu_f^2/Q^2)$$

For a given factorization (eg $\overline{\text{MS}}$) have *two* sources of MHOU for each observable:

- PDF evolution : ‘factorization scale’ μ_f : MHOU in splitting functions
- hard coeff fn : ‘renormalization scale’ μ_r : MHOU in coeff fns / hard xsecs

Variations of μ_f and μ_r must be performed

- **independently** of each other (two sources of MHOU distinct)
- **symmetrically** (two sources of MHOU equally important)
- μ_f variations correlated across all processes (PDFs **universal**)
- μ_r variations correlated within a process, uncorrelated for different processes

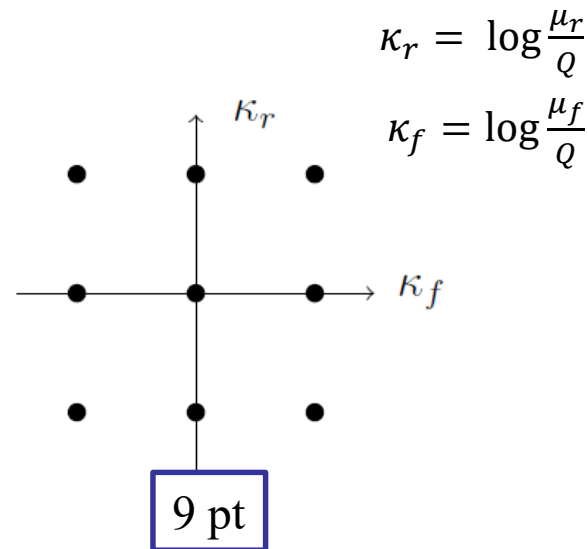
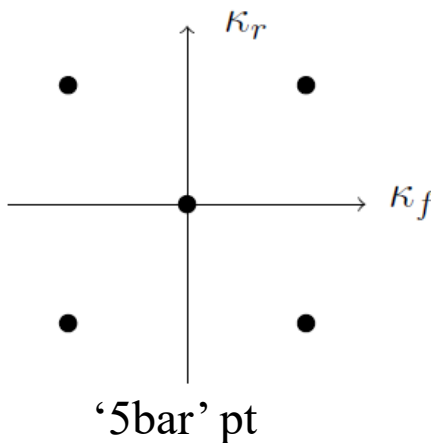
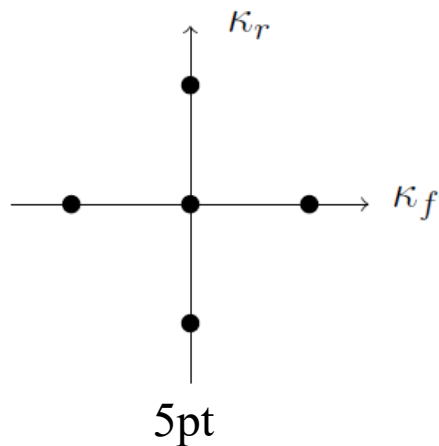
In this first study, we assume

- a single μ_f (though singlet/nonsinglet/valence in principle independent)
- five different μ_r (corresponding to DIS NC, DIS CC, DY, jets, top)
- all variations by factor of two either way

Symmetric Prescriptions

$$S_{ij} = \mathcal{N} \sum \Delta_i(\mu_f, \mu_r) \Delta_j(\mu_f, \mu_r) \quad \Delta_i(\mu_f, \mu_r) = T_i(\mu_f, \mu_r) - T_i(Q, Q)$$

e.g.



Same process:

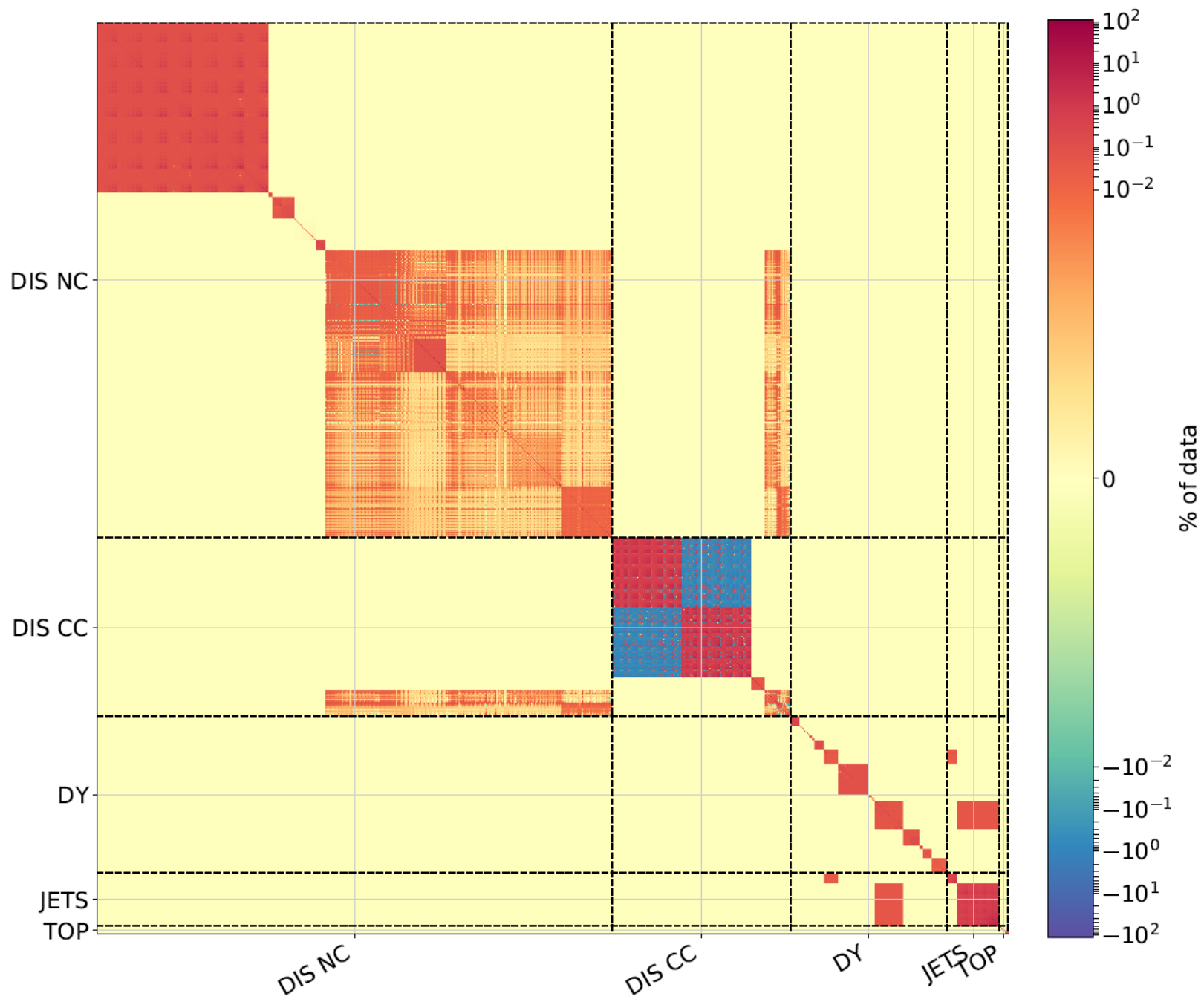
$$S_{ij}^{(9\text{pt})} = \frac{1}{4} \{ \Delta_i^{+0} \Delta_j^{+0} + \Delta_i^{-0} \Delta_j^{-0} + \Delta_i^{0+} \Delta_j^{0+} + \Delta_i^{0-} \Delta_j^{0-} \\ + \Delta_i^{++} \Delta_j^{++} + \Delta_i^{+-} \Delta_j^{+-} + \Delta_i^{-+} \Delta_j^{-+} + \Delta_i^{--} \Delta_j^{--} \} . \quad i, j \in \pi$$

Different processes:

$$S_{i_1 j_2}^{(9\text{pt})} = \frac{1}{24} \{ 2(\Delta_{i_1}^{+0} + \Delta_{i_1}^{++} + \Delta_{i_1}^{+-})(\Delta_{j_2}^{+0} + \Delta_{j_2}^{++} + \Delta_{j_2}^{+-}) \\ + 2(\Delta_{i_1}^{-0} + \Delta_{i_1}^{-+} + \Delta_{i_1}^{--})(\Delta_{j_2}^{-0} + \Delta_{j_2}^{-+} + \Delta_{j_2}^{--}) \} \\ + 3(\Delta_{i_1}^{0+} + \Delta_{i_1}^{0-})(\Delta_{j_2}^{0+} + \Delta_{j_2}^{0-}) \} . \quad i_1 \in \pi_1, j_2 \in \pi_2$$

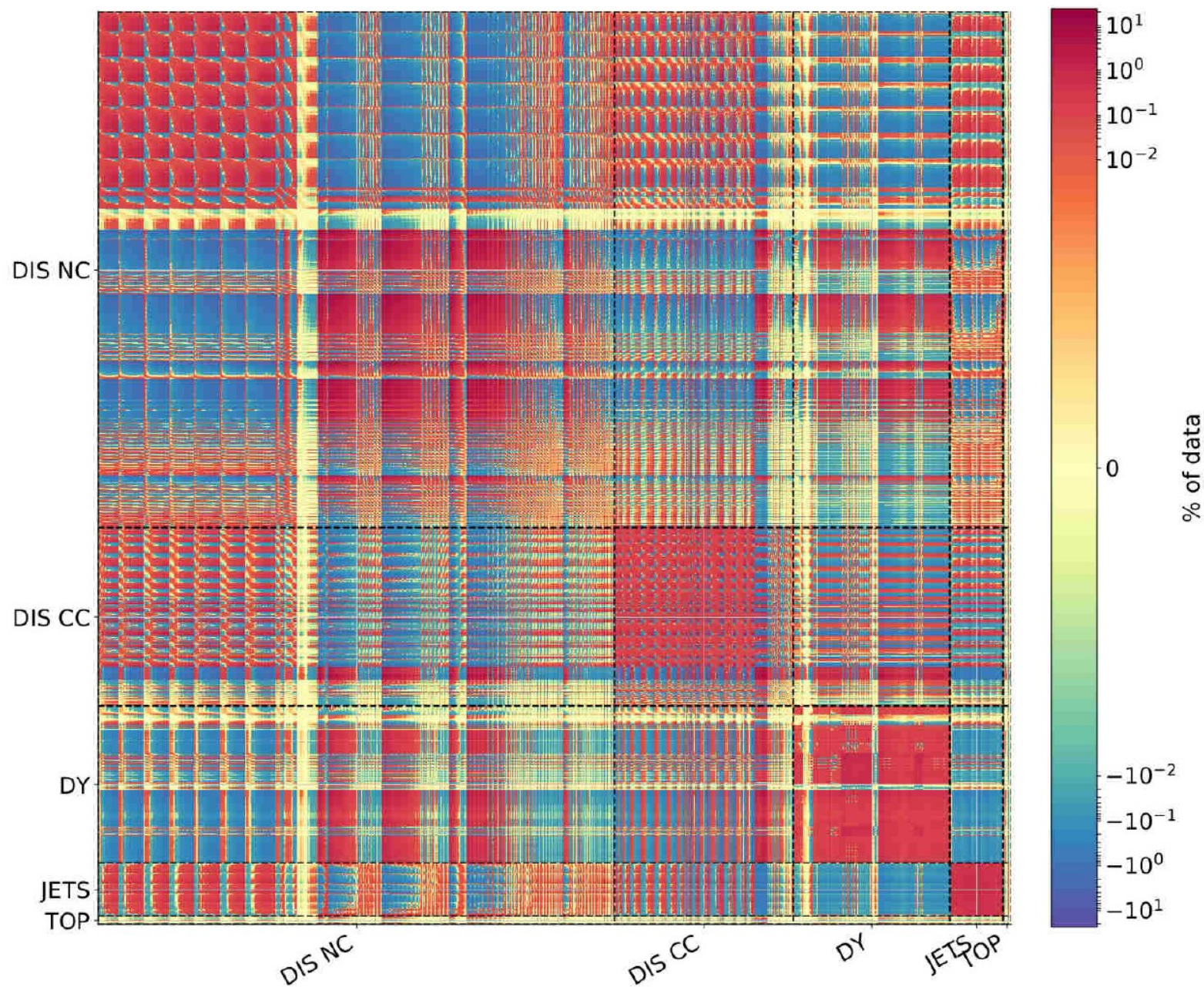
$$\mu = \begin{cases} 2Q & + \\ Q & 0 \\ \frac{1}{2}Q & - \end{cases}$$

Experimental Covariance Matrix

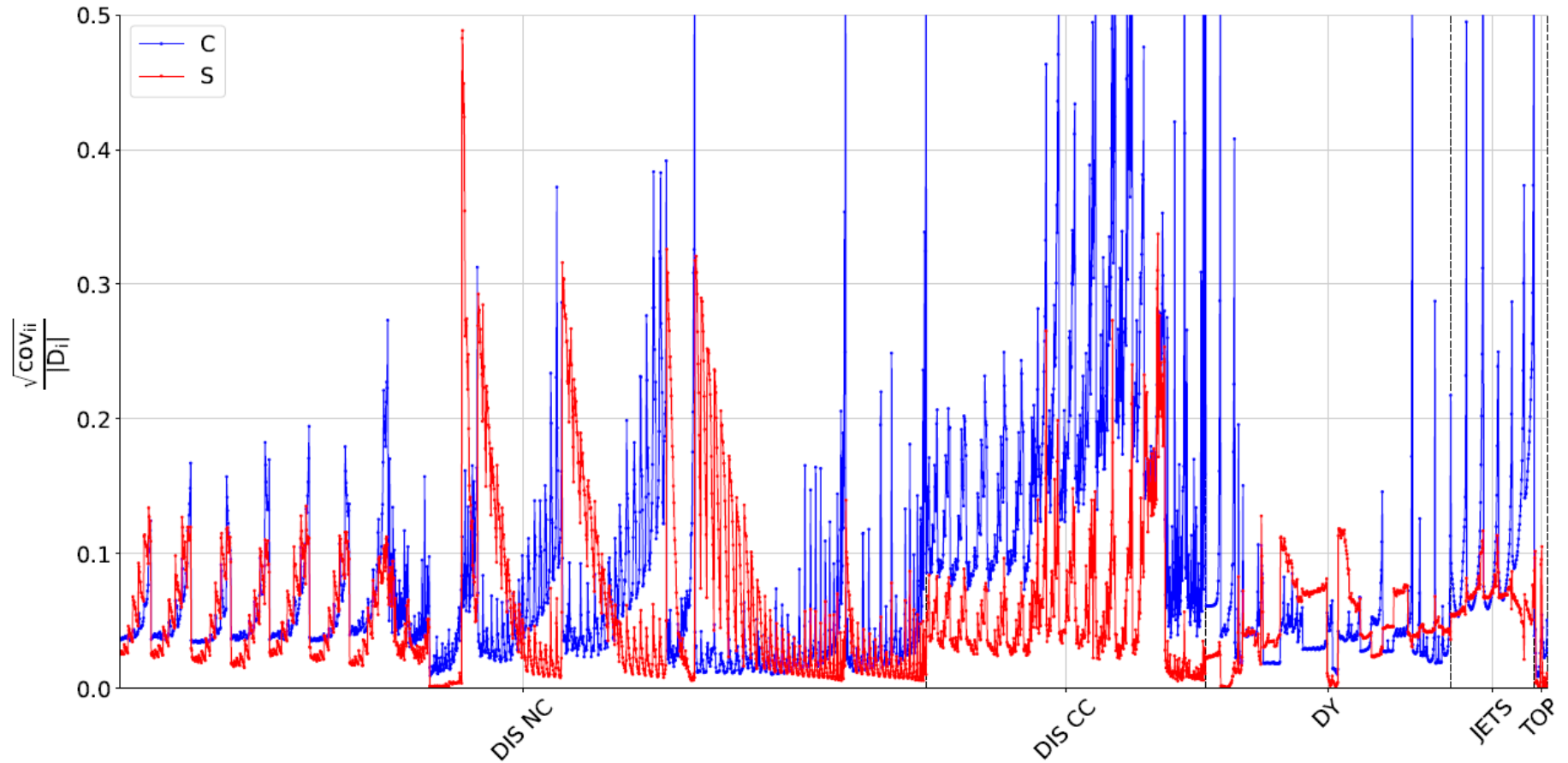


Theory Covariance matrix (9 pt)

NLO



MHOU ($\sqrt{S_{ii}}$) vs Exp Unc ($\sqrt{C_{ii}}$)



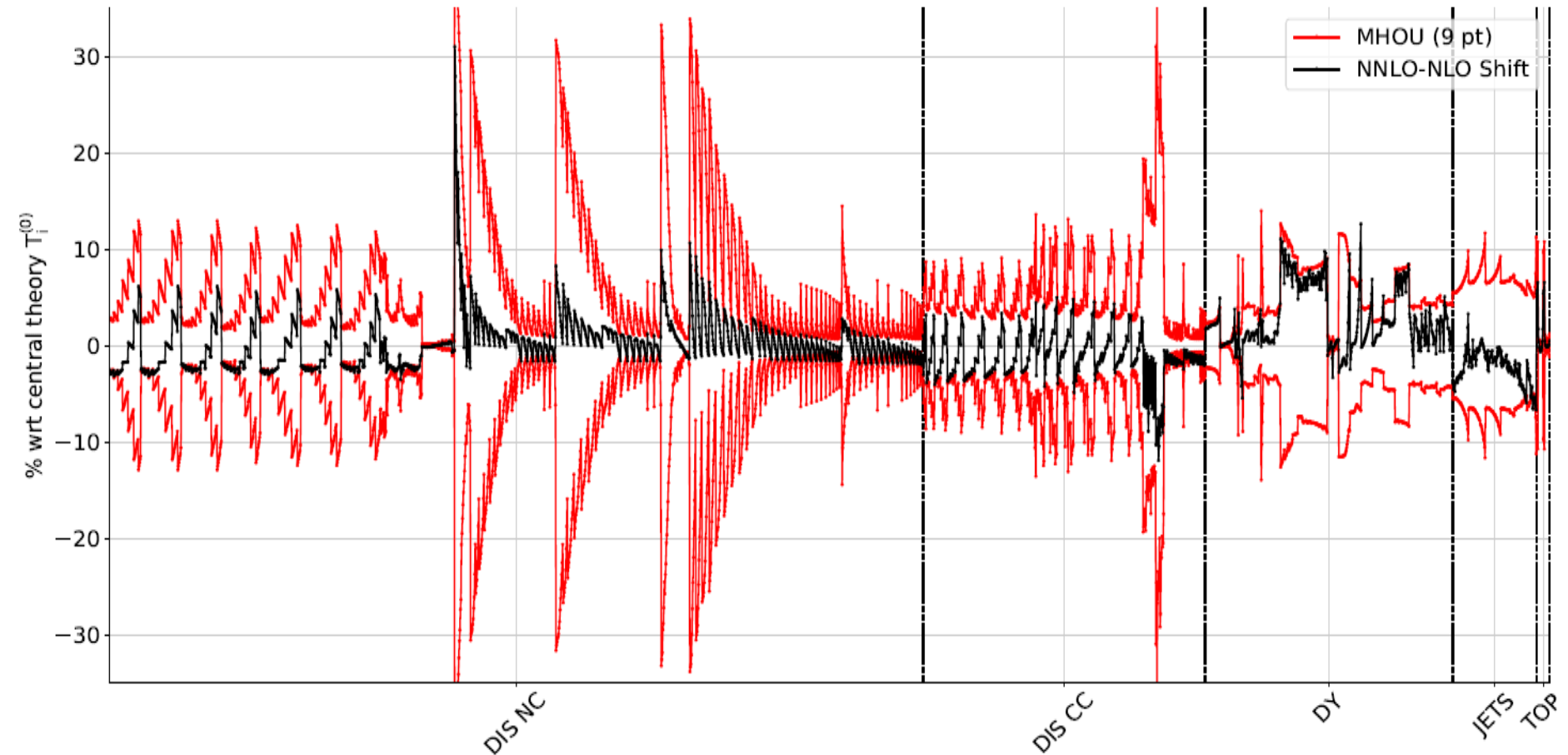
- if $S_{ii} \gg C_{ii}$ data points suppressed in fit
- if $S_{ii} \ll C_{ii}$ MHOU has no effect

NLO

Validation : diagonals

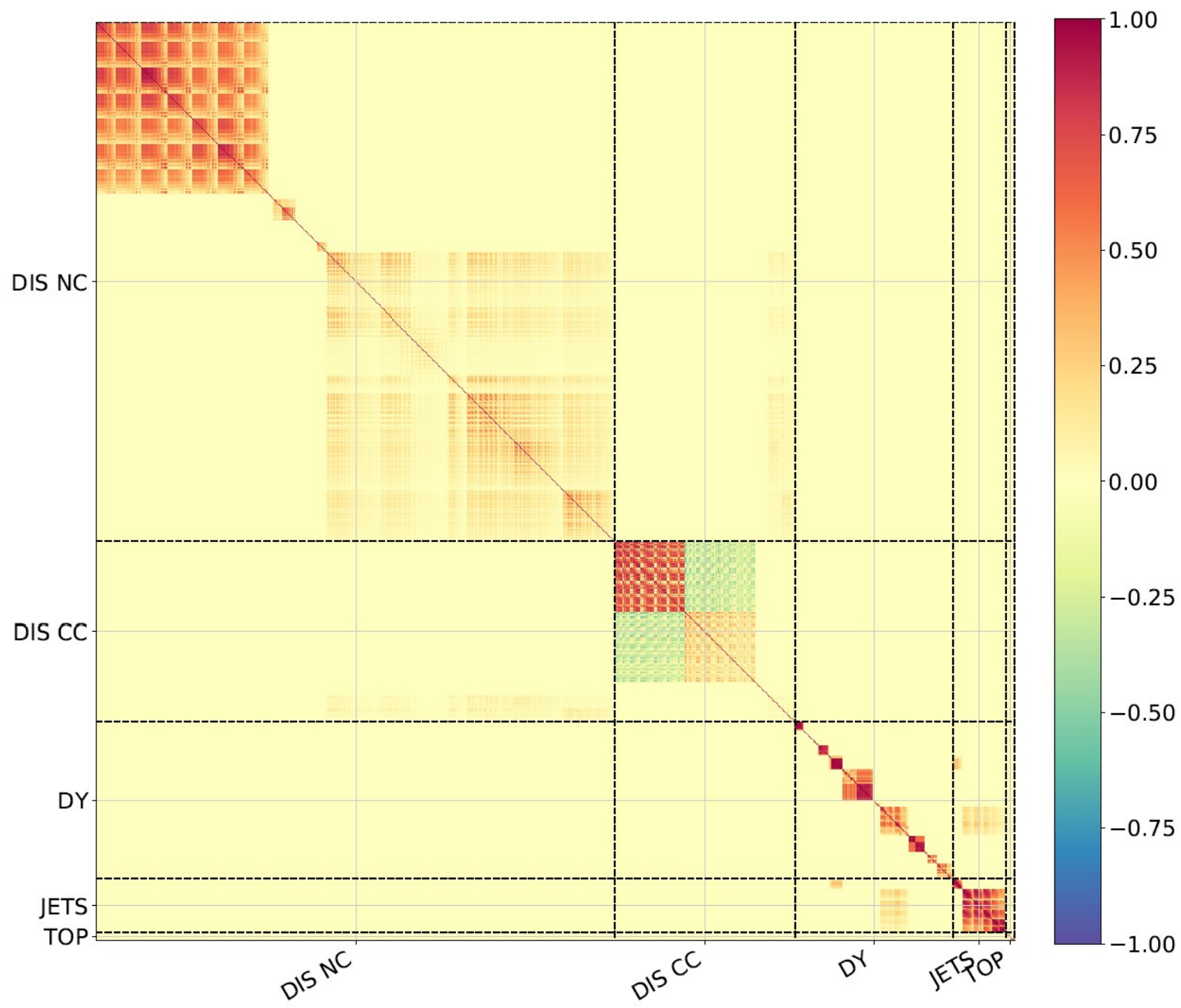
Compare diagonal **MHOU** ($\sqrt{S_{ii}}$) to 'shift'

$$\delta_i = (T_i^{\text{NNLO}} - T_i^{\text{NLO}}) / T_i^{\text{NLO}}$$



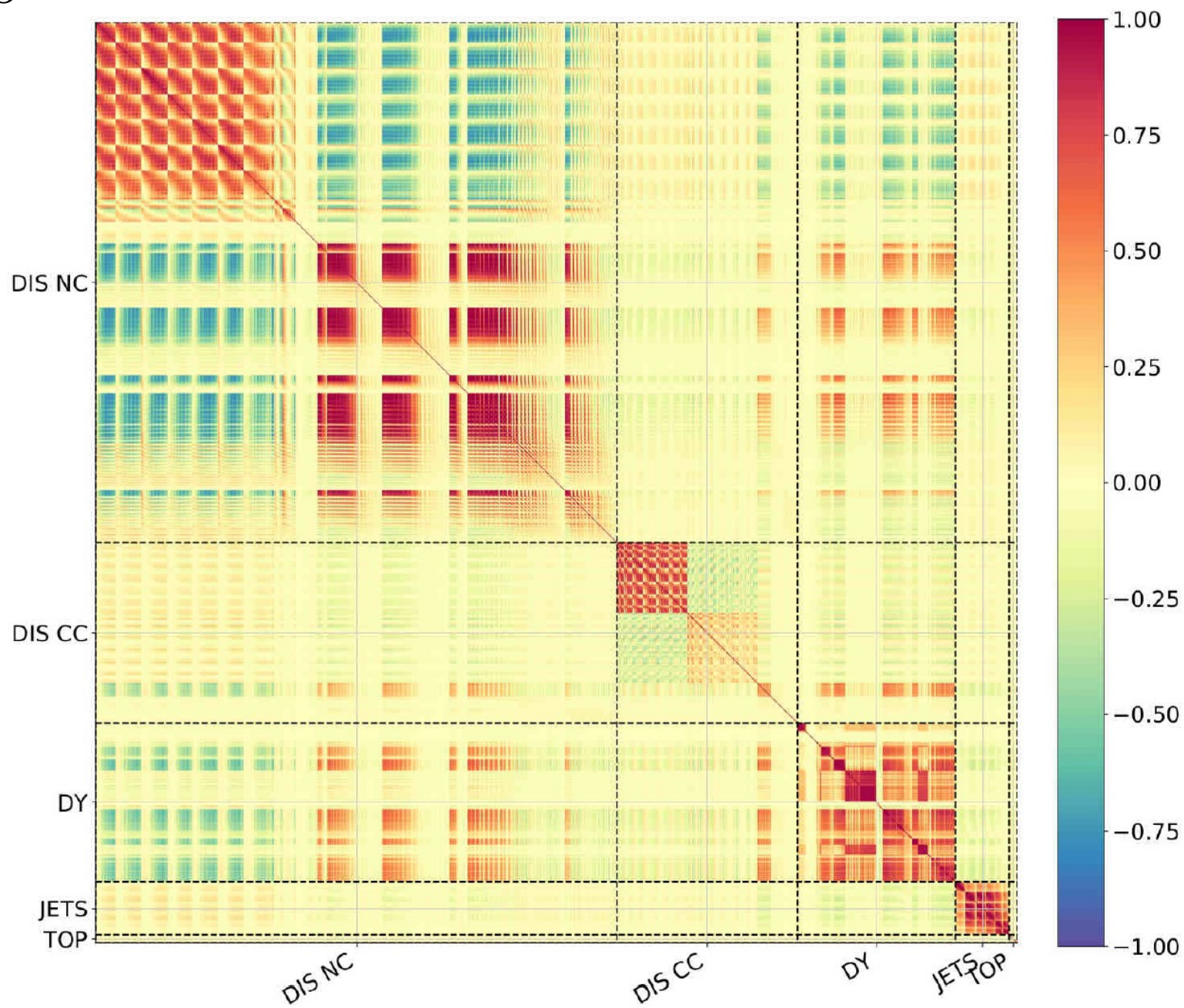
$\delta_i < \sqrt{S_{ii}}$ for all data points : **conservative**

Experimental Correlation Matrix



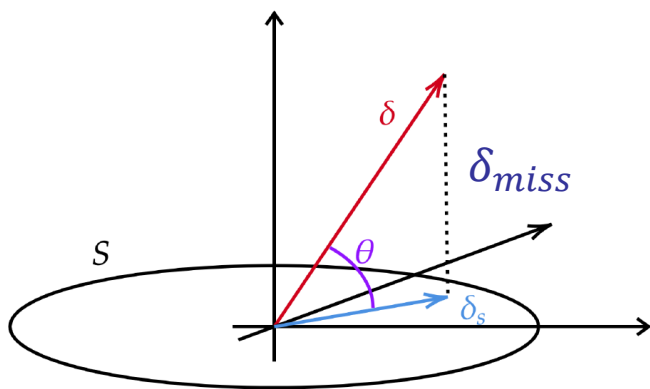
NLO

Experimental + Theory Correlation Matrix (9 pt)



Validation : correlations

- view shift δ_i as a vector in ~ 3000 dimensional space
- diagonalize S_{ij} in this space: gives ellipsoid S in < 30 dim subspace
- project δ_i into S : how much of δ_i lies in S ???

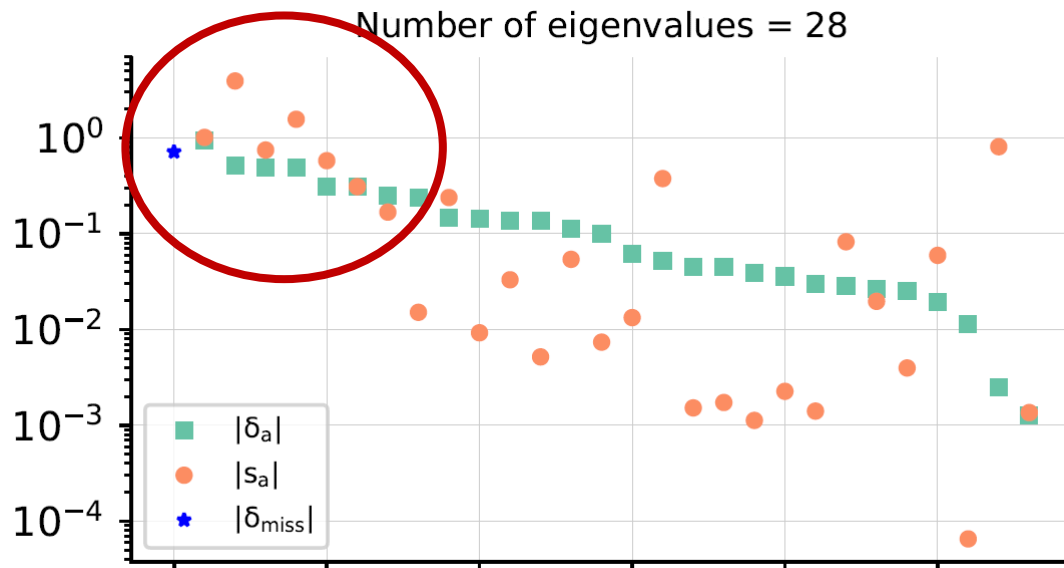


smaller angles θ mean that S_{ij} gives a better estimate of δ_i

Presc.	N_{sub}	Global	DIS NC	DIS CC	DY	JET	TOP
		2819	1593	552	484	164	26
5-pt	8	33°	39°	21°	25°	17°	11°
5-pt	12	31°	38°	17°	23°	22°	10°
9-pt	28	26°	32°	16°	22°	14°	3°
3-pt	6	52°	54°	36°	39°	24°	12°
7-pt	14	29°	35°	17°	22°	16°	3°

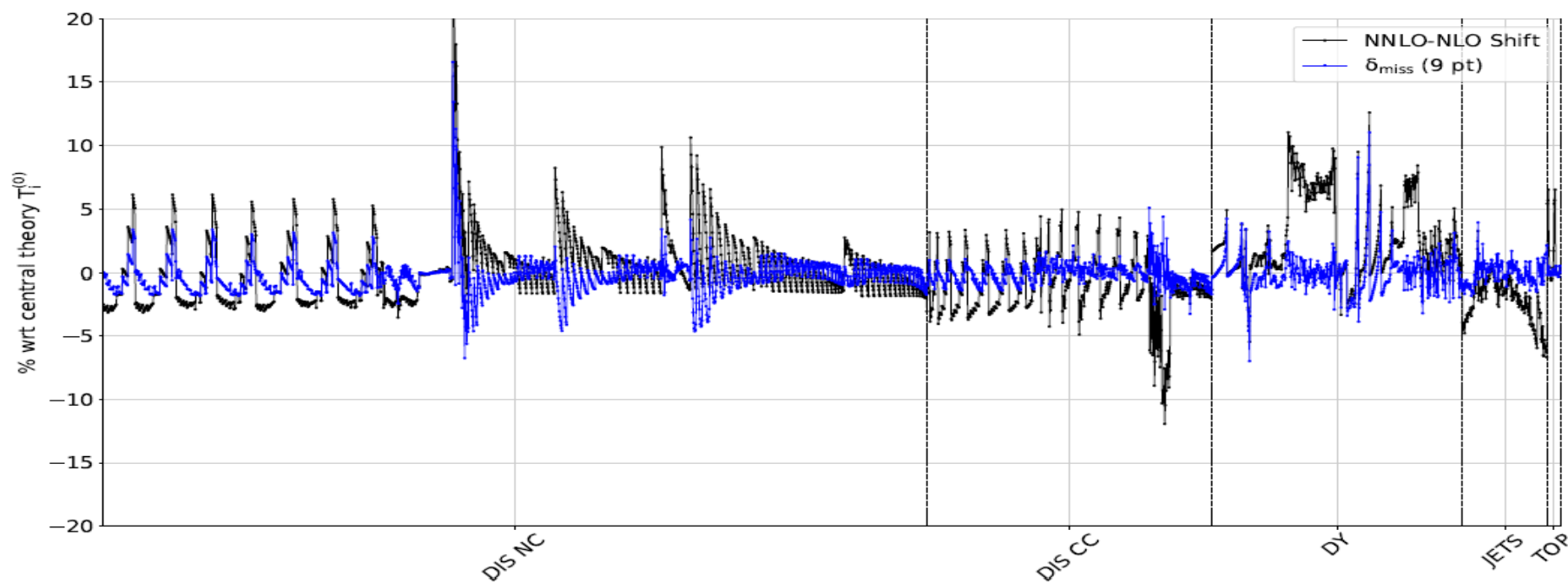
9-pt works best!

NLO, 9pt



For larger projections δ ,
evals $s > \delta$

$$\delta_{\text{miss}}^i \ll \delta^i$$



NLO global fits with MHOU

- Generate data replicas: $\lim_{N_{\text{rep}} \rightarrow \infty} \frac{1}{N_{\text{rep}}(N_{\text{rep}} - 1)} \sum_{k=1}^{N_{\text{rep}}} \left(D_i^{(k)} - \langle D_i \rangle \right) \left(D_j^{(k)} - \langle D_j \rangle \right) = C_{ij} + S_{ij},$
- Fit PDF replicas: $\chi^2[f^{(k)}] = \frac{1}{N_{\text{dat}}} \sum_{i,j=1}^{N_{\text{dat}}} (T_i[f^{(k)}] - D_i^{(k)})(C + S)_{ij}^{-1} (T_j[f^{(k)}] - D_j^{(k)})$

Process	n_{dat}	χ^2/n_{dat} in the NNPDF3.1 global fits				
		NLO				NNLO
		C	$C + S^{(9\text{pt})}$	$C + S^{(7\text{pt})}$	$C + S^{(3\text{pt})}$	C
DIS NC	1593	1.088	1.079	1.086	1.095	1.084
DIS CC	552	1.012	0.928	0.933	0.960	1.079
DY	484	1.486	1.447	1.485	1.483	1.231
JETS	164	0.907	0.839	0.858	0.901	0.950
TOP	26	1.260	1.012	1.016	1.077	1.068
Total	2819	1.139	1.109	1.129	1.139	1.105

MHOU improves χ^2 :
9pt does best!
(~ NNLO)

NLO global fits with MHOU

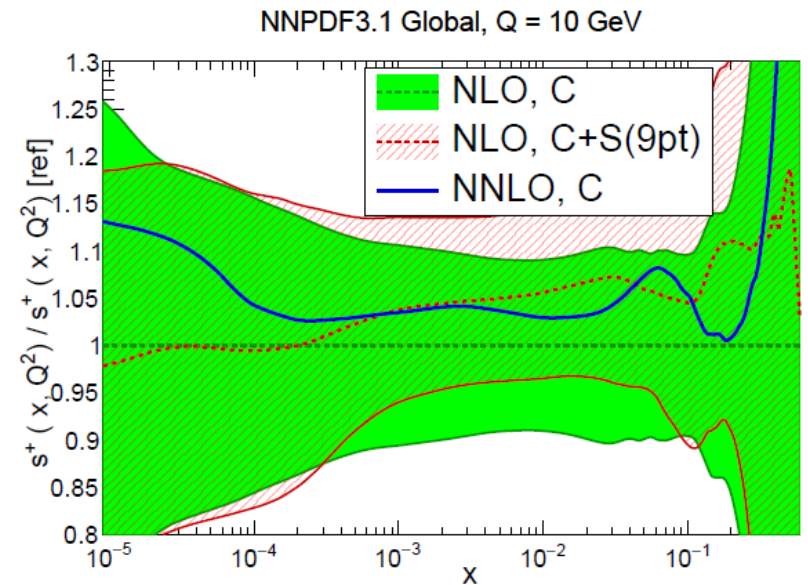
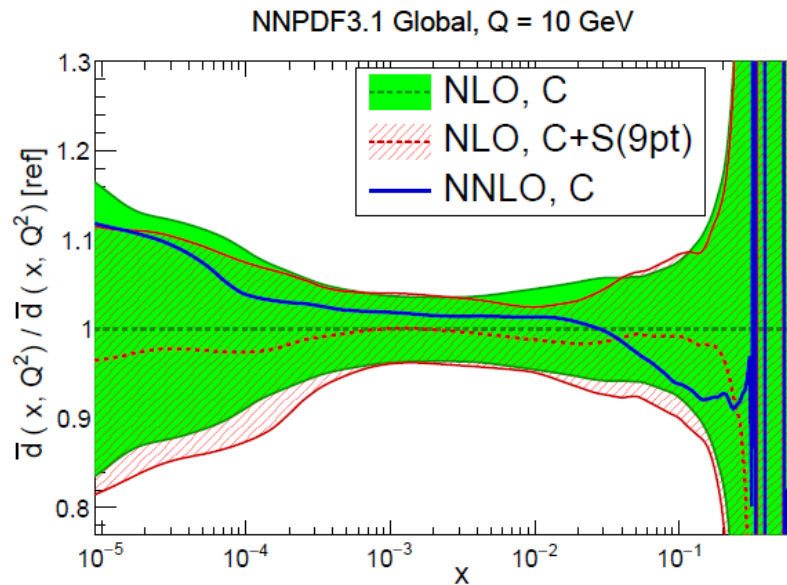
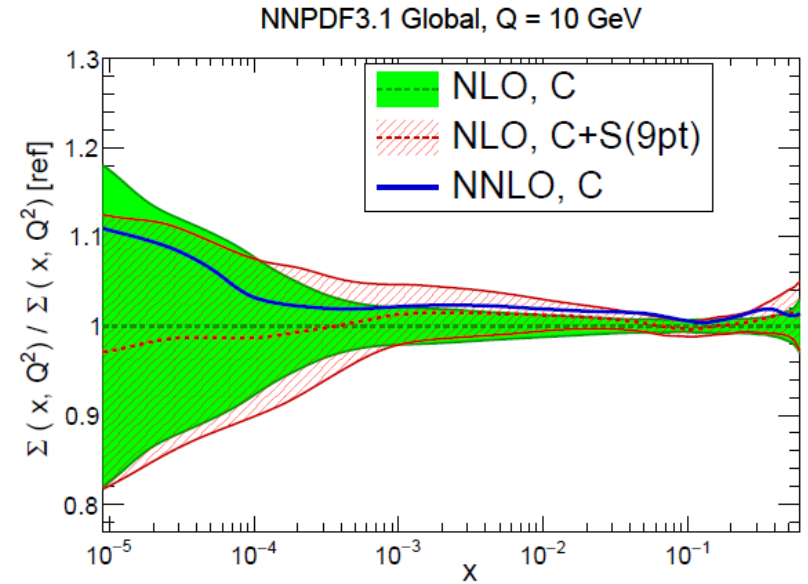
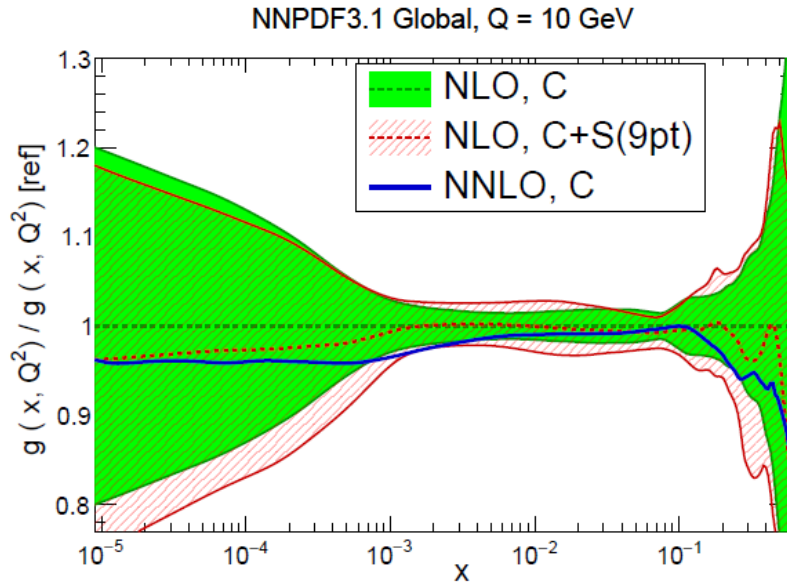
- Generate data replicas: $\lim_{N_{\text{rep}} \rightarrow \infty} \frac{1}{N_{\text{rep}}(N_{\text{rep}} - 1)} \sum_{k=1}^{N_{\text{rep}}} \left(D_i^{(k)} - \langle D_i \rangle \right) \left(D_j^{(k)} - \langle D_j \rangle \right) = C_{ij} + S_{ij},$
- Fit PDF replicas: $\chi^2[f^{(k)}] = \frac{1}{N_{\text{dat}}} \sum_{i,j=1}^{N_{\text{dat}}} (T_i[f^{(k)}] - D_i^{(k)})(C + S)_{ij}^{-1} (T_j[f^{(k)}] - D_j^{(k)})$

Process	ϕ in the NNPDF3.1 global fits				
	NLO				NNLO
	C	$C + S^{(9\text{pt})}$	$C + S^{(7\text{pt})}$	$C + S^{(3\text{pt})}$	C
DIS NC	0.266	0.412	0.393	0.384	0.305
DIS CC	0.389	0.408	0.427	0.442	0.471
DY	0.361	0.377	0.369	0.379	0.380
JETS	0.295	0.359	0.327	0.333	0.392
TOP	0.375	0.443	0.387	0.405	0.363
Total	0.314	0.405	0.394	0.394	0.362

$$\phi \sim \left\langle \frac{\sigma_{PDF}}{\sigma_{exp}} \right\rangle$$

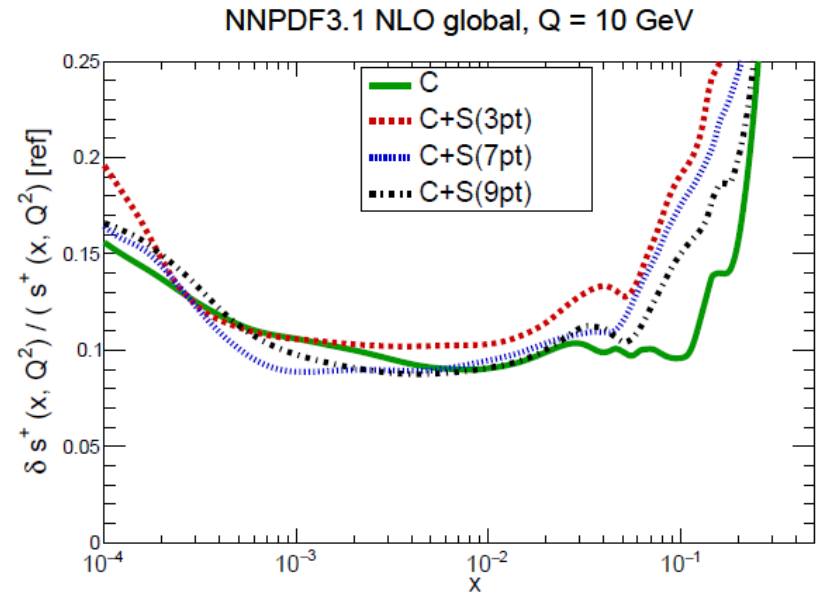
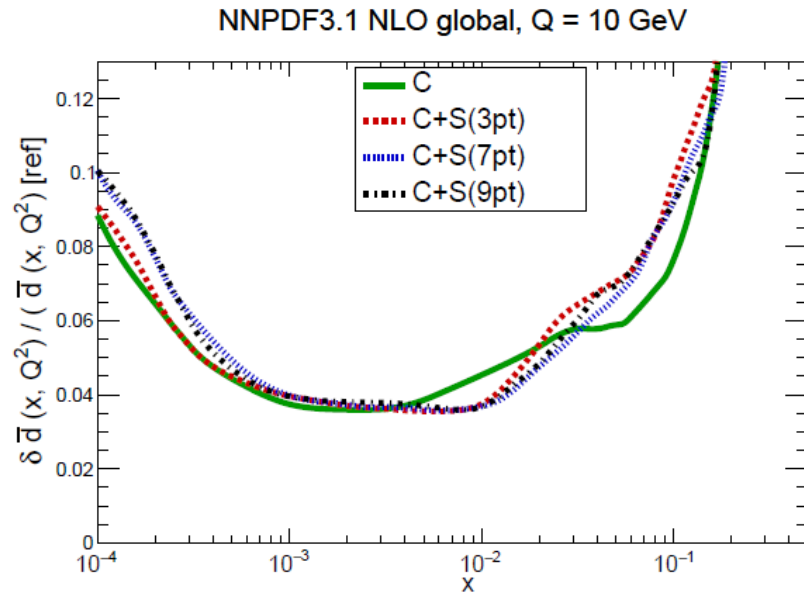
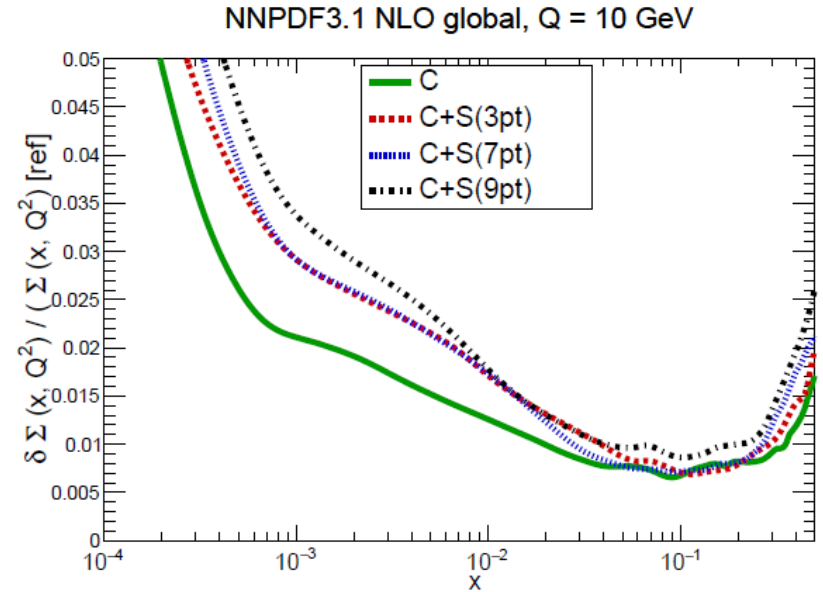
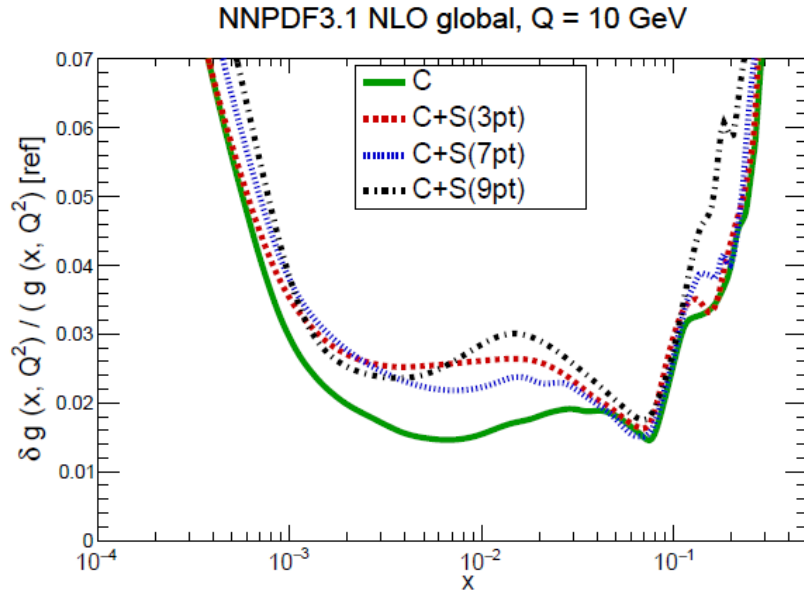
MHOU increase unc:
but only by $\sim 30 \%$
($\sim$ NNLO)

NLO PDFs with and without MHOU



MHOU moves NLO PDF towards NNLO

NLO PDFs with MHOU: uncertainties

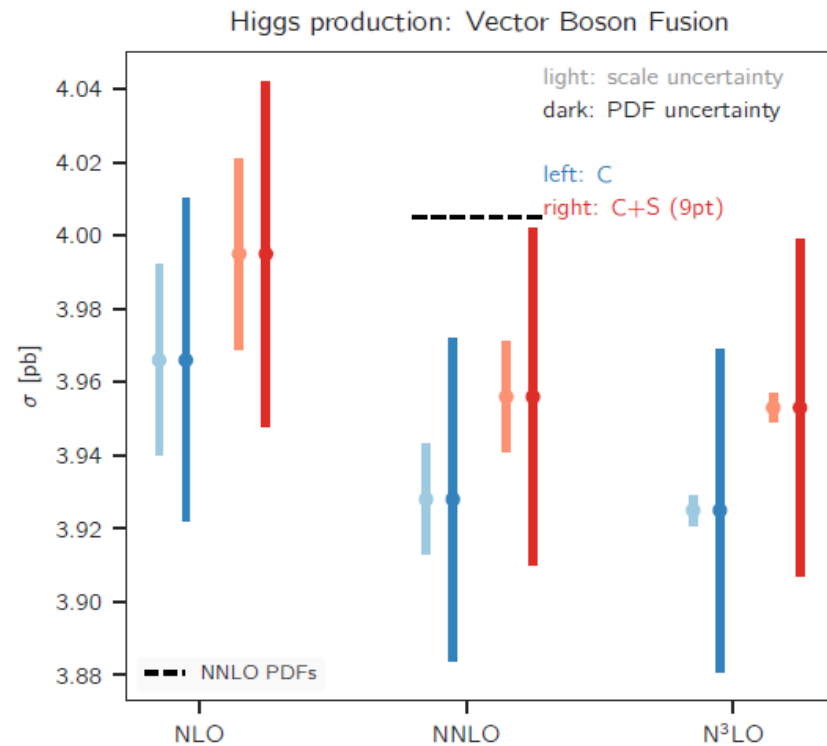
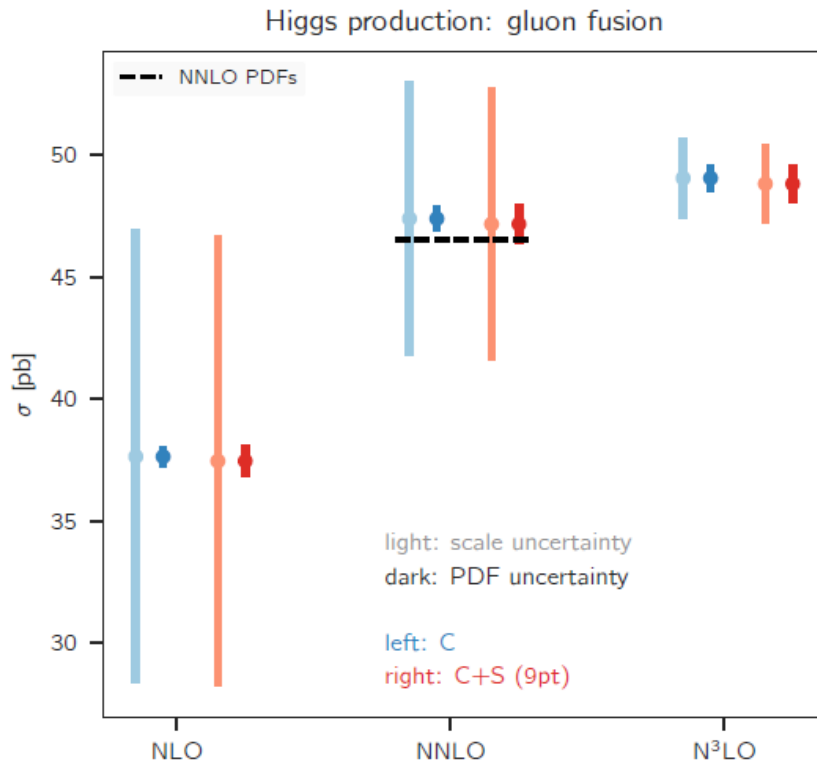


Increase in uncertainties $< 30\%$ in data region

Predictions

$$\sigma_i^0 = \underbrace{\langle \sigma_i[f] \rangle_f}_{\text{Av over reps}} \quad \text{Var}(\sigma)_{ij} = \underbrace{\langle (\sigma_i[f] - \sigma_i^0)(\sigma_j[f] - \sigma_j^0) \rangle_f}_{\text{PDF unc}} + \underbrace{S_{ij}^{9\text{pt}}}_{\text{MHOU}}$$

Add PDF unc and MHOU in quadrature



MHOU moves NLO prediction towards NNLO result

New PDF4LHC combination?

$$\text{PDF4LHC 15} = \text{NNPDF3.0} \oplus \text{CT14} \oplus \text{MMHT14}$$

$$\text{PDF4LHC 20} = \text{NNPDF3.1} \oplus \text{CT18} \oplus \text{'MHTXXX19' ???}$$

NNPDF3.1: Eur.Phys.J. C77 (2017) 663, on LHAPDF

CT18: (arXiv: 1908.11394 + ?), some PDFs now available

MHTXXX19: (arXiv: 1907.08147 - latest update of MMHT14), no new release

Towards NNPDF4.0

- new NN architecture, fitted preprocessing, hyperopt (Juan Cruz-Martinez, next talk)
- new faster fitting technology (N3fit) (closure tested)
- new evolution code (replaces Apfel, includes N3LO)
- **theory uncertainties at NLO and NNLO**
- photon PDF, and electroweak corrections to all datasets
- lots of new datasets (and new processes: prompt photon, dijets, single top)

Summer 2020?

New data for NNPDF4.0

Upgrades

ELECTROWEAK

- * ATLAS high-mass Drell-Yan double-differential distributions at 8 TeV
- * ATLAS W/Z total xsec at 13 TeV (81pb-1)
- * ATLAS triple-differential Z production at 8 TeV (20.2 fb-1)
- * ATLAS W+jets differential distributions at 8 TeV
- * CMS differential distributions in Z production at 13 TeV
- * LHCb W $\rightarrow$ e ν rapidity dist, 8 TeV (2 fb-1)
- * LHCb Z rapidity distribution, 13 TeV
- * CMS W pt distribution, 8 TeV (18.4 fb-1)
- * CMS Z+charm at 8 TeV, 19.7 fb-1
- * CMS W+charm differential distributions at 13 TeV

- * ATLAS W/Z production, 7 TeV (4.6 fb-1) $\Rightarrow$ added the off-peak and forward Z prod bins
- * Final combination of charm and beauty str fns from HERA (Runs I+II): replaces HERA-I charm comb and H1, ZEUS structure functions

JETS and PHOTONS

- * ATLAS isolated photon production 8 TeV, 20 fb-1
- * ATLAS isolated photon production, 13 TeV, 3.2 fb-1
- * ATLAS dijet cross-sections at 7 TeV
- * ATLAS inclusive jet cross-sections at 8 TeV from the 2012 dataset
- * CMS dijet cross-sections at 7 TeV
- * CMS inclusive jet production at 8 TeV, 19.6 fb-1
- * CMS triple differential dijet cross-sections at 8 TeV (19.6 fb-1)
- * CMS double-differential dijet distributions at 5 TeV
- * Inclusive jet and di-jet production in neutral-current DIS from H1 and ZEUS (HERA DIS jets)

prompt photons (at NNLO)

Dijets (at NNLO)

DIS jets (at NNLO)

TOP QUARK

- * CMS total xsec of top-pair production at 5.02 TeV, 27.4 pb-1
- * CMS double differential distributions top-quark production 8 TeV, 19.7 fb-1
- * CMS single differential distributions in top-pair production (lepton+jets) at 13 TeV, L=35.8 fb-1(2016)
- * CMS single differential distributions in top-pair production (dilepton) at 13 TeV, 35.8 fb-1(2016)
- * CMS single top t-channel total cross section ratio at 7 TeV
- * CMS single top t-channel total cross section ratio at 8 TeV
- * CMS single top t-channel total cross section ratio at 13 TeV
- * ATLAS single top t-channel total cross section ratio and diff. distributions at 7 TeV
- * ATLAS single top t-channel total cross section ratio at 8 TeV
- * ATLAS single top t-channel total cross section ratio at 13 TeV

single top (at NNLO)

Cutoff date for new data:
end of 2019

Summary & Outlook

- PDFs with MHOU using scale variation
- Modest increase in PDF unc at NLO
- Increases precision by resolving tensions
- Global fit NNLO + MHOU to follow
- NNPDF4.0: well on its way

Bayesian Theory Uncertainties

data D theory T

‘truth’

 $\mathcal{T}$

- Gaussianity:

$$P(D|\mathcal{T}) \propto \exp \left(-\frac{1}{2}(\mathcal{T}_i - D_i)C_{ij}^{-1}(\mathcal{T}_j - D_j) \right),$$

 C_{ij} : exp covmat

$$P(T|\mathcal{T}) \propto \exp \left(-\frac{1}{2}(\mathcal{T}_i - T_i)S_{ij}^{-1}(\mathcal{T}_j - T_j) \right),$$

 S_{ij} : th covmat

- Bayes Thm:

$$P(\mathcal{T}|DT)P(D|T) = P(D|\mathcal{T}T)P(\mathcal{T}|T).$$

- Independence:

$$P(D|\mathcal{T}T) = P(D|\mathcal{T}).$$

‘theorists don’t bias experiments’

Marginalise on $\mathcal{T}$ (we can never know the ‘truth’):

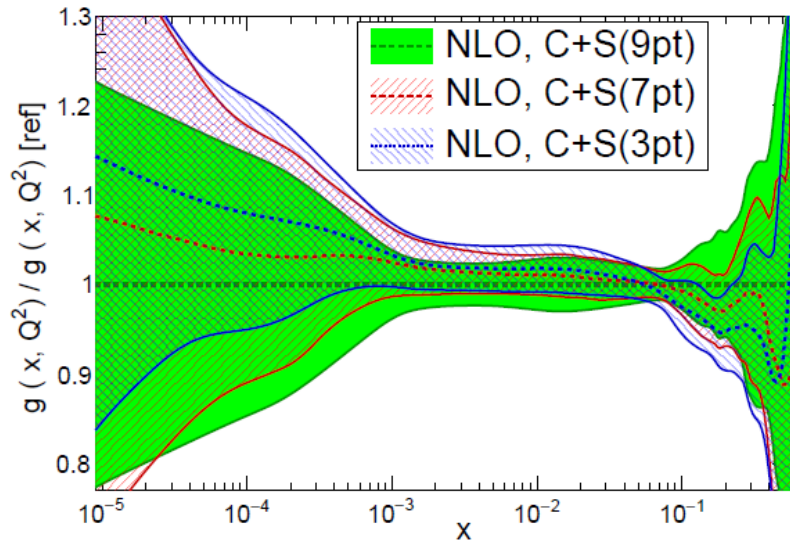
$$P(T|D) \propto \exp \left(-\frac{1}{2}(D_i - T_i)(C + S)_{ij}^{-1}(D_j - T_j) \right).$$

Result: $C_{ij} \rightarrow C_{ij} + S_{ij}$

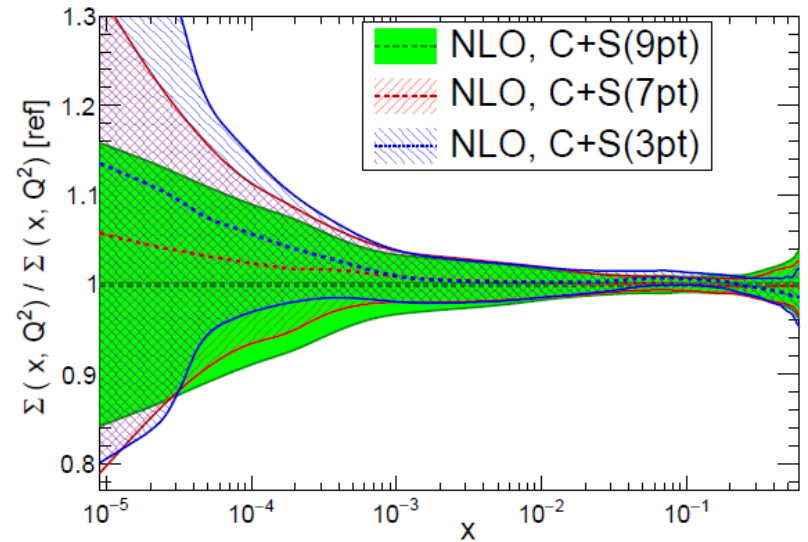
theory uncertainties $\leftrightarrow$ exp systematics: add in quadrature because uncorrelated

NLO PDFs with MHOU with various prescriptions

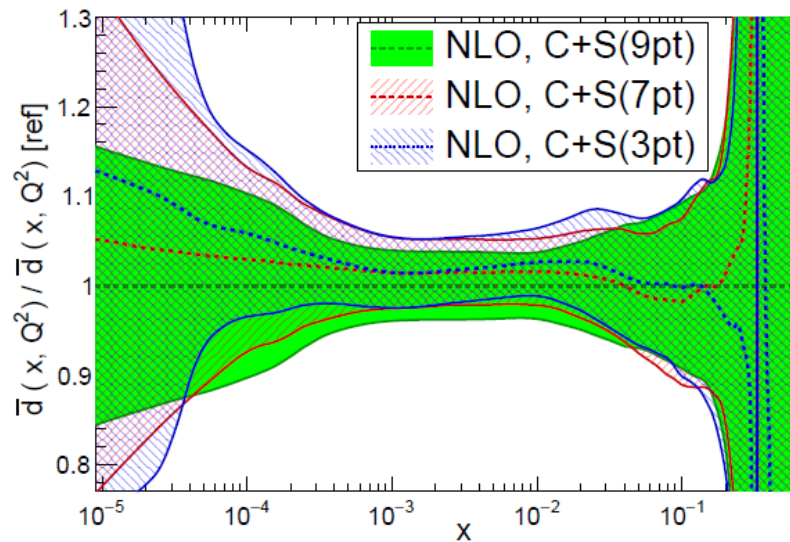
NNPDF3.1 Global, $Q = 10$ GeV



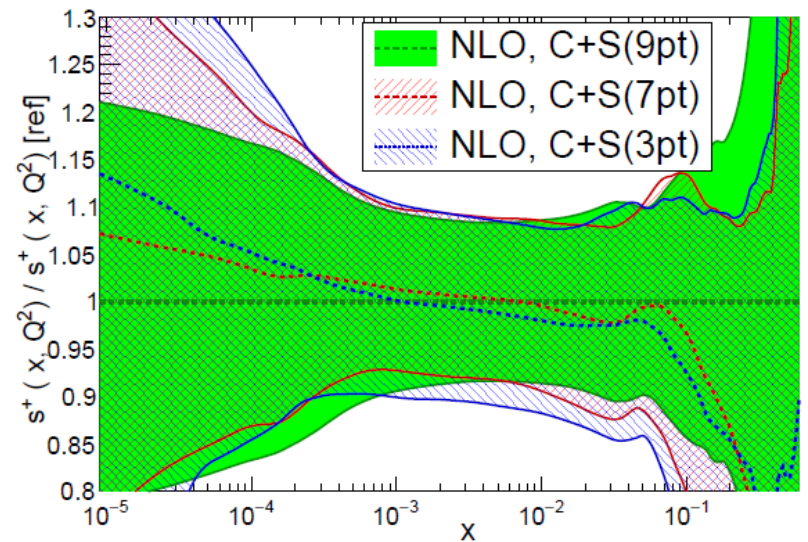
NNPDF3.1 Global, $Q = 10$ GeV



NNPDF3.1 Global, $Q = 10$ GeV

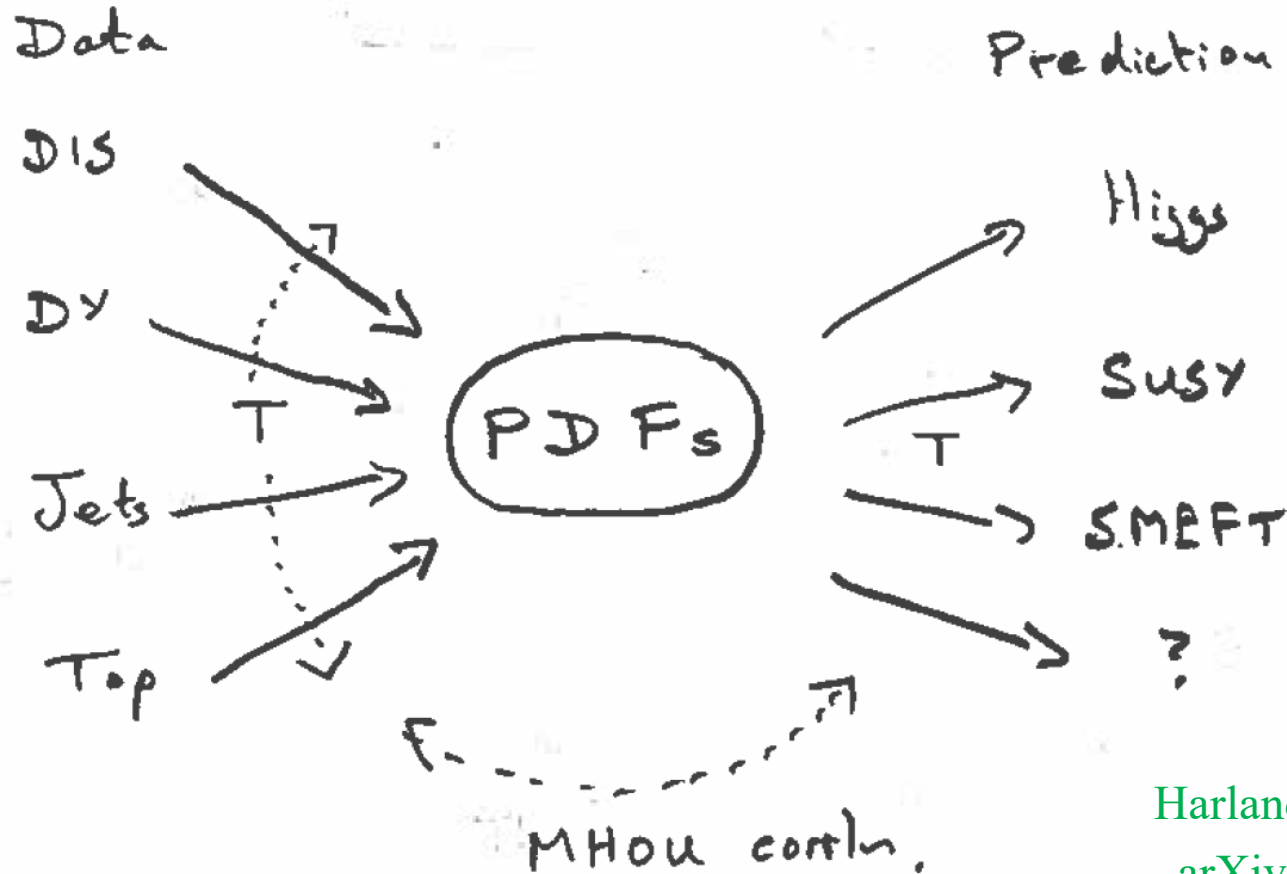


NNPDF3.1 Global, $Q = 10$ GeV



Stable in data region

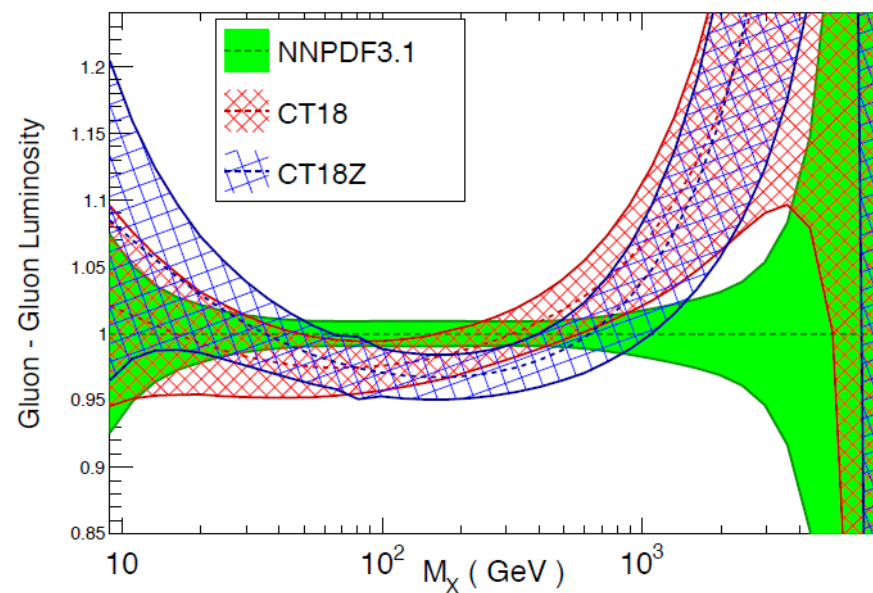
Missing MHOU Correlations



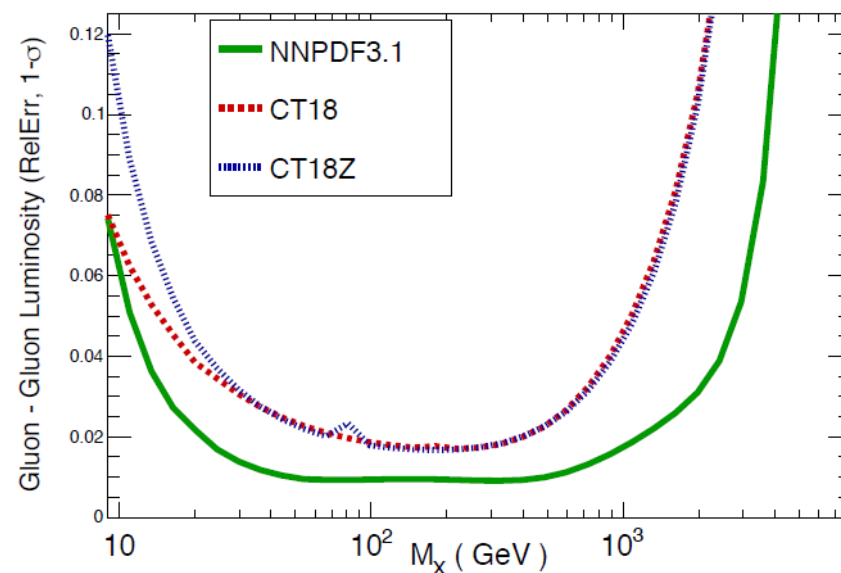
Harland-Lang Thorne
arXiv: 1811.08434

Including correlations between MHOU in PDFs and MHOU in predicted xsec may reduce overall MHOU. But requires extra deliverable beyond PDFs.

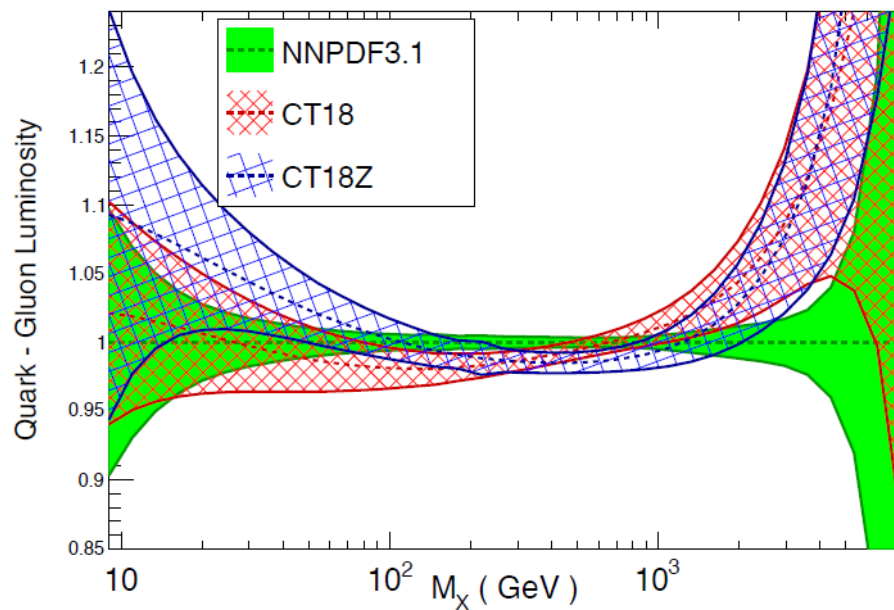
LHC 13 TeV, NNLO



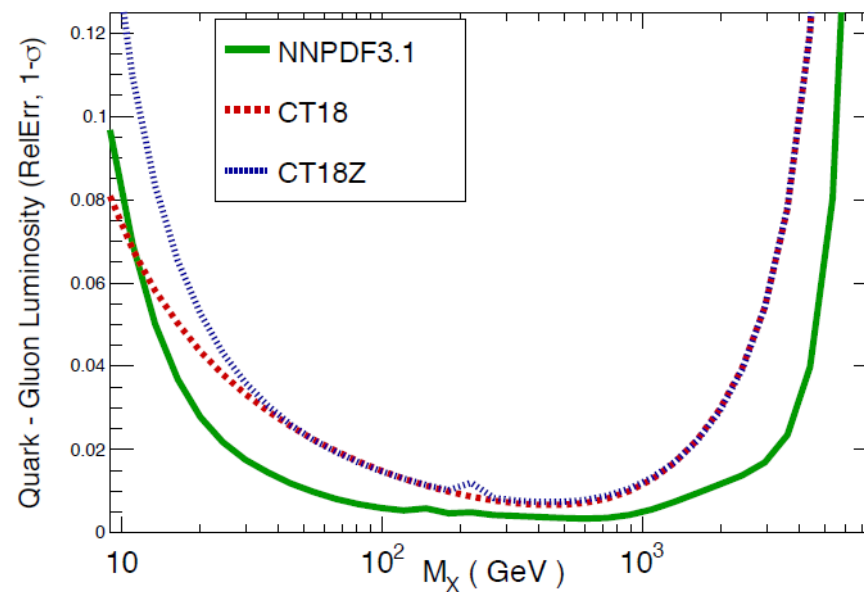
LHC 13 TeV, NNLO



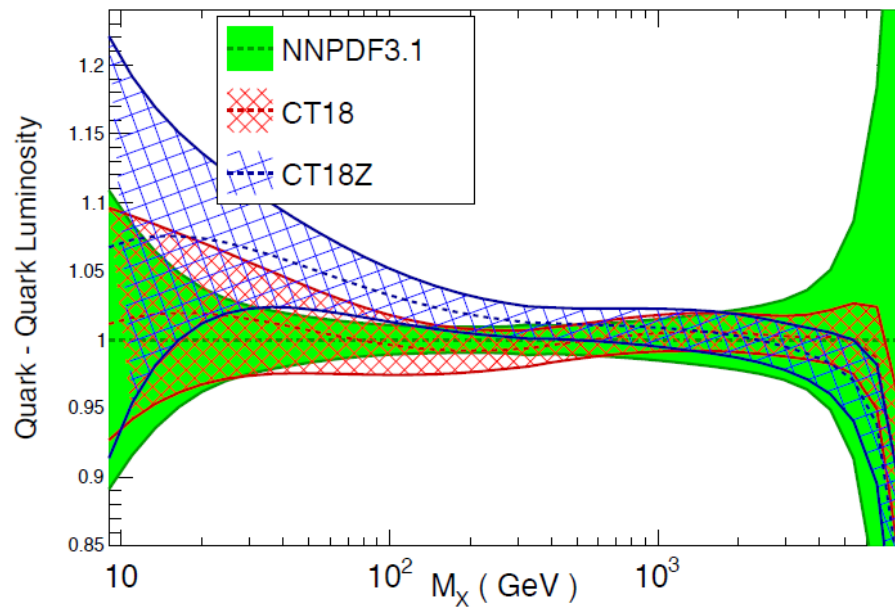
LHC 13 TeV, NNLO



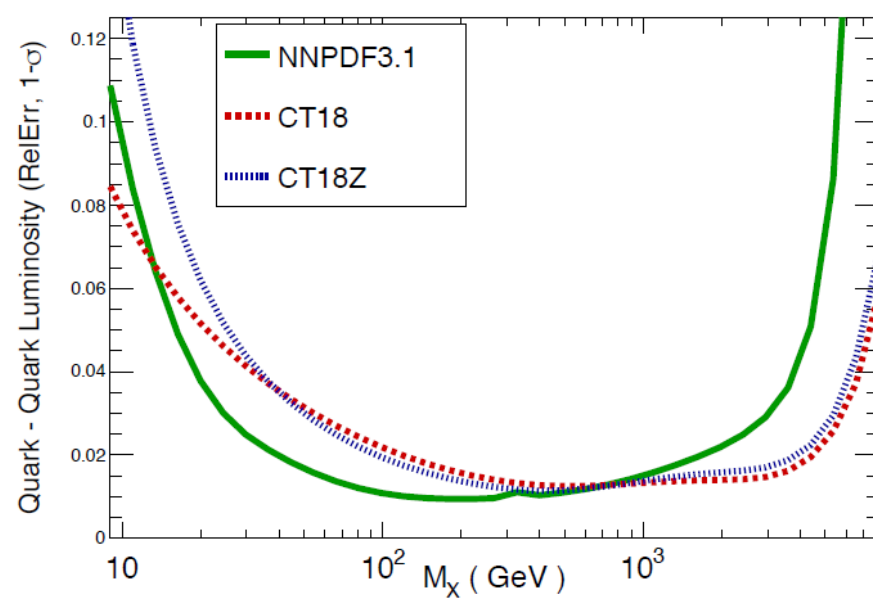
LHC 13 TeV, NNLO



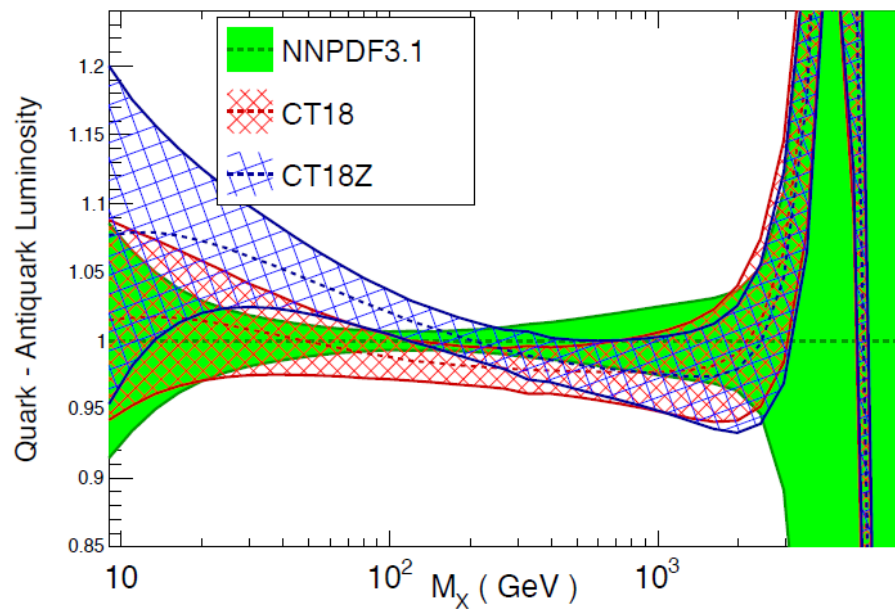
LHC 13 TeV, NNLO



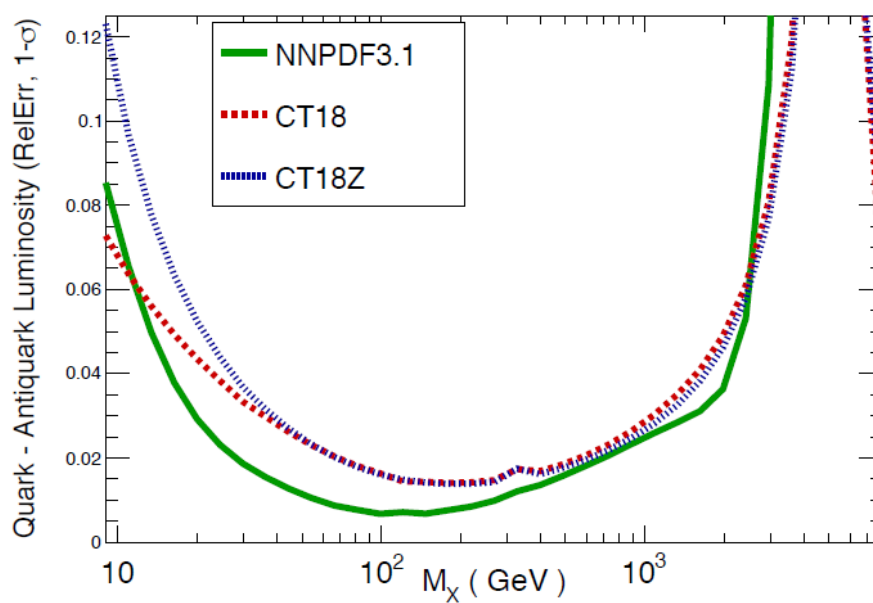
LHC 13 TeV, NNLO



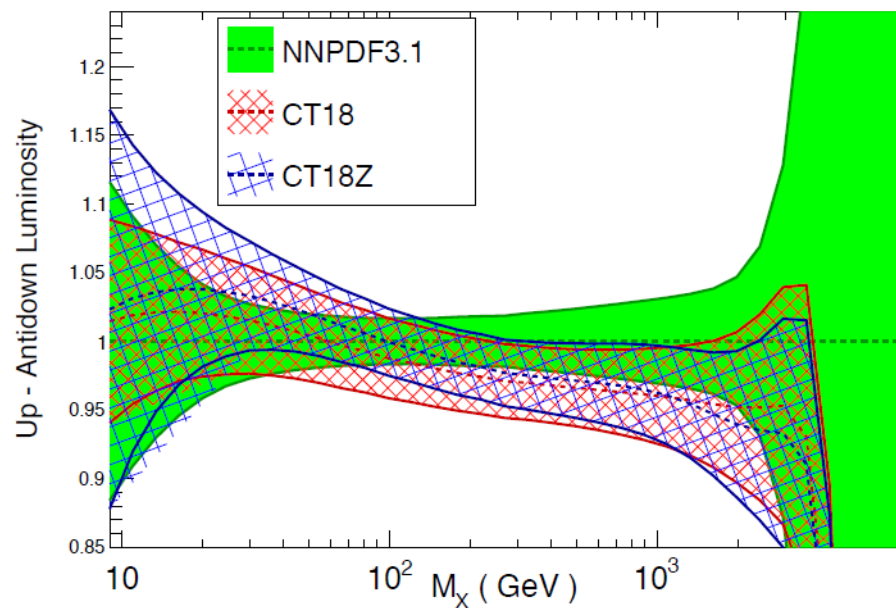
LHC 13 TeV, NNLO



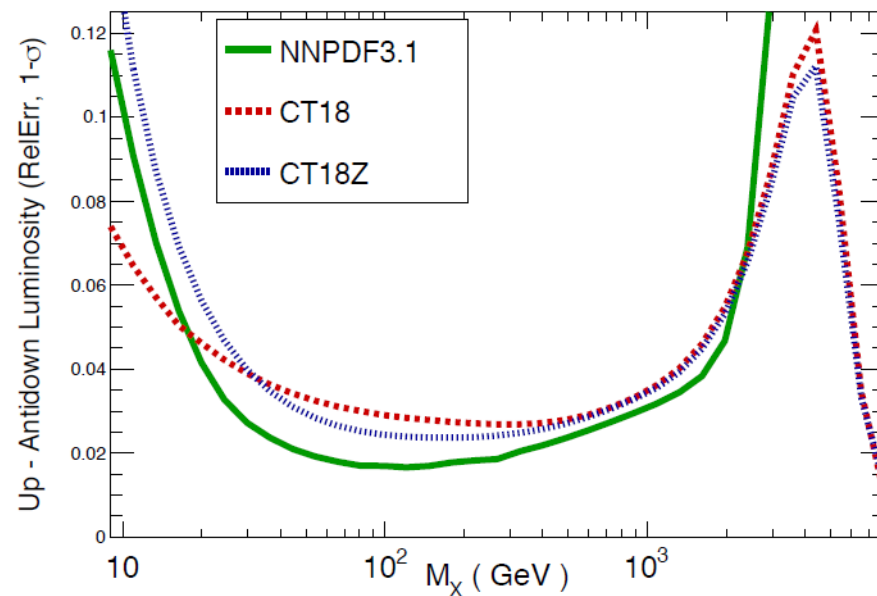
LHC 13 TeV, NNLO



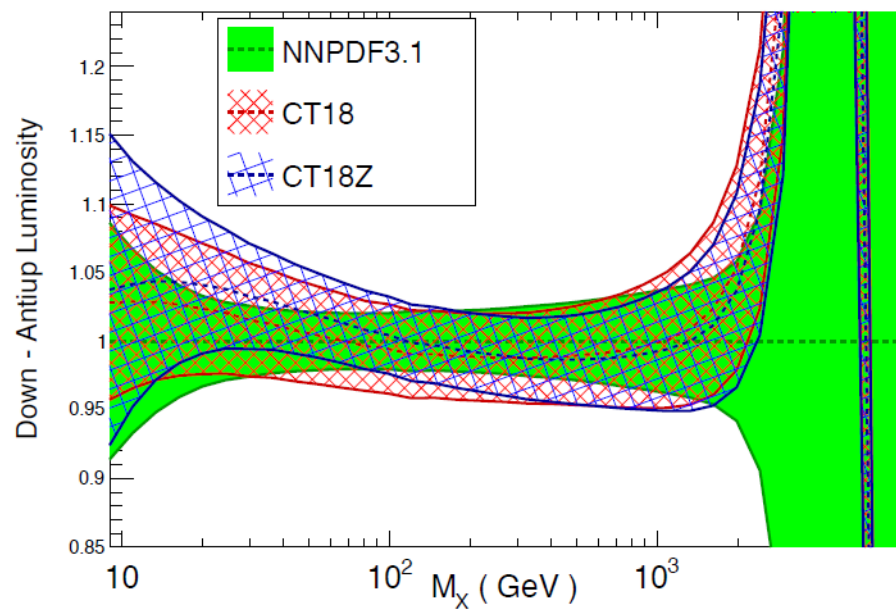
LHC 13 TeV, NNLO



LHC 13 TeV, NNLO



LHC 13 TeV, NNLO



LHC 13 TeV, NNLO

