QCD coherence and how it fails

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In collaboration with Jeff Forshaw

This talk is based on work to come [arXiv:2001.XXXX]
and slightly on *Parton branching at amplitude level* [arXiv:1905.08686].

Coherence recap

What is coherence?

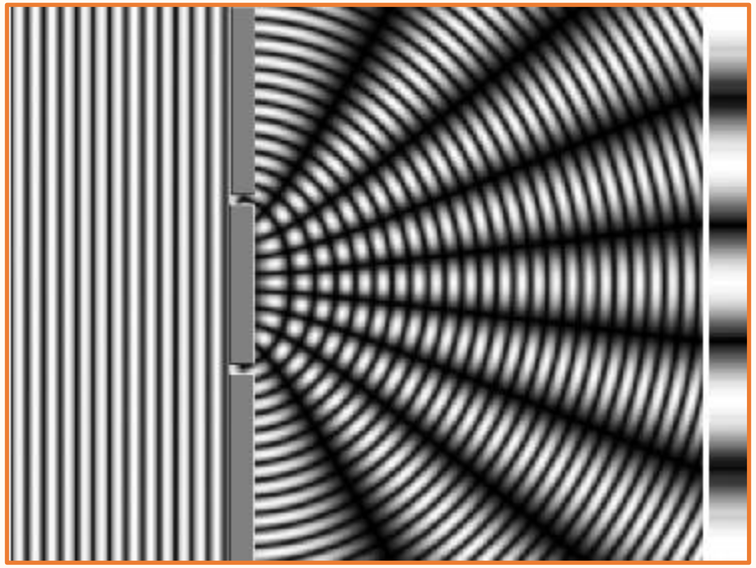
Coherence is sometimes a vague concept. It can have multiple definitions depending on who you speak to:

Coherence recap

What is coherence?

Coherence is sometimes a vague concept. It can have multiple definitions depending on who you speak to:

- Waves with a constant phase relationship are coherent.



Young and Friedman

Coherence recap

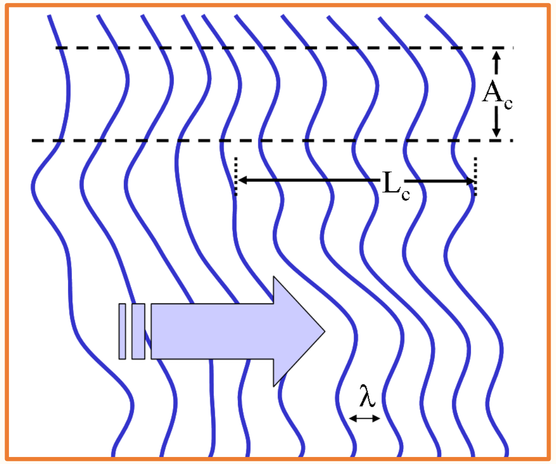
What is coherence?

Coherence is sometimes a vague concept. It can have multiple definitions depending on who you speak to:

- Fields that are mth order coherent have correlation functions that factorise as

$$G_n(x_1, \dots, x_n; x_1, \dots, x_n) = \sqrt{\prod_i G(x_i; x_i)} \quad \forall n \leq m$$

  This is a powerful macroscopic description.



RJ Glauber '63

Coherence recap

What is coherence?

Coherence is sometimes a vague concept. It can have multiple definitions depending on who you speak to:

- Wave functions are have quantum coherence if their amplitudes sum linearly.

$$\mathcal{A}_{\text{system}} = \sum_{i \in \{\text{paths}\}} \mathcal{A}_{\text{i}}$$

This is what I will be using.

(Note, this definition is closely related to Glauber's and this description plus Glauber's ∞th order coherence gives the simple first definition)

Quantum coherence recap

Wave functions are have quantum coherence if their amplitudes sum linearly.

$$\mathcal{A}_{\mathrm{system}} = \sum_{i \in \{\mathrm{paths}\}} \mathcal{A}_{\mathrm{i}}$$

is a coherent sum.

$$P_{\mathrm{event}} = \sum_{i \in \{systems\}} P_{\mathrm{i}}$$

is an incoherent sum (or replace the sum with a product for incoherent product).

It can be intuitive to view processes that fundamentally rely on quantum coherence as processes dependent on entanglement:

$$\Psi = \psi_+ \otimes \phi_- + \psi_+ \otimes \phi_+ = \psi_+ \otimes (\phi_- + \phi_+)$$

$$|\Psi|^2 = |\psi_+|^2 \otimes |\phi_- + \phi_+|^2 \equiv P_\psi P_\phi$$

Fully described with ψ and ϕ treated incoherently

Quantum coherence recap

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It can be intuitive to view processes that fundamentally rely on quantum coherence as processes dependent on entanglement:

$$\Psi = \psi_+ \otimes \phi_- + \psi_- \otimes \phi_+ \equiv \Psi_{+-} + \Psi_{-+}$$

Description fundamentally requires coherence

Quantum coherence recap

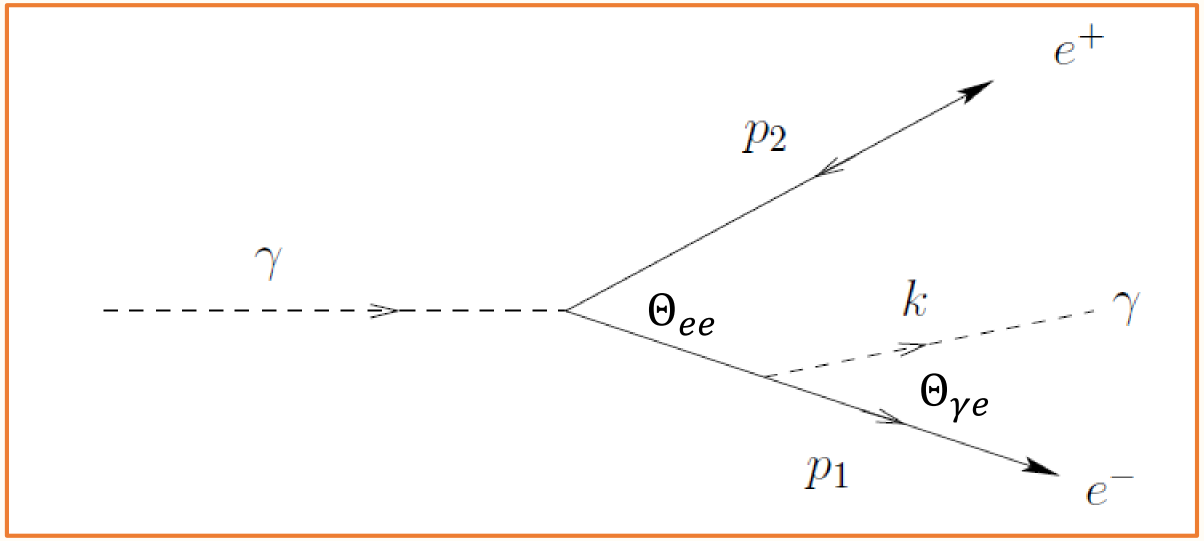
An amplitude must be summed coherently with other amplitudes if it describes a process that can probe the quantum phenomena of the other amplitudes (i.e. interference effects or entanglement).

We expect incoherent physics to describe all systems that interact in a classical limit or are independent events.

QCD coherence

What is QCD coherence?

Electrodynamics example:
To probe a charge distribution of size d then we need $\lambda \lesssim d$.



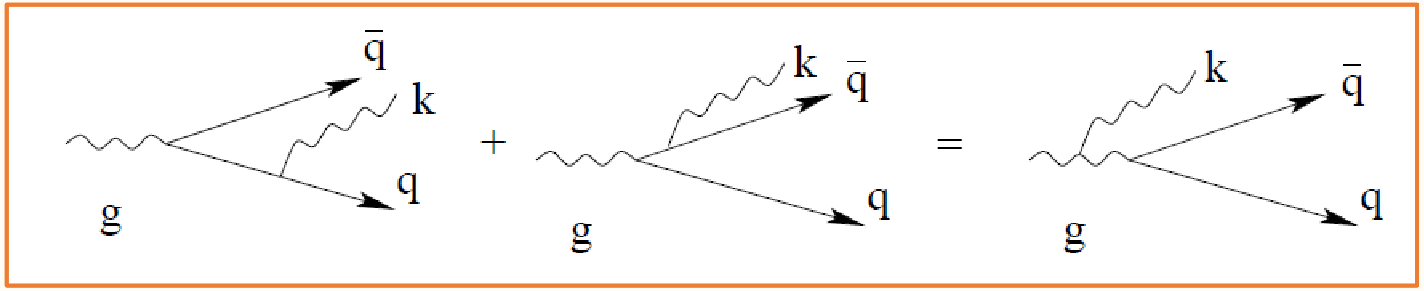
$$\Theta_{\gamma e} < \Theta_{ee}$$

QCD coherence

What is QCD coherence?

More generally we expect physics at large scales (or long wavelength) to be insensitive to physics at short scales (or short wavelength).

This manifests in multiple ways in QCD. The previous example now appears as



when $\Theta_{qq} < \Theta_{qg}$.

Let’s look at a more subtle example…

QCD coherence

What is QCD coherence?

Let's consider an 'accelerating' proton. We are going to take it from some low scale μ and to some high scale Q and study how its parton distribution function (PDF) changes. As it accelerates, it will radiate gluons and quarks. Lets focus on radiating gluons. This is described by DGLAP evolution '72 '77.

$$\frac{\mathrm{d}f_q(x,\mu)}{\mathrm{d}\ln\mu} = \frac{\alpha_s}{2\pi}\int_0^1 \frac{\mathrm{d}z}{z} P_{qq}(z)\left(f_q\left(\frac{x}{z},\mu\right)\Theta(z \geq x) - z^2 f_q(x,\mu)\right)$$

Parton distribution function: 'likelihood for finding a quark of scale $\sim x\mu$ in a proton'

$$P_{qq}(z) = C_F \frac{1+z^2}{1-z}$$

Splitting function: squared matrix element for $q \to qg$.
z is momentum faction.

QCD coherence

What is QCD coherence?

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$$P_{qq}(z) = C_\mathrm{F}\frac{1+z^2}{1-z}$$

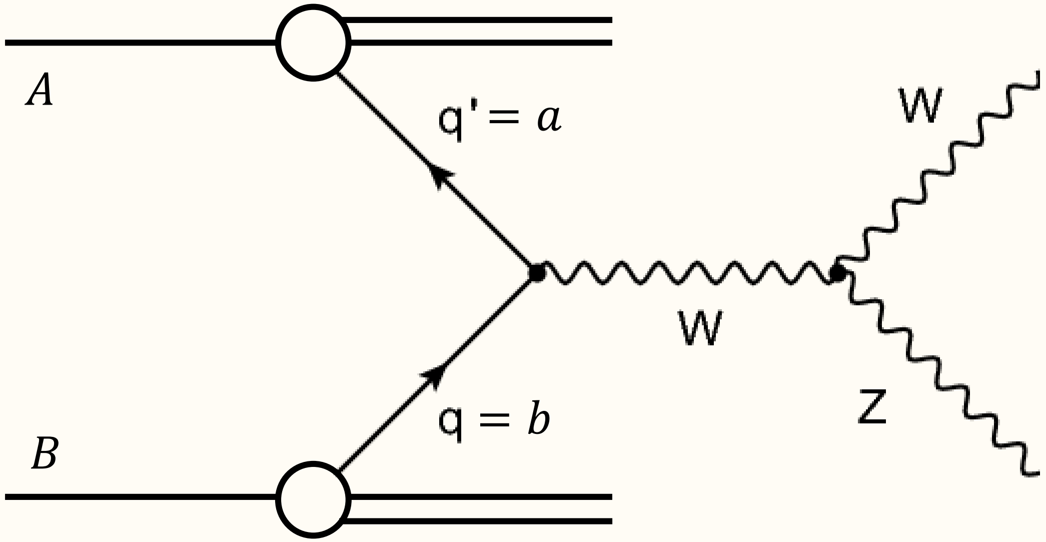
Note that as $z \to 1$ this whole equation goes to 0. This limit is the soft gluon limit (relative to the proton’s scale). So a high energy proton is only sensitive to high energy radiation. This makes intuitive sense.

QCD coherence

What is QCD coherence?

So let's calculate something we could see at the LHC.
The born cross-sections interface with PDFs in the following simple way:

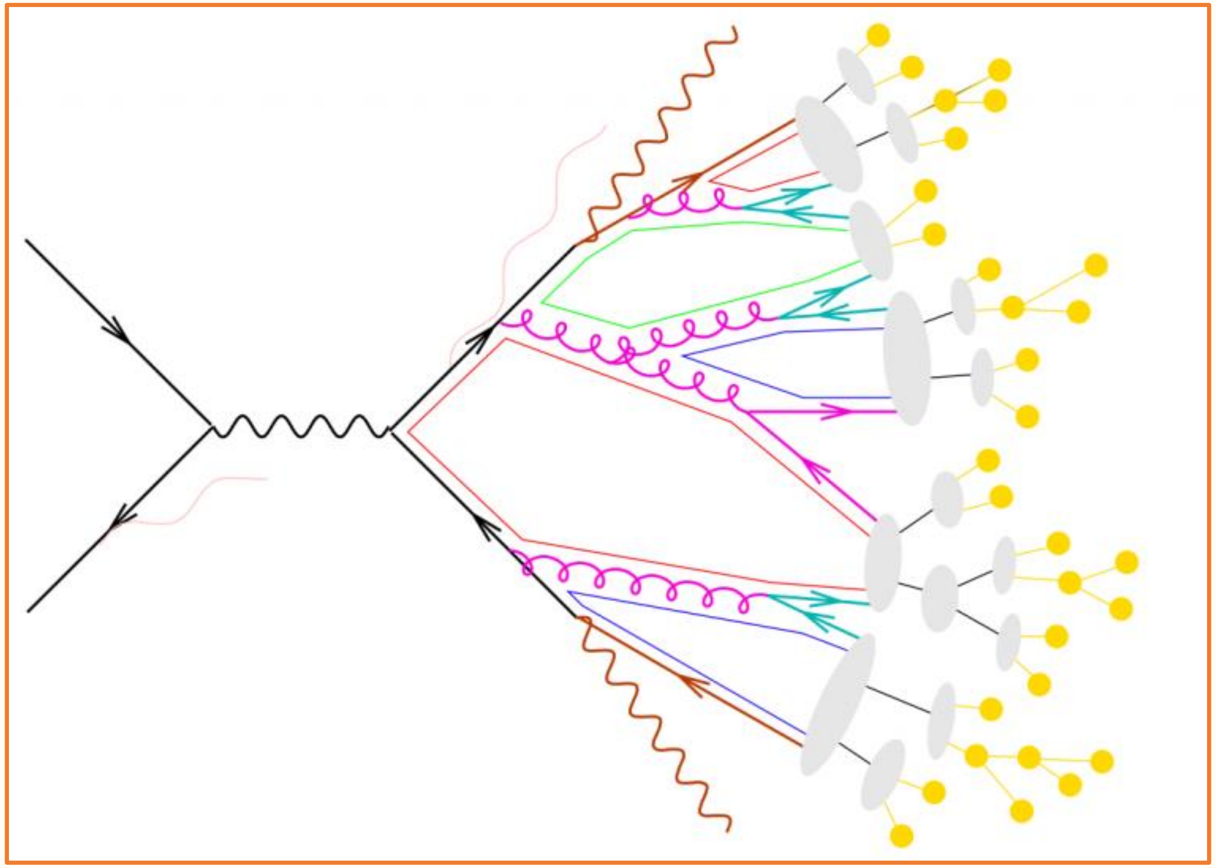
$$\frac{\mathrm{d}\sigma(Q)}{\mathrm{d}x_a\mathrm{d}x_b} = f_A(x_a, Q) f_B(x_b, Q) \frac{\mathrm{d}\sigma_{\mathrm{partonic}}(\sqrt{x_a x_b} Q)}{\mathrm{d}x_a\mathrm{d}x_b}$$



QCD coherence

What is QCD coherence?

What about beyond the Born level?



QCD factorisation

What is QCD coherence?

So now we include all the radiation that might dress the final state. This is often dominated by soft gluons, so they are what we will study. Coherence causes us to expect that we can still write the following to all orders with soft radiation

$$\frac{\mathrm{d}\sigma(Q)}{\mathrm{d}x_a \mathrm{d}x_b} = f_A(x_a, Q) f_B(x_b, Q) \frac{\mathrm{d}\sigma_{\text{partonic}}(\sqrt{x_a x_b} Q)}{\mathrm{d}x_a \mathrm{d}x_b}$$

i.e. using an incoherent product, as protons only care about hard gluons. Interface should be semi-classical.

This is correct for ‘sufficiently inclusive’ processes
– Colins, Soper and Sterman ‘83-’89.

QCD factorisation

What is QCD coherence?

Forshaw, Kyrieleis, Seymour '06 '07

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QCD factorisation

How is QCD coherence violated?

Need to unpack our previous formulas more:

Measurement function, functions on the parton phase space

$$\Sigma(\mu, \{x_i\}, \{p\}_0, \{v\}) = \int \sum_n \mathrm{d}\sigma_n \star \left\{ \prod_{i \in \mathrm{initial}} f_{v_i}\left(\frac{x_i}{z_{i_1} z_{i_2} \dots}; \mu\right)\right\} u_n(\{p\}_n, \{v\}),$$

Observable

Incoherent sum over gluon multiplicities

PDF with renormalisation scale below the inclusivity scale.

We will need to resum soft parts of this and compute parts to α_s^4. Warning maths coming…

QCD factorisation

How is QCD coherence violated?

Gaps between jets observable

$$\Sigma(\mu, \{x_i\}, \{p\}_0, \{v\}) = \int \sum_n \mathrm{d}\sigma_n \star \left\{ \prod_{i \in \mathrm{initial}} f_{v_i}\left(\frac{x_i}{z_{i_1} z_{i_2} \dots}; \mu\right)\right\} u_n(\{p\}_n, \{v\}),$$

One collinear gluon from proton term in Σ

Collinear splitting function

Colour charge operator for collinear emission, 'trapped' behind the soft Sudakov factors.

$$\frac{\mathrm{d}\sigma_1}{\mathrm{d}x_a \mathrm{d}x_b\, \mathrm{d}\mathcal{B}} = \frac{\alpha_s}{\pi}\int_{Q_0}^{Q} \frac{\mathrm{d}k_T}{k_T} \int_{k_T^2/2Q^2}^{1-k_T^2/2Q^2} \frac{\mathrm{d}z}{z} P_{qq}(z)\, f_B(x_b, Q)$$
$$\times \left[\Theta(z - x_a)\, f_A(x_a/z, Q_0) \frac{1}{\mathbf{T}_a^2} \mathrm{Tr}(\mathbf{V}_{Q_0,k_T} \mathbf{T}_a \mathbf{V}_{k_T,Q} \mathbf{H} \mathbf{V}^\dagger_{k_T Q} \mathbf{T}_a^\dagger \mathbf{V}^\dagger_{Q_0,k_T}) - z^2 f_A(x_a, Q_0) \mathrm{Tr}(\mathbf{V}_{Q_0,Q} \mathbf{H} \mathbf{V}^\dagger_{Q_0,Q})\right].$$

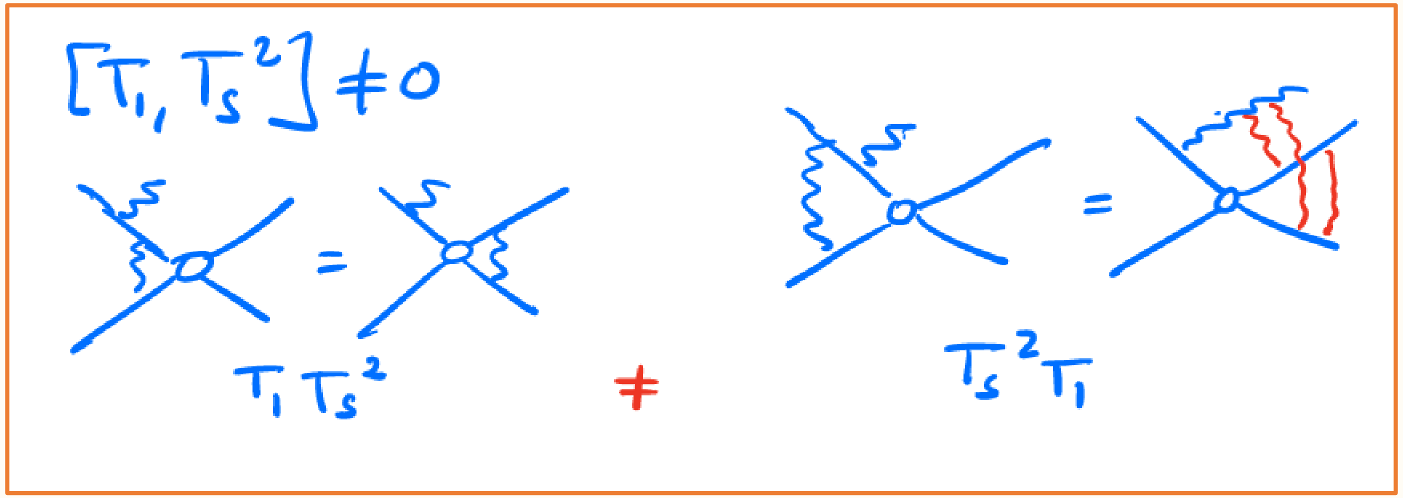
Resummed soft gluons described by Sudakov factors (cusp anomalous dimensions), modified they are only integrated over a IR finite portion of phase space.

PDF with renormalisation scale below the inclusivity scale.

Coherence violating logarithms

How is QCD coherence violated?

$$\frac{\mathrm{d}\sigma_1}{\mathrm{d}x_a \mathrm{d}x_b\, \mathrm{d}\mathcal{B}} = \frac{\alpha_s}{\pi}\int_{Q_0}^{Q}\frac{\mathrm{d}k_T}{k_T}\int_{k_T^2/2Q^2}^{1-k_T^2/2Q^2}\frac{\mathrm{d}z}{z}\,P_{qq}(z)\, f_B(x_b,Q)$$
$$\times\left[\Theta(z-x_a)\, f_A(x_a/z,Q_0)\frac{1}{\mathbf{T}_a^2}\mathrm{Tr}(\mathbf{V}_{Q_0,k_T}\mathbf{T}_a\mathbf{V}_{k_T,Q}\mathbf{H}\mathbf{V}^\dagger_{k_T Q}\mathbf{T}_a^\dagger\mathbf{V}^\dagger_{Q_0,k_T}) - z^2 f_A(x_a,Q_0)\mathrm{Tr}(\mathbf{V}_{Q_0,Q}\mathbf{H}\mathbf{V}^\dagger_{Q_0,Q})\right] .$$



Coherence violating logarithms

How is QCD coherence violated?

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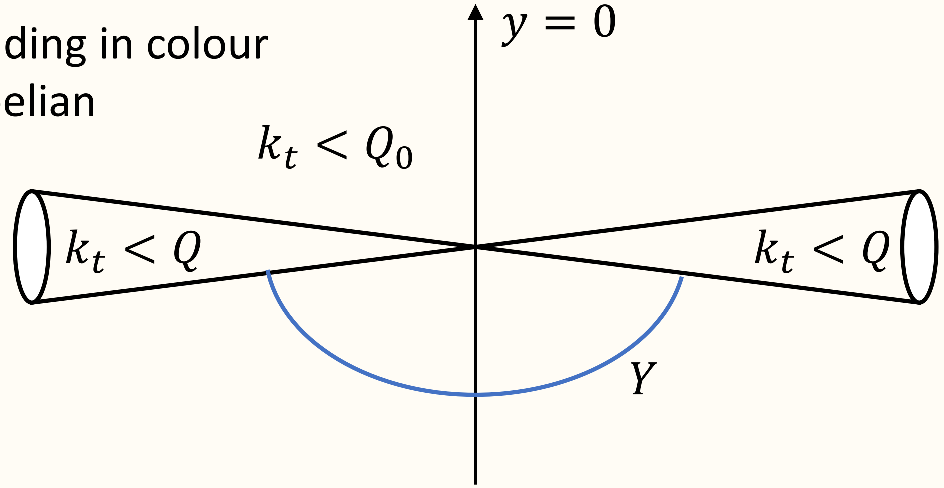
$$\times \left[\Theta(z - x_a)\, f_A(x_a/z, Q_0) \frac{1}{\mathbf{T}_a^2} \mathrm{Tr}(\mathbf{V}_{Q_0,k_T} \mathbf{T}_a \mathbf{V}_{k_T,Q} \mathbf{H} \mathbf{V}^\dagger_{k_T Q} \mathbf{T}_a^\dagger \mathbf{V}^\dagger_{Q_0,k_T}) - z^2 f_A(x_a, Q_0) \mathrm{Tr}(\mathbf{V}_{Q_0,Q} \mathbf{H} \mathbf{V}^\dagger_{Q_0,Q}) \right] .$$

Evaluate at fixed order

Super leading!

Sub-leading in colour
Non-abelian

$$\frac{\mathrm{d}\Sigma_1}{\mathrm{d}x_A \mathrm{d}x_B} \approx -\sigma_0 \left(\frac{\alpha_\mathrm{s}}{\pi}\right)^4 \ln^5\left(\frac{Q}{Q_0}\right) \pi^2 Y \frac{3N_c^2 - 4}{15} f_A\, f_B$$
$$\times \frac{\mathrm{d(L.I.P.S.)}\delta^4\left(P_\mathrm{final} - P_\mathrm{initial}\right)}{2s} + \mathcal{O}(\alpha_\mathrm{s}^n L^n) + \mathcal{O}(\alpha_\mathrm{s}^5),$$



Coherence violating logarithms

$$\frac{\mathrm{d}\sigma_1}{\mathrm{d}x_a \mathrm{d}x_b\, \mathrm{d}\mathcal{B}} = \frac{\alpha_s}{\pi}\int_{\mu_\mathrm{F}}^{Q}\frac{\mathrm{d}k_T}{k_T}\int_{k_T^2/2Q^2}^{1-k_T^2/2Q^2}\frac{\mathrm{d}z}{z}\,P_{qq}(z)\,u_0(k)\,f_B(x_b,Q)$$
$$\times\left[\Theta(z-x_a)\,f_A(x_a/z,\mu_\mathrm{F})\frac{1}{\mathbf{T}_a^2}\mathrm{Tr}(\mathbf{V}_{\mu_\mathrm{F},k_T}\mathbf{T}_a\mathbf{V}_{k_T,Q}\mathbf{H}\mathbf{V}^\dagger_{k_TQ}\mathbf{T}_a^\dagger\mathbf{V}^\dagger_{\mu_\mathrm{F},k_T}) - z^2 f_A(x_a,\mu_\mathrm{F})\mathrm{Tr}(\mathbf{V}_{\mu_\mathrm{F},Q}\mathbf{H}\mathbf{V}^\dagger_{\mu_\mathrm{F},Q})\right],$$

where

$$\mathbf{V}_{x,y} \approx \mathrm{Pexp}\left(\frac{\alpha_\mathrm{s}}{\pi}\sum_{i\neq j|i,j\in\{n\}}\mathbf{T}_i\cdot\mathbf{T}_j\int_x^y\frac{\mathrm{d}k_\perp^{(ij)}}{k_\perp^{(ij)}}\left[\int_{\text{on-shell}}\frac{\mathrm{d}y_k^{(ij)}\mathrm{d}\phi_k^{(ij)}}{4\pi}(1-u_n(k)) + \frac{i\pi}{2}\tilde{\delta}_{ij}\right]\right),$$

Very general. Studying this allowed us to observe that coherence is always violated once a computation is performed to high enough order.

5 gluons will always ensure coherence is violated, often 4 is sufficient.

Coherence violating logarithms

$$\frac{d\sigma_1}{dx_a dx_b d \cdots}$$

$$\times \Bigg[\Theta(z \cdots \cdots \mathbf{V}_{\mu_F,Q}\mathbf{H}\mathbf{V}^\dagger_{\mu_F,Q})\Bigg] ,$$

where

$$\mathbf{V} \cdots + \frac{i\pi}{2}\tilde{\delta}_{ij}\Big]\Big) ,$$

Very ge… e is
always … h order.

5 gluon… icient.