Prospects for theory calculations for high-precision collider physics

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What do the next 10 years have in store?

Max-Planck-Institut für Physik

European Research Council
Established by the European Commission

How do theorists keep up with precision of LHC experiments?

Image credit: D. Dominguez
'The two-loop explosion', G. Zanderighi, CERN courier

Outline

- Motivation
- Examples of state-of-the-art methods and results
- Outlook

We are entering an era of precision LHC measurements

LHC timeline data collected

2011	2012	2013	2014	2015	2016	2017	2018	2019	2020	2021	2022	2023	2024	2025	2026	…	2037
Run 1		Long Stop 1		Run 2				Long Stop 2		Run 3				Long Stop 3		HL-Run	
7,8 TeV, 30 fb^{-1}				13 TeV, 150 fb^{-1}						13.5-14 TeV 300 fb^{-1}						14 TeV 3000 fb^{-1}	

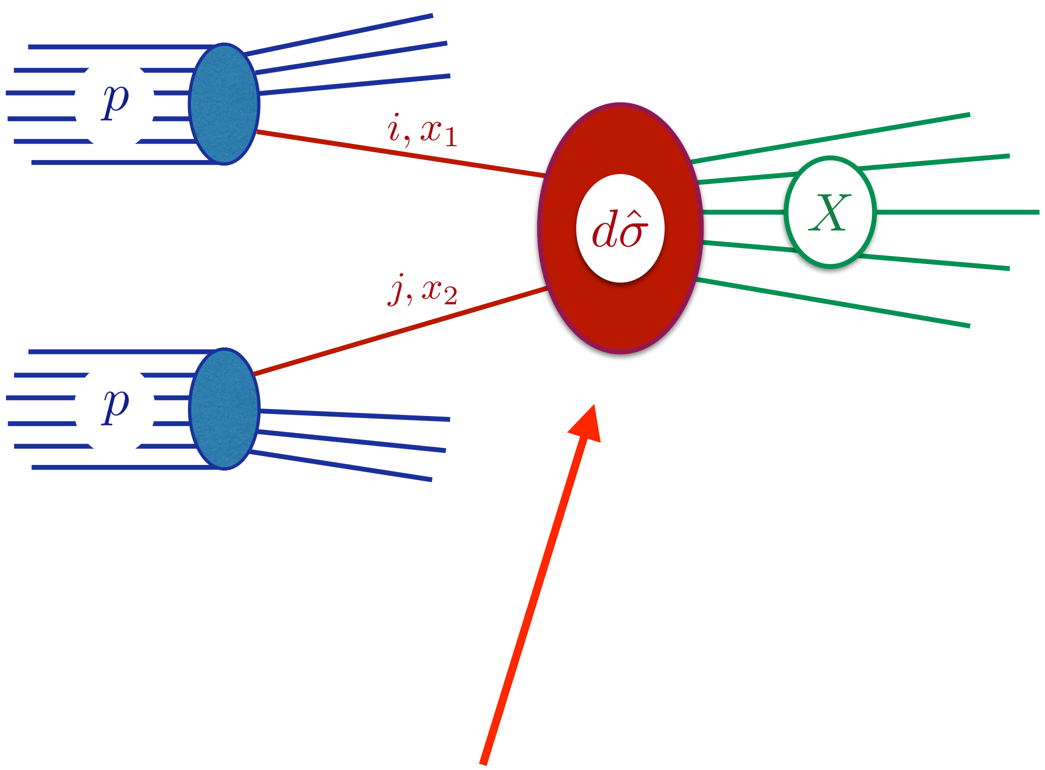
- Precision measurements of fundamental parameters of Nature
- Guide to new physics beyond Standard Model

New effort to produce theoretical predictions at the level of precision of upcoming data is needed.

Proton collisions and factorization

Parton Distribution Functions

- non-perturbative
- describe structure of proton



Cross-section of partons (e.g. quarks and gluons)

- computable in perturbation theory
- methods for scattering amplitudes

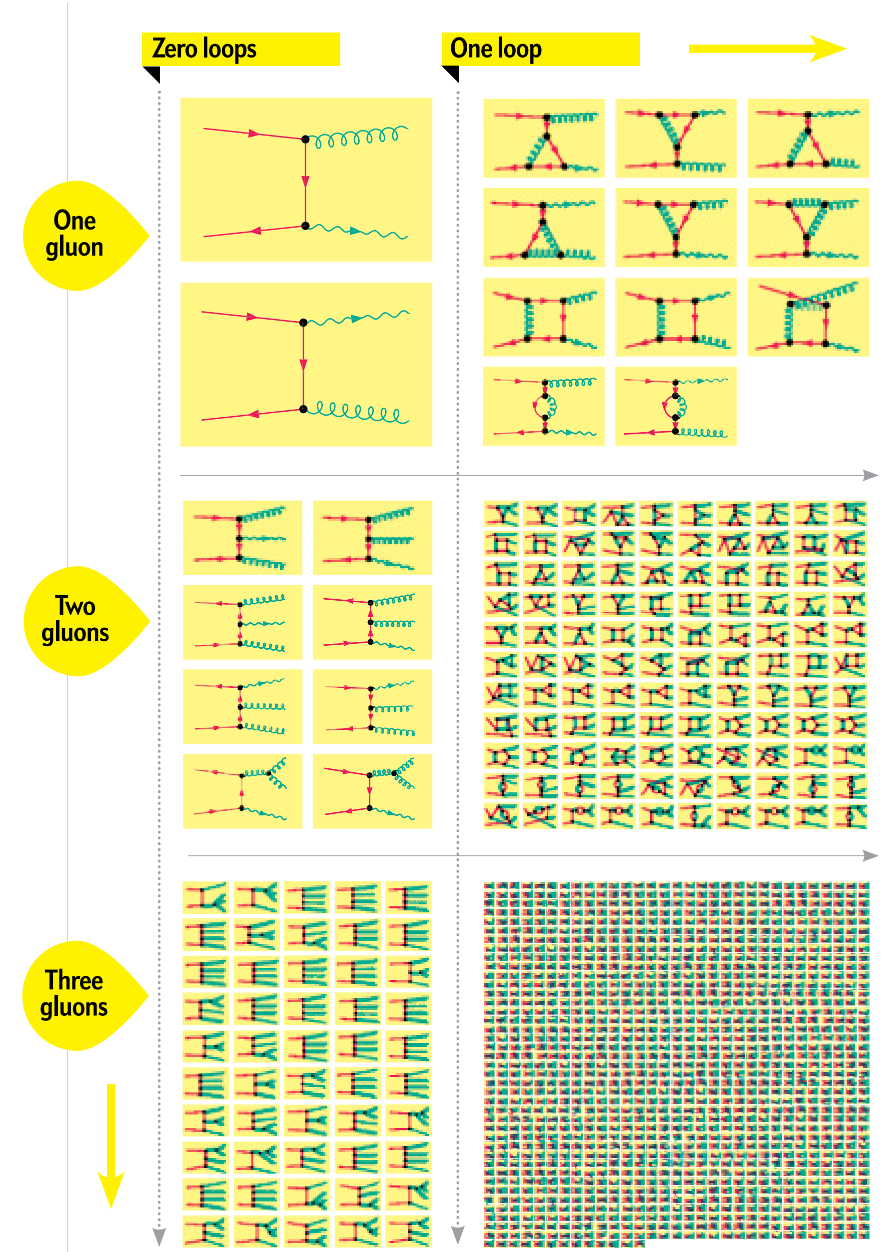
Scattering amplitudes

Feynman diagrams

- each diagram translates into an analytic formula
- perturbative expansion ~ number of loops

Cross sections related to square of amplitudes

$$\sigma \sim |\mathcal{A}|^2$$



Scientific American (2012)
Bern, Dixon, Kosower

For Many QCD Processes, Next-to-Leading Order is Insufficient

E.g. strong coupling from 3-jet/2-jet ratio:

[CMS Collaboration, Eur.Phys.J. C73 (2013) no.10, 2604]

$$\alpha_S(M_Z) = 0.1148 \pm 0.0014 \pm 0.0018 \pm 0.0050$$

(exp) (PDF) (theory)

Large theoretical uncertainty!

Next-to-Next-to-Leading Order (NNLO), i.e. two loop theory predictions needed.

Challenges for future perturbative calculations

Dramatic recent progress, 'NNLO revolution'

⟶ Most 2 to 2 standard model processes available

e.g. top pair, dijet, V+V', production

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Many processes with 3 and more final states needed

(cf. Les Houches theory wishlist)

Bigger computers help, but novel ideas are better!

Challenges force us to understand better QFT!

Hidden Structures in Quantum Field Theory

The QCD Lagrangian is deceptively (?) simple:

$$\mathcal{L}_{QCD} = \bar{\Psi}(i\gamma_\mu \mathcal{D}^\mu - m)\Psi - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}\,.$$

…yet, working out cross sections is very complicated

Final results are often simple: e.g. closed formulas for scattering of n gluons at tree level

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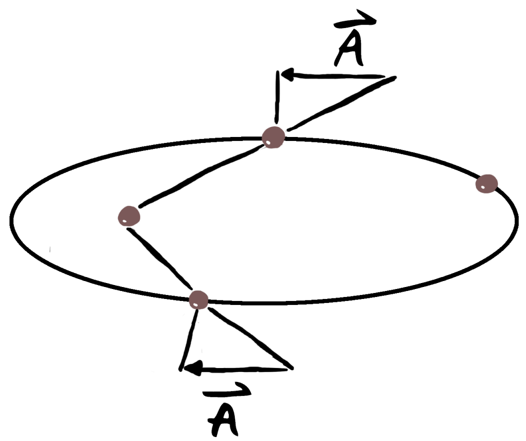
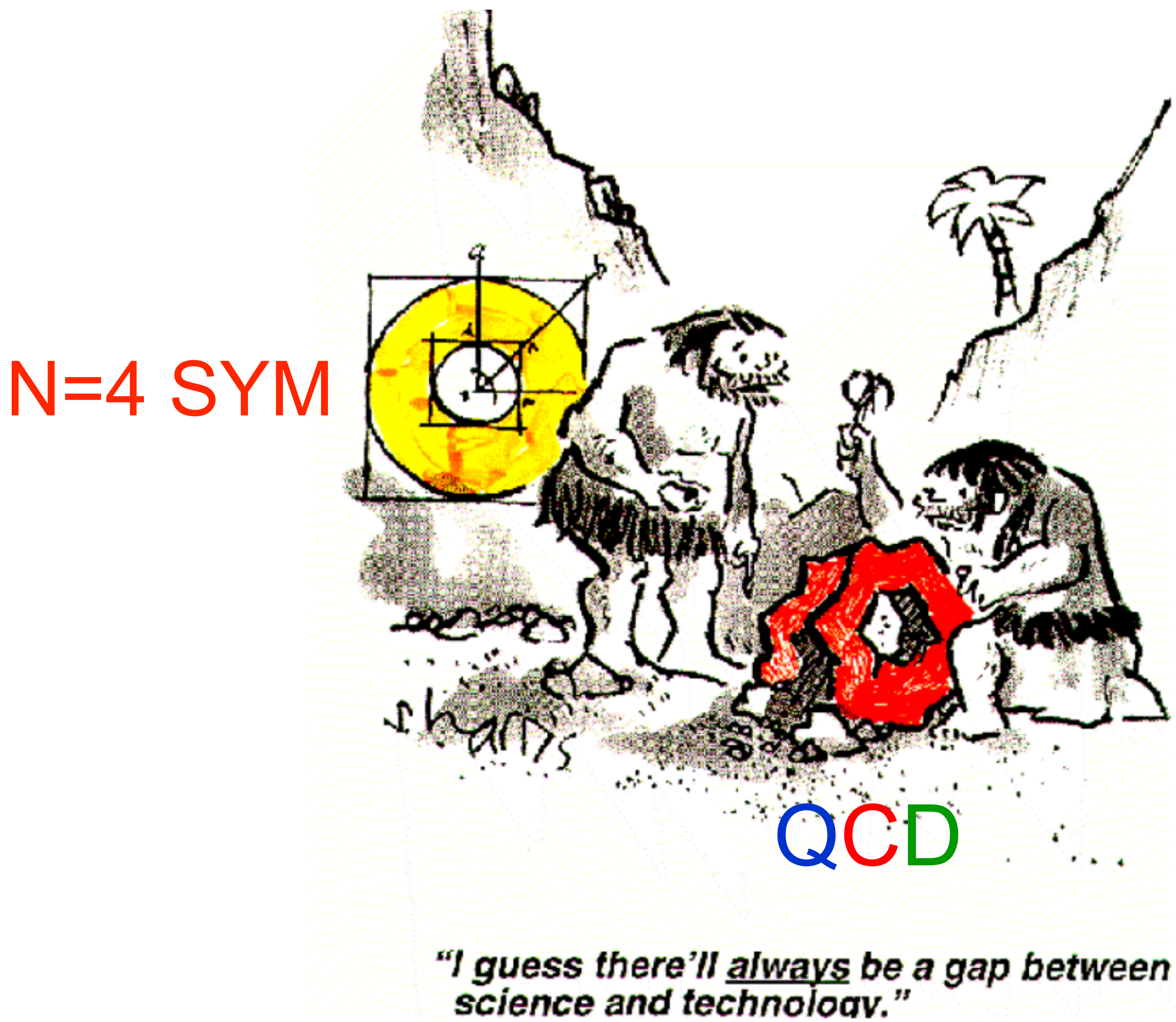
Symmetry explains simplicity:



- Yang-Mills theory has hidden Laplace-Runge-Lenz symmetry!
- The same symmetry is responsible for stability of planetary orbits, and simplicity of spectrum of hydrogen!

Ideal laboratory: 'N=4 super Yang-Mills is the Hydrogen atom of QFT'

Many new methods that inspired QCD advances



[picture from Lance Dixon, talk at EPS HEP11]

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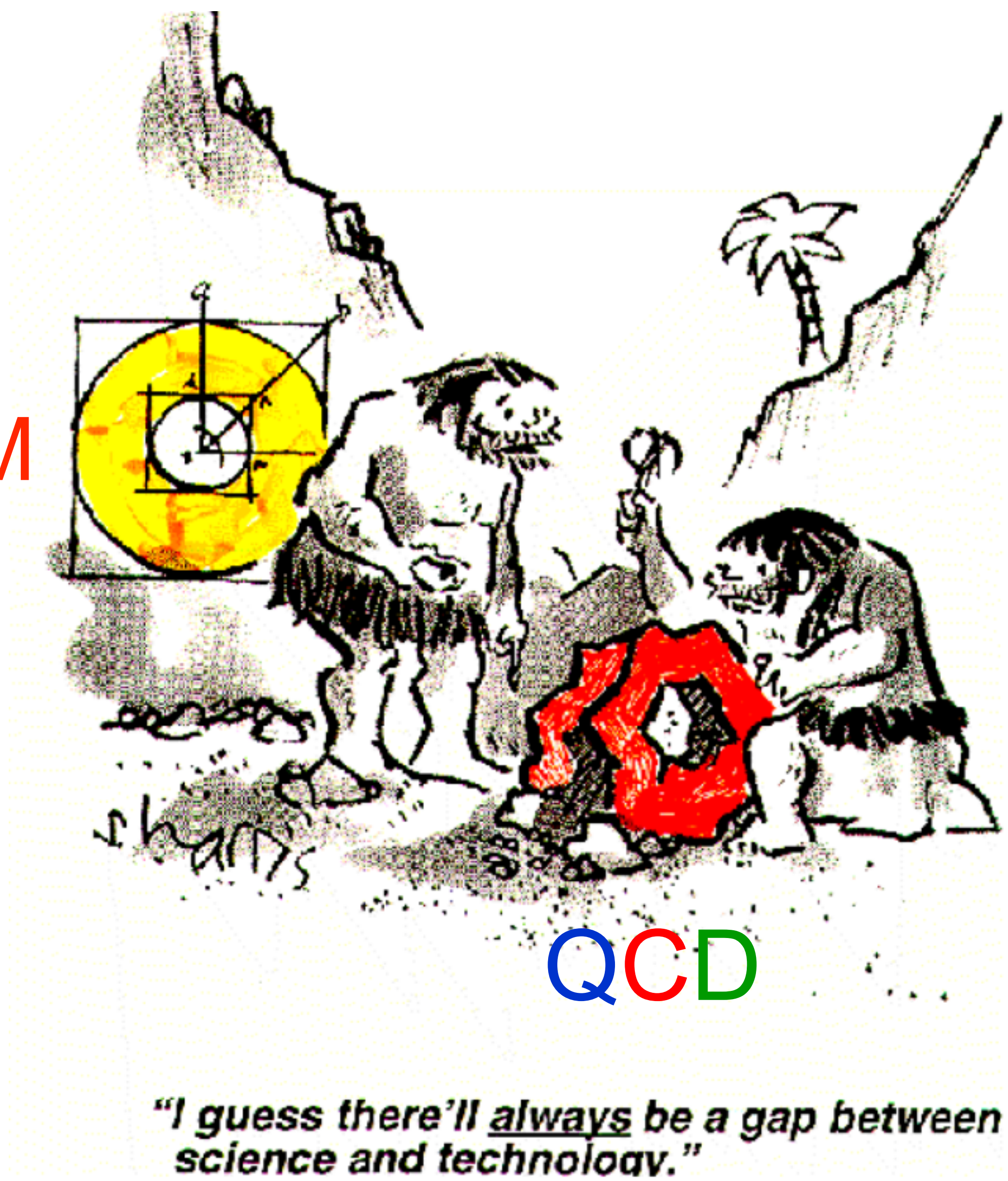
N=4 SYM



[picture from Lance Dixon, talk at EPS HEP11]

Twistor space

generalized unitarity

Structure of infrared divergences

Leading singularities

Amplituhedron

on-shell recursion relations

BCJ relations

Symbol and coproduct

Scattering equations

Canonical differential equations

Numerical vs analytical methods

Numerical

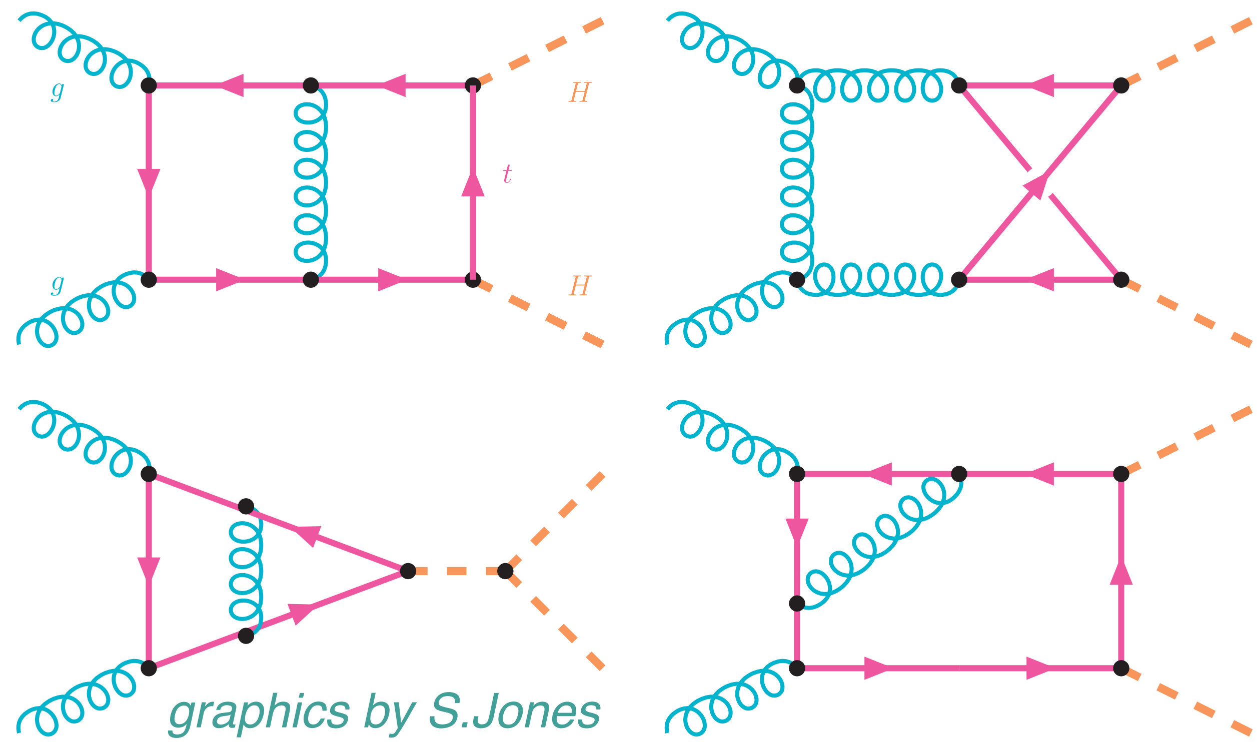
+ Very versatile, applies to large classes of integrals
- Cancellations only numerically
- Slow, large computing resources required

Analytical

- Each Feynman integral is different, might need new methods
+ Exact calculations, analytic limits
+ Fast evaluation

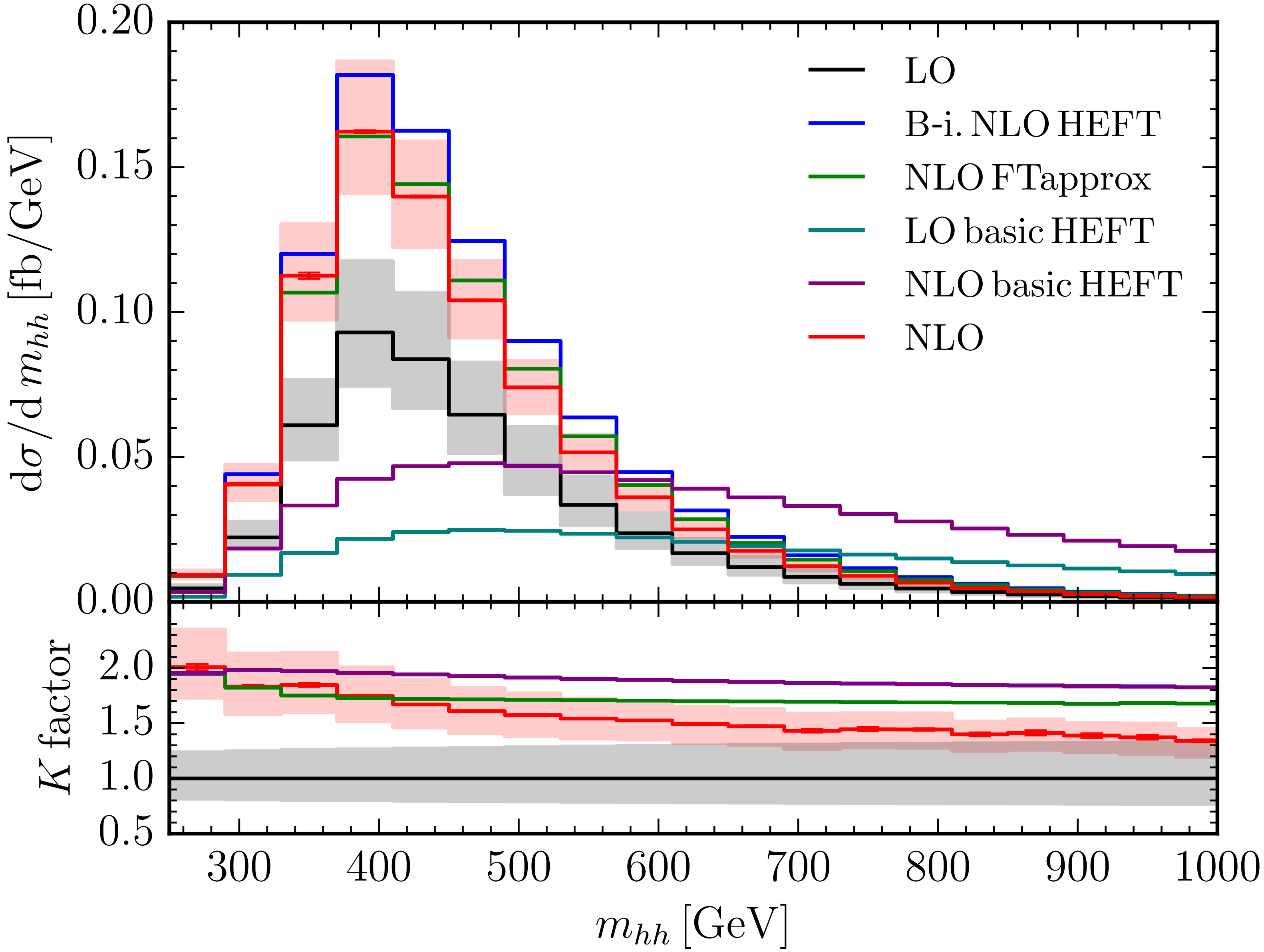
Example: Two-loop calculation of HH production with full top quark dependence

Feynman integrals evaluated numerically with *SecDec*



[*SecDec*: Borowka, Heinrich, Kerner, Schlenk, Zirke '15; +Jahn '17,'19]

Invariant mass of Higgs boson pair



New: full NLO result

[Borowka, Greiner, Heinrich, Jones, Kerner, Schlenk, Schuber, Zirke '16]

40 - 50% difference

Heavy top approximation is not good beyond top quark pair production threshold

Challenges for multi-particle calculations

Algebraic complexity

- many different scales
- huge algebraic expressions O(GigaByte), very hard to handle for computer algebra

Complexity of special functions

- multi-dimensional Feynman integrals
- complicated special functions of many variables

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Finite field and rational reconstruction techniques

[Schabinger, von Manteuffel '15; Peraro '16,'19]

Complexity of special functions

- multi-dimensional Feynman integrals
- Multi varied functions of many variables

Better understanding of how to perform integration.

Special functions from canonical differential equations. [Henn, 2013]

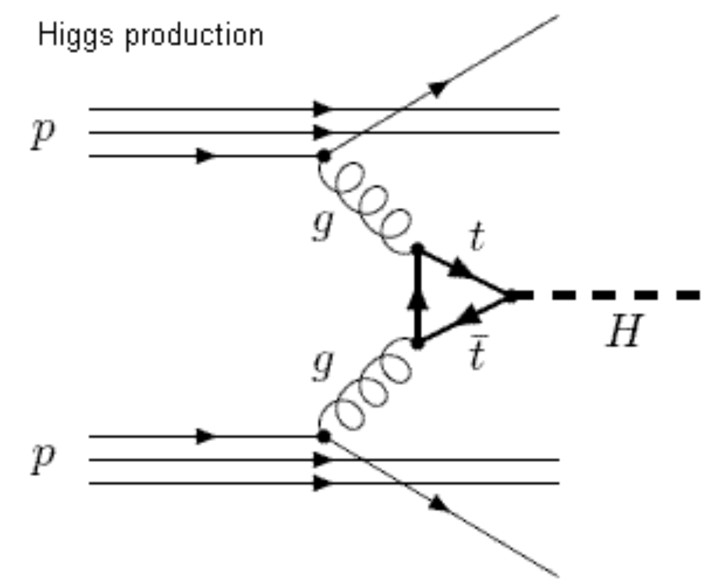
Special functions appearing in Feynman integrals

One-loop example: Higgs through gluon fusion

$$\int \frac{d^4k}{i\pi^2} \frac{1}{(m_t^2 - k^2)(m_t^2 - (k+p_1)^2)(m_t^2 - (k-p_2)^2)} =$$

$$= -\frac{1}{2s} \log^2 \left(\frac{\sqrt{1 - 4m_t^2/s} - 1}{\sqrt{1 - 4m_t^2/s} + 1} \right)$$

$$s = (p_1 + p_2)^2$$



Multivalued function; two-particle threshold

More generally: Laurant series in D=4-2 ϵ

Special functions appearing in Feynman integrals

One loop: logarithms and dilogarithm sufficient

$$\log z = \int_1^z \frac{dt}{t} \qquad \mathrm{Li}_2(z) = -\int_0^z \frac{dt}{t} \log(1-t)$$

Natural generalization: 'Hyperlogarithms' cover large classes of multi-loop Feynman integrals

$$G_{a_1,\dots a_n}(z) = \int_0^z \frac{dt}{t-a_1} G_{a_2,\dots,a_n}(t)$$

At two loops, also new functions related to elliptic integrals

$$K(z) = \int_0^1 \frac{dt}{\sqrt{(1-t^2)(1-zt^2)}}$$

Multiple elliptic polylogarithms

Canonical differential equation method

Feynman integrals satisfy n-th order partial differential equations (DE)

Equivalently, system of 1st order DE

Typically complicated

Idea: (rational) loop integrand contains key information on special functions appearing after integration

Special functions defined from 'canonical' DE

Very simple

Integrals involving multiple elliptic polylogarithms

Appear e.g. in the following processes

Top quark pair production

Higgs plus jet production, finite top quark mass

Typical for mixed QCD-electroweak corrections

Energy-energy correlators at NNLO

…

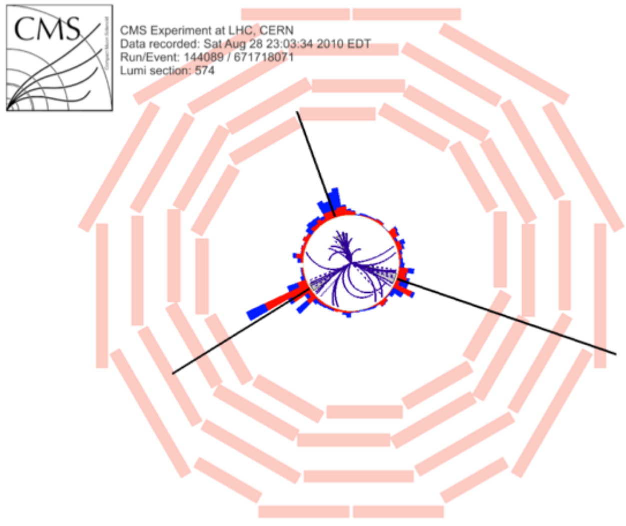
[Bloch, Vanhove; Duhr et al; Adams, Bogner, Chaubey, Schweitzer, Weinzierl; Brödel, Duhr, Dulat, Marzucca, Tancredi, Penante; Hiddings, Maestri, Moriello …
(2013-2019)]

Lots of progress, yet a lot still to be done!

Example: Five-particle processes

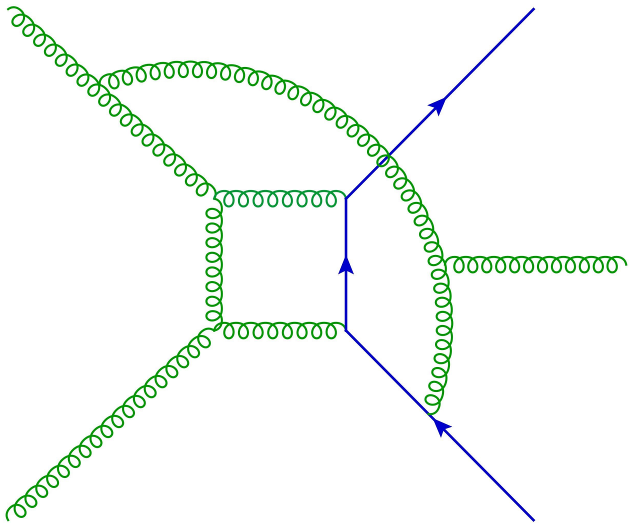
Multi-jet processes important for phenomenology

- Determination of strong coupling constant α_s
- tests of Standard Model
- search for new physics



Major challenge:

virtual two-loop amplitudes



Dramatic recent progress in two-loop five-particle amplitudes

We know all integrals needed for 5-parton scattering!

[Gehrmann, Henn, Lo Presti (2015); Papadoloulos, Tommasini, Wever (2016), Gehrmann, Henn, Wasser, Zhang, Zoia (2018)]

Two-loop results for 2 to 3 scattering within reach

- All QCD amplitudes known analytically in the *planar* limit
  [Badger, Brønnum-Hansen, Hartanto, Peraro '18; Abreu, Dormans, Febres Cordero, Ita, Page '18; Abreu, Dormans, Febres Cordero, Ita, Page, Sotnikov '19]
- Full-color amplitude for special helicity configuration
  [Badger, Chicherin, Gehrmann, Heinrich, Henn, Peraro, Wasser, Zhang, Zoia '19]
- Fist results for W+2 jets two-loop amplitude
  [Badger, Brønnum-Hansen, Hartanto, Peraro, '19]

Conclusion

- We are entering an era of precision physics, thanks to future LHC data

- Challenges for theorists: both conceptual and practical advances needed

- Opportunity to understand better quantum field theory