Quantum Gravity

No Strings Attached!

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Why Gravity is Quantum

- Semi-classical approach does not make sense

$$G_{\mu\nu} \neq 8\pi G \langle \hat{T}_{\mu\nu} \rangle$$

- Need a UV complete theory
  - General relativity valid @ large distances / low energy
- Resolution of singularities in GR
  - Black holes
  - Big bang

Why No Strings?

- String theory generally predicts
  - new particles
  - extra dimensions
  - more symmetries

  } Unobserved!
- No cosmological constant problem (Λ is just a parameter)
- No de Sitter in the swampland?

Quantum Gravity in a Nutshell

- Quantisation: Classical $\to$ Quantum
  - Canonical, Deformation, Geometric, Path integral, ...
- This talk: Path integral quantisation

$$Z = \int [\mathrm{d}g][\mathrm{d}\phi] \mathrm{e}^{\mathrm{i}(S_{\text{gravity}}[g] + S_{\text{matter}}[\phi, g])}$$

- Z formal / undefined
  - Diffeomorphisms $g \to L_X g$ are gauge symmetry
  - $S_{\text{gravity}} = S_{\text{Einstein}-\text{Hilbert}}$ perturbatively non-renormalisable

“Observables” in Quantum Gravity

- Coordinates have no meaning
- Need gauge-invariant quantities

2-point function for scalar fields

$$\langle(\phi\phi)(R)\rangle = \int [\mathrm{d}g][\mathrm{d}\phi]\mathrm{e}^{\mathrm{i}S[g,\phi]} \times \int \mathrm{d}^D x \int \mathrm{d}^D y\, \phi(x)\phi(y)\delta\left(R - d_{xy}[g]\right)$$

$d_{xy}[g]$ ... geodesic distance

“Observables” in Quantum Gravity

Hausdorff / Fractal dimension

Scaling of balls of size R

$$d_{\mathrm{H}}(R) = \frac{\mathrm{d}}{\mathrm{d}\log R}\log V(R)$$

$$V(R) = \int_{\|x\|\leq R} \mathrm{d}^D x \sqrt{g}$$

In $\mathbb{R}^D$: $V(R) = \frac{\pi^{D/2}}{\Gamma\left(\frac{D}{2}+1\right)} R^D \Rightarrow d_{\mathrm{H}}(R) = D$

“Observables” in Quantum Gravity

Spectral dimension

Scaling of length of random walk

$$d_S(R) = -2 \frac{\mathrm{d}}{\mathrm{d}\log\sigma} \log P(\sigma)$$

$P(\sigma)$ . . . Return probability as function of “diffusion time” σ

$$P(\sigma) \propto \langle \| x_f - x_i \| (\sigma) \rangle^{-1}$$

In $\mathbb{R}^D$: $\langle \| x_f - x_i(\sigma) \| \rangle \propto \sigma^{D/2} \Rightarrow d_S(R) = D$

Asymptotic Safety

Gravity as a QFT

- Ultra-conservative approach: "Quantum Einstein Gravity"
  - Field: Metric g
  - Symmetries: Diffeomorphisms $g \to L_X g$
- Einstein–Hilbert action for GR

$$S_{\text{EH}} = \frac{1}{16\pi G} \int \mathrm{d}^d x \sqrt{|g|}(R - 2\Lambda)$$

- Well-known that GR is perturbatively non-renormalisable
  - Mass dimension of G is -2
  - Need infinite number of counterterms
- Stelle gravity is perturbatively renormalisable

$$S_{\text{Stelle}}[g] = \int \mathrm{d}^d x \sqrt{|g|}(\alpha R + \beta R^2 + \gamma R_{\mu\nu} R^{\mu\nu})$$

  - But not unitary... (Higher-derivative theory)

Average Effective Action

- Wilsonian point of view: Couplings depend on the (energy) scale of interest
- Renormalisation group flow describes how couplings change with energy scale k
- Effective action at scale k (Infinitely many terms!)

$$\Gamma_k[\phi] = \sum_{a=1}^{\infty} g_a(k) O_a[\phi]$$

  - $g_a(k)$ ... parameters of theory
  - $O_a[\phi]$ ... basis functionals that respect symmetry

Asymptotic Safety

- RG flow obtained by solving

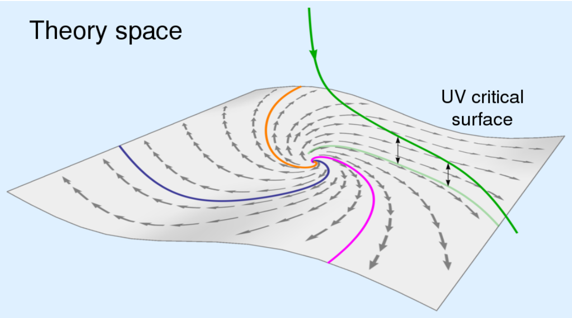
$$\frac{\mathrm{d}}{\mathrm{d}\log k}\Gamma_k[\phi] = \ldots$$

- Fixed point (vanishing beta functions)

$$\frac{\mathrm{d}}{\mathrm{d}\log k} g_a(k) = 0\,, \quad g_a = g_a^*$$

- Two cases for directions in parameter space
  - UV attractive: Relevant coupling
  - UV repulsive: Irrelevant coupling
- The AS scenario requires
  1. Existence of (non-Gaussian) fixed point for $k \to \infty$
  2. Finite dimensionality of critical surface ($= \#$ of relevant directions)

Asymptotic Safety Cartoon



[Andreas Nink — http://www.scholarpedia.org/article/Asymptotic_Safety_in_quantum_gravity]

Main results

- 3 relevant directions (so far)
- Dimensional reduction in the UV

$$d_{\mathrm{S}}(k \to \infty) = 2$$

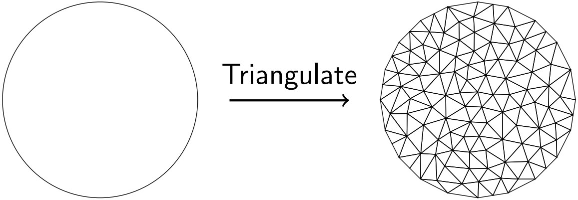
Main problem

- Need truncation of action
- Unclear how many relevant directions

Discrete Approaches

Discretise

- Lesson from QCD: “Be wise, discretise!”
- Approximate smooth manifold by triangulation (i.e. D-simplices)



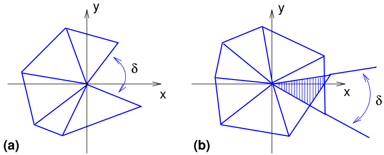
$(M, g_{\mu\nu})$ $(T, \{\ell_i\}_i)$

Geometric degrees of freedom:

- Connectivity of T
- Edge lengths ℓ_i

Regge Action

- Piecewise-flat manifold
  - Building blocks: D-simplices
  - Curvature located @ $(D-2)$-simplices



[Ambjørn, Jurkiewicz, Loll — arXiv:hep-th/0509010]

- Two options for path integral
  1. Regge calculus: Fix triangulation

$$Z = \int [\mathrm{d}\{\ell_i\}] \mathrm{e}^{\mathrm{i}S_{\mathrm{Regge}}(\{\ell_i\})}$$

  2. Dynamical Triangulation: Fix edge lengths

$$Z = \sum_T \frac{1}{C(T)} \mathrm{e}^{\mathrm{i}S_{\mathrm{Regge}}(T)}$$

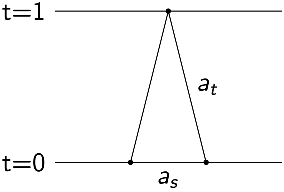
Failure of Euclidean Dynamical Triangulation

$$Z_{\mathrm{EDT}} = \sum_{T} \frac{1}{C(T)} \mathrm{e}^{-S_{\mathrm{Regge}}^{\mathrm{Euclidean}}(T)}$$

- "Naïve" approach of (Euclidean) dynamical triangulation does not work in $D \geq 3$
  - Proliferation of discrete geometries with no continuous interpretation
    - $d_{\mathrm{H}} \approx 2$ (polymer phase)
    - $d_{\mathrm{H}} \approx \infty$ (crumpled phase)

Causal Dynamical Triangulations

- Idea: Impose causal structure
  - Enforce topology $M \simeq \mathbb{R} \times \Sigma$
- Building blocks: D-simplices with
  - Spatial edges equilateral $a_s^2 \equiv a^2$
  - Temporal edges $a_t^2 \equiv -\alpha a^2$ $(\alpha > 0)$
  - E.g. in $D = 2$



- "Wick rotation": $\alpha \to -\alpha$
  - Gives Eucildean action
  - Interpretation as statistical system

CDT – Main Results in $D = 4$

- 4 phases with both first and second order transitions
- $\exists$ phase with de Sitter evolution
- Spatial reduction for short distances

Matrix Models

- Matrix model defined by action

$$S_{\mathrm{mat}} = \frac{1}{2}\,\mathrm{tr}\,M^2 - \frac{g}{\sqrt{N}}\,\mathrm{tr}\,M^3$$

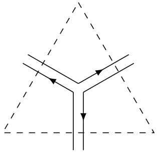
where M is a $N \times N$ Hermitian matrix

- Perturbative expansion of partition function

$$Z = \sum_{\Gamma} \frac{1}{|\,\mathrm{Sym}(\Gamma)|}\mathcal{A}(\Gamma) \simeq \sum_{T} \frac{1}{|\,\mathrm{Sym}(T)|}\mathrm{e}^{-S^{\mathrm{Euclidean}}_{\mathrm{Regge}}(T)}$$

Γ: Feynman graph, T: Dual graph (Triangulation)

- Matrix model $\simeq$ 2D gravity



Matrix Models (contd.)

- Main feature: Double scaling limit
- Simultaneously: $g \to g_c$, $N \to \infty$
- Area of triangle goes to zero, while number of triangles goes to infinity
- Continuum limit!

Tensor Models

- Matrix model $\simeq$ 2-dimensional gravity
- Rank D Tensor model $\simeq$ D-dimensional gravity (?)
- Already $D = 3$ very problematic
  - Naïve models do not give "good" geometries
- Progress being made
  - Decorate graph with extra labels (Colors, Group theoretic data)
  - $1/N$ expansion available for 3-tensor models

Closing Remarks

Building Blocks for Spacetime

- Quantising continuous gravity theory might be like quantising hydrodynamics
- Fundamentally discrete/combinatorial structure?
- Observed continuous spacetime emergent?

Conclusion

- Quantum gravity is hard!
- Many approaches with some common trends
- Relations need more exploration