Can you regularise gravity on a supermanifold?

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Outline

- Introduction and motivation

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- Outline of $SU(N|N)$ case

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- We are inspired by, e.g., arXiv: hep-th/0106258 (Arnone, Kubyshin, Morris, Tighe) where something similar is done in gauge theory.
- This is done by extending the $SU(N)$ group to the supergroup $SU(N|N)$.
- The analogous thing to do in gravity then is to extend the diffeomorphism group (Diff) to the superdiffeomorphism group (SDiff). This involves extending the spacetime to include Grassmann directions.

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- Here, $\mathcal{A}^i_\mu$ is bosonic and B_μ is fermionic.
- $\mathcal{A}^0_\mu$ parametrises a $U(1)$ subgroup but decouples from everything else, so is ignored here.

Breaking $SU(N|N)$

- We introduce a superscalar field

$$\mathcal{C} = \begin{pmatrix} C^1 & D \\ \bar{D} & C^2 \end{pmatrix}$$

with C^i bosonic and D fermionic.

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  with C^i bosonic and D fermionic.
- This picks up an expectation value $\langle \mathcal{C} \rangle \propto \sigma_3$ and thus spontaneously breaks $SU(N|N)$ to $SU(N) \times SU(N)$.

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- Then we see that the lowest order coupling is

$$\mathrm{str}\left((\mathcal{F}^1)^2\right) \mathrm{str}\left((\mathcal{F}^2)^2\right)$$

which is irrelevant, so can be ignored.

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$$\mathrm{tr}(\mathbb{I}_N) = N \quad \text{vs.} \quad \mathrm{str}(\mathbb{I}_{2n}) = \mathrm{tr}(\sigma_3 \mathbb{I}_{2n}) = 0.$$

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- Begs the question: how to define a supermanifold?
- Mathematicians tend to define these in terms of sheafs, categories...

Supermanifolds

- We take a more pragmatic approach. We write points on our (4,4)-supermanifold as
$$x^A = \begin{pmatrix} x^\mu \\ \theta^a \end{pmatrix}$$
where $\mu, a = 1, 2, 3, 4$ and $A = 1, \ldots, 8$, and θ^a fermionic.

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where $\mu, a = 1, 2, 3, 4$ and $A = 1, \ldots, 8$, and θ^a fermionic.
- This is probably best thought of as a principal bundle over the "base" manifold $\mathcal{M}$. That is, like $\mathcal{M}$ but with some "fuzziness" around it.

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- Lesson: BE CAREFUL - can't assume formulae carry over in a simple way.

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- Regularise the theory, solve quantum gravity, win the lottery, world peace etc...

Defining the theory

- We take our supermetric to be

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- Choose a background metric

$$\bar{\delta}_{AB} = \begin{pmatrix} \delta_{\mu\nu} & 0 \\ 0 & \epsilon_{ab} \end{pmatrix}$$

with ϵ_{ab} arbitrary (potential for symmetry breaking?)

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- Calculating R is significantly harder... use FORM.

Results from FORM (1/2)

$$\mathcal{L}_{bb} = \frac{1}{4}\partial_\rho h^\mu{}_\mu \partial_\rho h^\nu{}_\nu + \frac{1}{2} h^\rho{}_\rho \partial_\mu \partial_\nu h^{\mu\nu} - \frac{1}{4}\partial_\rho h_{\mu\nu}\partial^\rho h^{\mu\nu} + \frac{1}{2}\partial^\nu h_{\mu\nu}\partial_\rho h^{\mu\rho} - \frac{1}{4}\partial_a h^\mu{}_\mu \partial^a h^\nu{}_\nu + \frac{1}{4}\partial_a h_{\mu\nu}\partial^a h^{\mu\nu}$$

$$\mathcal{L}_{bm} = -h^\mu{}_\mu \partial_\nu \partial_a h^{\nu a} - \partial^\nu h_{\mu\nu}\partial_a h^{\mu a}$$

$$\mathcal{L}_{bf} = \frac{1}{2}\partial_\rho \partial^\rho h^\mu{}_\mu h^a{}_a + \frac{1}{2} h^\mu{}_\mu \partial^b \partial_b h^a{}_a - \frac{1}{2} h^a{}_a \partial_\mu \partial_\nu h^{\mu\nu} - \frac{1}{2} h^\mu{}_\mu \partial_a \partial_b h^{ab}$$

Results from FORM (2/2)

$$\begin{aligned}
\mathcal{L}_{mm} &= -\frac{1}{2}\partial_\nu h_{\mu a}\partial^\nu h^{\mu a} - \frac{1}{2}\partial_\mu h^{\mu a}\partial^\nu h_{\nu a} - \frac{1}{2}\partial_b h_{\mu a}\partial^b h^{\mu a} + \frac{1}{2}\partial^a h_{\mu a}\partial_b h^{\mu b} \\
\mathcal{L}_{mf} &= h^a{}_a \partial_\mu \partial_b h^{\mu b} + \partial^\mu h_{\mu a}\partial_b h^{ab} \\
\mathcal{L}_{ff} &= \frac{1}{4}\partial_\mu h^a{}_a \partial^\mu h^b{}_b - \frac{1}{4}\partial_c h^a{}_a \partial^c h^b{}_b + \frac{1}{2} h^a{}_a \partial_c \partial_d h^{cd} + \frac{1}{4}\partial_\mu h_{ab}\partial^\mu h^{ab} \\
&\quad - \frac{1}{4}\partial_c h_{ab}\partial^c h^{ab} + \frac{1}{2}\partial^b h_{ab}\partial_c h^{ac}
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24 terms - looks long, but tractable. BUT this was simplified by hand from around 100 terms. How do we know there's not been a mistake in either the code or the simplification?

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- Unfortunately, we want to see how the theory behaves on our base manifold $\mathcal{M}$. This means performing the $d^4\theta$ part of the integration in the action.
- Each field in our 8-dimensional action is actually 5 fields from the point of view of the 4-dimensional theory:

$$h(x,\theta) = h(x) + M\theta^a h_{,a} + M^2\theta^a\theta^b h_{,ab} + M^3\theta^a\theta^b\theta^c h_{,abc} + M^4\theta^a\theta^b\theta^c\theta^d h_{,abcd}$$

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We will not show (x,θ) or (x) arguments unless it is particularly instructive to do so.

Dealing with the θ's

- We take the convention that

$$\int d^4\theta\, \theta^a\theta^b\theta^c\theta^d = M^{-4}\epsilon^{abcd}.$$

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This ensures that starting with dimension-1 fields means ending with dimension-1 fields.
- It is also useful to define a "dual" in θ-space:
$$*h = \epsilon^{abcd} h_{,abcd}$$
$$*h^{,a} = \epsilon^{abcd} h_{,bcd}$$
$$*h^{,ab} = \frac{1}{2}\epsilon^{abcd} h_{,cd}$$
and we organise terms to have the lowest number of indices possible (to make cancellations clearer).

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- Some fields left then act as Lagrange multipliers and enforce certain conditions, which removes some more terms.

Final Lagrangian

$$\mathcal{L}_e = -\frac{1}{2}\partial_\rho h_{\mu\nu}\partial^\rho * h^{\mu\nu} - \frac{1}{2}\partial_\rho h_{\mu\nu,ab}\partial^\rho * h^{\mu\nu,ab} + \partial^\nu h_{\mu\nu,ab}\partial_\rho * h^{\mu\rho,ab} + M^2\epsilon^{ab}h_{\mu\nu,ab} * h_{\mu\nu}$$

$$\mathcal{L}_o = *\varphi^{,a}\partial_\mu\partial_\nu h^{\mu\nu}{}_{,a} + \partial_\rho h_{\mu\nu,a}\partial^\rho * h^{\mu\nu,a} + M^2\epsilon^{ab}\epsilon_{abcd} * \varphi^{,c} * \varphi^{,d} - \frac{1}{4}M^2\epsilon^{ab}\epsilon_{abcd} * h_{\mu\nu}{}^{,c} * h^{\mu\nu,d}$$

The propagator

- Focus on the even part only. Define

$$D = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & p \end{pmatrix} (= D^{-1}) \quad \text{and} \quad A = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & -1 \\ 1 & -1 & 0 \end{pmatrix}$$

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- Success! We've generated a mass, right? Well, not quite...

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- It turns out that DA is *nilpotent*. That is, $(DA)^4 = 0$.
- Therefore, the Taylor series terminates, and we are left with only have a pole at $p = 0$.
- So, there are no massive excitations in this theory (as it stands), and so the procedure used for $SU(N|N)$ does not apply.

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- It is entirely possible to construct a linearised theory of gravity on a supermanifold.
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- After performing the $d^4\theta$ integration, gauge fixing and applying Lagrange multipliers, we end up with a (relatively) simple Lagrangian.

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- It is entirely possible to construct a linearised theory of gravity on a supermanifold.
- Supermanifolds have many issues around how to carefully define geometric objects such as vectors and derivatives.
- After performing the $d^4\theta$ integration, gauge fixing and applying Lagrange multipliers, we end up with a (relatively) simple Lagrangian.
- However, the “mass” M that appeared in the expansion of superfields does not behave like a mass at the level of the propagator.

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- The key thing we did not do on the supermanifold - that was done in $SU(N|N)$ - was introduce a superscalar fields to break SDiff, so perhaps doing so would be more useful.
- One thing is clear however: supermanifolds have interesting properties in their own right.

Any Questions?

Thanks for your attention.