Grand Covariance in Quantum Gravity

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with Apostolos Pilaftsis and Sotirios Karamitsos

Wednesday 18$^{\mathrm{th}}$ December 2019

arXiv:1910.06661

Grand Covariance in Quantum Gravity

Outline

- Motivational Example
- Covariance
- Covariance in Gravity
- Grand Covariance in Gravity
- Grand Covariance in Quantum Gravity
- Summary

Complex Scalar Field

$$\mathcal{L} = \partial_\mu \phi^* \partial^\mu \phi - m^2 |\phi|^2 - \lambda |\phi|^4$$

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$$\mathcal{L} = \partial_\mu \phi^* \partial^\mu \phi - m^2 \left|\phi\right|^2 - \lambda \left|\phi\right|^4, \qquad \phi = \frac{1}{\sqrt{2}}\left(\phi_1 + i\phi_2\right)$$

Real and Imaginary Parts

$$\mathcal{L} = \tfrac{1}{2}\partial_\mu \phi_1 \partial^\mu \phi_1 + \tfrac{1}{2}\partial_\mu \phi_2 \partial^\mu \phi_2 - \tfrac{1}{2} m^2(\phi_1^2 + \phi_2^2) - \tfrac{\lambda}{4}(\phi_1^2 + \phi_2^2)^2$$

$$\langle \phi \rangle = \rho_0 \equiv \sqrt{\frac{-m}{\lambda}}, \qquad m_1 = m^2 + 3\lambda\rho_0^2$$

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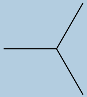
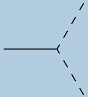
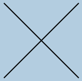
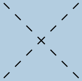
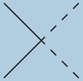
Feynman Rules

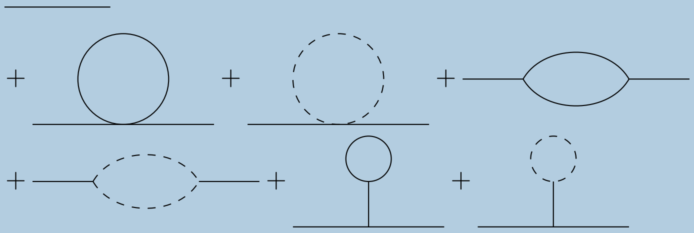
 $= \dfrac{i}{p^2 - m_1^2}$,  $= \dfrac{i}{p^2}$,

 $= -6i\lambda\rho_0$,  $= -2i\lambda\rho_0$,

 $= -6i\lambda$,  $= -6i\lambda$,  $= -2i\lambda$

Higgs Self-energy

$i\Gamma_{HH}(p) =$



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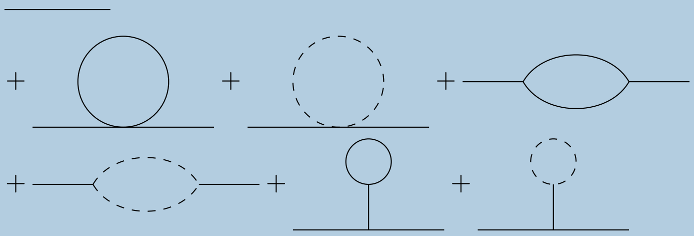

$$\Gamma_{HH}(p) = (p^2 - m_1^2) + \frac{\lambda m_1^2}{4\pi^2} \ln\left(\frac{p^2}{\mu^2}\right) + \frac{\lambda m_1^2}{(4\pi)^2}\left[4C_{\rm UV} - 4 - 9\int_0^1 dx \ln\left(\frac{x(x-1)p^2 + m_1^2}{\mu^2}\right)\right]$$

$$C_{UV} \equiv \frac{2}{4-D} - \gamma_{\rm E} + \ln(4\pi)$$

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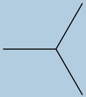
Modulus and Argument

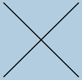
$$\mathcal{L} = \frac{1}{2} \partial_\mu \rho \partial^\mu \rho + \frac{1}{2} \left(\frac{\rho}{\rho_0} \right)^2 \partial_\mu \sigma \partial^\mu \sigma - \frac{1}{2} m^2 \rho^2 - \frac{\lambda}{4} \rho^4$$

Feynman Rules: Modulus and Argument

$$= \frac{i}{p^2 - m_1^2},$$



$$= -6 i \lambda \rho_0$$



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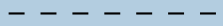
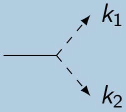
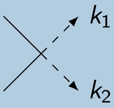


$$= \frac{i}{p^2}$$



$$= -\frac{2i}{\rho_0} k_1 \cdot k_2,$$



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Feynman Rules: Real and Imaginary Parts


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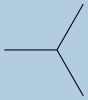
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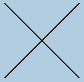
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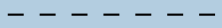
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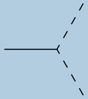
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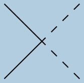
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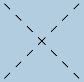
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Feynman Rules: Modulus and Argument


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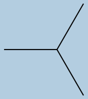
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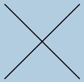
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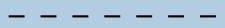
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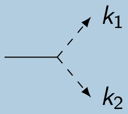

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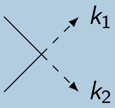

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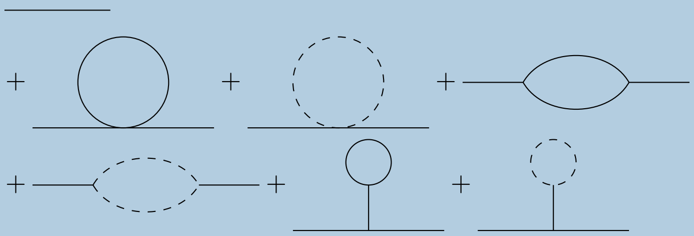

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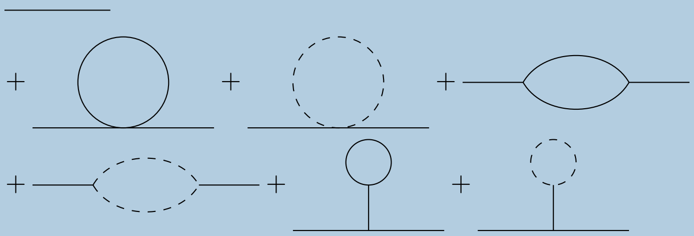

Higgs Self-Energy: Modulus and Argument

$$\Gamma_{HH}(p) = (p^2 - m_1^2) + \frac{3\lambda m_1^2 + \lambda \frac{p^4}{m_1^4}}{(4\pi)^2} \ln\left(\frac{p^2}{\mu^2}\right) + \frac{\lambda m_1^2}{(4\pi)^2}\left(3 + \frac{p^4}{m_1^4}\right) C_{\mathrm{UV}}$$
$$+ \frac{\lambda m_1^2}{(4\pi)^2}\left[2\frac{p^4}{m_1^4} - 6 - 9\int_0^1 dx \ln\left(\frac{x(x-1)p^2 + m_1^2}{\mu^2}\right)\right].$$

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Field Space Covariance

Field Theory

- Fields: ϕ^A
- Field Redefinitions: $\phi^A \to \tilde{\phi}^A(\boldsymbol{\phi})$

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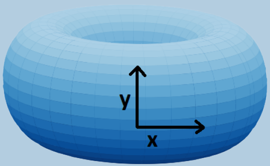

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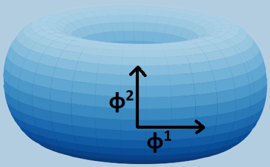

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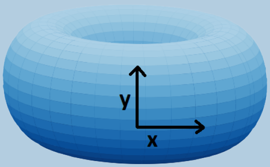

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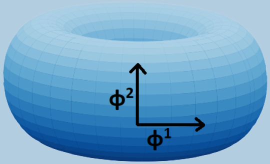

- Field-space tensors: S, V^A, T^{AB}

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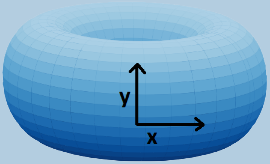


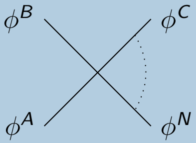
- Spacetime tensors: S, V^μ, $T^{\mu\nu}$

Are Feynman Diagrams Covariant?

Propagators

$$= \left(\frac{\partial^2 \mathcal{L}}{\partial \phi^A \partial \phi^B} \bigg|_{\langle \phi \rangle} \right)^{-1}$$

Vertices



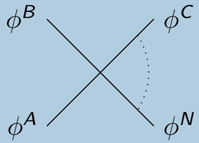
$$= \frac{\partial^N \mathcal{L}}{\partial \phi^A \dots \partial \phi^N} \bigg|_{\langle \phi \rangle}$$

Are Feynman Diagrams Covariant?

Propagators

$$\phi^A \bullet\!\!\longrightarrow\!\!\!\!-\!\!\bullet\, \phi^B \;=\; \left(\nabla_A \nabla_B \mathcal{L}\big|_{\langle\phi\rangle}\right)^{-1}$$

Vertices



$$= \nabla_A \ldots \nabla_N \mathcal{L}\big|_{\langle\phi\rangle}$$

Field Space Covariant Derivatives

- $\nabla_C X^A = \frac{\partial X^A}{\partial \phi^C} + \Gamma^A_{CD} X^D, \qquad \nabla_C X_A = \frac{\partial X_A}{\partial \phi^C} - \Gamma^D_{CA} X_D, \qquad \text{etc.}$
- Field-space Christoffels: $\Gamma^A_{BC} \equiv \frac{1}{2} G^{AD} \left[\frac{\partial G_{BD}}{\partial \phi^C} + \frac{\partial G_{DC}}{\partial \phi^B} - \frac{\partial G_{BC}}{\partial \phi^D} \right]$

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- Field-space Metric: $G_{AB} \equiv \frac{g_{\mu\nu}}{D} \frac{\partial^2 \mathcal{L}}{\partial(\partial_\mu \phi^A) \partial(\partial_\nu \phi^B)}$

Example: Scalar field Theory

$$\mathcal{L} = \frac{1}{2} k_{AB} \partial^\mu \phi^A \partial_\mu \phi^B - V(\phi)$$

$$\Downarrow$$

$$G_{AB} = k_{AB}$$

Complex Scalar Field

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$$\mathcal{L} = \tfrac{1}{2}\partial_\mu\phi_1\partial^\mu\phi_1 + \tfrac{1}{2}\partial_\mu\phi_2\partial^\mu\phi_2 - \tfrac{1}{2}m^2(\phi_1^2 + \phi_2^2) - \tfrac{\lambda}{4}(\phi_1^2 + \phi_2^2)^2$$

Field space Metric

$$G_{AB} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

Field space Christoffels

$$\Gamma^A_{BC} = 0$$

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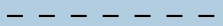
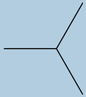
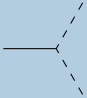
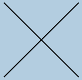
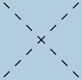
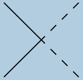
Covariant Feynman Rules: Real and Imaginary Parts

 $= \dfrac{i}{p^2 - m_1^2}$,  $= \dfrac{i}{p^2}$,

 $= -6i\lambda\rho_0$,  $= -2i\lambda\rho_0$,

 $= -6i\lambda$,  $= -6i\lambda$,  $= -2i\lambda$

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Modulus and Argument

$$\mathcal{L} = \tfrac{1}{2}\partial_\mu \rho \partial^\mu \rho + \tfrac{1}{2}\left(\frac{\rho}{\rho_0}\right)^2 \partial_\mu \sigma \partial^\mu \sigma - \tfrac{1}{2} m^2 \rho^2 - \tfrac{\lambda}{4}\rho^4$$

Field space Metric

$$G_{AB} = \begin{pmatrix} 1 & 0 \\ 0 & \left(\frac{\rho}{\rho_0}\right)^2 \end{pmatrix}$$

Field space Christoffels

$$\Gamma^\rho_{\sigma\sigma} = -\frac{\rho}{\rho_0^2},$$

$$\Gamma^\sigma_{\rho\sigma} = \frac{1}{\rho}$$

Complex Scalar Field

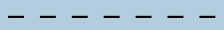
$$\mathcal{L} = \partial_\mu \phi^* \partial^\mu \phi - m^2 |\phi|^2 - \lambda |\phi|^4, \qquad \phi = \frac{1}{\sqrt{2}} \rho e^{i\frac{\sigma}{\rho_0}}$$

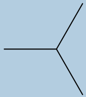
Modulus and Argument

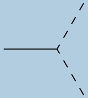
$$\mathcal{L} = \frac{1}{2}\partial_\mu \rho \partial^\mu \rho + \frac{1}{2}\left(\frac{\rho}{\rho_0}\right)^2 \partial_\mu \sigma \partial^\mu \sigma - \frac{1}{2} m^2 \rho^2 - \frac{\lambda}{4}\rho^4$$

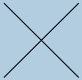
Covariant Feynman Rules: Modulus and Argument

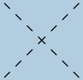
 $= \dfrac{i}{p^2 - m_1^2}$,

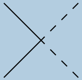
 $= \dfrac{i}{p^2}$,

 $= -6i\lambda\rho_0$,

 $= -2i\lambda\rho_0$,

 $= -6i\lambda$,

 $= -6i\lambda$,

 $= -2i\lambda$

Gravity

Scalar-Tensor Gravity

$$S = \int d^4x\sqrt{-g}\left[-\frac{f(\phi)}{2}R + \frac{1}{2}g^{\mu\nu}k_{AB}(\phi)\partial_\mu\phi^A\partial_\nu\phi^B - V(\phi)\right]$$

Field Redefinitions

- $g_{\mu\nu} \to \tilde{g}_{\mu\nu}(g_{\mu\nu}, \phi)$
- $\phi^A \to \tilde{\phi}^A(g_{\mu\nu}, \phi)$

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Frame Transformations

- $g_{\mu\nu} \to \Omega^2(\phi)g_{\mu\nu}$
- $\phi^A \to \tilde{\phi}^A(\phi)$

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Grand Field Space

- $\Phi^I = \begin{pmatrix} g^{\mu\nu} \\ \phi^A \end{pmatrix}$

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Spacetime in the Grand Field Space

Spacetime Line Element

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu$$

Conformal Transformation

$$g_{\mu\nu} \to \Omega^2(\phi) g_{\mu\nu} \quad \Rightarrow \quad ds^2 \to \Omega^2(\phi) ds^2$$

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Spacetime in the Grand Field Space

Spacetime Line Element

$$ds^2 = \bar{g}_{\mu\nu} dx^\mu dx^\nu$$

$$\bar{g}_{\mu\nu} = \frac{1}{\ell^2(\phi)} g_{\mu\nu}$$

Conformal Transformation

$$\begin{aligned} g_{\mu\nu} &\to \Omega^2 g_{\mu\nu} \\ \ell &\to \Omega \ell \end{aligned} \quad \Rightarrow \quad ds^2 \to ds^2$$

Grand Covariant Quantum Gravity

Action

$$S = \int d^4x \sqrt{-g} \left[-\frac{f(\phi)}{2} R + \frac{1}{2} g^{\mu\nu} k_{AB}(\phi) \partial_\mu \phi^A \partial_\nu \phi^B - V(\phi) \right] \equiv \int \sqrt{-g} \mathcal{L} d^4x$$

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Metric

- $G_{IJ} = \frac{\bar{g}_{\mu\nu}}{D} \frac{\partial^2 \bar{\mathcal{L}}}{\partial(\partial_\mu \Phi^I)\partial(\partial_\nu \Phi^J)} = \begin{pmatrix} \frac{1}{2}\ell^2 f\, P_{\mu\nu\rho\sigma} & -\frac{1}{2}\ell^2 f_{,B} g_{\mu\nu} \\ -\frac{1}{2}\ell^2 f_{,A} g_{\rho\sigma} & \ell^2 k_{AB} \end{pmatrix}$
- $P_{\mu\nu\rho\sigma} = \frac{1}{2}\left(g_{\mu\rho} g_{\sigma\nu} + g_{\mu\sigma} g_{\rho\nu} - g_{\mu\nu} g_{\rho\sigma} \right)$

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- $P_{\mu\nu\rho\sigma} = \frac{1}{2}\left(g_{\mu\rho}g_{\sigma\nu} + g_{\mu\sigma}g_{\rho\nu} - g_{\mu\nu}g_{\rho\sigma}\right)$

Invariant Path Integral Measure

$$\sqrt{\det\left(G_{IJ}\right)}\,[\mathcal{D}\Phi]$$

Summary

- The standard approach to quantum field theory is parametrisation dependent
- This can be rectified using field-space covariance
- The inclusion of gravity introduces a new subtlety regarding the spacetime metric
- We must therefore introduce a new model function $\ell(\phi)$ and redefine the spacetime metric as $\bar{g}_{\mu\nu} = \frac{1}{\ell^2} g_{\mu\nu}$
- We can then construct a fully grand covariant theory of quantum gravity