Soft Anomalous Dimension in QCD

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THE UNIVERSITY of EDINBURGH

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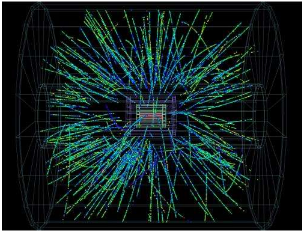
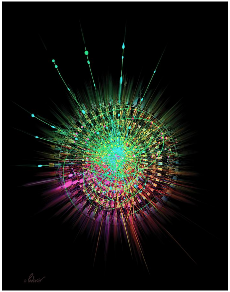
Overview



Credit: Alice/CERN

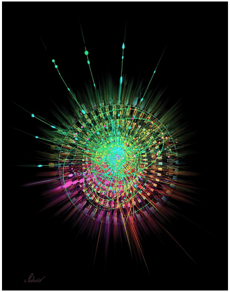
- Terminology
- Motivation
- Factorisation of Amplitudes
- Bootstrapping soft anomalous dimension
- Dipole Constraints and other limits

Terminology



Credit: Jean-francois Podevin

Terminology



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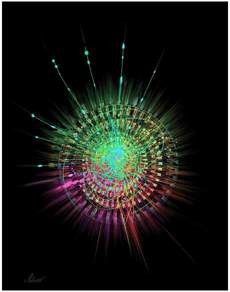
- Soft

Terminology



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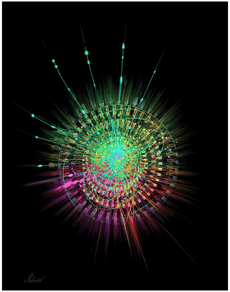
- Soft
- Anomalous Dimension

Terminology



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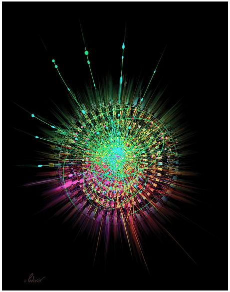
- Soft
- Anomalous Dimension

$$\mu \frac{d}{d\mu} \mathcal{S}_{IK} \left(\beta_i \cdot \beta_j, \alpha_s \left(\mu^2 \right), \epsilon \right) = -\Gamma^S_{IJ} \mathcal{S}_{JK}$$

Terminology



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- QCD

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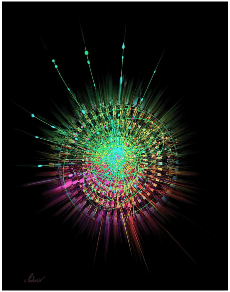

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- Soft
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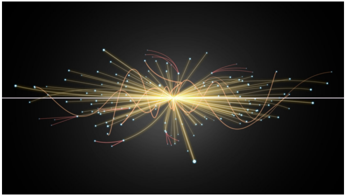
- QCD
- Massless Partons

Motivation

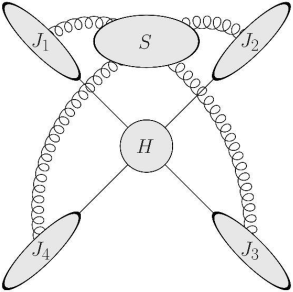


Credit:general-fmv/Shutterstock

- Infrared singularities
- Cross section σ and precision
- Mathematical relations for kinematics functions with bootstrapping

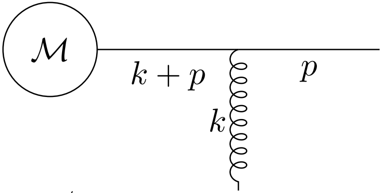
Amplitude Factorisation
Gardi, Magnea 0901.1091

Credit:Øyvind Almelid



$$\mathcal{M}_L(\tfrac{p_i}{\mu}, \alpha_s(\mu^2), \epsilon) =$$
$$S_{LK}(\beta_i \cdot \beta_j, \alpha_s(\mu^2), \epsilon) \times$$
$$H_K\left(\frac{2p_i \cdot p_j}{\mu^2}, \frac{(2p_i \cdot n_i)^2}{n_i^2 \mu^2}, \alpha_s(\mu^2)\right) \times$$
$$\prod_{i=1}^{n} \frac{J_i\left(\frac{(2p_i \cdot n_i)^2}{n_i^2 \mu^2}, \alpha_s(\mu^2), \epsilon\right)}{\mathcal{J}_i\left(\frac{2(\beta_i \cdot n_i)^2}{n_i^2}, \alpha_s(\mu^2)\epsilon\right)}$$

Singularities

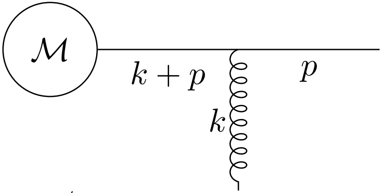


$\overline{u}(p)(-igT^{(a)})\gamma^{\mu}\frac{i(\not{p}+\not{k})}{(p+k)^2+i0}\mathcal{M}$

$(p+k)^2 = p^2 + 2p.k + k^2 = 2p.k$

$2p_0 k_0(1 - \cos\,\theta_{pk})$

Singularities



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$2p_0k_0(1-\cos\theta_{pk})$

IR: $k_0 = 0$ **Collinear**: $\cos\theta_{pk} = 1$

Eikonal approximation

When the momentum of the soft gluon k is considered negligible compared to the momentum p of the parton leg

$$\overline{u}(p)\,T^{(a)}\gamma^{\mu}\frac{p^{\mu}}{p\cdot k+i0}$$

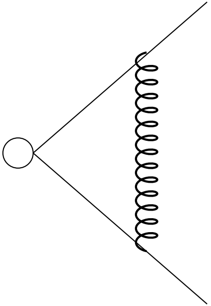
$$T^{(a)}\frac{p^{\mu}}{p\cdot k+i0} = T^{(a)}\frac{\beta^{\mu}}{\beta\cdot k+i0},\ S_{LK}(\beta_i\cdot\beta_j,\alpha_s(\mu^2),\epsilon)$$

Wilson Lines

Non-abelian exponentiation: Gardi, Smilie, White 1304.7040

$$\Phi_\beta(0,\infty) = \mathcal{P}\left(\exp\left[i\int_C dx^\mu \beta_i A^a_\mu(\beta_i x) T^a\right]\right)$$

$$\mathcal{S}\left(\{\beta_i\},\{\mathbf{T}_i\},\epsilon,\alpha_s\right) = \langle 0\,|\,T\left[\Phi_{\beta_1}\Phi_{\beta_2}\ldots\Phi_{\beta_n}\right]|\,0\rangle$$



$$\mu \frac{d}{d\mu} \mathcal{S}_{IK}\left(\beta_i \cdot \beta_j, \alpha_s\left(\mu^2\right), \epsilon\right) = -\Gamma^S_{IJ} \mathcal{S}_{JK}$$

Webs
1304.7040

$$\mu \frac{d}{d\mu}\mathcal{S}_{IK}\left(\beta_i \cdot \beta_j, \alpha_s\left(\mu^2\right), \epsilon\right) = -\Gamma^S_{IJ}\mathcal{S}_{JK}$$

$$S = \mathcal{P}\exp\left(-\frac{1}{2}\int\limits_0^{\mu^2}\frac{d\lambda^2}{\lambda^2}\Gamma_n(\beta_i \cdot \beta_j, \alpha_s(\lambda^2, \epsilon)\right)$$

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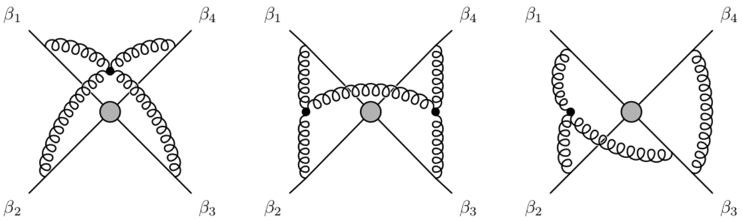
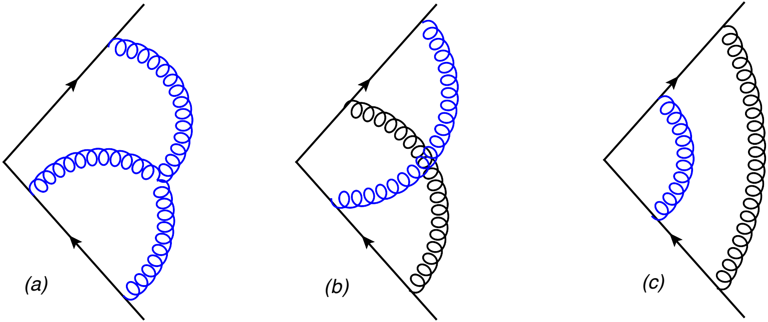


Credit:Øyvind Almelid

Webs

Gardi, Laenen, Stavenga Gerben, White 1008.0098



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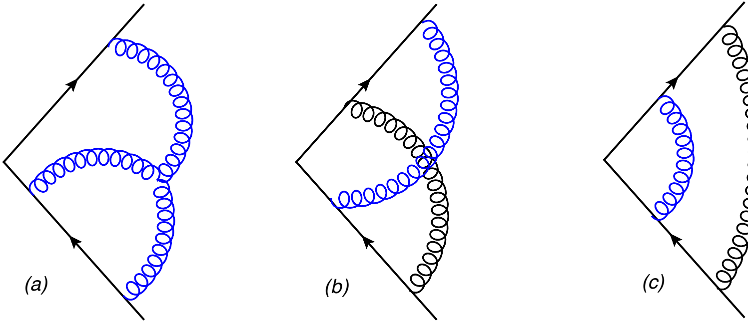

$$\boldsymbol{T}_1^a \boldsymbol{T}_1^b \boldsymbol{T}_2^a \boldsymbol{T}_2^b - \boldsymbol{T}_1^a \boldsymbol{T}_1^b \boldsymbol{T}_2^b \boldsymbol{T}_1^a.$$

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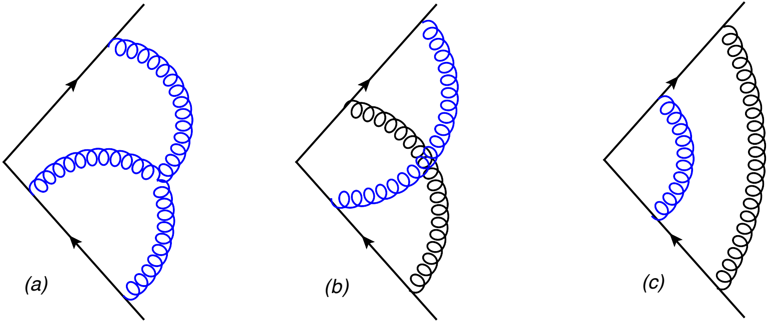

$$\mathbf{T}_1^a \mathbf{T}_1^b \mathbf{T}_2^a \mathbf{T}_2^b - \mathbf{T}_1^a \mathbf{T}_1^b \mathbf{T}_2^b \mathbf{T}_1^a.$$

$$\mathbf{T}_1^a \mathbf{T}_1^b [\mathbf{T}_2^a, \mathbf{T}_2^b] = \mathbf{T}_1^a \mathbf{T}_1^b i f^{abc} \mathbf{T}_2^c$$

Step 1: Choose a loop level L

Step 2: Write down all possible fully connected diagrams up to L loops $\rightarrow$ each has a colour structures of $SU(N)$ multiplying a kinematic function that depends only on external legs

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Bootstrapping the Soft Anomalous Dimension

Almelid, Duhr, Gardi, McLeod, White 1706.10162

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Step 6: Check the high energy limit and see what constraints it puts on the kinematic functions

Dipole Constraint equation

0901.1091

$$\sum_{j\neq i}^{n} \frac{\partial}{\partial \ln \rho_{ij}} \Gamma_{IJ}^{\overline{S}}\left(\rho_{ij}, \alpha_s\right) = 0, \quad \forall i, \quad I \neq J,$$

$$\sum_{j\neq i}^{n} \frac{\partial}{\partial \ln \rho_{ij}} \Gamma_{IJ}^{\overline{S}}\left(\rho_{ij}, \alpha_s\right) = \frac{1}{4}\gamma_K^{(i)}\left(\alpha_s\right), \quad \forall i, \quad I = J$$

$$\rho_{ij} \equiv \frac{n_i^2 n_j^2 \left(\beta_i \cdot \beta_j\right)^2 \mathrm{e}^{2\pi \mathrm{i} \lambda_{ij}}}{4(\beta_i \cdot n_i)^2 \left(\beta_j \cdot n_j\right)^2} \qquad \bar{S} = \frac{S}{\mathcal{J}}$$

Cusp anomalous dimension

Henn,Korchemsky, Mistlberger 1911.10174

UV divergences of Wilson loops

Known for QCD and $\mathcal{N} = 4sYM$

Contains quadratic and quartic Casimirs

$$
\begin{aligned}
\Gamma^{\mathrm{QCD}}_{\text{cusp },R} =& C_R \left[\left(\frac{\alpha_s}{\pi}\right) \right. \\
& + \left(\frac{\alpha_s}{\pi}\right)^2 (1.03864\, C_A - 0.555556 n_f\, T_F) \\
& + \left(\frac{\alpha_s}{\pi}\right)^3 \left(1.52982\, C_A^2 - 1.45614\, C_A n_f\, T_F + \ldots\right) \\
& + \left(\frac{\alpha_s}{\pi}\right)^4 \left(2.38379\, C_A^3 - 1.44271\, C_A^2 n_f\, T_F + \ldots\right) \\
& + \left(\frac{\alpha_s}{\pi}\right)^4 \left(-\frac{d_R^{b-d} d_A^{ab}}{N_R} 1.97915 - n_f \frac{d_R^{ncd} d_F^{abcd}}{N_R} 0.483964 \right)
\end{aligned}
$$

Collinear Limit

$$\Gamma_{\mathbf{Sp}}\left(p_1, p_2, \mathbf{T}_1, \mathbf{T}_2, \mu_f, \alpha_s\left(\mu_f^2\right)\right)$$
$$\equiv \Gamma_n\left(p_1, p_2, \{p_j\}, \mathbf{T}_1, \mathbf{T}_2, \{\mathbf{T}_j\}, \mu_f, \alpha_s\left(\mu_f^2\right)\right) -$$
$$\Gamma_{n-1}\left(P, \{p_j\}, \mathbf{T}, \{\mathbf{T}_j\}, \mu_f, \alpha_s\left(\mu_f^2\right)\right)$$

Constraints and Limits

0901.1091

Collinear Limit

$$\Gamma_{\mathbf{Sp}}\left(p_1, p_2, \mathbf{T}_1, \mathbf{T}_2, \mu_f, \alpha_s\left(\mu_f^2\right)\right)$$
$$\equiv \Gamma_n\left(p_1, p_2, \left\{p_j\right\}, \mathbf{T}_1, \mathbf{T}_2, \left\{\mathbf{T}_j\right\}, \mu_f, \alpha_s\left(\mu_f^2\right)\right) -$$
$$\Gamma_{n-1}\left(P, \left\{p_j\right\}, \mathbf{T}, \left\{\mathbf{T}_j\right\}, \mu_f, \alpha_s\left(\mu_f^2\right)\right)$$

High Energy (Regge) Limit

$s \gg -t$

Analytic continuation from Euclidean region required and momentum conservation.

Four loops

Becher, Neubert 1908.11379

$$
\begin{aligned}
\Gamma(\{\underline{s}\},\mu) = & \sum_{(i,j)} \frac{\boldsymbol{T}_i \cdot \boldsymbol{T}_j}{2} \gamma_{\text{cusp}} \left(\alpha_s\right) \ln \frac{\mu^2}{-s_{ij}} + \sum_i \gamma^i \left(\alpha_s\right) \mathbf{1} \\
& + f\left(\alpha_s\right) \sum_{(i,j,k)} \mathcal{T}_{iijk} + \sum_{(i,j,k,l)} \mathcal{T}_{ijkl} F\left(\beta_{ijlk}, \beta_{iklj}; \alpha_s\right) \\
& + \sum_R g^R \left(\alpha_s\right) \left[\sum_{(i,j)} \left(\mathcal{D}^R_{iijj} + 2\mathcal{D}^R_{iiij} \right) \ln \frac{\mu^2}{-s_{ij}} \right] \\
& + + \sum_R \sum_{(i,j,k)} \mathcal{D}^R_{ijkk} \ln \frac{\mu^2}{-s_{ij}} + \sum_{(i,j,k,l)} \mathcal{D}^R_{ijkl} G^R \left(\beta_{ijlk}, \beta_{iklj}; \alpha_s\right) \\
& + \sum_{(i,j,k,l)} \mathcal{T}_{ijkli} H_1 \left(\beta_{ijlk}, \beta_{iklj}; \alpha_s\right) \\
& + \sum_{(i,j,k,l,m)} \mathcal{T}_{ijklm} H_2 \left(\beta_{ijkl}, \beta_{ijmk}, \beta_{ijml}, \beta_{jlmi}; \alpha_s\right) + \mathcal{O}\left(\alpha_s^5\right)
\end{aligned}
$$

Conclusion

- Scattering amplitudes factorise and with greater precision can contribute to theoretical cross section values
- Discussed the singularities in the soft anomalous dimension
- The steps for bootstrapping the soft anomalous dimension and limits

Any questions?

References

- 0901.1091
- 1008.0098
- 1304.7040
- 1908.11379
- 1911.10174