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Sub-Planckian ϕ^2 Inflation with an R^2 term in Palatini Gravity

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ARXIV: IN PREPARATION

Introduction and Motivation

The aim of this work is to investigate further the ϕ^2 inflation model with an R^2 term in the Palatini formalism as proposed by Enckell et al (arXiv: 1810.05536) and Antoniadis et al (arXiv: 1810.10418).

The objectives of this work are as follows:

- Investigate whether the Palatini $\phi^2 R^2$ model can also be consistent with a sub-Planckian inflaton and Planck-suppressed potential corrections.
- More generally, study the dynamics of inflation to show that the model is a viable model of cosmology which is consistent with observations, specifically successful reheating and bounds on the scalar spectral index.

What is ϕ^2 Inflation?

- Originally considered in a chaotic inflation framework in conventional gravity
- Minimal potential:

$$V(\phi) \approx \frac{1}{2} m^2 \phi^2$$

- Good prediction of scalar spectral index, $n_s = 1 - \frac{2}{N} = 0.964$
- Unacceptably large prediction of tensor to scalar ratio, $r = \frac{8}{N} = 0.15$
- Super-Planckian inflaton field needed to successfully complete inflation ($N = 55$), $\phi \approx 15 M_{pl}$

Palatini Formulation of Gravity

- In General relativity we define the Levi-Civita connection, an object which depends on the metric and derivatives of the metric

$$\Gamma^{\sigma}_{\mu\rho} = \frac{1}{2} g^{\sigma\nu} \left(\partial_{\mu} g_{\nu\rho} + \partial_{\rho} g_{\mu\nu} - \partial_{\nu} g_{\mu\rho} \right)$$

- In the Palatini formalism, the metric and the connection are defined independently as variables in the action.
- In a minimal gravity framework, the Palatini formulation is equivalent to GR at the level of the equations of motion and the connection has Levi-Civita form.
- Anything beyond minimal gravity and the two formalisms are distinctly different.

The $\phi^2 R^2$ Model

- We start with the following action in the Jordan frame

$$S = \int d^4x\sqrt{-g}\left[\frac{M_{pl}^2}{2}R + \frac{\alpha}{4}R^2 - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi)\right]$$

$$V(\phi) = \lambda_2 M_{pl}^2\phi^2$$

- The Jordan frame action can be written in terms of an auxiliary field χ

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{2}R(M_{pl}^2 + \alpha\chi^2) - \frac{\alpha}{4}\chi^4 + \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi)\right]$$

- The minimal gravity term can be rewritten as:

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{2}M_{pl}^2\Omega^2 R - \frac{\alpha}{4}\chi^4 - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi)\right]$$

$$\Omega^2 = 1 + \frac{\alpha\chi^2}{M_{pl}^2}$$

Conformal transformation

- We can think of the Jordan frame as the "model" frame where the structure and symmetries of the model are defined. The Einstein frame is the "physics" frame, where conventional physics applies and we perform our calculations.
- The transformation from the Jordan frame to the Einstein frame is made via a conformal transformation on the metric

$$g_{\mu\nu} \longrightarrow \tilde{g}_{\mu\nu} = \Omega^2 g_{\mu\nu}$$

$$g^{\mu\nu} \longrightarrow \tilde{g}^{\mu\nu} = \frac{1}{\Omega^2} g^{\mu\nu}$$

- The conformal factor in this case is: $\Omega^2 = 1 + \frac{\alpha\chi^2}{M_{pl}^2}$

The significance of Metric vs Palatini

- Transforming the metric also means that anything in the action which depends on the metric must also transform
- This includes the Ricci scalar, which we still have in the minimal coupling to gravity.
- The Ricci scalar comes from the Riemann tensor
- The Riemann tensor is built from products of the connection and its derivatives

$$R^{\sigma}_{\mu\rho\nu} = R^{\sigma}_{\mu\rho\nu}\left(\Gamma, \partial\Gamma\right)$$

The significance of Metric vs Palatini

- This dependence is carried through into the Ricci tensor, and subsequently the Ricci scalar

$$R^{\sigma}_{\mu\rho\nu} \longrightarrow R^{\sigma}_{\mu\sigma\nu} = R_{\mu\nu}\left(\Gamma, \partial\Gamma\right)$$

$$g^{\mu\nu} R_{\mu\nu} = R^{\nu}_{\nu}\left(\Gamma, \partial\Gamma\right) \equiv R$$

- The Ricci tensor is transformed from the Jordan frame to the Einstein frame in the following way

$$g^{\mu\nu} R_{\mu\nu} \longrightarrow \tilde{g}^{\mu\nu} \tilde{R}_{\mu\nu} = \frac{1}{\Omega^2} g^{\mu\nu} \tilde{R}_{\mu\nu}\left(\tilde{\Gamma}, \partial\tilde{\Gamma}\right) = \frac{1}{\Omega^2} \tilde{R}^{\nu}_{\nu} = \frac{1}{\Omega^2} \tilde{R}\left(\tilde{\Gamma}, \partial\tilde{\Gamma}\right)$$

The significance of Metric vs Palatini

- In the metric formalism we have that the connection is the Levi-Civita connection, therefore it depends on the metric and derivatives of the metric

$$\Gamma = \Gamma\left(g_{\mu\nu}, \partial g_{\mu\nu}\right) \qquad \Gamma^{\sigma}_{\mu\rho} = \frac{1}{2} g^{\sigma\nu}\left(\partial_{\mu} g_{\nu\rho} + \partial_{\rho} g_{\mu\nu} - \partial_{\nu} g_{\mu\rho}\right)$$

- The Levi-Civita connection must therefore transform

$$\tilde{\Gamma}^{\sigma}_{\mu\rho} = \Gamma^{\sigma}_{\mu\rho} + \frac{1}{\Omega} g^{\sigma\nu}\left[g_{\nu\mu}\partial_{\rho}\Omega + g_{\rho\nu}\partial_{\mu}\Omega - g_{\mu\rho}\partial_{\nu}\Omega\right] \qquad R^{\sigma}_{\mu\rho\nu}\left(\Gamma, \partial\Gamma\right) \longrightarrow \tilde{R}^{\sigma}_{\mu\rho\nu}\left(\tilde{\Gamma}, \partial\tilde{\Gamma}\right)$$

- In four dimensions the transformation on the Ricci scalar is (Kaiser, arXiv: 1003.1159)

$$\tilde{R} = \frac{1}{\Omega^2}\left[R - \frac{6}{\Omega}\partial_{\mu}\partial^{\mu}\Omega\right]$$

- As a result of this transformation, in the metric formalism we get an additional kinetic term in the Einstein frame action.

The significance of Metric vs Palatini

- In the Palatini formalism the connection does not depend on the metric

$$\Gamma \neq \Gamma\left(g_{\mu\nu}, \partial g_{\mu\nu}\right) \qquad \therefore \Gamma = \tilde{\Gamma}$$

- This means that the only dependence we have to worry about during the transformation on the Ricci tensor is the metric factor used in the contraction

$$\tilde{R}^{\sigma}_{\mu\rho\nu}\left(\tilde{\Gamma}, \partial\tilde{\Gamma}\right) = R^{\sigma}_{\mu\rho\nu}\left(\Gamma, \partial\Gamma\right) \qquad \tilde{R}_{\mu\nu} \longrightarrow \tilde{g}^{\mu\nu}\tilde{R}_{\mu\nu} = \frac{1}{\Omega^2} g^{\mu\nu}\tilde{R}_{\mu\nu} = \frac{1}{\Omega^2}\tilde{R}^{\nu}_{\nu}$$

- Hence in Palatini the transformation on the Ricci scalar is as follows

$$\tilde{R} = \frac{1}{\Omega^2} R$$

- And we get no additional kinetic term in the Einstein frame action.

Jordan frame transformation

- The Jordan frame action from earlier is

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{2}M_{pl}^2\Omega^2 R - \frac{\alpha}{4}\chi^4 - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi)\right]$$

- Transforming to the Einstein frame gives

$$S_E = \int d^4x\sqrt{-\tilde{g}}\left[\frac{M_{pl}^2}{2}\tilde{R} - \frac{\alpha\chi^4}{4\Omega^4} - \frac{1}{2\Omega^2}\partial_\mu\phi\partial^\mu\phi - \frac{V(\phi)}{\Omega^4}\right]$$

Einstein frame action

- Eliminating χ via its equations of motion gives the following Einstein frame action

$$S_E = \int d^4x\sqrt{-\tilde{g}}\left[\frac{1}{2}M_{pl}^2\tilde{R} - \frac{1}{2}\frac{\partial^\mu\phi\partial_\mu\phi}{\left(1+\frac{4\alpha V(\phi)}{M_{pl}^4}\right)} + \frac{\alpha}{4M_{pl}^4}\frac{(\partial_\mu\phi\partial^\mu\phi)^2}{\left(1+\frac{\alpha V(\phi)}{M_{pl}^4}\right)} - \frac{V(\phi)}{\left(1+\frac{4\alpha V(\phi)}{M_{pl}^4}\right)}\right]$$

- The ∂^4 terms can be neglected for now until we examine unitarity violation later.
- We rewrite the remaining kinetic term in terms of the canonically normalised scalar

$$\left(\frac{d\sigma}{d\phi}\right)^2 = \frac{1}{1+\frac{4\alpha\lambda_2\phi^2}{M_{pl}^2}} \Rightarrow \frac{d\sigma}{d\phi} = \pm\frac{1}{\sqrt{1+\frac{4\alpha\lambda_2\phi^2}{M_{pl}^2}}}$$

$$\sigma = \frac{M_{pl}}{\sqrt{4\alpha\lambda_2}}\ln\left(\frac{2\sqrt{4\alpha\lambda_2}\phi}{M_{pl}}\right)$$

Einstein frame potential

- The Einstein frame potential is given by:

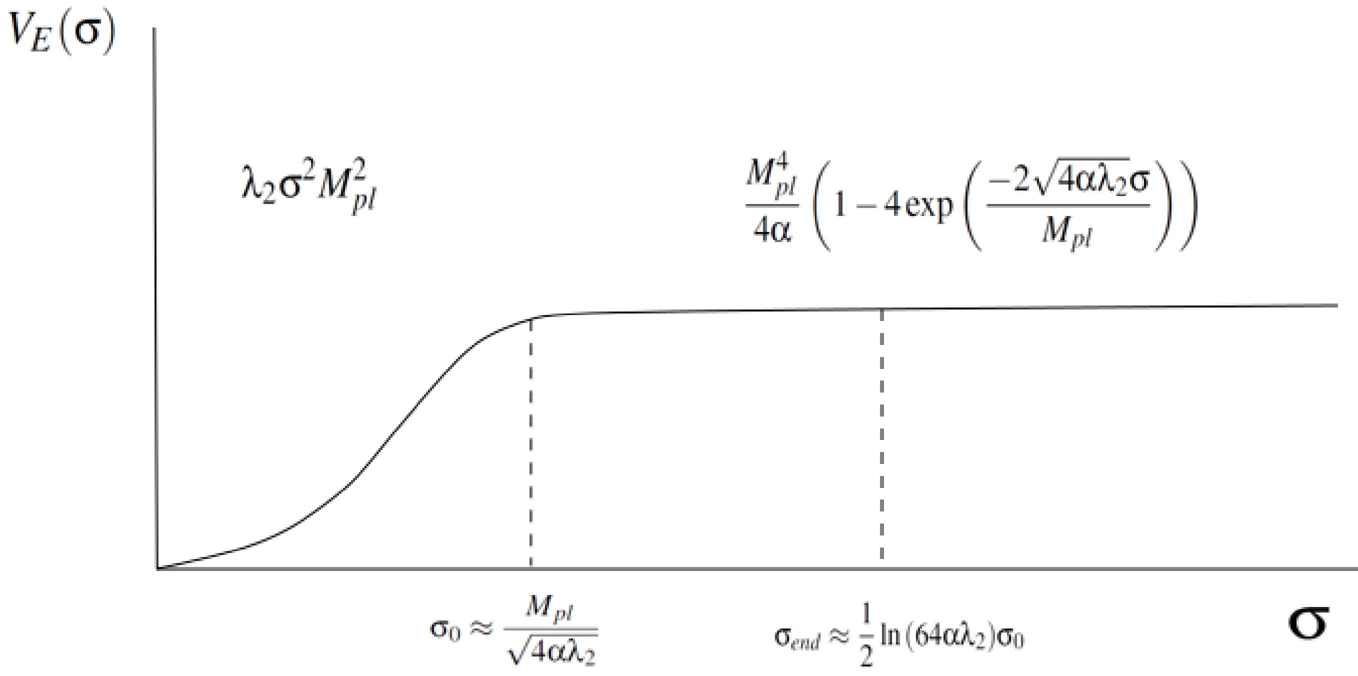
$$V_E(\phi) = \frac{V(\phi)}{1 + \frac{4\alpha V(\phi)}{M_{pl}^4}}$$

- Working in the limit

$$\frac{4\alpha V}{M_{pl}^4} >> 1 \quad \Rightarrow \quad V_E(\phi) \approx \frac{M_{pl}^4}{4\alpha}\left(1 - \frac{M_{pl}^2}{4\alpha\lambda_2\phi^2}\right)$$

- In terms of the canonically normalised field this is

$$V_E(\sigma) = \frac{M_{pl}^4}{4\alpha}\left(1 - 4\exp\left(\frac{-2\sqrt{4\alpha\lambda_2}\sigma}{M_{pl}}\right)\right)$$



Schematic illustrating the general shape and plateau region of the Einstein frame potential

Inflationary Analysis

- The number of e-folds of inflation in the Einstein frame is given by

$$N(\sigma) = -\frac{1}{M_{pl}^2}\int_{\sigma}^{\sigma_{end}} \frac{V_E}{V_E'}\,d\sigma = \frac{1}{64\alpha\lambda_2}\exp\left(\frac{2\sqrt{4\alpha\lambda_2}}{M_{pl}}\sigma\right)$$

- The field can be written as

$$\sigma(N) = \frac{M_{pl}}{2\sqrt{4\alpha\lambda_2}}\ln(64\alpha\lambda_2 N) \quad \Rightarrow \quad \phi(N) = 2\sqrt{N}M_{pl}$$

- This allows the slow roll parameters and inflationary observables to be calculated, as well as constraints to be made on the model.

Slow roll parameters and observables

- The slow roll parameters can all be calculated in terms of the number of e-folds:

$$\varepsilon = \frac{M_{pl}^2}{2}\left(\frac{\frac{\partial V_E}{\partial \sigma^2}}{V_E}\right)^2 \Rightarrow \varepsilon = \frac{1}{32\alpha\lambda_2}\frac{1}{N^2} \qquad \eta = M_{pl}^2\frac{\frac{\partial^2 V_E}{\partial \sigma^2}}{V_E} \Rightarrow \eta = -\frac{1}{N}$$

- Scalar spectral index: $n_s = 1 + 2\eta - 6\varepsilon = 1 - \frac{2}{N} - \frac{3}{16\alpha\lambda_2}\frac{1}{N^2}$

- Tensor to scalar ratio: $r = 16\varepsilon = \frac{1}{2\alpha\lambda_2}\frac{1}{N^2} \quad (\ll 1)$

- Power Spectrum: $P_R = \frac{V_E}{24\pi^2 \varepsilon M_{pl}^4} = \frac{\lambda_2 N^2}{3\pi^2} \quad (\Rightarrow \lambda_2 \simeq 1.7\times 10^{-11})$

Sub-Planckian Inflation

- In addition to an inflation model consistent with the observed tensor to scalar ratio we wanted to check whether the model would remain consistent, and be successful if

1. The inflaton were constrained to remain sub-Planckian during inflation
2. Non-renormalisable corrections to the potential-such as those expected to appear in a UV completion of quantum gravity-were introduced

Sub-Planckian inflaton

In order for σ to remain sub-Planckian we require that

$$\sigma(N) < M_{pl}$$

Substituting our expression for σ(N) $\Rightarrow$

$$\frac{M_{pl}}{\sqrt{4\alpha\lambda_2}} \ln\left(4\sqrt{4\alpha\lambda_2 N}\right) < M_{pl} \Rightarrow \sqrt{4\alpha\lambda_2} > \frac{1}{2}\ln(64\alpha\lambda_2 N)$$

From here we substitute our values of N and $\lambda_2 \simeq 1.7 \times 10^{-11}$ to find a constraint on α

$$\alpha \gtrsim 10^{12}$$

Potential corrections

- In addition to the issue of a super-Planckian inflaton we must also consider how any potential corrections due to some UV completion of quantum gravity would affected inflationary observables.
- To investigate this we consider non-renormalizable corrections to the potential of the form

$$\Delta V_E = \sum_n \frac{k_n \sigma^n}{M_{pl}^{n-4}}$$

- Assuming a $\sigma \to -\sigma$ symmetry carried over from the $\phi \to -\phi$ symmetry of the Jordan frame, the lowest order correction we consider is

$$\Delta V_E = \frac{k\sigma^6}{M_{pl}^2}$$

where $k \sim 1$.

η-shift from unsuppressed Planck corrections

- The following quantum corrections are made to the potential

$$V_{TOT} \equiv V_E + \Delta V_E = V_E(\sigma) + \frac{k\sigma^6}{M_{pl}^2}$$

- In order to derive a constraint on α from this, we calculate the shift on η due to the addition to the extra corrections to the potential

$$\Delta\eta \approx M_{pl}^2 \frac{\Delta V_E''}{V_E} \Rightarrow \Delta\eta \approx \frac{120k\alpha}{M_{pl}^4}\sigma^4.$$

- In order to retain agreement with the scalar spectral index measurements we require that

$$|\Delta\eta| \leq 0.01$$

- This constrains α to be

$$\alpha \gtrsim 1.2 \times 10^{30}$$

η-shift with a shift symmetry

- We also consider the constraint on α from the η shift in the case of a shift symmetry, $\phi \rightarrow \phi + \text{constant}$, which is broken by the m^2 term (where $m^2 = 2\lambda_2 M^2{}_{pl}$ is the inflaton mass) in the renormalisable potential (for a review see Baumann & McAllister, arXiv:1404.2601).

- Non-renormalisable corrections should vanish in the limit that $m^2 \rightarrow 0$, so any non-renormalisable potential corrections should be proportional to the mass squared

$$\Delta V_E = \frac{k\sigma^6}{M_{pl}^2}$$

- Previously $k{\sim}1$ we now have:

$$k \approx \frac{m^2}{M_{pl}^2} \quad \Rightarrow \quad \Delta V_E \approx \left(\frac{m^2}{M_{pl}^2}\right)\frac{\sigma^6}{M_{pl}^2}$$

- In order to be consistent with predictions on n_s in this case we require that: $\alpha \gtrsim 10^{19}$

Sub-Planckian Inflation Conclusion

For sufficiently large α:

- The inflaton in the Palatini $\phi^2 R^2$ model can be sub-Planckian
- The model can be completely consistent with the effect of Planck-suppressed potential corrections.

Post-inflation cosmology

- In order to be a complete model of cosmology, the model must reheat successfully and be consistent with the observed scalar spectral index
- An upper bound on the reheating temperature is given by the instantaneous reheating temperature

$$T_{R\,max} = \left(\frac{30}{\pi^2 g\left(T_{R\,max}\right)}\right)^{\frac{1}{4}} \left(3M_{pl}^2 \tilde{H}^2\right)^{\frac{1}{4}} = \left(\frac{15}{2\pi^2 g(T_{R\,max})}\right)^{1/4} \frac{M_{pl}}{\alpha^{1/4}}$$

- In order to calculate n_s we need the number of e-folds at the Planck pivot scale ($k = 0.05Mpc^{-1}$), which is given by

$$N = \ln\left[\left(\frac{g_s\left(T_0\right)}{g_s\left(T_{end}\right)}\right)^{\frac{1}{3}} \left(\frac{\pi^2 g\left(T_{end}\right)}{1080\alpha}\right)^{\frac{1}{4}} T_0 \lambda_0\right] \qquad \lambda_0 = \frac{2\pi}{k}$$

- Substituting in some numbers gives the following relation for N(α)

$$N = \ln\left[\frac{1.6 \times 10^{27}}{\alpha^{\frac{1}{4}}}\right] = 62.63 - \frac{1}{4}\ln\left(\alpha\right)$$

Results

- Below are the results calculated for each α

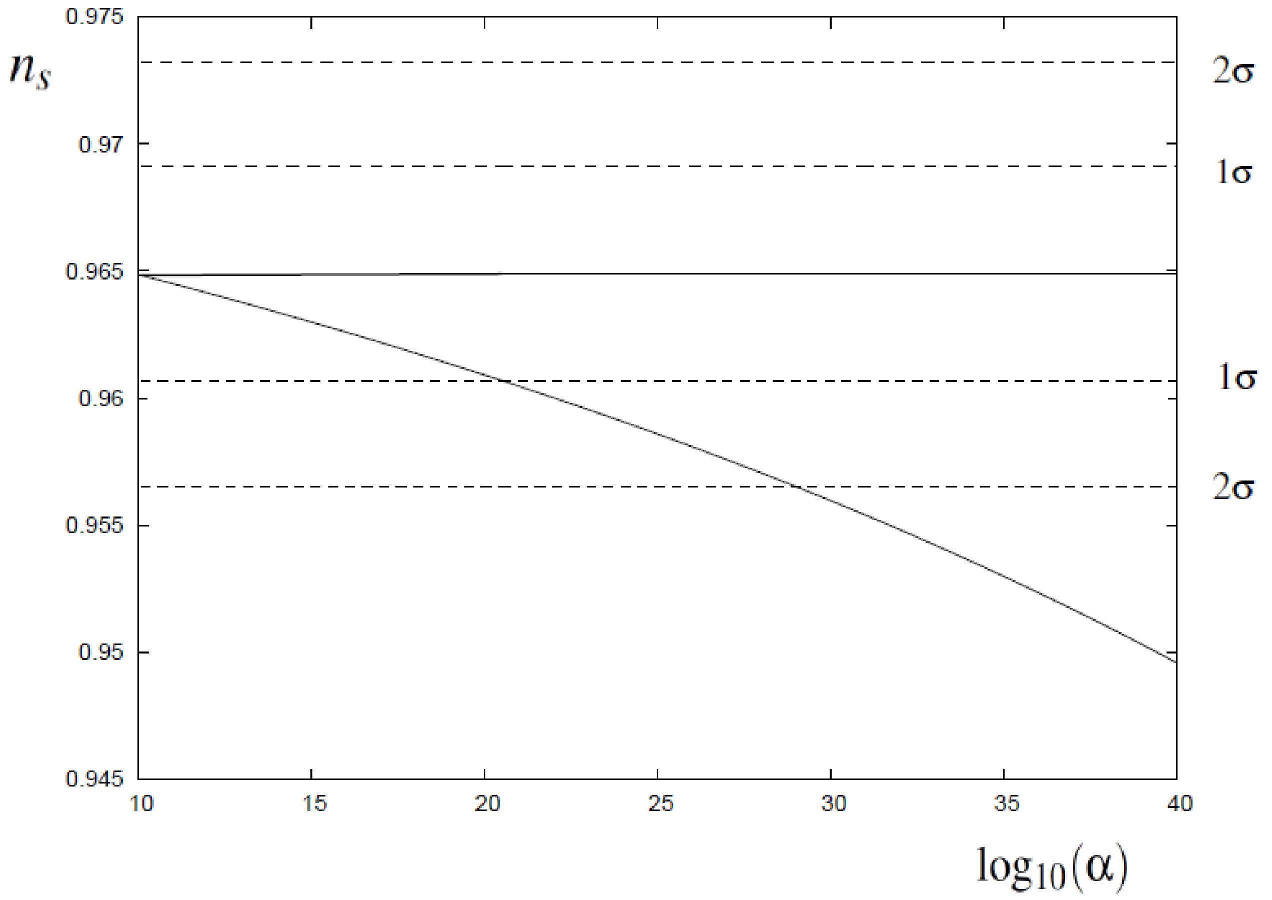
α	n_s	r	N	T_R/GeV
1.0×10^{12}	0.9641	8.33×10^{-6}	55.7	7.22×10^{14}
1.0×10^{19}	0.9613	9.6×10^{-13}	51.7	1.28×10^{13}
1.25×10^{30}	0.9558	9.83×10^{-24}	45.3	2.17×10^{10}

- The tensor to scalar ratio is suppressed, the highest value $\sim10^{-6}$ is much smaller than the observable limit of future experiments ($r\sim10^{-3}$).
- Scalar spectral index values are in agreement with Planck bounds, the first two to within 1σ

$$n_s = 0.9649 \pm 0.0042 \qquad (1\sigma)$$

- The model gives an adequately large estimate on reheating temperature for BBN, $T_R \gg MeV$.
- These results show that the Palatini $\phi^2 R^2$ model can be a viable model of inflation which is consistent with quantum gravity.

Plot of n_s vs α



Unitarity Violation

- The term in the action responsible for unitarity violation in the Einstein frame is

$$\frac{\alpha}{4}\frac{(\partial_\mu\sigma\partial^\mu\sigma)^2}{M_{pl}^4}\left(1+\frac{4\alpha V(\phi)}{M_{pl}^4}\right)$$

- Expanding this for the fluctuations of the canonical scalar field about a classical inflaton background

$$\sigma=\bar{\sigma}(t)+\delta\sigma$$

- We find that the dimensional scattering amplitude for $\delta\sigma\delta\sigma\rightarrow\delta\sigma\delta\sigma$ scattering is

$$|\mathcal{M}|\sim\frac{\alpha}{4}\frac{\tilde{E}^4}{M_{pl}^4}\left(1+\frac{4\alpha V(\bar{\phi})}{M_{pl}^4}\right)$$

Unitarity Violation continued

- Unitarity is violated in the Einstein frame when

$$|\mathcal{M}| \gtrsim 1.$$

- This occurs when the interaction energy exceeds the unitarity cutoff, given by

$$\tilde{\Lambda} \approx \frac{M_{pl}}{\alpha^{1/2}(4\lambda_2 N)^{1/4}}$$

- The basic condition for unitarity conservation during inflation is given by

$$\tilde{H} < \tilde{\Lambda}$$

$$\tilde{H} = \left(\frac{V_E}{3M_{pl}^2}\right)^{\frac{1}{2}} = \frac{M_{pl}}{\sqrt{12\alpha}}$$

- This translates to the constraint:

$$\lambda_2 N \lesssim 36$$

- $\lambda_2 \approx 10^{-11}$ in this model, so this constraint is easily satisfied

Conclusion

We conclude that the Palatini $\phi^2 R^2$ inflation model, based on a minimal ϕ^2 potential, can successfully account for inflation, and for sufficiently large α can provide

- Sufficiently small predicted tensor to scalar ratio
- Agreement with Planck bounds on scalar spectral index, two results to within 1σ
- Adequate reheating temperature for BBN
- Successful inflation with a sub-Planckian inflaton.
- Complete consistency in the presence of Planck-suppressed potential corrections from a quantum gravity UV completion.
- Unitarity safety from $\delta\sigma\delta\sigma \rightarrow \delta\sigma\delta\sigma$ scattering processes during inflation

Future Work

Study of reheating, specifically into different mechanisms

- Reheating via Higgs portal coupling

$$\frac{\lambda_{\phi H}}{2}\phi^2|H|^2.$$

- Reheating via decay to right handed neutrinos $\frac{\lambda_{\phi N}}{2}\phi\bar{N}_R^c N_R.$

- Consistency of the model with each reheating channel

- Condensate Fragmentation

If you are interested in the original Palatini $\phi^2 R^2$ model, see:

- Enckell et al; arXiv: 1810.05536

- Antoniadis et al; arXiv: 1810.10418

Thanks for listening! Please feel free to ask questions

Reheating and Quantum corrections

- In this model consider two reheating channels. The Higgs portal coupling and the coupling to an extension of right handed neutrinos

$$\frac{\lambda_{\phi H}}{2}\phi^2|H|^2 + \frac{\lambda_{\phi N}}{2}\phi \bar{N}_R^c N_R$$

- In order to check whether the addition of these couplings affects the consistency of the model we consider the 1-loop Coleman-Weinberg corrections to the effective potential

$$V_{TOT} = V(\phi) + \Delta V_{CW}(\phi) \quad \Rightarrow \quad \Delta V_{CW}(\phi) = \sum_i \pm \frac{M_i^4(\phi)}{64\pi^2} \ln\left(\frac{M_i(\phi)^2}{\mu^2}\right)$$

- As with the Planck-suppressed potential corrections we check the η-shift due to these corrections to ensure that the reheating dynamics of the model remain compatible with the predictions on n_s.

Potential Regimes

- Integrating the canonical scalar relation gives the following exact solution for σ

$$\sigma = \frac{1}{\sqrt{K}} \ln\left(\sqrt{1+K\phi^2} + \sqrt{K}\phi\right) + C \;\; ; \;\; K = \frac{4\alpha\lambda_2}{M_{pl}^2} \;, \qquad \Longrightarrow \qquad \sigma = \phi_0 \ln\left(\sqrt{1+\frac{\phi^2}{\phi_0^2}} + \frac{\phi}{\phi_0}\right)$$

- σ is scaled such that σ≈ϕ in the limit where $\phi < \phi_0$.
- ϕ_0 is defined as:

$$\phi_0 = \frac{M_{pl}}{\sqrt{4\alpha\lambda_2}}$$

- This corresponds to an era close to the end of inflation where $\Omega \approx 1$.

Potential Regimes

- For $\phi < \phi_0$:

$$\sigma \approx \phi$$

- For $\phi > \phi_0$:

$$\sigma \approx \phi_0 \ln\left(\frac{2\phi}{\phi_0}\right) = \frac{M_{pl}}{\sqrt{4\alpha\lambda_2}} \ln\left(\frac{2\sqrt{4\alpha\lambda_2}\phi}{M_{pl}}\right)$$