Resurgence in the Bi-Yang-Baxter Model

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Previous work by Demulder, Dorigoni, Thompson (2016) and Cherman, Dorigoni, Dunne, Ünsal (2014).

Outline

The Principal Chiral Model (PCM)

- Non-linear, **integrable** (i.e. there exists a flat Lax connection) σ-model on the world-sheet.
- Fields are maps from the worldsheet into a Lie group: $g : \Sigma \to G = SU(N)$.
- Action:

$$S = \frac{1}{2\pi t} \int dz^2 \mathrm{Tr} \left(g^{-1} \partial_+ g g^{-1} \partial_- g \right) .$$

- $z_\pm = t \pm ix$ light-cone coordinates on the worldsheet $\partial_\pm = \partial_{z_\pm}$.
- $\partial_+ g \in \mathfrak{g}$ Lie algebra valued.
- $G^L \times G^R$ global symmetry.

Yang-Baxter Deformations

- Let R be the Yang-Baxter (YB) Operator on $\mathfrak{g}$ that satisfies

$$[RA, RB] - R([RA, B] + [A, RB]) = [A, B].$$

- On SU(2), in the basis $\sigma_i/\sqrt{2}$:

$$R = \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

- bi-Yang-Baxter (bYB) deformed PCM:

$$S = \frac{1}{2\pi t} \int dz^2 \mathrm{Tr} \left(g^{-1} \partial_+ g \frac{1}{1 - \eta R - \zeta R^g} g^{-1} \partial_- g \right).$$

- $\mathrm{Ad}_g(u) = gug^{-1}$, $R^g = \mathrm{Ad}_{g^{-1}} \circ R \circ \mathrm{Ad}_g$.
- $\eta, \zeta \in \mathbb{R}$ parameters that squish $S^3 \simeq SU(2)$.
- **preserves integrability** (Klimcik, 2014).

Unitons

- Parametrise $g \in SU(2)$ by Hopf coordinates:

$$g = \begin{pmatrix} \cos\theta e^{i\phi_1} & i\sin\theta e^{i\phi_2} \\ i\sin\theta e^{-i\phi_2} & \cos\theta e^{-i\phi_1} \end{pmatrix}.$$

- There exists a solution to the equation of motions in terms of a holomorphic function $f(z)$:

$$\phi_1 = \frac{\pi}{2}, \quad \phi_2 = \pi + \frac{i}{2}\log\left(\frac{f}{\bar{f}}\right),$$

$$\cos\theta(|f|^2) = \frac{\sqrt{1+(\eta-\zeta)^2}(|f|^2-1)}{\sqrt{(1+|f|^2)^2+(\eta-\zeta)^2(1-|f|^2)^2}}.$$

- **“Uniton”**.

Unitons (continued)

- Finite, quantised action configuration:

$$S = \frac{2k}{t(1+(\zeta+\eta)^2)} S_I,$$

$$S_I = \frac{(1+(\zeta+\eta)^2)}{2\zeta\eta}[(\zeta-\eta)\arctan(\zeta-\eta) - (\zeta+\eta)\arctan(\zeta+\eta)].$$

- $k \in \mathbb{R}$ order of the polynomial $f(z)$.

$\rightarrow$ Analogous complex uniton.

Dimensional Reduction

- Compactify along the x-direction by a twisted boundary condition

$$g(t, x + L) = e^{iH_L} g(t, x) e^{-iH_R}, \quad H_L = H_R = \begin{pmatrix} \frac{\pi}{2} & 0 \\ 0 & \frac{\pi}{2} \end{pmatrix}.$$

$\rightarrow$ Integrate out heavy Kaluza-Klein modes.

$\rightarrow$ Obtain an effective Quantum Mechanics for θ.

Schrödinger System

- Schrödinger Equation for θ:

$$H = \frac{g^2}{4} p_\theta^2 + \frac{1}{g^2} V(\theta), \quad g = t(1 + (\zeta + \eta)^2),$$

with

$$V(\theta) = \frac{\pi^2 \mathrm{sn}^2(\theta)(1 + (\zeta - \eta)^2 \mathrm{sn}^2(\theta))}{\mathrm{dn}^2(\theta)}, \quad m = \frac{4\zeta\eta}{1 + (\zeta + \eta)^2}.$$

- $\mathrm{sn}(z)$, $\mathrm{cn}(z)$ and $\mathrm{dn}(z)$ are the **Jacobi elliptic functions** depending on the elliptic modulus m.
- $\zeta = 0 \implies m = 0 \implies \mathrm{sn}(z) = \sin(z), \mathrm{dn}(z) = 1.$

Borel Analysis

- Use the WKB method to obtain a perturbative expansion for the ground state energy:

$$E(g^2) = \sum_{n=0}^{\infty} a_n g^{2n} \quad a_n = a_n(\zeta, \eta).$$

- This series is divergent: $a_n \sim n!$. **asymptotic expansion**.
- Borel transform:

$$B[E](g^2) = \sum_{n=0}^{\infty} \frac{a_n}{n!} g^{2n}.$$

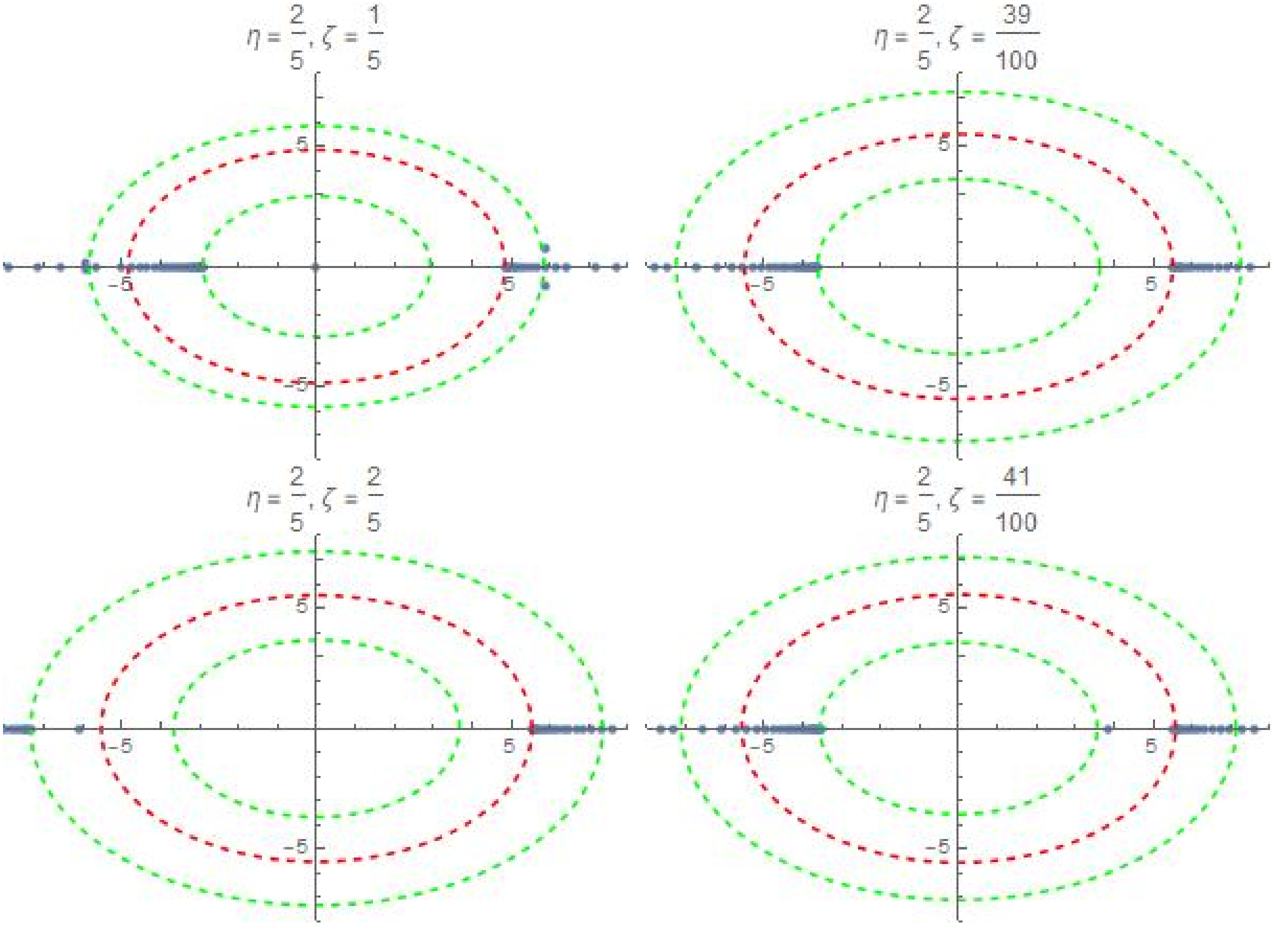
- Borel resummation

$$S[E](g^2) = \int_0^{\infty} dz B[E](g^2) e^{-z/g^2} \sim E(g^2).$$

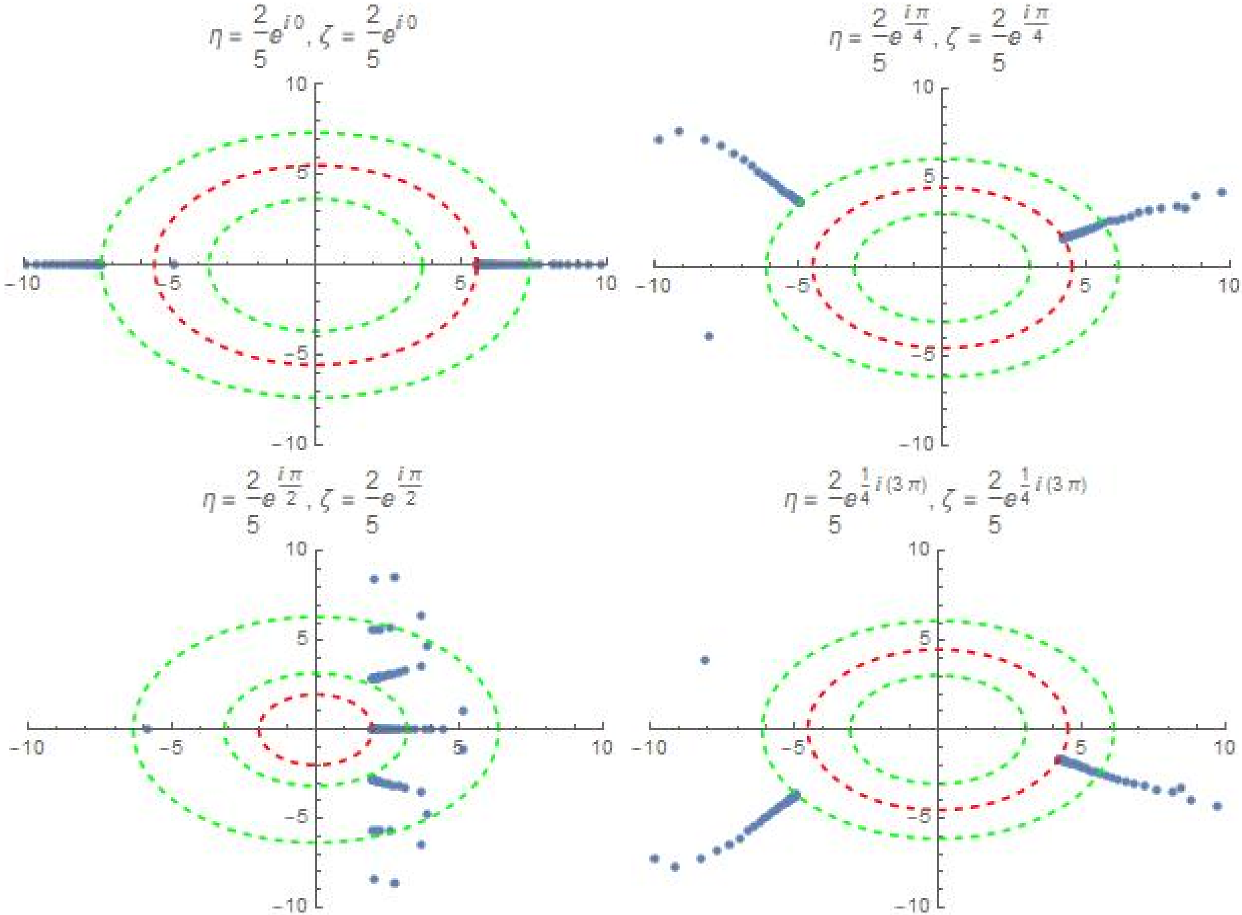
Borel Analysis (Continued)

- Poles in the Borel plane obstruct this integral along certain rays and are due to **non-perturbative** effects: Unitons!
- Non-perturbative effects go as e^{-2S_i/g^2}.
- Use Bender-Wu package (Sulejmanpasic and Ünsal, 2016) to obtain 150-200 orders of perturbation theory ($\sim$ 10 minutes per plot).
- Use Padé approximant to find poles and accumulation points.

Poles in the Borel Plane (1)



Poles in the Borel Plane (2)



Seiberg-Witten (SW) Curve

- Consider the quadratic differential of the low-energy effective theory of $\mathcal{N} = 2$ Seiberg-Witten theory.
- $SU(2)$ with $N_f = 4$ and pairwise equal masses.
- Trigonometric limit of $SU(2) \times SU(2)$ elliptic quiver theory with $N_f = 2$.
- Coincides with our WKB-differential!
- BPS states = Unitons?
- Connection with integrable Hitchin system through Gaudin Models??

Questions