AN INTRODUCION TO

HIGH ENERGY JETS

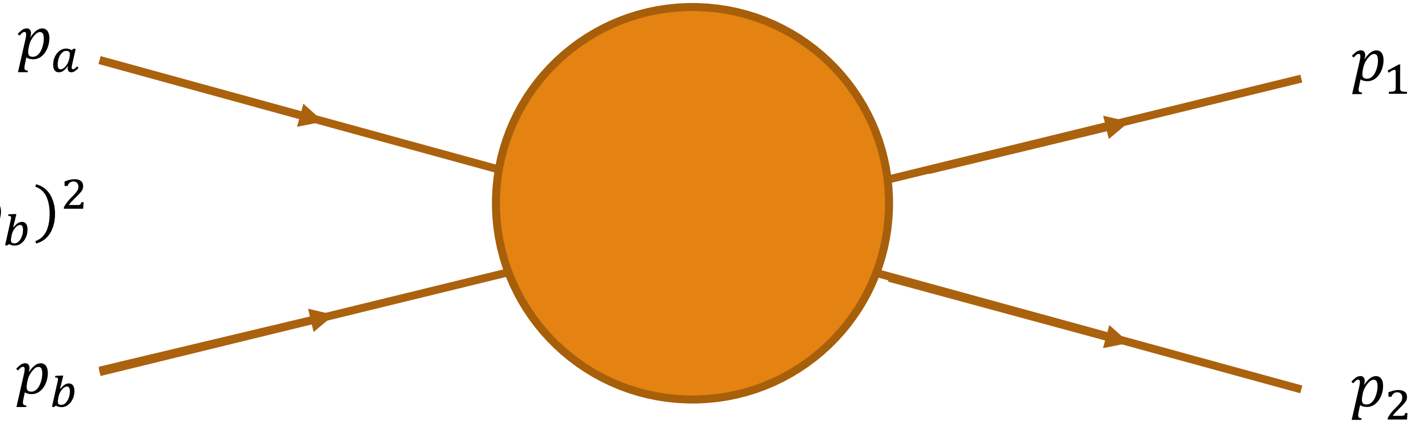
AND CURRENT WORK ON NLO MATCHING

EMMET BYRNE

THE REGGE LIMIT

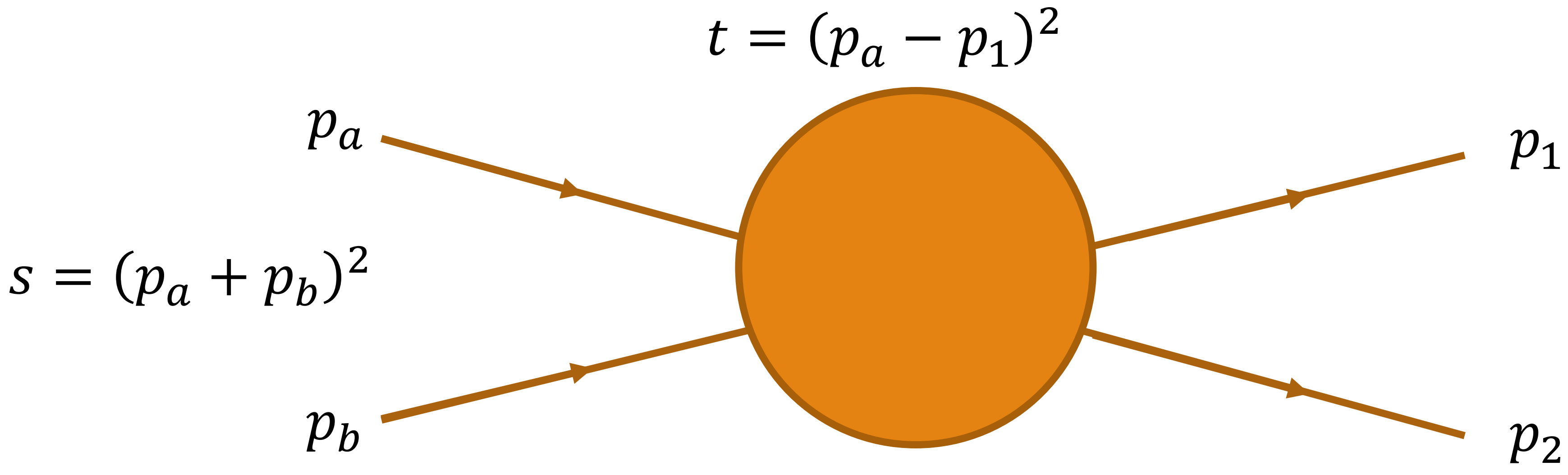
$$t = (p_a - p_1)^2$$

$$s = (p_a + p_b)^2$$



[1] T. Regge. Il Nuovo Cimento, 14:951, 1959.

THE REGGE LIMIT



In the 1950's Tulio Regge studied the quantum mechanical amplitude for 2→2 scattering in the limit of $s \to \infty$ while keeping t fixed.

This is now known as the *Regge Limit*.

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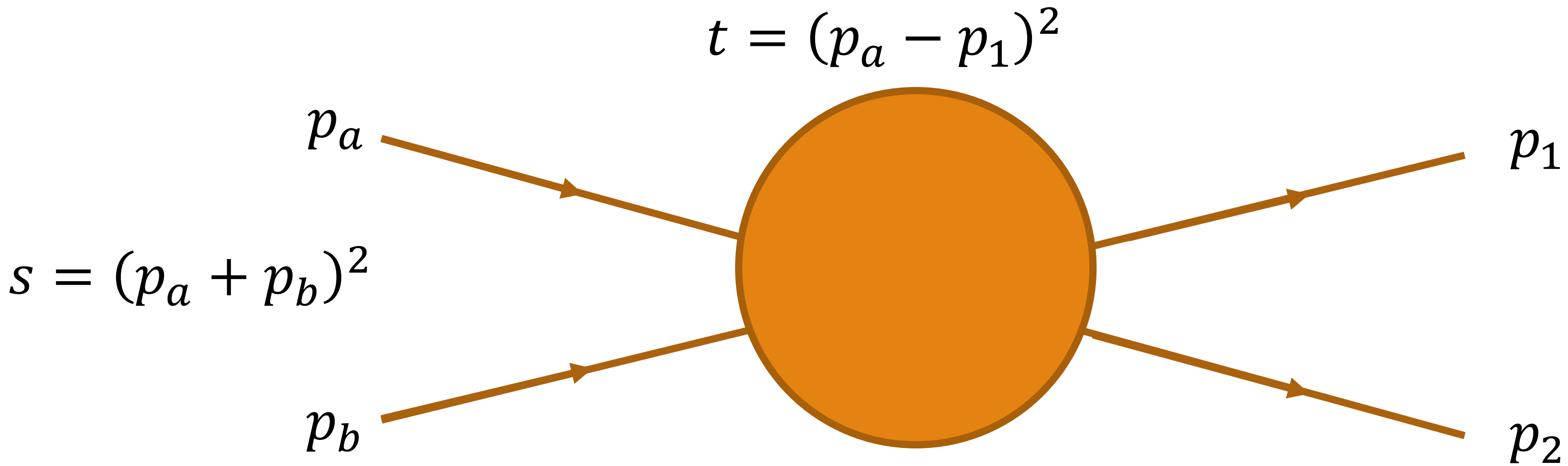

In the 1950's Tulio Regge studied the quantum mechanical amplitude for 2→2 scattering in the limit of $s \to \infty$ while keeping t fixed.

This is now known as the *Regge Limit*.

In this limit, the s dependence takes a simple form:

$$\mathcal{M}(s,t) \xrightarrow[|\frac{s}{t}|\to\infty]{} f(t)s^{\alpha(t)}$$

This phenomenon is now known as *Reggezation.*

[1] T. Regge. Il Nuovo Cimento, 14:951, 1959.

A POTENTIAL PROBLEM

When we have a large separation between two scales, we generically get large logarithms of their ratio at all orders in fixed order perturbative QFT.

$$\sigma_{LO} \equiv \sigma^{(0)}$$

$$\sigma^{(1)} = \sigma^{(0)}\left(\alpha_s c_0^{(1)} \log\left(\frac{s}{t}\right) + \alpha_s c_1^{(1)}\right)$$

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$$\sigma^{(3)} = \sigma^{(0)}\left(\alpha_s^3 c_0^{(3)} \log^3\left(\frac{s}{t}\right) + \alpha_s^3\, c_1^{(3)} \log^2\left(\frac{s}{t}\right) + \alpha_s^2 c_2^{(3)} \log\left(\frac{s}{t}\right) + \alpha_s^2\, c_3^{(3)}\right)$$

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For large energy collisions we need to sum these logarithmic contributions to all orders in order to ensure our perturbative series converges.

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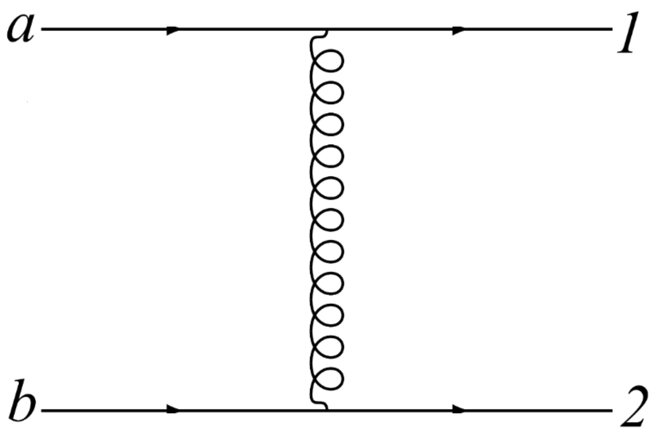
LEADING ORDER IN THE REGGE LIMIT



$$\mathcal{M}^{(0)qQ\to qQ} = [\bar{u}^{\lambda_1}(p_1) i g_s \gamma^\mu T^A_{c_1 c_a} u^{\lambda_a}(p_a)] \frac{-i g_{\mu\nu}}{t} [\bar{u}^{\lambda_2}(p_2) i g_s \gamma^\nu T^A_{c_2 c_b} u^{\lambda_b}(p_b)]$$

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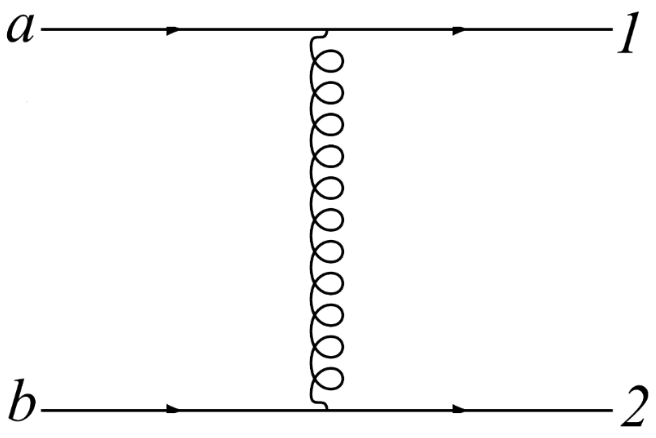

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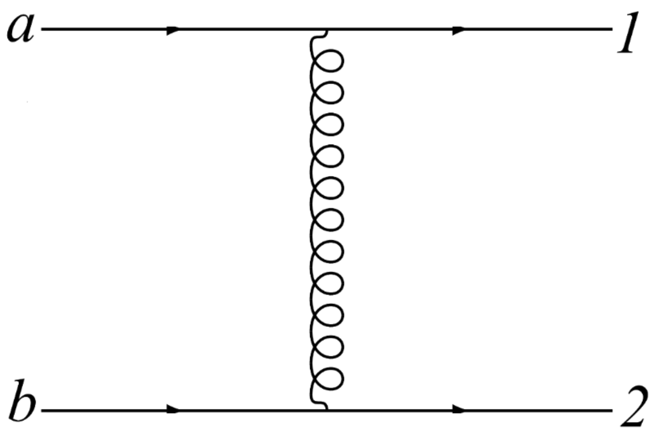

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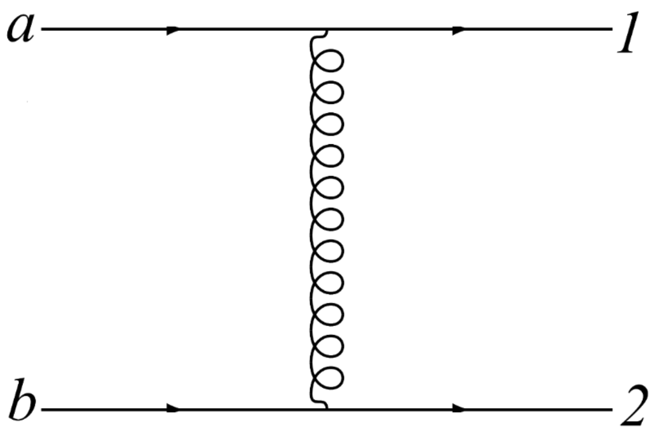
This result is independent of the type of particle scattering: For gluon scattering the only difference is a change in the color factors.

LEADING ORDER IN THE REGGE LIMIT



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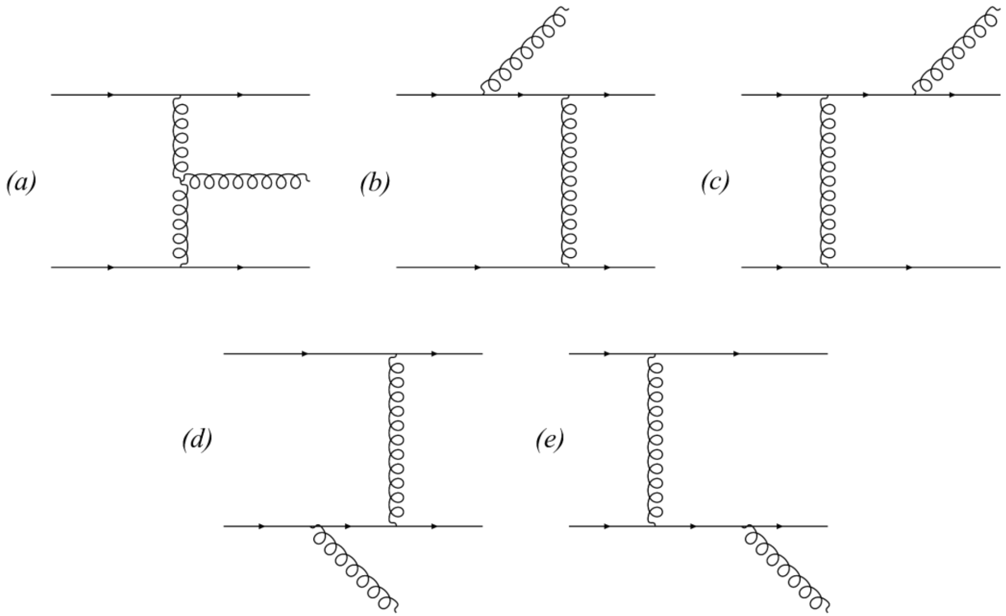
This result is independent of the type of particle scattering: For gluon scattering the only difference is a change in the color factors.

For later use we introduce the notation S to mean the spinor-string content of the leading order amplitude:

$$\mathcal{M}^{(0)qQ\to qQ} = g_s^2 T^{A}_{c_1 c_a} T^{A}_{c_2 c_b} \frac{\mathcal{S}^{(0)qQ\to qQ}}{t}$$

ONE REAL EMISSION



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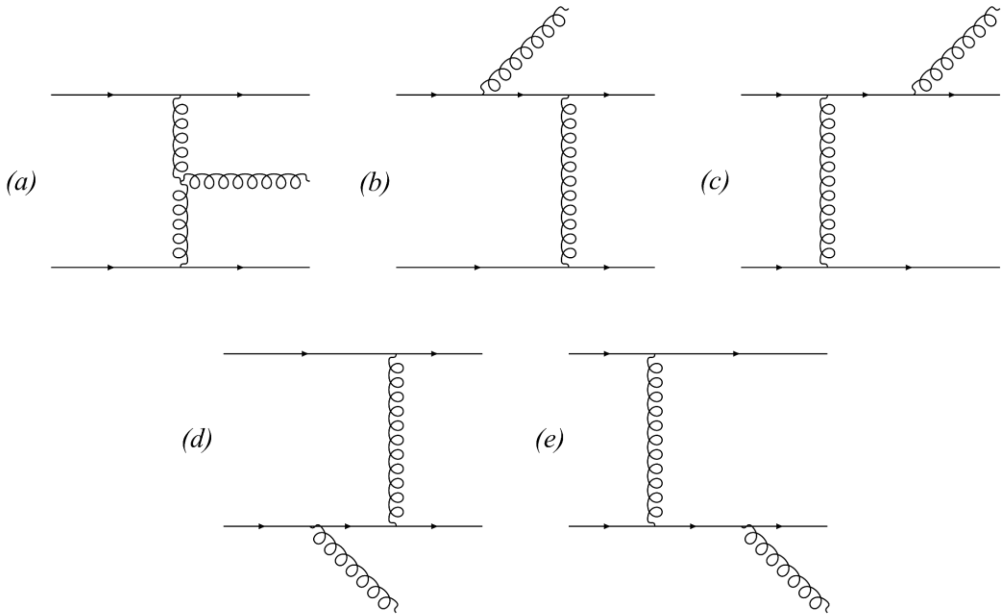

$$\mathcal{M}^{(0)qQ\to qgQ}_{\text{Fig. 4 (a)}} \xrightarrow[\text{MRK}]{} -g_s^3 \frac{\mathcal{S}^{(0)qQ\to qQ}_{\text{MRK}}}{\mathbf{q}^2_{1\perp}\mathbf{q}^2_{2\perp}} \epsilon^*_\rho \left(-2p_a^\rho \frac{s_{23}}{s_{ab}} + 2p_b^\rho \frac{s_{12}}{s_{ab}} + (\mathbf{q}_{1\perp}+\mathbf{q}_{2\perp})^\rho\right) T^A_{c_1 c_a} T^B_{c_3 c_b} f^{ABG}$$

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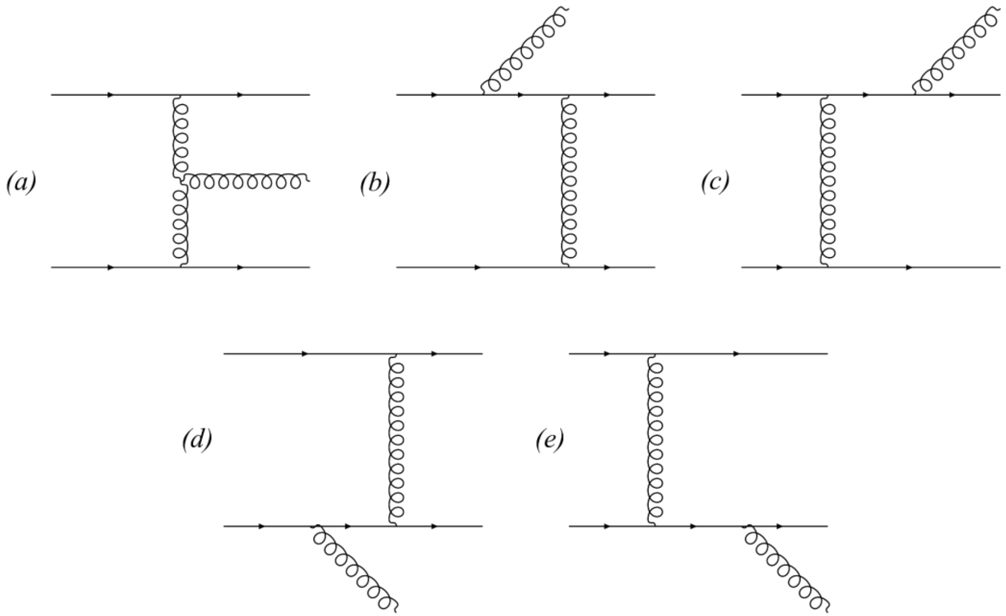

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$$\mathcal{M}^{(0)qQ\to qgQ}_{\text{Fig. 4 (b)}} + \mathcal{M}^{(0)qQ\to qgQ}_{\text{Fig. 4 (c)}} \xrightarrow[\text{MRK}]{} -ig_s^3 \frac{\mathcal{S}^{(0)qQ\to qQ}_{\text{MRK}}}{\mathbf{q}^2_{2\perp}} \epsilon^*_\rho\left(\frac{2p_1^\rho}{s_{12}} T^G_{c_1 c_m} T^A_{c_m c_a} - \frac{2p_a^\rho}{s_{a2}} T^A_{c_1 c_m} T^G_{c_m c_a}\right) T^A_{c_2 c_b}$$

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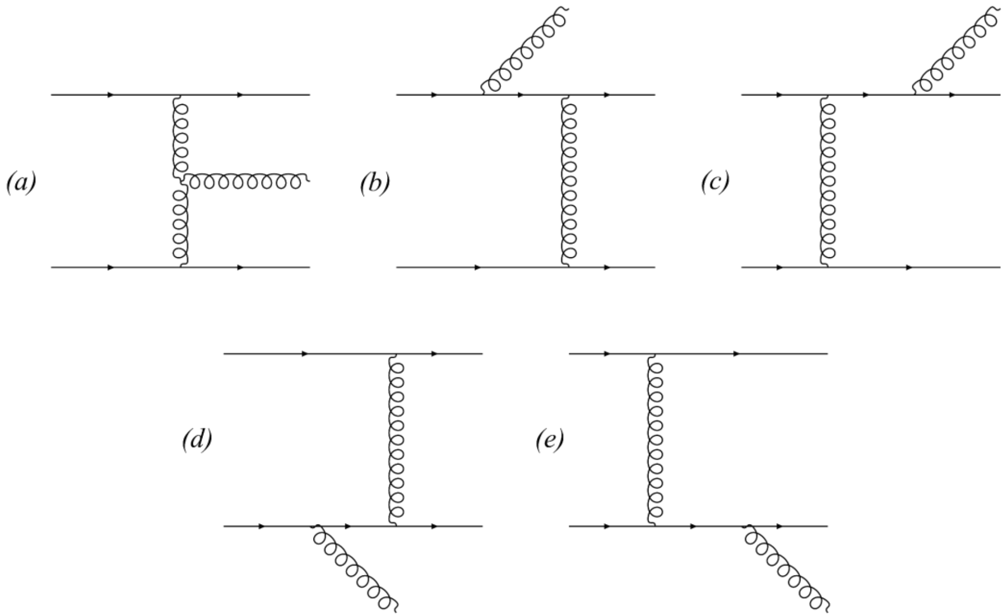

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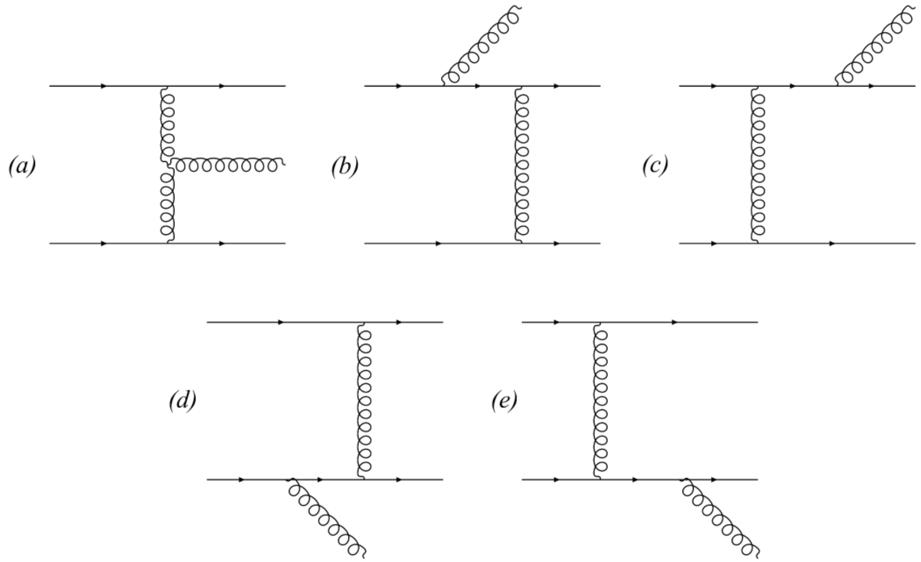
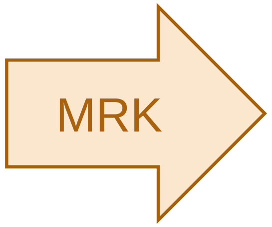
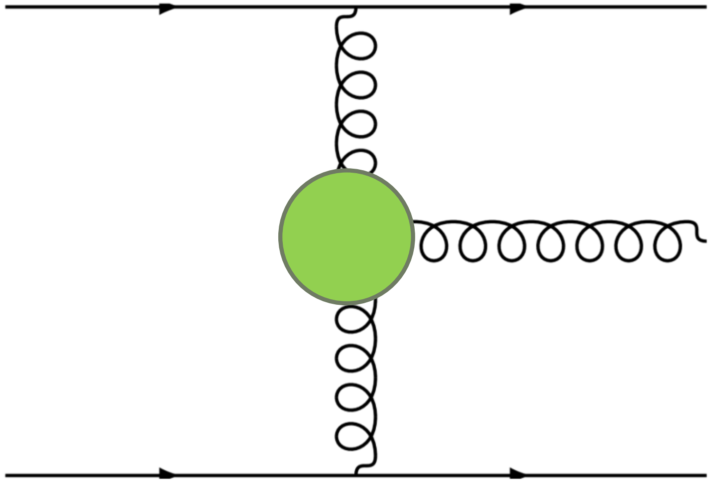
$$\mathcal{M}^{(0)qQ\to qgQ}_{\text{Fig. 4 (b)}} + \mathcal{M}^{(0)qQ\to qgQ}_{\text{Fig. 4 (c)}} \xrightarrow[\text{MRK}]{} -ig_s^3 \frac{\mathcal{S}^{(0)qQ\to qQ}_{\text{MRK}}}{\mathbf{q}^2_{2\perp}} \epsilon^*_\rho \left(\frac{2p_1^\rho}{s_{12}} T^G_{c_1 c_m} T^A_{c_m c_a} - \frac{2p_a^\rho}{s_{a2}} T^A_{c_1 c_m} T^G_{c_m c_a}\right) T^A_{c_2 c_b}$$

$$\mathcal{M}^{(0)qQ\to qgQ}_{\text{Fig. 4 (d)}} + \mathcal{M}^{(0)qQ\to qgQ}_{\text{Fig. 4 (e)}} \xrightarrow[\text{MRK}]{} -ig_s^3 \frac{\mathcal{S}^{(0)qQ\to qQ}_{\text{MRK}}}{\mathbf{q}^2_{1\perp}} \epsilon^*_\rho \left(\frac{2p_3^\rho}{s_{23}} T^G_{c_2 c_m} T^A_{c_m c_b} - \frac{2p_b^\rho}{s_{23}} T^A_{c_2 c_m} T^G_{c_m c_b}\right) T^A_{c_1 c_a}$$

ONE REAL EMISSION IN THE MRK LIMIT



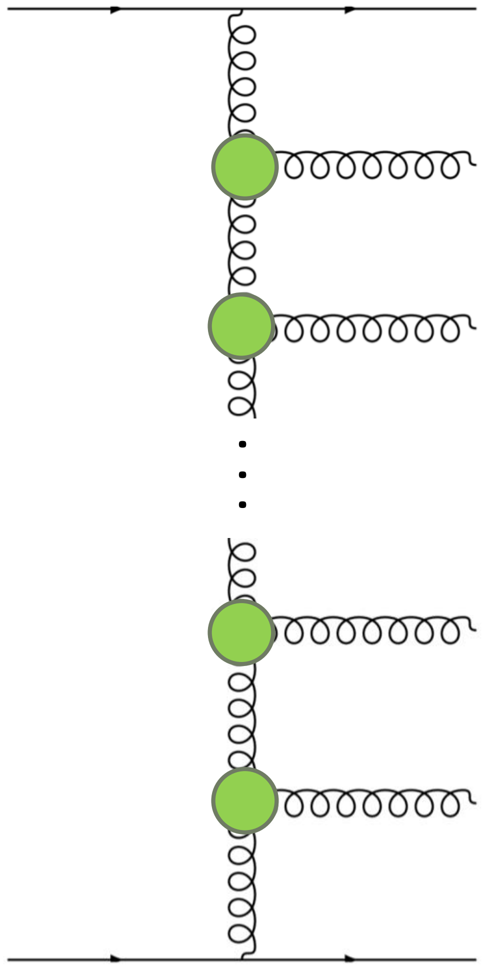




$$\mathcal{M}^{(0)qQ\to qgQ} \xrightarrow[\text{MRK}]{} g_s^3 T^A_{c_1 c_a} T^B_{c_3 c_b} f^{ABG} \frac{S^{(0)qQ\to qQ}_{\text{MRK}}}{\mathbf{q}_{1\perp}^2 \mathbf{q}_{2\perp}^2} \epsilon^*_\rho \boxed{V^\rho(\mathbf{q}_{1\perp}, \mathbf{q}_{2\perp})}$$

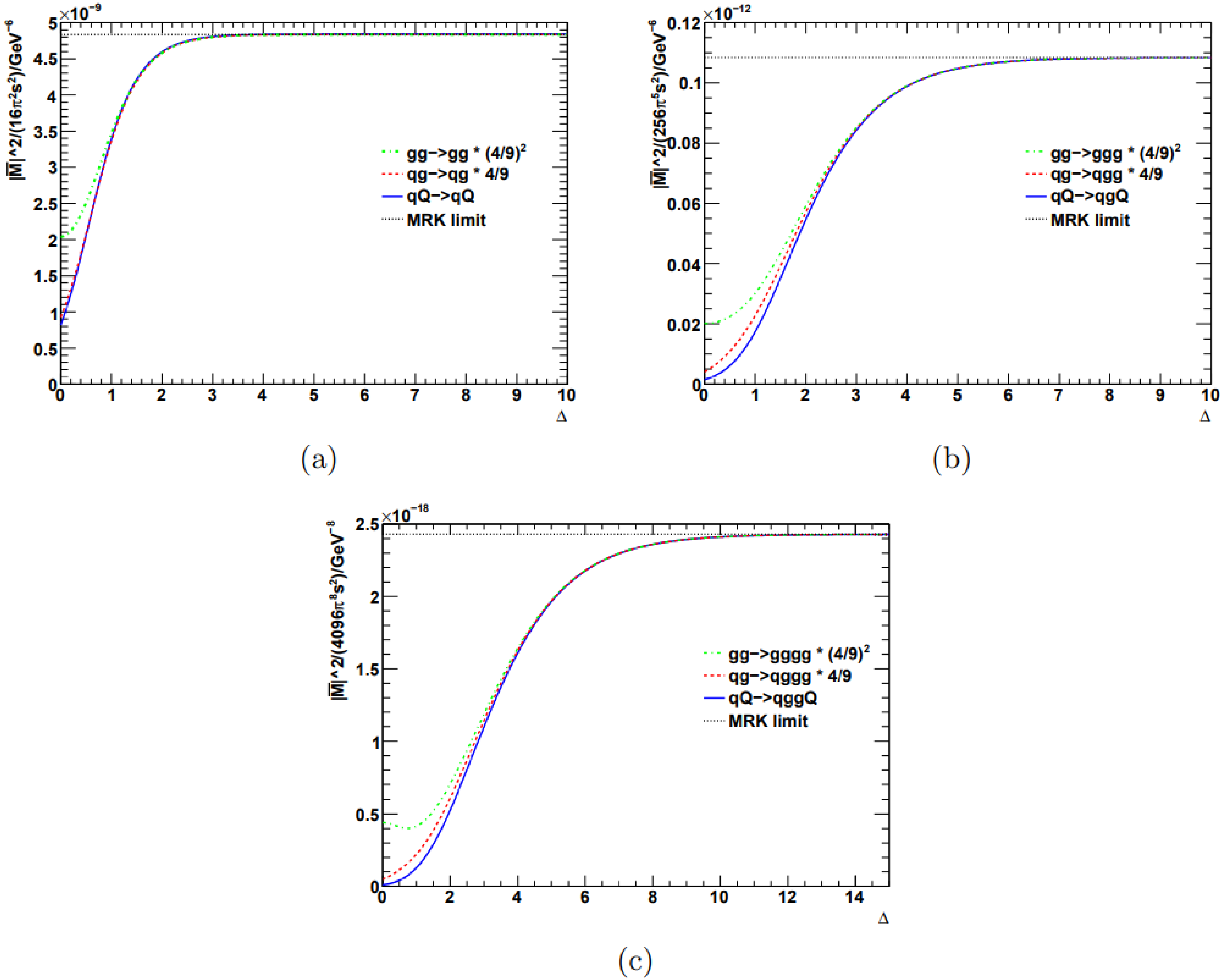
$$\boxed{V^\rho(\mathbf{q}_{i\perp}, \mathbf{q}_{i+1\perp}) = -(\mathbf{q}_{i\perp} + \mathbf{q}_{i+1\perp})^\rho + 2p_a^\rho \left(\frac{\mathbf{q}_{i\perp}^2}{s_{ai}} + \frac{s_{ib}}{s_{ab}} \right) - 2p_b^\rho \left(\frac{\mathbf{q}_{(i+1)\perp}^2}{s_{ib}} + \frac{s_{ai}}{s_{ab}} \right)}$$

n REAL EMISSIONS IN THE MRK LIMIT



$$|\mathcal{M}^{(0)qQ\to qg\cdots gQ}_{\mathrm{MRK}}|^2 = \frac{N^2}{N^2-1}|\mathcal{S}^{(0)qQ\to qQ}_{\mathrm{MRK}}|^2 \left(\frac{g_s^2 C_F}{\mathbf{q}_1^2}\right)\boxed{\left(\prod_{i=1}^{n-2}\frac{4g_s^2 C_A}{\mathbf{p}_i^2}\right)}\left(\frac{g_s^2 C_F}{\mathbf{q}_{n+2}^2}\right)$$

APPLICABILITY OF MRK LIMIT?



THE APPROACH OF HEJ TO REAL EMISSIONS

Why not reinstate the full $S^{(0)}$ instead of using $S_{MRK}^{(0)}$?

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Why not reinstate 4-momenta in the Lipatov vertex, and restore symmetry?

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\begin{aligned}
V^{\rho}(q_1, q_2) = &-(q_1 + q_2)^{\rho} \\
&+ \frac{p_a^{\rho}}{2}\left(\frac{q_1^2}{p_2 \cdot p_a} + \frac{p_2 \cdot p_b}{p_a \cdot p_b} + \frac{p_2 \cdot p_3}{p_a \cdot p_3}\right) + p_a \leftrightarrow p_1 \\
&- \frac{p_b^{\rho}}{2}\left(\frac{q_2^2}{p_2 \cdot p_b} + \frac{p_2 \cdot p_a}{p_b \cdot p_a} + \frac{p_2 \cdot p_1}{p_b \cdot p_1}\right) - p_b \leftrightarrow p_3
\end{aligned}
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These are the steps taken by the High Energy Jets framework in order to better describe collisions at the large but finite energies of the LHC:

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$$
|\mathcal{M}_{\text{HEJ}}^{(0)qQ\to qg\cdots gQ}|^2 = \frac{N^2}{N^2-1}|\mathcal{S}^{(0)qQ\to qQ}|^2 \left(\frac{C_F}{t_1^2}\right)\left(\prod_{i=1}^{n-2}\frac{-g_s^2 C_A}{t_i t_i + 1}\boxed{V^\rho(q_i, q_{i+1})V_\rho(q_i, q_{i+1})}\right)\left(\frac{C_F}{t_2^2}\right)
$$

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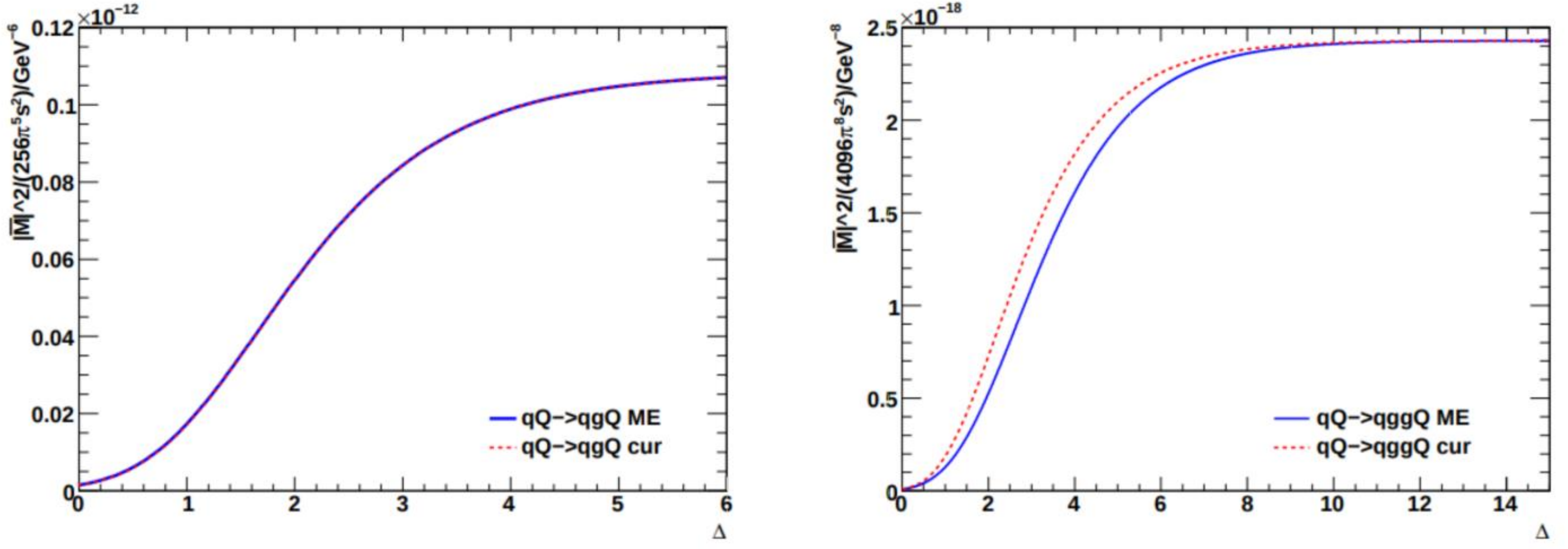
$$\boxed{\begin{aligned} V^\rho(q_1, q_2) = &- (q_1 + q_2)^\rho \\ &+ \frac{p_a^\rho}{2}\left(\frac{q_1^2}{p_2 \cdot p_a} + \frac{p_2 \cdot p_b}{p_a \cdot p_b} + \frac{p_2 \cdot p_3}{p_a \cdot p_3}\right) + p_a \leftrightarrow p_1 \\ &- \frac{p_b^\rho}{2}\left(\frac{q_2^2}{p_2 \cdot p_b} + \frac{p_2 \cdot p_a}{p_b \cdot p_a} + \frac{p_2 \cdot p_1}{p_b \cdot p_1}\right) - p_b \leftrightarrow p_3 \end{aligned}}$$

These are the steps taken by the High Energy Jets framework in order to better describe collisions at the large but finite energies of the LHC:

$$|\mathcal{M}^{(0)qQ\to qg\cdots gQ}_{\text{HEJ}}|^2 = \frac{N^2}{N^2-1}|\mathcal{S}^{(0)qQ\to qQ}|^2 \left(\frac{C_F}{t_1^2}\right)\left(\prod_{i=1}^{n-2}\frac{-g_s^2 C_A}{t_i t_i + 1}\boxed{V^\rho(q_i, q_{i+1})V_\rho(q_i, q_{i+1})}\right)\left(\frac{C_F}{t_2^2}\right)$$

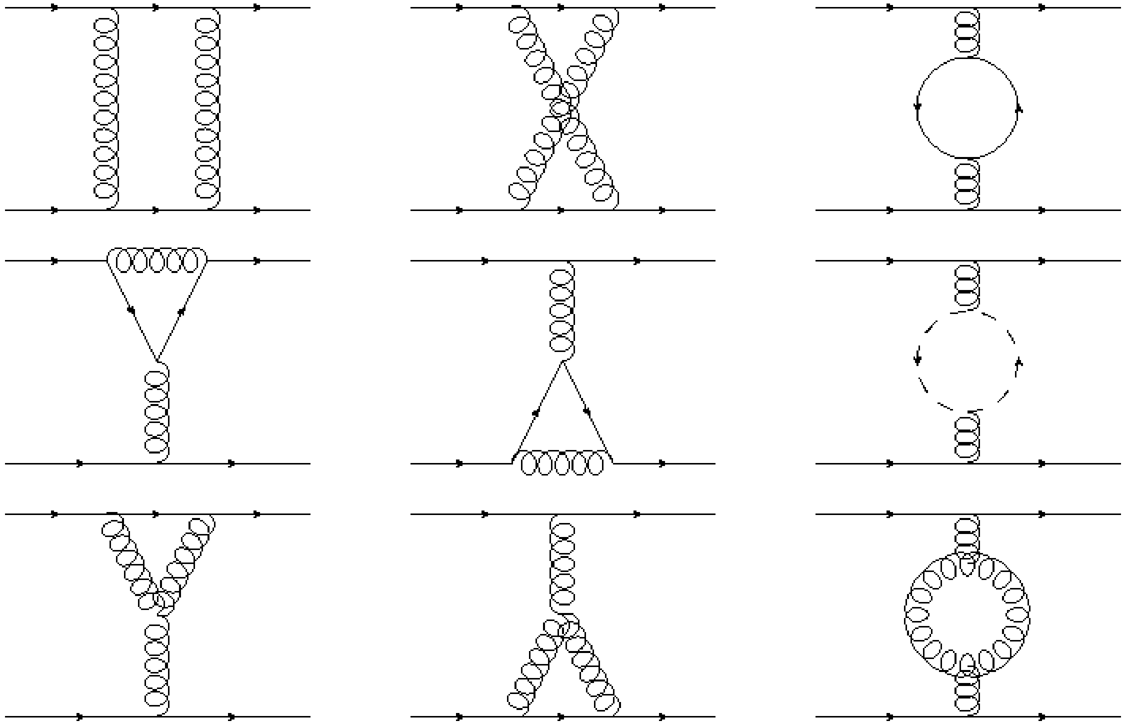
What effect do these changes have?

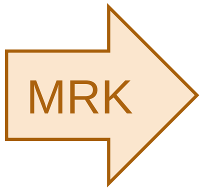
AN IMPROVED DESCRIPTION



(a) (b)

VIRTUAL CORRECTIONS IN THE REGGE LIMIT





$$\mathcal{M}^{(1)qQ\to qQ}_{\mathrm{MRK}} = \mathcal{M}^{(0)qQ\to qQ}_{\mathrm{MRK}} \left(\boxed{\alpha(\mathbf{q}_\perp)} \log\left(\frac{s}{\mathbf{q}_\perp^2}\right)\right)$$

$$\boxed{\alpha(\mathbf{q}_\perp) = \alpha_s C_A \int \frac{d^2\mathbf{k}_\perp}{(2\pi)^2} \frac{-\mathbf{q}_\perp^2}{\mathbf{k}_\perp^2(\mathbf{k}_\perp - \mathbf{q}_\perp)^2}}$$

[4] Fadin, Kuraev, and Lipatov, Phys. Lett. B60 (1975) 50–52

N^{n}LO VIRTUAL CORRECTIONS IN THE REGGE LIMIT

The pattern continues to higher orders:

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Fadin, Kuraev and Lipatov found that to all orders, the LL corrections do give:

$$\mathcal{M}^{qQ\to qQ}_{\text{LL}} = 2s[g_s T^A_{c_1 c_a}]\left(\frac{1}{t}\exp\left(\alpha(\mathbf{q}_\perp)\log\left(\frac{s}{\mathbf{q}_\perp}\right)\right)\right)[g_s T^A_{c_2 c_b}]$$

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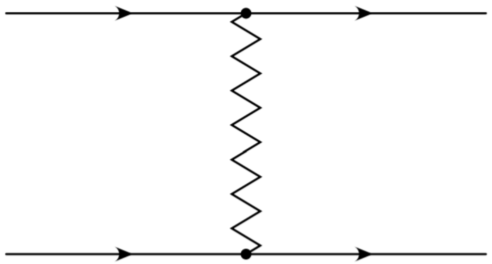

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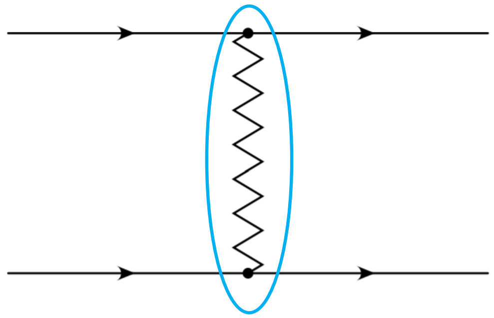
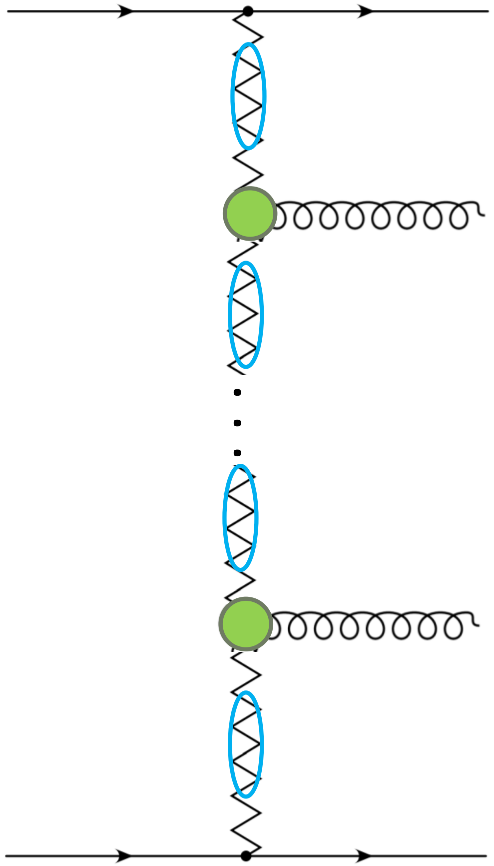


VIRTUAL CORRECTIONS IN HEJ

We apply the exponential factor to each *t*-channel gluon:



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REGULARISATION OF DIVERGENCES

The factor α is infra-red divergent:

$$\alpha(\mathbf{q}_{i\perp}) = -g_s^2 C_A \frac{\Gamma(1-\epsilon)}{(4\pi)^{2+\epsilon}} \frac{2}{\epsilon} \left(\frac{\mathbf{q}_{i\perp}^2}{\mu^2} \right)^{\epsilon} + \mathcal{O}(\epsilon)$$

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But consider the case where one emitted gluon is soft:

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Integrating over the soft phase space gives

$$\int_{y_{i-1}}^{y_{i+1}} \int_0^{\lambda} \frac{d^{2+2\epsilon}\mathbf{p}\mu^{-2\epsilon}}{(2\pi)^{2+\epsilon}} \frac{4g_s^2 C_A}{\mathbf{p}_i^2} = \frac{g_s^2 C_A}{(2\pi)^{2+2\epsilon}} (y_{i+1} - y_{i-1}) \left[\frac{\pi^{\epsilon}}{\Gamma(1+\epsilon)} \frac{1}{\epsilon} \left(\frac{\lambda^2}{\mu^2} \right)^{\epsilon} \right]$$

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Meanwhile, the hard matrix element $|\mathcal{M}_{\text{HEJ}}^{(0)qQ\to qg_1\cdots g_{i-1}g_{i+1}\cdots g_nQ}|^2$ contains

$$\exp(2\alpha(q_i)(y_{i+1} - y_{i-1})) = \frac{-g_s^2 C_A}{(2\pi)^{2+2\epsilon}} (y_{i+1} - y_{i-1}) \left[\frac{\pi^{\epsilon}\Gamma(1-\epsilon)}{\epsilon} \left(\frac{\mathbf{q}_i^2}{\mu^2}\right)^{\epsilon}\right] + \mathcal{O}(\epsilon)$$

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- Thanks for your attention!