Perturbatively renormalizable quantum gravity and the WRG

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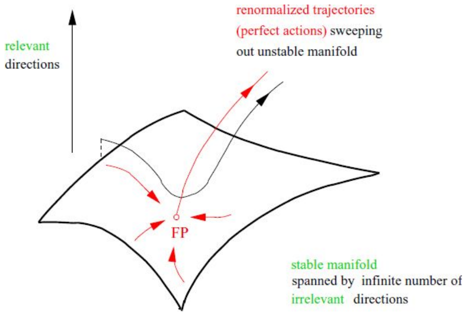
Outline

- We know gravity is difficult to quantize, not just practically but also conceptually

- From the renormalization group perspective the coupling $\kappa \propto G^{\frac{1}{2}}$ is irrelevant, $[\kappa] = -1$



- This is particularly evident when looking for interactions of the graviton:
  - $\mathcal{L}_{int} \supset H^n \partial H \partial H$
- These form irrelevant operators of dimension n + 4.
- In a minimal theory then the trick is to look for interacting operators of the graviton which are relevant (even though I just said you can't do this)

Outline

- The trick to resolving this impasse is the decomposition of the graviton into it's trace and non-trace parts:

$$H_{\mu\nu} = h_{\mu\nu} + \frac{1}{2}\delta_{\mu\nu}\phi$$

- ϕ is our 'dilaton', a non-dynamical field contributing to the background

$$\mathcal{L}_{EH} = \frac{1}{2\kappa}R\sqrt{-g}$$

$$\mathcal{L}_{EH} \supset \frac{1}{2}(\partial_\lambda h_{\mu\nu})^2 - \frac{1}{2}(\partial_\lambda \phi)^2$$

- Typically this is brushed under the rug since ϕ is non-dynamical and the minus sign is removed by setting $\phi \to i\phi$
- We choose not to do this and instead define our hilbert space with this minus sign, this changes the sign in our Sturm-Liouville weight
- This restricts the operators that are allowed within this Hilbert space, Σ^-

Outline

- Typically the Sturm-Liouville weight has a negative sign for a positive kinetic sign, leading to a space of function square integrable under $e^{-(\frac{\chi}{\Lambda_0})^2}$:

$$\int_{-\infty}^{\infty} \mathcal{D}\chi \; e^{-(\frac{\chi}{\Lambda_0})^2} \mathcal{O}_n(\chi)\mathcal{O}_m(\chi) \propto \delta_{nm}$$

- The situation is subtly but drastically different for a negative kinetic sign, leading to a change in the sign of the weight;

$$\int_{-\infty}^{\infty} \mathcal{D}\phi \; e^{+(\frac{\chi}{\Lambda_0})^2} \mathcal{O}_n(\phi)\mathcal{O}_m(\phi) \propto \delta_{nm}$$

- As a consequence of this change in sign we are no longer permitted to use polynomials as they do not converge. Our Hilbert space is restricted and permits only the below 'tower operator'

$$\delta^{(n)}_{\Lambda_0}(\phi) \equiv \frac{\partial^n}{\partial\phi^n}\delta^{(0)}_{\Lambda_0} \quad \text{with} \quad \delta^{(0)}_{\Lambda_0} = \frac{1}{\sqrt{2\pi\Omega_{\Lambda_0}}} e^{-\frac{\phi^2}{2\Omega_{\Lambda_0}}}$$

- This effervescent tower operator has dimension

$$[\delta^{(n)}_{\Lambda_0}(\phi)] = -1 - n \text{ with } \Omega_{\Lambda_0} \propto \hbar\Lambda_0^2$$

- Hence it is non-perturbative in $\hbar$

Outline

- So far this hasn't been gravity, we need our operators to respect diffeomorphism invariance for this to be gravity
- We introduce this here via the Batalin-Vilkovisky formalism, an extension to BRST.
- BRST introduces a nilpotent operator Q which is associated to a symmetry, if an action is invariant under the operator Q then it respects the associated symmetry

$$Q^2 = 0 \qquad QS = 0$$

- We extend this operator under BV formalism to account for field redefinitions

$$s = Q + Q^{-} - \Delta^{-} - \Delta^{=}$$

- We also introduce the quantum master equation

$$\frac{1}{2}(S,S) - \Delta S = 0$$

$$(X,Y) = \frac{\partial_r X}{\partial \Phi^A}\frac{\partial_l Y}{\partial \Phi^*_A} - \frac{\partial_r X}{\partial \Phi^*_A}\frac{\partial_l Y}{\partial \Phi^A} \qquad \Delta X = (-)^A \frac{\partial_l}{\partial \Phi^A}\frac{\partial_l}{\partial \Phi^*_A} X.$$

- We introduce as part of this a non-dynamical anti-field which sources our BRST transformations

$$Q_0 H_{\mu\nu} = 2\, \partial_{(\mu} c_{\nu)}$$

$$S \to S + (Q\Phi^A)\Phi^*_A$$

- We can grade these anti-fields by the 'anti-ghost number', with agn = 2 corresponding to the Lie algebra, agn = 1 to the BRST trasnformations and agn = 0 corresponding to the interactions between fields

Outline

- We have at our disposal a series of gradings; dimension of operator, ghost number, anti-ghost number, number of fields etc, to use to constrain our action

$$\mathcal{L}_1 = \mathcal{L}_1^2 + \mathcal{L}_1^1 + \mathcal{L}_1^0$$

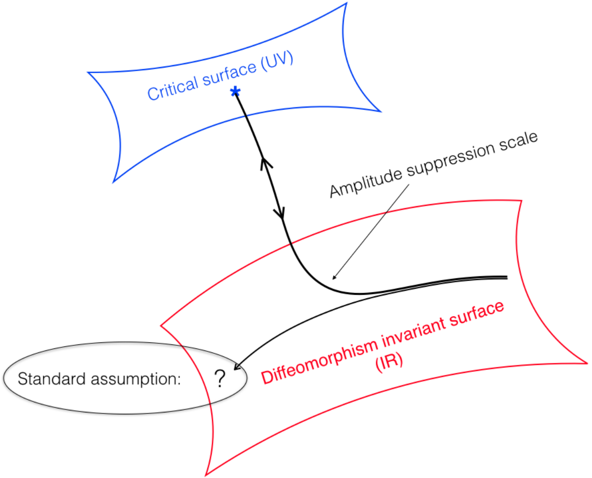
- In particular we find a series of 'cascade equations' one can use to produce an action from a highly constrained agn = 2 part

$$Q_0 \mathcal{L}_1^2 = 0, \qquad Q_0 \mathcal{L}_1^1 + Q_0^- \mathcal{L}_1^2 = 0, \qquad Q_0 \mathcal{L}_1^0 + Q_0^- \mathcal{L}_1^1 = 0.$$

- With this we recover the 13 terms one finds from expanding out $\mathcal{L}_{EH}$ as well as the tadpole correction i.e. recover diffeomorphism invariant operators that are within Σ^-

- This is done in a way which simultaenously solves the QME and the renormalization group equations

- In the limit of $\Lambda_\sigma \to \infty$ we recover the standard non-trivial interactions


Critical surface (UV)
Amplitude suppression scale
Standard assumption: ?
Diffeomorphism invariant surface
(IR)

Outline

- We are free to add exact terms to produce alternative expressions of the BRST transformations

$$\mathcal{L}^1_1 \to \mathcal{L}'^1_1 = \mathcal{L}^1_1 + Q_0 K^1_1$$

- Using this we find a more convenient expression for $\mathcal{L}^1_1$

$$\mathcal{L}'^1_1 = \left(-\partial_\gamma c_\alpha H_{\alpha\beta} - \partial_\beta c_\alpha H_{\alpha\gamma} - c_\alpha \partial_\alpha H_{\gamma\beta}\right) H^*_{\gamma\beta}$$

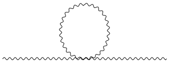
- This is used to find convenient expressions at the second order

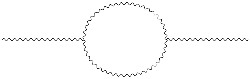
$$\mathcal{L}_2^2 = \mathcal{L}_2^1 = 0 \qquad \mathcal{L}_2^0 = 36 \ \textit{terms}$$

- This will make calculations at the 2nd order far easier to do, it also means our Lie algebra and BRST transformations are complete statements

Outline

- In this more useful parametrization we can more easily calculate loop diagrams





$$\dot{\Gamma}_{2,f} = -\frac{1}{2}\int d\!\!\!^{-}{}^4 p \; d\!\!\!^{-}{}^4 p_1 \;\; \dot{C}^{\Lambda}(p) \;\; h_{\alpha\beta}(p_1) h_{\alpha\beta}(-p_1)(1 + \frac{(p_1)^2}{p^2})$$

- To conclude we may have been working on a minimal route to quantizing gravity
- We resolve issues of irrelevancy using our tower operator and have incorporating diffeomorphism invariance into the theory using the BV formalism
- We use the anti-field cascade to re-express the first order action of the BRST charge on the graviton, find the second order part of the action and have begun using this to calculate loop diagrams
- In addition to this we will also be able to begin investigating the running of couplings and the nature of marginal operators

Thanks for listening!

Any questions?