Dualities for QED in 2+1 dimensions

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Patterns from the past

QFT developments

- $\mathcal{N} = 1$ Seiberg duality
- $\mathcal{N} = 2$ Seiberg-Witten solution,
- d=3 mirror symmetry

String theory realisation

- Elitzur-Giveon-Kutasov trajectory
- Witten's M-theory uplift
- Hanany-Witten interpretation

Lesson: String theory seems to be aware of deep facts about QFT. But it is a huge framework and so QFT input is crucial.

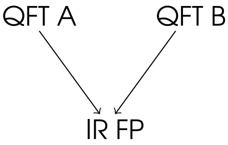
Recent developments

Many interesting results in non-SUSY gauge theories in the recent past. One particularly interesting theory is QED_3

$$\int d^3x \frac{1}{4e^2} f_{\mu\nu}^2 + \frac{K}{4\pi} \epsilon^{\mu\nu\rho} a_\mu \partial_\nu a_\rho + i\bar{\psi} \left(\partial\!\!\!/ + qa \right) \psi$$

- Strongly coupled in the IR: $\beta < 0$, $[e^2] = \text{mass}$
- It has relevance to brane dynamics in string theory

Duality



$$\{\mathcal{O}_A\} \longleftrightarrow \{\widetilde{\mathcal{O}}_B\}$$

- Gauge group, matter representation are not fundamental notions
- But honest global symmetries better agree.
- 't Hooft anomalies must match
- Physical observables (gauge invariant operators) must match

Example: Maxwell theory

$$Z_A = \int \mathcal{D}a_\mu e^{\int d^3x \frac{1}{4e^2} f_{\mu\nu}^2} = \int \mathcal{D}f_{\mu\nu}\mathcal{D}\sigma e^{\int d^3x \left[\frac{1}{4e^2} f_{\mu\nu}^2 - \frac{i}{4\pi}\sigma\epsilon^{\mu\nu\rho}\partial_\mu f_{\nu\rho}\right]}$$

$$Z_B = \int \mathcal{D}\sigma e^{\int d^3x - \frac{e^2}{8\pi^2}(\partial_\mu\sigma)^2}$$

Symmetries: $U(1)_M \times U(1)_{\text{1-form}}$

$$J_\mu = \frac{1}{4\pi}\epsilon_{\mu\nu\rho}F^{\nu\rho} \qquad \widetilde{J}_\mu = \frac{e^2}{8\pi^2}\partial_\mu\sigma$$

$$J_{\mu\nu} = \frac{1}{2e^2} f_{\mu\nu} \qquad \widetilde{J}_{\mu\nu} = \frac{1}{4\pi}\epsilon_{\mu\nu\rho}\partial^\rho\sigma$$

Note: No gauge symmetry on the dual side

Bosonisation

Central result in 3d gauge theories

$$U(N)_{K,K\mp N} \oplus N_f \text{ fermions} \leftrightarrow U(K)_{-N,-N\pm K} \oplus N_f \text{ WF scalars} \qquad (1)$$

Evidence

- Large N checks
- Quantum numbers of dual operators match
- Anomaly matching
- String theory embeddings
- Consistent with other dualities

Consider $N = 0$, $N_f = |K| = 1$, i.e.

$$\bar{\psi} i \not{D}_B \psi \mp \frac{1}{8\pi} BdB \longleftrightarrow |D_a \phi|^2 \pm \frac{1}{4\pi} ada \pm \frac{1}{2\pi} adB$$

A web of dualities

$$\bar{\psi} i \not{D}_B \psi \mp \frac{1}{8\pi} BdB \longleftrightarrow |D_a \phi|^2 \pm \frac{1}{4\pi} ada \pm \frac{1}{2\pi} adB$$

Starting from this we can generate a web of dualities, We first promote $B \to b$

$$\bar{\psi} i \not{D}_b \psi \longleftrightarrow |D_a \phi|^2 + \frac{1}{4\pi} ada + \frac{1}{2\pi} adb + \frac{1}{8\pi} bdb + \frac{1}{4\pi} bdC$$

Next, integrate out b; $db = -d(C + 2a)$

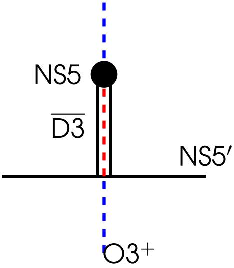
$$\bar{\psi} i \not{D}_b \psi \longleftrightarrow |D_a \phi|^2 - \frac{1}{4\pi} ada - \frac{1}{2\pi} adC - \frac{1}{8\pi} CdC$$

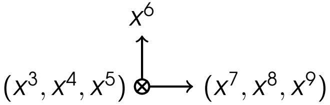
Using the lower sign duality in (3) we have

$$\bar{\psi} i \not{D}_b \psi \longleftrightarrow \bar{\chi} i \not{D}_C \chi$$

"particle-vortex duality"

Charge 2 QED_3





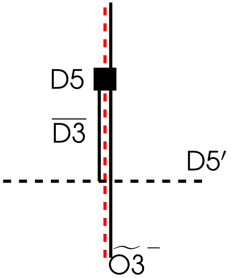
D3	:				6
O3	:				6
NS5	:	3	4	5	
NS5′	:	$\begin{bmatrix}3\\7\end{bmatrix}_{\theta_1}$	$\begin{bmatrix}4\\8\end{bmatrix}_{\theta_2}$	$\begin{bmatrix}5\\9\end{bmatrix}_{\theta_3}$	

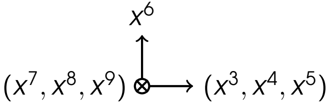
Tower of vectors, scalars and fermions with masses

$$M_{\mathrm{V}}^{(n)} = \frac{n\pi}{L}, \quad M_{\mathrm{s}_6}^{(n)} = \frac{n\pi}{L}, \quad M_{\mathrm{s}_k}^{(n)} = \frac{\theta_k + n\pi}{L}, \quad M_{\mathrm{f}_i}^{(n)} = \frac{\theta_i + n\pi}{L},$$

Effective theory at scales $E \ll \frac{1}{L} \ll \frac{1}{l_s}$ is charge 2 QED_3:

$$-\frac{1}{4e^2} f_{\mu\nu}^2 + \bar{\psi} \mathrm{i} \not{D} \psi$$





$$
\begin{array}{ll}
\text{D3} : & 6 \\
\text{O3} : & 6 \\
\text{D5} : & 7 \quad 8 \quad 9 \\
\text{D5}' : & \begin{bmatrix}7\\3\end{bmatrix}_{\theta_1} \begin{bmatrix}8\\4\end{bmatrix}_{\theta_2} \begin{bmatrix}9\\5\end{bmatrix}_{\theta_3}
\end{array}
$$

$$\frac{1}{2}\partial_\mu\phi\partial^\mu\phi + \bar{\chi}i\partial\!\!\!/\chi + 2y\phi\bar{\chi}\chi \,,$$

Flavour current $\tilde{J}_\mu = \bar{\chi}\gamma_\mu\chi$ mapped to the magnetic symmetry of QED_3

Puzzle: charge 2 QED_3 also has a $\mathbb{Z}_2^{\text{1-form}}$ centre.

Summary & Outlook

- Many gauge theories in (2+1)d exhibit dualities
- The weakly coupled descriptions are radically different
- Duality for charge 2 QED_3 from a brane construction
- confusion about the centre symmetry
- To compare with existing conjecture must include CS terms
- In our model this can be done by turning on RR flux for the axion