Recursion Relations for Anomalous Dimensions in the 6d (2,0) Theory

Theresa Abl

Durham University

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based on [1902.00463] with Paul Heslop and Arthur Lipstein

Motivation

- M-theory – combines all five 10-dimensional superstring theories
- M2 and M5 branes
- Low-energy limit: 11d supergravity

Goal: Find M-theory corrections to the low-energy effective action

- Strongly coupled $\rightarrow$ AdS/CFT correspondence
- M-theory in $AdS_7 \times S^4$ dual to 6d (2,0) theory
- No Lagrangian known $\rightarrow$ conformal bootstrap
- Derive recursion relations for anomalous dimensions of double-trace operators in the conformal block expansion of stress tensor multiplets in 6d (2,0) theory
- anomalous dimensions contain information about higher-derivative corrections

M-Theory

- Low-energy effective action

$$S \sim \int d^{11}x\sqrt{-g}\,\frac{1}{G_N^{11}}\left(\mathcal{R}\qquad\qquad\qquad\qquad\qquad\qquad\right)$$

- Worldvolume theory of single M5-brane: abelian $(2,0)$ tensor multiplet: two-form gauge field, five scalars and eight fermions

N coincident M5-branes – 6d $(2,0)$ theory $\xleftrightarrow{AdS/CFT}$
M-theory in $AdS_7 \times S^4$ with N units of flux through S^4

- Large-N limit $\leftrightarrow$ low-energy limit

- Away from large N in CFT – higher-derivative corrections to 11d supergravity

M-Theory

- Low-energy effective action

$$S \sim \int d^{11}x\sqrt{-g}\,\frac{1}{G_N^{11}}\left(\mathcal{R} + \text{higher-derivative corrections}\right)$$

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6d $(2,0)$ Theory

- 6d (2,0) is manifestly strongly coupled and no Lagrangian description is known yet
- Conformal bootstrap techniques
- Calculate 4-point stress tensor correlators
- Superconformal primary T_{IJ} of half-BPS multiplet: dimension-4 scalar of 2-index symmetric traceless representation of SO(5)
- AdS_7-dual are scalars with mass $m^2 = -8$
- CFT 4-point functions are in one-to-one correspondence with local quartic interactions of this massive scalar field in AdS

  [Heemskerk,Penedones,Polchinski,Sully]

Operator-Product Expansion

- Operator-product expansion (OPE)

$$\phi_1(x)\phi_2(0)\,|0\rangle = \sum_{\mathcal{O}\ \text{primaries}} \lambda_{12\mathcal{O}}\, C_{\mathcal{O}}(x,\partial_y)\ \mathcal{O}(y)\big|_{y=0}\ |0\rangle$$

- CFT data: list of scaling dimensions Δ_i of all local primaries of the theory together with all OPE coefficients λ_{ijk} for any three primaries

- 4-point function in terms of OPEs of two pairs of primaries:

$$\begin{aligned}
&\langle\phi_1(x_1)\phi_2(x_2)\phi_3(x_3)\phi_4(x_4)\rangle \\
&= \sum_{\mathcal{O}_\Delta} \lambda_{12\mathcal{O}}\,\lambda_{34\mathcal{O}}[C_{\mathcal{O}}(x_{12},\partial_y)\,C_{\mathcal{O}}(x_{34},\partial_z)\langle\mathcal{O}(y)\,\mathcal{O}(z)\rangle] \\
&= \sum_{\mathcal{O}_\Delta} \lambda_{12\mathcal{O}}\,\lambda_{34\mathcal{O}}\frac{G_{\mathcal{O}}(u,v)}{(x_{12})^{2\Delta}(x_{34})^{2\Delta}}
\end{aligned}$$

where $u = \frac{x_{12}^2 x_{34}^2}{x_{13}^2 x_{24}^2} = z\bar{z}, \quad v = \frac{x_{14}^2 x_{23}^2}{x_{13}^2 x_{24}^2} = (1-z)(1-\bar{z})$

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conformal blocks

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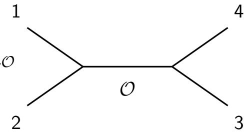
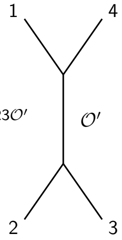
Crossing equation

$$\sum_{\mathcal{O}_\Delta} \lambda_{12\mathcal{O}}\, \lambda_{34\mathcal{O}} \frac{G_{\mathcal{O}}(u,v)}{(x_{12})^{2\Delta}(x_{34})^{2\Delta}} = \sum_{\mathcal{O}_\Delta} \lambda_{14\mathcal{O}}\, \lambda_{23\mathcal{O}} \frac{G_{\mathcal{O}}(v,u)}{(x_{14})^{2\Delta}(x_{23})^{2\Delta}}$$

$\sum_{\mathcal{O}} \lambda_{12\mathcal{O}}\lambda_{34\mathcal{O}}$ 

$= \sum_{\mathcal{O}'} \lambda_{14\mathcal{O}'}\lambda_{23\mathcal{O}'}$ 

Definition: A CFT is a set of CFT data which satisfies the OPE associativity for all 4-point functions

4-point Functions in the 6d (2,0) Theory

- Superconformal symmetry constrains 4-point function in terms of prepotential $F(z, \bar{z})$ [Heslop]

$$\lambda^4 \left(g_{13} g_{24}\right)^{-2} \langle T_1 T_2 T_3 T_4 \rangle = \mathcal{D}\left(\mathcal{S} F\left(z, \bar{z}\right)\right) + \mathcal{S}_1^2 F\left(z, z\right) + \mathcal{S}_2^2 F\left(\bar{z}, \bar{z}\right),$$

where $\mathcal{D} = -\left(\partial_z - \partial_{\bar{z}} + \lambda \partial_z \partial_{\bar{z}}\right) \lambda$ and $\lambda = z - \bar{z}$.

- Crossing symmetry: $\boxed{F(u, v) = F(v, u)}$

$$F(z, \bar{z}) = \frac{A}{u^2} + \frac{g(z) - g(\bar{z})}{u\,\lambda} + \lambda\, G(z, \bar{z})$$

- Schematic form of double-trace operators in OPE $T \partial^l \Box^n T$ with $n \geq 0$ and $\Delta = 2n + l + 8 + \mathcal{O}(1/N^3)$

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unit operator — protected operators — unprotected double trace operators

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Conformal Block Expansion

- Expand in conformal blocks

$$\lambda^2 G(z,\bar{z}) = \sum_{n,l\geq 0} A_{n,l}\, G_{\Delta,l}(z,\bar{z})$$

- Expand the OPE data in $1/N$:

$$A_{n,l} = A_{n,l}^{(0)} + \frac{1}{N^3} A_{n,l}^{(1)} + \dots, \qquad \Delta = \underbrace{2n+l+8}_{\Delta_0} + \frac{1}{N^3}\gamma_{n,l} + \dots$$

- $A_{n,l}^{(0)}$ from free disconnected contribution:

$$F(u,v)_{\text{free-disc}} = 1 + \frac{1}{u^2} + \frac{1}{v^2}$$

- Perform conformal block expansion and solve for leading contribution to OPE coefficients:

$$A_{n,l}^{(0)} = \frac{(l+2)\,(n+3)!\,(n+4)!\,(l+2n+9)\,(l+2n+10)\,(l+n+5)!\,(l+n+6)!}{72\,(2n+5)!\,(2l+2n+9)!}$$

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Conformal Block Expansion

- Higher-derivative corrections:

$$A_{n,l} = A_{n,l}^{(0)} + \frac{1}{N^3} A_{n,l}^{(1)} + \dots \qquad \Delta = \underbrace{2n + l + 8}_{\Delta_0} + \frac{1}{N^3} \gamma_{n,l} + \dots$$

$$G_{\Delta,l} = G_{\Delta_0,l} + \frac{1}{N^3} \gamma_{n,l} \frac{\partial}{\partial \Delta} G_{\Delta,l}|_{\Delta=\Delta_0} + \dots$$

- Expand crossing equation to order $1/N^3$

$$\sum_{n,l \geq 0} \left[A_{n,l}^{(1)} \, G_{\Delta_0,l}(z,\bar{z}) + \frac{1}{2} A_{n,l}^{(0)} \;\; \gamma_{n,l} \;\; \partial_n G_{\Delta_0,l}(z,\bar{z}) \right] + (u \leftrightarrow v) = 0$$

- Conformal blocks [Dolan,Osborn]

$$G_{\Delta,l}(z,\bar{z}) \sim \sum u^n h_\alpha(z) h_\beta(\bar{z}),$$

where

$$h_\beta(z) = {}_2F_1\left(\beta/2, \beta/2 - 1, \beta, z\right)$$

- Goal: Find anomalous dimensions $\gamma_{n,l}$ for any n and l

Conformal Block Expansion

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$\sim \log(u)$

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- Goal: Find anomalous dimensions $\gamma_{n,l}$ for any n and l

Recursion Relations

$$\sum_{n,l\geq 0}\left[A^{(1)}_{n,l}\,G_{\Delta_0,l}(z,\bar{z})+\frac{1}{2}A^{(0)}_{n,l}\,\gamma_{n,l}\,\underbrace{\partial_n G_{\Delta_0,l}(z,\bar{z})}_{\sim\log(z\bar{z})}\right]+\underbrace{(u\leftrightarrow v)}_{\sim\log((1-z)(1-\bar{z}))}=0$$

- Follow closely method in [Heemskerk,Penedones,Polchinski,Sully; Alday,Bissi,Lukowski]
- Isolate terms containing $\gamma_{n,l}$ by taking $z\to 0$ and $\bar{z}\to 1$
- Use $h_\beta(\bar{z})=\log(1-\bar{z})\,(1-\bar{z})\,\tilde{h}_\beta(1-\bar{z})+$ holomorphic at $\bar{z}=1$, where $\tilde{h}_\beta(z)=\frac{\Gamma(\beta)}{\Gamma(\beta/2)\Gamma(\beta/2-1)}{}_2F_1\left(\beta/2+1,\beta/2,2,z\right)\to h_\alpha(z)\tilde{h}_\beta(1-\bar{z})$ and $h_\alpha(1-\bar{z})\tilde{h}_\beta(z)$

$$\sum_{n,l\geq 0}A^{(0)}_{n,l}\,\gamma_{n,l}\left(\partial_n G_{\Delta_0,\,l}(z,\bar{z})\right)\Big|_{\log z\log(1-\bar{z})}=$$

$$-\sum_{n,l\geq 0}A^{(0)}_{n,l}\,\gamma_{n,l}\left(\partial_n G_{\Delta_0,\,l}(1-z,1-\bar{z})\right)\Big|_{\log z\log(1-\bar{z})}$$

Recursion Relations

$$\sum_{n,l\geq 0}\left[A^{(1)}_{n,l}\,G_{\Delta_0,l}(z,\bar z)+\frac{1}{2}A^{(0)}_{n,l}\,\boxed{\gamma_{n,l}}\,\underbrace{\partial_n G_{\Delta_0,l}(z,\bar z)}_{\sim\log(z\bar z)}\right]+\underbrace{(u\leftrightarrow v)}_{\sim\log((1-z)(1-\bar z))}=0$$

- Follow closely method in [Heemskerk,Penedones,Polchinski,Sully; Alday,Bissi,Lukowski]
- Isolate terms containing $\gamma_{n,l}$ by taking $z\to 0$ and $\bar z\to 1$
- Use $h_\beta(\bar z)=\log(1-\bar z)\,(1-\bar z)\,\tilde h_\beta(1-\bar z)+$ holomorphic at $\bar z=1$, where

$$\tilde h_\beta(z)=\frac{\Gamma(\beta)}{\Gamma(\beta/2)\Gamma(\beta/2-1)}\,{}_2F_1\left(\beta/2+1,\beta/2,2,z\right)\to h_\alpha(z)\tilde h_\beta(1-\bar z)\text{ and }h_\alpha(1-\bar z)\tilde h_\beta(z)$$

$$\sum_{n,l\geq 0}A^{(0)}_{n,l}\,\gamma_{n,l}\left(\partial_n G_{\Delta_0,\,l}(z,\bar z)\right)\big|_{\log z\log(1-\bar z)}=$$

$$\times\frac{h_{-2q}(z)}{z^q\,(1-z)}\;\frac{h_{-2p}(1-\bar z)}{(1-\bar z)^p\,\bar z}$$

$$-\sum_{n,l\geq 0}A^{(0)}_{n,l}\,\gamma_{n,l}\left(\partial_n G_{\Delta_0,\,l}(1-z,1-\bar z)\right)\big|_{\log z\log(1-\bar z)}$$

Recursion Relations

- Orthogonality of hypergeometric functions:

$$\delta_{m,m'} = \oint \frac{dz}{2\pi i} \frac{z^{m-m'-1}}{1-z} h_{2m+4}(z)\, h_{-2m'-2}(z)$$

- Define inner product

$$\mathcal{I}_{m,m'} = \oint \frac{dz}{2\pi i} \frac{(1-z)^{m-3}}{z^{m'-1}} \tilde{h}_{2m}(z)\, h_{-2m'}(z)$$

- Recursion relation:

$$\begin{aligned} 0 = \sum_{l=0}^{L} \sum_{n=0}^{\infty} A^{(0)}_{n,l} \gamma_{n,l} \Big[& P_{n,l} \left(\delta_{q,n} \mathcal{I}_{n+l+6,p+2} - \delta_{q,n+l+3} \mathcal{I}_{n+3,p+2} \right) \\ & + Q_{n,l} \left(\delta_{q,n+2} \mathcal{I}_{n+l+6,p+2} - \delta_{q,n+l+3} \mathcal{I}_{n+5,p+2} \right) \\ & + R_{n,l} \left(\delta_{q,n+l+2} \mathcal{I}_{n+4,p+2} - \delta_{q,n+1} \mathcal{I}_{n+l+5,p+2} \right) \\ & + S_{n,l} \left(\delta_{q,n+l+4} \mathcal{I}_{n+4,p+2} - \delta_{q,n+1} \mathcal{I}_{n+l+7,p+2} \right) - (q \leftrightarrow p) \Big] \end{aligned}$$

- Truncate to spin L

- Choose appropriate values of p, q to solve for all $\gamma_{n,l}$ with $n \leq \min(2n-2, L)$ in terms of $(L+2)(L+4)/8$ free parameters

L=0 Solution

- Recursion relation can be solved for all $\gamma_{n,0}$ in terms of $\gamma_{0,0}$ $\rightarrow$

- Spin-0 solution

$$\gamma_{n,0}^{\text{spin}-0} = \gamma_{0,0} \frac{11\,(n+1)_8\,(n+2)_6}{2304000\,(2n+7)\,(2n+9)\,(2n+11)}$$

- Large-n behaviour gives information about higher-derivative corrections to effective M-theory action

- $\gamma_{n,0}^{\text{spin}-0} \sim n^{11} \sim n^6 \gamma_{n,0}^{\text{sugra}}$ $\rightarrow$ bulk interaction vertex has six more derivatives than supergravity $(\mathcal{R})$ $\rightarrow$ $\mathcal{R}^4$ correction

- Massive scalar field in AdS with local quartic interactions – spin-0 interaction vertex Φ^4 [Heemskerk,Penedones,Polchinski,Sully]

- Agrees with results from conformal block expansion of functions satisfying the crossing equation and whose block expansion truncates to spin-L [Heslop,Lipstein]

L=2 Solutions

- Spin-L truncation: $\frac{1}{8}(L+2)(L+4)$ solutions
- L=2 solutions:

$$\gamma_{n,0}^{\text{spin-2}} = \frac{\gamma_{n,0}^{\text{spin-0}}}{\gamma_{0,0}} \left(\gamma_{0,0} + \gamma_{0,2}\, f_1(n) + \gamma_{1,2}\, f_2(n)\right),$$

$$\gamma_{n,2}^{\text{spin-2}} = -\frac{\gamma_{n,0}^{\text{spin-0}}}{\gamma_{0,0}} \frac{845\,(n-1)\,(n+5)\,(n+6)\,(n+8)\,(n+9)^2\,(n+10)\,(n+12)}{4064256\,(2n+13)\,(2n+15)} \times \left(\gamma_{0,2} - \gamma_{1,2}\,\frac{51n\,(n+11)}{364\,(n-1)\,(n+12)}\right)$$

where

$$f_1(n) = \frac{325n\,(n+9)\left(13n^6 + 351n^5 + 6201n^4 + 64233n^3 + 385476n^2 + 1251666n + 1512620\right)}{1016064\,(2n+5)\,(2n+13)},$$

$$f_2(n) = -\frac{1105n\,(n+9)\left(5n^6 + 135n^5 + 2157n^4 + 20601n^3 + 117468n^2 + 370494n + 441700\right)}{9483264\,(2n+5)\,(2n+13)}.$$

- Large-n behaviour $\sim n^{15}$ and $\sim n^{17}$ $\rightarrow$ interaction vertices of form $D^4\mathcal{R}^4$ and $D^6\mathcal{R}^4$
- Spin-2 interaction vertices $\Phi^2(\nabla_\mu\nabla_\nu\Phi)^2$ and $\Phi^2(\nabla_\mu\nabla_\nu\nabla_\rho\Phi)^2$

Conclusions

- Derived recursion relations for anomalous dimensions of double-trace operators in the conformal block expansion of 4-point stress tensor correlators in the 6d (2,0) theory
- Large-n behaviour of $\gamma_{n,l}$ encodes higher-derivative corrections to 11d supergravity action
- We cannot fix the coefficients of these corrections

Outlook:

- Develop methods for fixing the coefficients
- [Chester,Perlmutter] apply chiral algebra conjecture [Beem,Rastelli,van Rees] to higher-charge correlators
- Extend our method to such operators and fix the coefficients of higher-derivative terms in the M-theory action
- Study loop-expansion in M-theory on $AdS_7 \times S^4$ using conformal bootstrap techniques

M-Theory Corrections to Effective Action

- Low-energy effective action:

$$\mathcal{L} \sim \frac{1}{G_N^{11}} \left(\mathcal{R} + \underbrace{c_1 \quad \mathcal{R}^4}_{Spin-0} + \underbrace{c_2 \quad D^4\mathcal{R}^4 + c_3 \quad D^6\mathcal{R}^4}_{Spin-2} + \dots \right)$$

  where $G_N^{11} \sim l_P^9 \sim N^{-3}$ and c_i unfixed
- 8-derivative correction comes with $G_N^{11}\, l_P^6 \sim N^{-5}$
- 12- and 14-derivative corrections come with $G_N^{11} l_P^{10} \sim N^{-19/3}$ and $G_N^{11} l_P^{12} \sim N^{-7}$ respectively
- Compare to 1-loop supergravity correction $\sim \left(G_N^{11}\right)^2 \sim N^{-6} \rightarrow$ loop correction is subleading compared to spin-0 correction of tree-level supergravity
- Can write and solve recursion relations for any spin-L truncation and get the structure of higher-derivative corrections

M-Theory Corrections to Effective Action

- Low-energy effective action:

$$\mathcal{L} \sim \frac{1}{G_N^{11}} \left(\mathcal{R} + \underbrace{c_1\, l_P^6\, \mathcal{R}^4}_{Spin-0} + \underbrace{c_2\, l_P^{10}\, D^4\mathcal{R}^4 + c_3\, l_P^{12}\, D^6\mathcal{R}^4}_{Spin-2} + \dots \right)$$

  where $G_N^{11} \sim l_P^9 \sim N^{-3}$ and c_i unfixed
- 8-derivative correction comes with $G_N^{11}\, l_P^6 \sim N^{-5}$
- 12- and 14-derivative corrections come with $G_N^{11} l_P^{10} \sim N^{-19/3}$ and $G_N^{11} l_P^{12} \sim N^{-7}$ respectively
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Conformal Blocks

[Dolan,Osborn]

$$G^{\text{DO}}(\Delta, l, \Delta_{12}, \Delta_{34}) = \mathcal{F}_{00} - \frac{l+3}{l+1}\mathcal{F}_{-11}$$
$$- \frac{\Delta-4}{\Delta-2}\,\frac{(\Delta+l-\Delta_{12})(\Delta+l+\Delta_{12})(\Delta+l+\Delta_{34})(\Delta+l-\Delta_{34})}{16(\Delta+l-1)(\Delta+l)^2(\Delta+l+1)}\mathcal{F}_{11}$$
$$+ \frac{(\Delta-4)(l+3)}{(\Delta-2)(l+1)}\,\frac{(\Delta-l-\Delta_{12}-4)(\Delta-l+\Delta_{12}-4)(\Delta-l+\Delta_{34}-4)(\Delta-l-\Delta_{34}-4)}{16(\Delta-l-5)(\Delta-l-4)^2(\Delta-l-3)}\mathcal{F}_{02}$$
$$+ 2(\Delta-4)(l+3)\,\frac{\Delta_{12}\Delta_{34}}{(\Delta+l)(\Delta+l-2)(\Delta-l-4)(\Delta-l-6)}\mathcal{F}_{01},$$

where $\Delta_{ij} = \Delta_i - \Delta_j$ and

$$\mathcal{F}_{ab} = \frac{(z\bar{z})^{\frac{1}{2}(\Delta-l)}}{\lambda^3}\Big\{ z^{l+a+3}\bar{z}^b \times {}_2F_1\left(\tfrac{1}{2}(\Delta+l-\Delta_{12})+a,\, \tfrac{1}{2}(\Delta+l+\Delta_{34})+a;\, \Delta+l+2a,\, z\right)$$
$$\times\, {}_2F_1\left(\tfrac{1}{2}(\Delta-l-\Delta_{12})-3+b,\, \tfrac{1}{2}(\Delta-l+\Delta_{34})-3+b;\, \Delta-l-6+2b;\, \bar{z}\right) - z \leftrightarrow \bar{z}\Big\}$$

6d $(2,0)$ theory: supersymmetric blocks [Heslop; Beem,Lemos,Rastelli,van Rees]

$$G_{\Delta,\,l}(z,\bar{z}) = \frac{4(l+1)}{(l+2)^2-\Delta^2}\,\frac{\lambda^3}{u^5}\,G^{\text{DO}}(\Delta+4, l, 0, -2)$$

Solutions

- Truncate to spin L
- Choose appropriate values of p, q to solve for all $\gamma_{n,l}$ with $n \leq \min(2n-2, L)$ in terms of $(L+2)(L+4)/8$ free parameters
- $L = 0$: setting $q = 0$ gives recursion relation in terms of p:

$$\frac{1}{15}\mathcal{I}_{6,p+2}A^{(0)}_{0,0}\gamma_{0,0} = \sum_{a=0}^{4} C_a A^{(0)}_{p-a,0}\gamma_{p-a,0}$$

$$C_0 = \frac{\mathcal{I}_{p+6,2}}{(p+3)(p+5)}$$

$$C_1 = -\frac{3\,\mathcal{I}_{p+4,2}}{(p+2)(p+4)} - \frac{(p+3)(p+6)\,\mathcal{I}_{p+6,2}}{4(p+2)(p+4)(2p+9)(2p+11)}$$

$$C_2 = \frac{3\,\mathcal{I}_{p+2,2}}{(p+1)(p+3)} + \frac{3(p+2)\,\mathcal{I}_{p+4,2}}{4(p+3)(2p+3)(2p+5)}$$

$$C_3 = -\frac{\mathcal{I}_{p,2}}{p(p+2)} - \frac{3(p+1)\,\mathcal{I}_{p+2,2}}{4(p+2)(2p+1)(2p+3)}$$

$$C_4 = \frac{p(p+3)\,\mathcal{I}_{p,2}}{4(p-1)(p+1)(2p+3)(2p+5)}$$