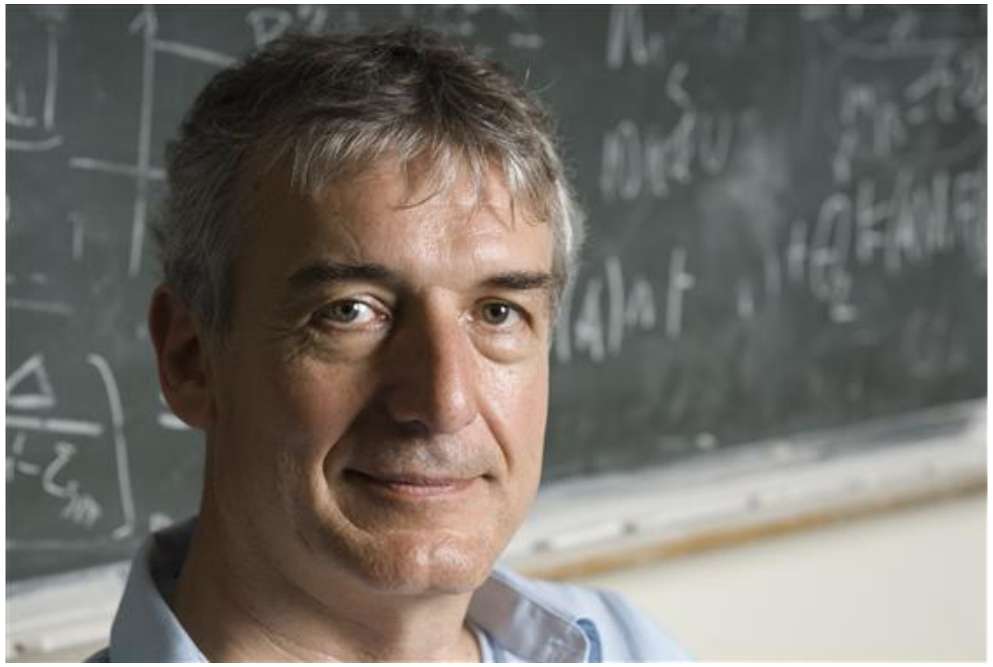
Sikivie-style resonant axion haloscopes - 1



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QSFP School 2020

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Strategy for my seminars

- Simplest pathways to understanding the principles of resonant cavity axion detectors. However, I have tried not to over-simplify and gloss over things that really are complex.
- Experimental focus (but I’m not shy about considering the theory where necessary).
- Classical rather than quantum models (but by all means introduce quantum mechanics where it matters).
- Use of analogies and simplified pictures.
- Aim is to give a clear picture of the pathway from axion phenomenology to an understanding of the detectors in practice.

Contents

Seminar 1

- Summary of relevant axion phenomenology, coupling to photons
- Equivalent circuit model of axion conversion in a cavity
- Signal power in a practical detector

Seminar 2

- Johnson noise
- The radiometer equation
- Amplifier noise and heterodyning
- Low noise amplifier technologies
- Matching and practical considerations
- Consequences of matching network imperfections

Seminar 3

- Techniques for background subtraction
- Combining data from overlapping power spectra
- Handling candidate 'events'
- Determining search sensitivity

The Axion in QCD Phenomenology

The Strong CP Problem

It is established by experiments that quantum chromodynamics conserves the discrete symmetry known as CP (charge conjugation and parity flip). Since CP is violated in the electroweak sector (K-mesons, for example), this is not expected, and, prior to Peccei and Quinn's work, was unexplained except by positing a zero mass quark.

The Peccei Quinn Mechanism

Robert Peccei and Helen Quinn proposed that QCD possessed a global U(1) symmetry broken at some high energy scale f_{PQ}. Once broken, the symmetry led to a CP conserving ground state for QCD.

The Axion

Frank Wilczek and Steve Weinberg both pointed out that oscillations about this ground state implied the existence of a new pseudoscalar particle, which Wilczek named the axion (after a washing powder he used - Weinberg wanted to name it the higglet).

QCD and CP Violation

How to construct a particle theory

Particle phenomenology is carried out by writing down all the Lagrangian terms in the fields of your theory consistent with the symmetries you think the theory should possess. For example, a Lorentz invariant QCD Lagrangian contains the following terms for the energy densities of the gluon field strength tensors:

$$\mathcal{L}_{\text{kinetic}} = \frac{1}{4}\sum_{\alpha=0}^{3}\sum_{\beta=0}^{3}\sum_{\text{flavours n}} F^{\alpha\beta}{}_{n} F_{\alpha\beta}{}_{n} = \frac{1}{4} g_{\alpha\gamma} g_{\beta\delta} F^{\alpha\beta}{}_{n} F^{\gamma\delta}{}_{n}$$

If you want a particle theory containing these fields that violates the CP symmetry, there is a second way of constructing a Lagrangian term out of the quark fields:

$$\mathcal{L}_{\text{CPV}} = \frac{\overline{\Theta}}{32\pi^2} \varepsilon_{\alpha\beta\gamma\delta} F^{\alpha\beta}{}_{n} F^{\gamma\delta}{}_{n}$$

where $\overline{\Theta}$ is, for now, a numerical coefficient expressing the extent of CP violation. The physical phenomena thought to contribute to CP violation include vacuum tunnelling (instanton) effects and a non-zero phase in the determinant of the quark mass matrix. But, in general, why should QCD conserve CP, when QED does not? This term shouldn’t just disappear.

The ~~Pool~~ Snooker table analogy to axion physics (Pierre Sikivie, arXiv:hep-ph/9506229)

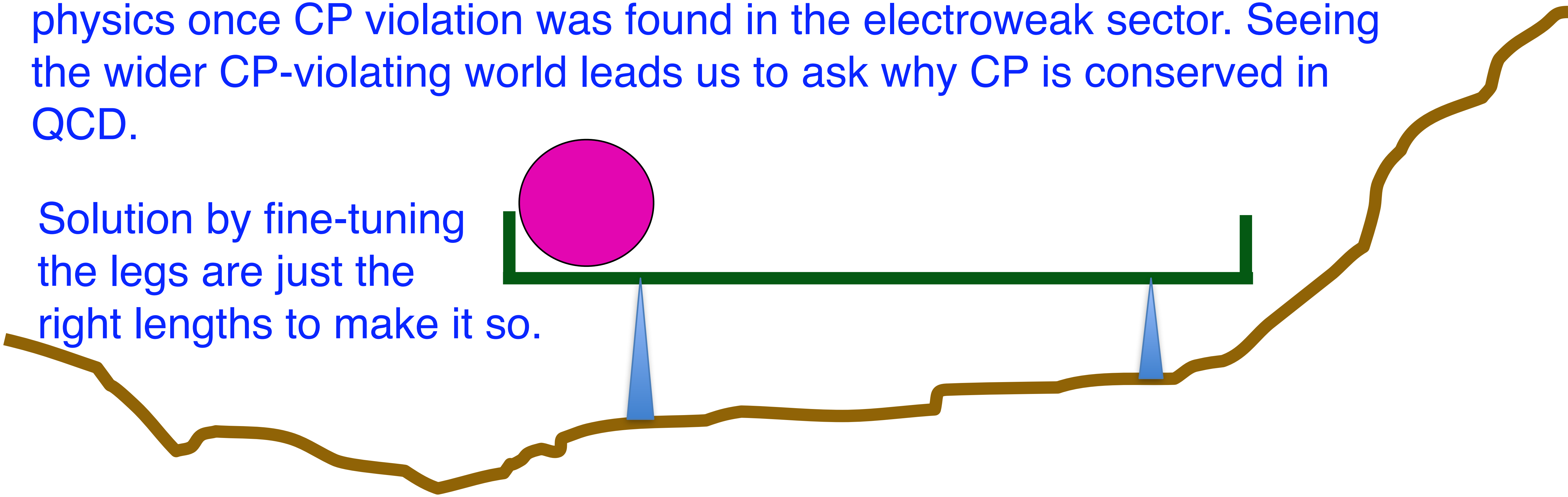
Consider a snooker ball on a flat table.

If you only consider physics on the table, there is translational symmetry - every horizontal position is the same as every other.

The snooker ball sees the wider world

A snooker table is an artificially flat landscape, with a symmetry imposed by special conditions. When you are not on the table, there is no symmetry with respect to translation. The analogy here is to particle physics once CP violation was found in the electroweak sector. Seeing the wider CP-violating world leads us to ask why CP is conserved in QCD.

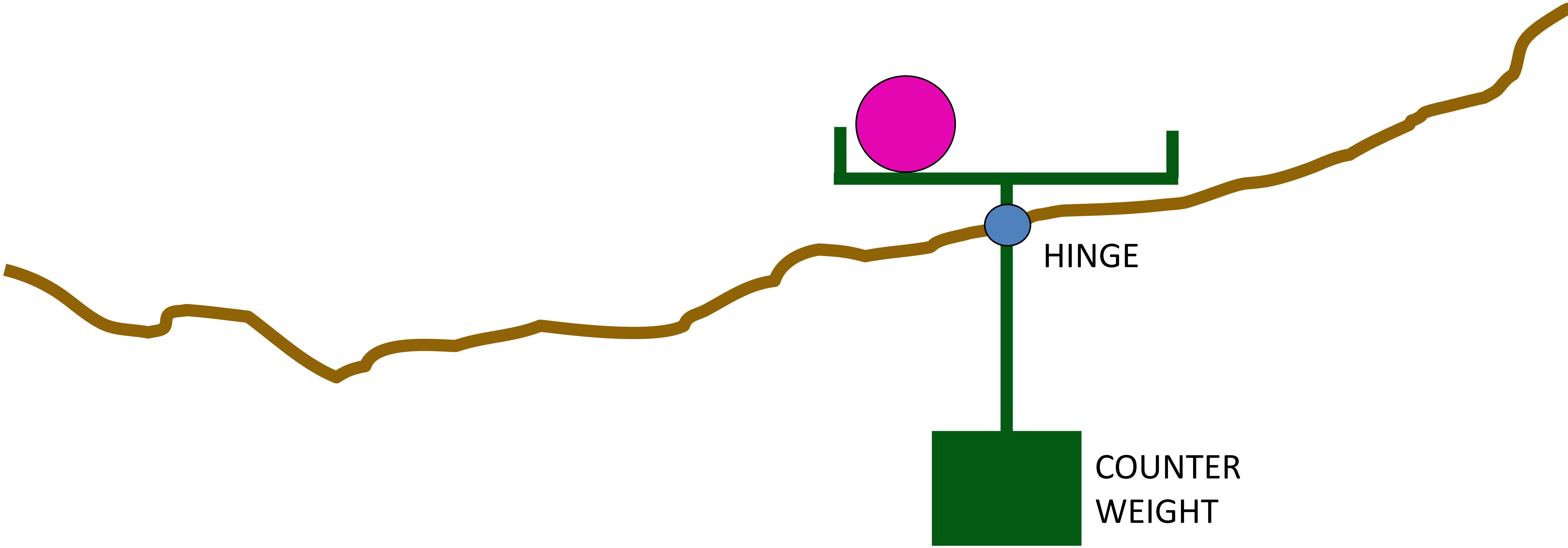
Solution by fine-tuning the legs are just the right lengths to make it so.



Fine tuning arguments are sometimes argued for - particularly by people who like anthropic arguments ‘if it wasn’t that way, we wouldn’t be here to write this paper’…or something like that. If you don’t like anthropic arguments, and I don’t, then you need a mechanism, a reason, for the observed symmetry.

A Pool Table Levelling Mechanism

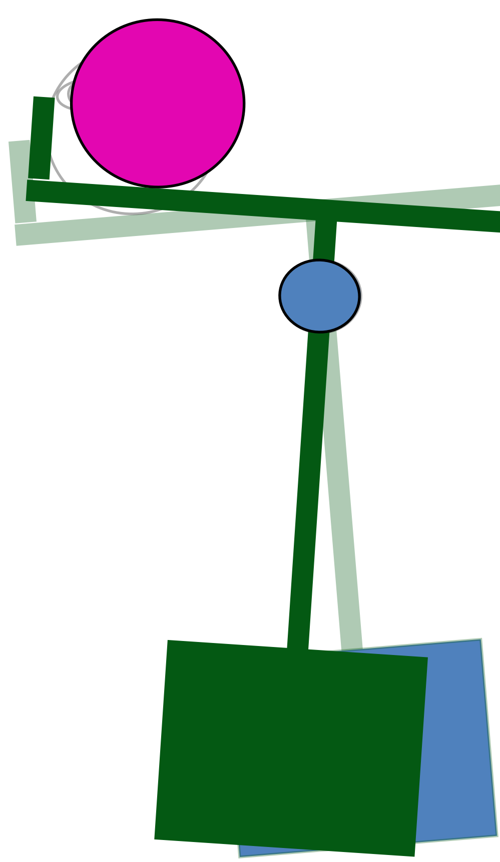
Here is a mechanism that forces the pool table to be flat. It only works in the absence of violent earthquakes that place the table and its contents in a state of violent excitement. A long time after the earthquake, oscillations in the mechanism are damped away, and you are left with a flat table and a restored translational symmetry.



The above language points to an analogy - a mechanism like this, and also the Peccei Quinn mechanism, can be made to work at low energies, but not at high energies. The early Universe was in a state of violent excitement, but expansion and redshift led to the lowering of energies of its constituents, and at some energy scale $f_{\rm PQ}$ the Peccei Quinn mechanism works to impose the CP symmetry on quantum chromodynamics.

Symmetry restoration at low energies

Suppose the snooker table started out at a arbitrary angle and the mechanism was activated to restore its flatness. There would be residual oscillations of the counterweight. As a result the table's orientation has small oscillations. These small oscillations have consequences for dynamics on the table. Analogously, residual oscillations about the low energy QCD vacuum correspond to residual axion field energy in our Universe. Quantum mechanics leads us to think of this residual field as pseudo scalar massive field quanta - a gas of massive (though very light) particles.



Analogies

Original arbitrary angle of snooker table	CP is not a symmetry of nature at high energies/early times.
Levelling mechanism	Peccei Quinn (PQ) mechanism forces CP conservation at low energies / late times.
Residual Table Oscillations	Axions !

Peccei and Quinn's mechanism

In Peccei and Quinn's solution to the strong CP problem, QCD possesses an additional global chiral U(1) symmetry. At low energies, this symmetry is spontaneously broken, in such a way as to introduce a new CP violating term in the QCD Lagrangian. The vacuum expectation value of this field exactly cancels the sum of all the other CP violating terms.

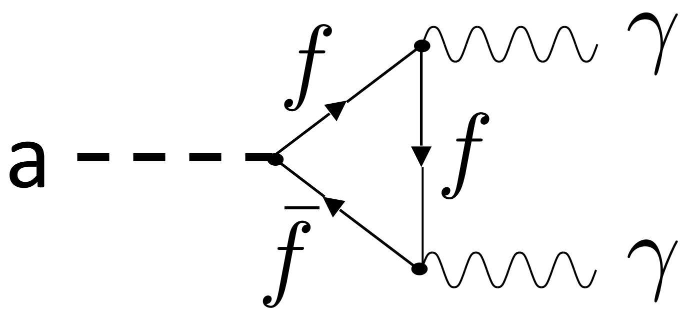
In fact, it is then possible to replace all the CP violating terms with a single term proportional to an axion field a having a zero vacuum expectation value at low energies.

$$\mathcal{L}_{\mathrm{PQ}} = \frac{g_{a\gamma\gamma}}{8\pi} a \varepsilon_{\alpha\beta\gamma\delta} F^{\alpha\beta}{}_{n} F^{\gamma\delta}{}_{n}$$

Of course this is new physics. It implies deep connections between nominally disparate sectors of QCD.

Axion interaction with photons

The axion field now yields other axial terms involving interactions with other sectors of the standard model. Because the axion is a pseudoscalar, it has the same perturbative interactions as another more familiar massive pseudoscalar, the π^0. In particular, an anomaly coupling to two photons:



This is the coupling used to detect axions in Sikivie-type resonant axion detectors. The interaction Lagrangian is here written in SI units where the dimensions of the action are $[S] = [\hbar] = ML^2T^{-1}$.

$$\mathcal{L}_{a\gamma\gamma} = \frac{g_{a\gamma\gamma}}{8} a\epsilon_0 c \varepsilon_{\alpha\beta\gamma\delta} F^{\alpha\beta}_{\mathrm{EM}} F^{\gamma\delta}_{\mathrm{EM}} = g_{a\gamma\gamma} a\epsilon_0 \vec{E} \cdot \vec{B}$$

This interaction term combines with the ordinary energy density term

$$\mathcal{L}_{\mathrm{EM}} = \frac{1}{4\mu_0 c} g_{\alpha\gamma} g_{\beta\delta} F^{\alpha\beta}_{\mathrm{EM}} F^{\gamma\delta}_{\mathrm{EM}} = \frac{1}{2c}\epsilon_0 E^2 + \frac{1}{2\mu_0 c} B^2$$

leading to modified Maxwells equations including the axion field. The modified behaviour of electromagnetic fields can be used to detect axions.

Maxwells equations with axions

Maxwell's equation for $\vec{\nabla} \times \vec{B}$ yields the effect of the axion field on the electromagnetic field in the cavity [2]

$$\vec{\nabla} \times \frac{\vec{B}}{\mu_0} = \frac{\partial(\epsilon_0 \vec{E})}{\partial t} + g_{a\gamma\gamma}\sqrt{\frac{\epsilon_0}{\mu_0}}\left(\frac{\partial}{\partial t}(a\vec{B}) + \vec{\nabla} \times (a\vec{E})\right)$$

The coupling $g_{a\gamma\gamma}$ is model dependent. The two canonical axion models are the more heavily coupled KSVZ axion, and the more slightly coupled DFSZ axion. $g_{a\gamma\gamma}$ can be written as [3]

$$g_{a\gamma\gamma} = \frac{g_\gamma \alpha}{\pi f_{PQ}} \quad \text{and} \quad g_\gamma = \frac{1}{2}\left(\frac{E}{N} - \frac{2(4+z)}{3(1+z)}\right)$$

Here f_{PQ} is the energy scale below which the PQ symmetry is broken. E and N are the electromagnetic and colour anomalies in the phenomenology under consideration, and z is the ratio of the up and down quark masses. For the KSVZ model $g_\gamma = -0.97$ and for DFSZ, $g_\gamma = 0.36$.

[1] M. E. Tobar, B. T. McAllister, M. Goryachev, *Modified axion electrodynamics as impressed electromagnetic sources through oscillating background polarisation and magnetisation.* Physics of the Dark Universe **26** (2019) 100339, arXiv:1809.01654
[2] L. D. Duffy and K. van Bibber, *Axions as dark matter particles,* New Journal of Physics, **11** (2009) 105008. References therein to the original KSVZ and DFSZ papers.

Coupling to a resonant cavity in a static magnetic field

Taking the term containing the time derivative of $a\vec{B}$ from the modified Maxwell equation, we obtain the source term exploited by resonant cavity axion detectors.

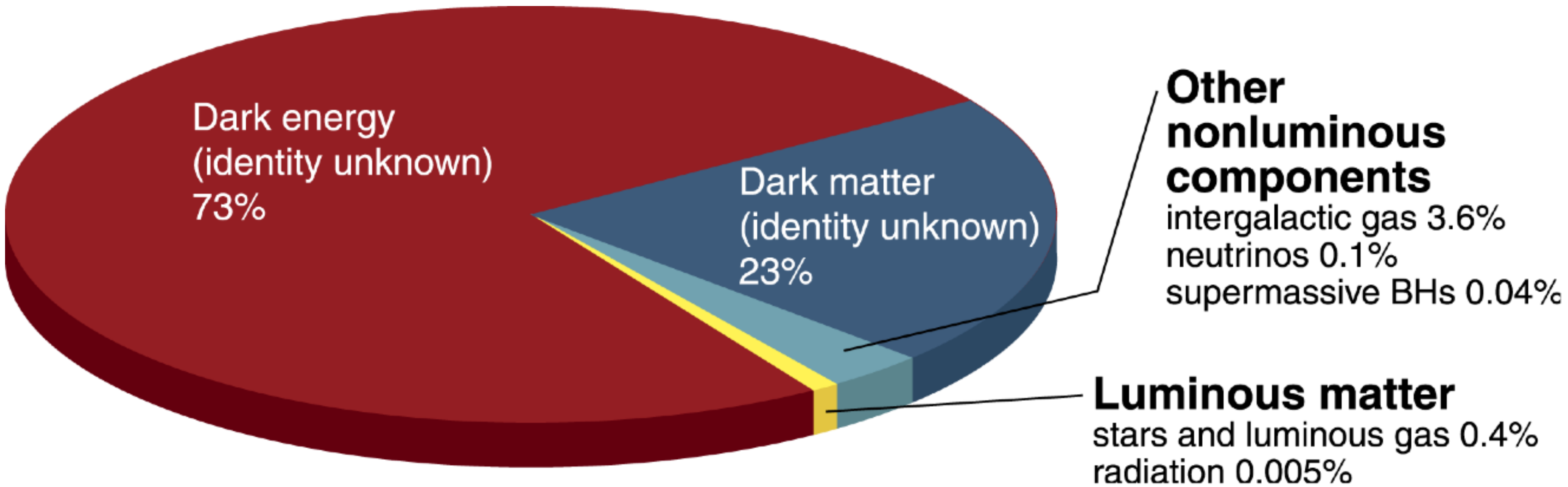
$$\vec{\nabla} \times \frac{\vec{B}}{\mu_0} = \frac{\partial(\epsilon_0 \vec{E})}{\partial t} + g_{a\gamma\gamma}\sqrt{\frac{\epsilon_0}{\mu_0}}\vec{B}\frac{\partial a}{\partial t}.$$

If a region is threaded by a static uniform magnetic field, and that region also contains an oscillating axion field, then a source term for oscillating electromagnetic fields at the same frequency is formed by the product of the magnetic field, the axion field, and the frequency of that axion field. Other source terms in the Maxwells equation can be neglected for this geometry.

It is possible to deduce the power converted into electromagnetic waves by integrating the solutions of the modified Maxwell equation over the volume of the detector and, in the case of a resonant cavity, incorporating the effects of the buildup of energy in the resonances. However, there is a short-cut to the correct answer for the signal power which shall take instead, later in this seminar.

Axions as dark matter

The picture of axions as residual oscillations in the axion field after spontaneous breaking of the PQ symmetry makes axions a good cold dark matter candidate.



It is thought that the matter budget of the Universe is dominated by unknown cold or warm dark matter (warm dark matter could be sterile neutrinos, for example). Another possibility is modified Newtonian dynamics in the limit of small accelerations. This idea is less popular after the bullet cluster observation.

Axions produced at the energy scale f_{PQ} will, as we shall see, have very faint couplings to baryonic matter; hence having been produced they do not reach thermal equilibrium with the other contents of the Universe, neither do they decay in the lifetime of our Universe. They have the required properties to form the structure that we see today, and could even dominate a cold dark matter halo.

Properties of axion dark matter

The properties of the axion relevant to dark matter searches are all related to the symmetry breaking scale. We've already seen that the coupling to two photons scales as $1/f_{PQ}$

$$g_{a\gamma\gamma} = \frac{g_\gamma \alpha}{\pi f_{PQ}}$$

The axion mass and its abundance in the Universe today can also be shown to scale with this same parameter.

$$m_a = 6\,\mu\mathrm{eV}\left(\frac{10^{12}\,\mathrm{GeV}}{f_{PQ}}\right)$$

Therefore axions produced at higher energies, or earlier times, are lighter and more faintly coupled. This led to the name 'invisible axions' for axions so light that they might never be detected as their couplings vanish. On the other hand, the abundance of axions in the Universe also scales with f_{PQ} [2].

$$\Omega_a \sim 0.15\left(\frac{f_{PQ}}{10^{12}\,\mathrm{GeV}}\right)^{\frac{7}{6}}$$

Axions that are too light may over-close the Universe, or, it can be argued, are produced during inflation where we don't understand physics.

Properties of axion dark matter - 2

Other axion properties are identical to those of WIMPs, since they derive from the observationally deduced properties of our halo. Though these assumptions are ‘vanilla’ ones, I will follow my colleagues in using them as a straw man model for the dark matter halo. Probably things are more complex in practice.

Local rest energy density of the dark matter halo in a spherically symmetric (flattened) model:

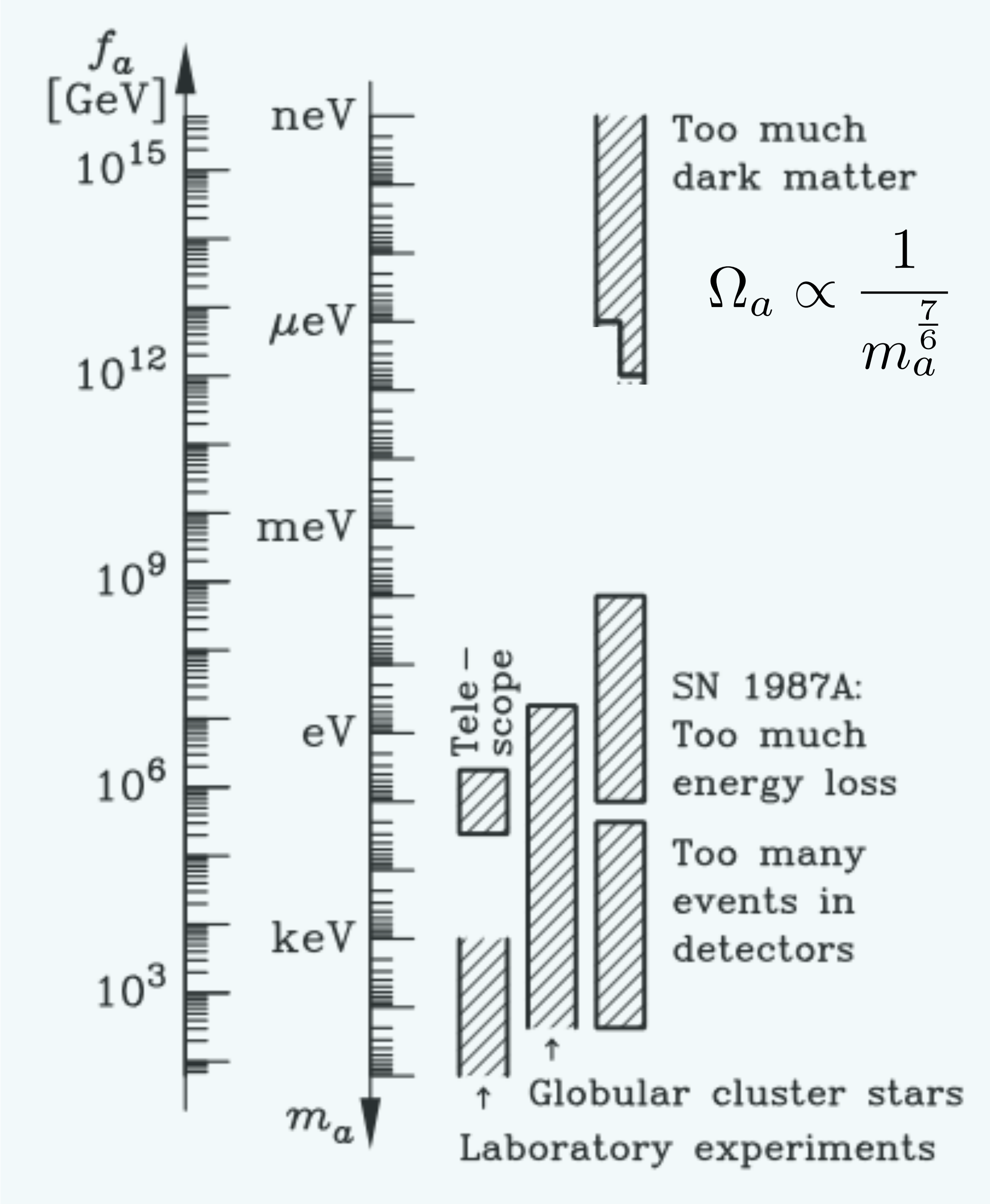
$$\rho_H = 0.3\,(0.45)\,\mathrm{GeV\,cm^{-3}}$$

RMS velocity of halo dark matter, equal to the virial velocity in the local galactic gravitational potential

$$v_0 = \sqrt{\overline{v^2}} \simeq 230\,\mathrm{km\,s^{-1}}$$

$6M question: The axion mass

Particle theory doesn't give us much clue, but a combination of cosmological and astrophysical constraints lead to some hints and for where to look.



Axions either over-close the Universe or their abundance is sub-overclosure but affected by new early-Universe physics.

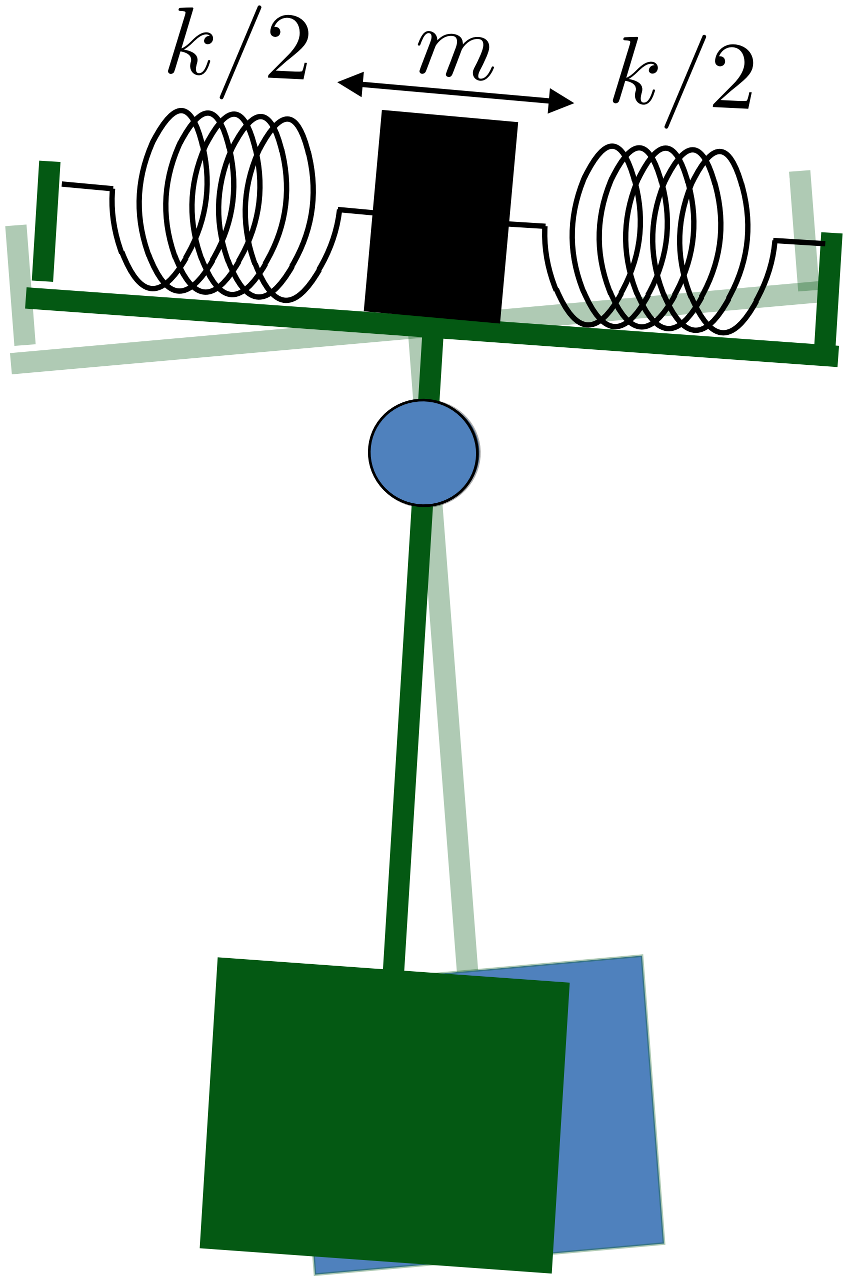
⇐Dark matter range:"axion window"

very hard to detect "invisible axions"

Heavy axions forbidden: else new pion-like particle

Rather like the WIMP miracle, there's the `Axion Coincidence'. It turns out that masses around a micro-eV are close to what is needed for the dark matter halo we think we have, and are not ruled out by existing observational limits.

Back to the snooker table analogy



The earthquake (big bang) was long ago, and the large oscillations of the levelling mechanism have damped out (due to expansion and red shift). There remain residual oscillations of the table surface at low amplitude. One approach to detection of these residual oscillations is to use a resonant detector - for example a mass on a spring.

The mass on the spring resonates with the frequency of the residual table oscillations when

$$2\pi f_{\mathrm{table}} = \sqrt{\frac{k}{m}}$$

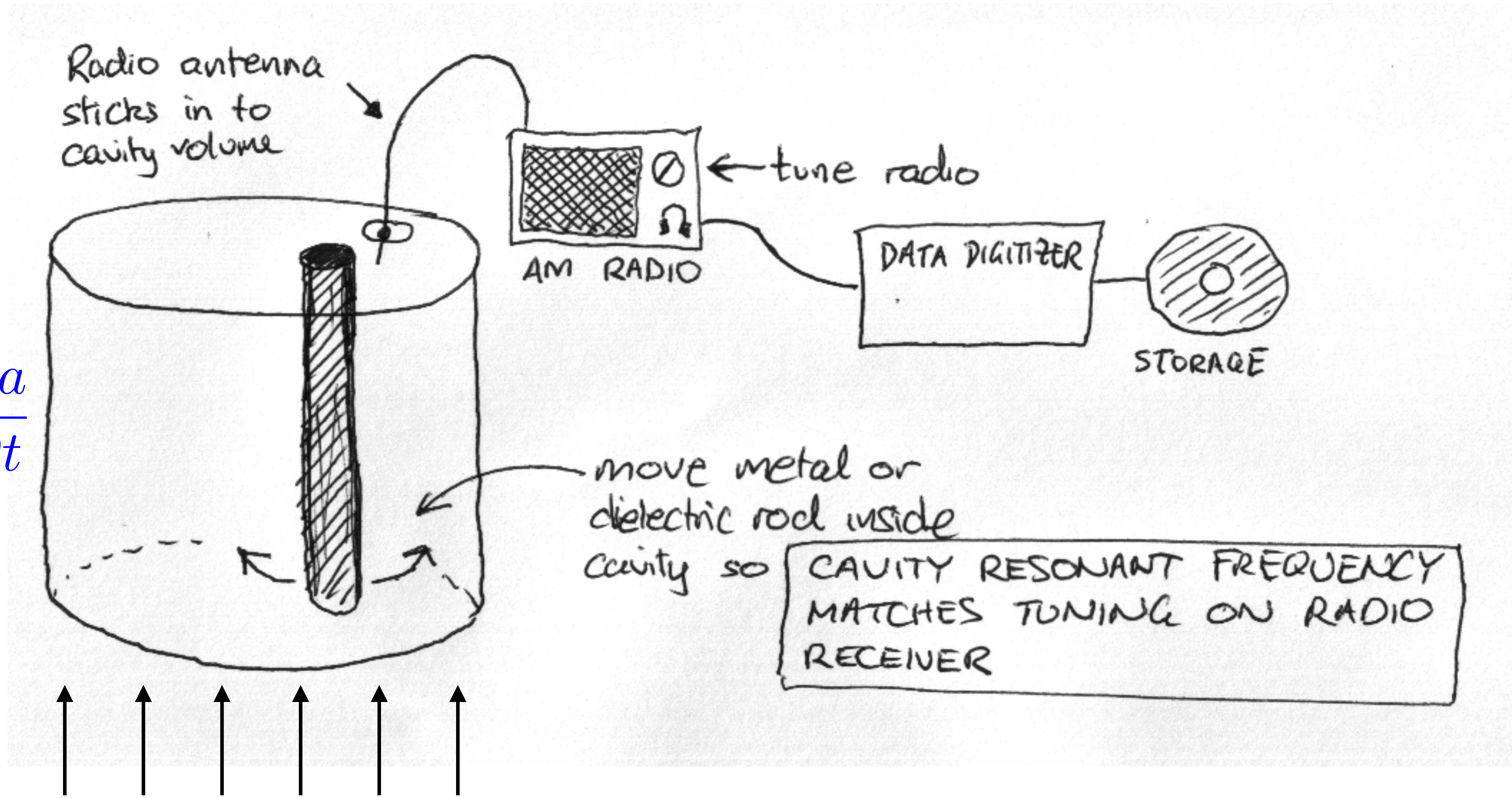
The amplitude of the mass' motion is larger than the displacement of the mass when the table is maintained at maximum slope by a factor of

$$Q = \frac{\sqrt{mk}}{\eta}$$

where η is the coefficient of Stokes (velocity) damping of the mass/spring mechanism.

This is the underlying principle of resonant detection. A high quality oscillator driven at resonance will have an amplitude larger than the drive amplitude by a factor of Q. The catch is, you have to know the resonant frequency.

Resonant axion detectors - cartoon edition



$$\vec{E}_a = g\vec{B}\frac{\partial a}{\partial t}$$

$\vec{B}$

The resonant detector is a mode of oscillation of a cylindrical copper plated metal box. The box is threaded by a static magnetic field. It contains moveable tuning rods, also copper coated, that may be moved to adjust the mode frequencies. When the frequency of the mode of interest is related to the axion mass by $h\nu \simeq m_a c^2$, power is deposited into the cavity mode. The cavity is read out by a radio receiver, for reasons that will be given presently.

More properties of axions, after some assumptions

To learn more about axions, we will make some assumptions. Assume that the axion mass is around that which would provide a good fraction of closure density. We'll choose $m_a c^2 = 4\,\mu\mathrm{eV}$. What are these axions like?

De Broglie wavelength
$$\lambda_{\mathrm{De\,B}} = \frac{h}{p} = \frac{2\pi\hbar c}{mc^2\,\frac{v}{c}}$$

The velocity is the local halo virial velocity of 230 km/s.
Using $\hbar c = 0.2\,\mathrm{GeV\,fm}$ we get

$$\lambda_{\mathrm{De\,B}} = \frac{6.28 \times 0.2 \times 10^9\,[\mathrm{eV\,fm}]}{4 \times 10^{-6}\mathrm{eV} \times \frac{230\,[\mathrm{km\,s^-1}]}{3.0\times 10^5\,[\mathrm{km\,s^{-1}}]}} \simeq 400\,\mathrm{m}$$

By contrast, the De Broglie wavelength of a WIMP is around 1 fm. What about the number density? If a single axion has a rest energy of $4\,\mu\mathrm{eV}$
The local halo density is around 0.4 GeV/cc, implying that each cubic centimetre of space would contain around 10^{14} axions. At this huge number density and with such a long De Broglie wavelength, it is clear that axion dark matter would have the characteristics of a massive pseudo-scalar field more than those of individual particles.

Properties of photons from axion conversion

Frequency

$$\nu_a = \frac{m_a c^2}{h} = \frac{4.0 \times 10^{-6} * \mathrm{e}\,[\mathrm{J}]}{6.6 \times 10^{-34}\,[\mathrm{Js}]} = 970\,\mathrm{MHz}$$

This is in the UHF band - roughly the same frequency band that is populated by mobile phone signals. 1GHz corresponds to a 30cm wavelength, so the resonant cavities that will respond to such photons will be of order 30cm diameter.

Bandwidth

Bandwidth is given by the ratio of the kinetic energy of axions to their rest energy, multiplied by the frequency. Here we have

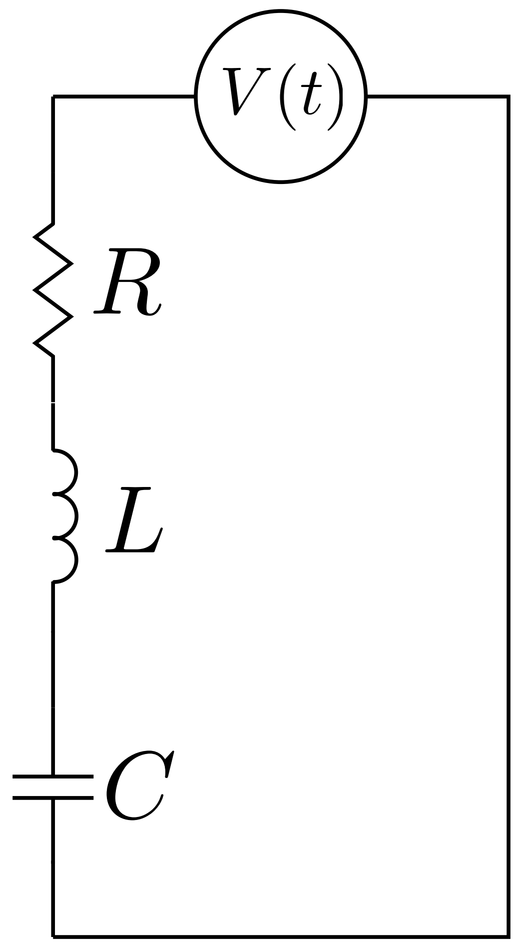
$$\frac{\Delta\nu_a}{\nu_a} \sim \frac{\frac{1}{2}mv_0^2}{mc^2} = \frac{v^2}{2c^2} = \frac{(230\,[\mathrm{km\,s}^-1])^2}{2 \times (3 \times 10^5\,[\mathrm{km\,s}^-1])^2} = 3 \times 10^{-7}$$

$$\Delta\nu_a = 290\,\mathrm{Hz}$$

This is quite a narrow line width. Our resonant cavity will be normal-conducting as it must be in a high magnetic field. Normal conducting cavities typically have $Q \sim 10^5$, corresponding to a line width of 9.7kHz. The axion signal will cover only a small fraction of the resonance width.

Equivalent circuit model of a resonant mode

To derive the signal power in a cavity mode from the fundamental axion-photon interaction, we take a short cut and model the mode as a series RLC circuit, shown below. The circuit is driven by a voltage source, which represents the axion signal driving the fields of the resonance.



The Lagrangian for the circuit in terms of the charge q on the capacitor plates, is

$$L = \frac{1}{2}\dot{q}^2 - \frac{1}{2C}q^2 + qV(t),$$

where the last term is the driving term from the axion source. To form the Euler-Lagrange equation, we add a dissipative term for the resistor,

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) - \frac{\partial L}{\partial Q} + \frac{\partial F}{\partial \dot{q}} = 0$$

where

$$F = \frac{1}{2}R\dot{q}^2$$

Lagrangian term for axion to photon conversion

The actual drive derives from the lagrange density for the interaction term between the axion field and the electric and magnetic fields.

$$\mathcal{L}_{a\gamma\gamma} = g_{a\gamma\gamma} a \epsilon_0 \vec{E} \cdot \vec{B}$$

We convert this Lagrangian density term into a drive term with energy units by multiplying by c and integrating over volume - here the volume of the cavity.

$$L_{a\gamma\gamma} = g_{a\gamma\gamma} a c \epsilon_0 \int_V \vec{E} \cdot \vec{B} \, dV$$

In a cavity detector, the integrand is interpreted as the dot product of the electric field of the cavity mode induced by the axion field with the static magnetic field threading the cavity. To simplify the argument (in practice this simplification is usually within about 20% of reality), we assume that the magnetic field is uniform, of magnitude B_0 and parallel to the z axis.

$$L_{a\gamma\gamma} = g_{a\gamma\gamma} a c \epsilon_0 B_0 \int_V \vec{E} \cdot \hat{z} \, dV$$

Cavity form factor

We define a form factor for the cavity mode as follows:

$$f_{nlm} = \frac{\left(\int_V \vec{E} \cdot \hat{z} \, dV \right)^2}{V \int_V E^2 \, dV}$$

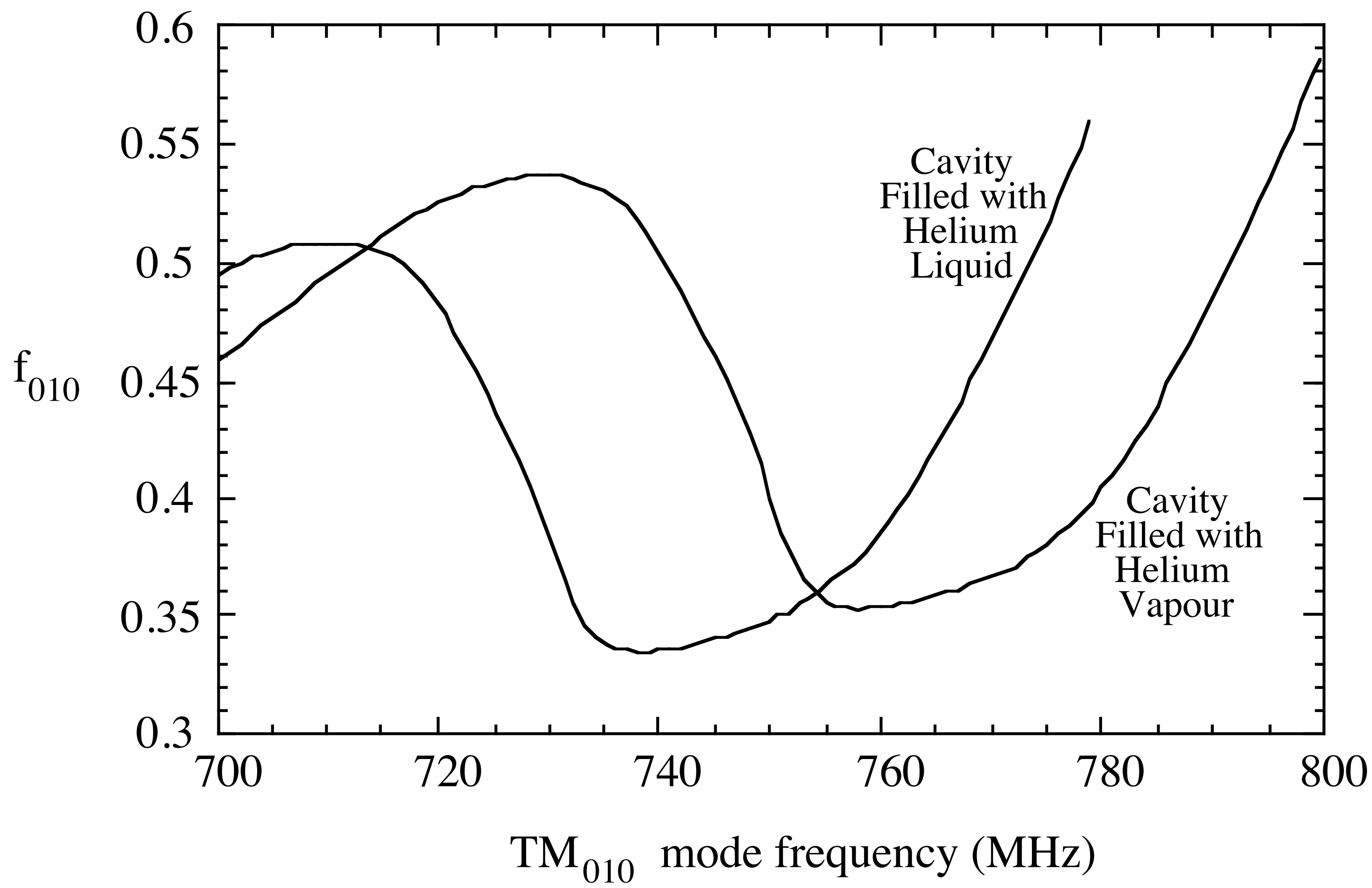
The indices [nlm] are there because resonant modes of cavities with cylindrical symmetry are classified using a mode type (TM, TE, or TEM) and a set of indices corresponding to the solutions of the partial differential equations into which the wave equation separates [4].

It turns out that only the very lowest order TM modes, and in particular the TM_{010} mode, have appreciable values of the form factor. The stored energy in the cavity mode is now equated with the stored energy in the capacitor of the equivalent circuit:

$$\frac{q^2}{2C} = \int_V \frac{1}{2} \epsilon_0 E^2 \, dV$$

[4] J.D. Jackson, *Classical Electrodynamics,* section 8.7

Typical TM_{010} mode form factor in practice



This form factor was calculated for a 50cm diameter cavity, 1 metre tall, containing a single tuning rod mounted parallel to the cavity axis. The tuning rod had approximately 8 cm diameter, and was mounted on a cam so that its displacement from the cavity symmetry axis could be varied, which changes the TM_{010} mode frequency. The form factor was calculated for a cavity filled with helium vapour, and for a cavity filled with liquid - filling with liquid moves the mode frequencies by about 3%, allowing the experiment to steer around mode crossings.

Relationship between axion field and equivalent drive voltage

$$\int_V \vec{E}\cdot\hat{z}\,dV = q\sqrt{\frac{f_{nlm}V}{\epsilon_0 C}}.$$

Substituting for the integral in the Lagrangian we obtain an equivalence between the voltage drive and the parameters of the driven cavity mode.

$$V(t) = g_{a\gamma\gamma}cB_0\sqrt{\frac{f_{nlm}V\epsilon_0}{C}}a(t)$$

The power deposited in the cavity mode is V^2/R

$$P = \frac{\langle V^2(t)\rangle}{R} = g_{a\gamma\gamma}^2 c^2\epsilon_0 B_0^2 V f_{nlm}\frac{1}{RC}\langle a^2(t)\rangle$$

It remains to remove the remaining R and C from the equivalent circuit model of the cavity. However, RC can be re-expressed in terms of the measurable parameters of the resonance ω_0 and Q.

$$Q = \frac{1}{R}\sqrt{\frac{L}{C}}$$ and $$\omega_0 = \frac{1}{\sqrt{LC}}$$ so $$\frac{1}{RC} = \omega_0 Q$$

How big is the axion field amplitude in the local halo?

A dimensional argument gets us started here. The dimensions of the axion field, deduced (again) starting from the dimensions the lagrange density and using either the mass or the kinetic energy terms, are

$$[a] = \mathrm{kg}^{\frac{1}{2}}\,\mathrm{s}^{-\frac{1}{2}}$$

Dimensional consistency between the axion field and the density of the halo in SI units lead us to conclude that

$$\left\langle a^2(t) \right\rangle = \frac{\rho_{\mathrm{H}} \hbar^2}{m_a^2 c}$$

We now exploit the fact that the product $a g_{a\gamma\gamma}$ is dimensionless. So, we form the square, also dimensionless,

$$g_{a\gamma\gamma}^2 \left\langle a(t)^2 \right\rangle = \frac{g_{a\gamma\gamma}^2 \rho_{\mathrm{H}} \hbar^2}{m_a^2 c}$$

We now substitute this expression and that for RC in terms of cavity resonance parameters into the axion conversion power formula

$$P = \left(\frac{g_{a\gamma\gamma}^2 \rho_{\mathrm{H}} \hbar^2}{m_a^2 c} \right) c^2 \epsilon_0 B_0^2 V \omega_0 Q f_{nlm}$$

Mixed units

$$P = \left(\frac{g_{a\gamma\gamma}^2 \rho_{\mathrm{H}} \hbar^2}{m_a^2 c} \right) c^2 \epsilon_0 B_0^2 V \omega_0 Q f_{nlm}$$

This entire equation is in SI units. The nice thing about that is that the power comes out in watts! The not-so-nice side is that the halo density and the coupling constant are in SI units too, which is painful.

However, we can play a trick, because the term in parentheses is dimensionless. Therefore, we can set $\hbar = c = 1$ in the brackets only, obtaining the wierd, but very convenient mixed units equation for the power

$$P = \left[\frac{g_{a\gamma\gamma}^2 \rho_{\mathrm{H}}}{m_a^2} \right]_{\mathrm{NAT}} c^2 \epsilon_0 B_0^2 V \omega_0 Q f_{nlm}$$

where it is understood that the halo density, coupling constant and axion mass are in calculated in natural units, all other terms are in SI units.

How much power from halo axions?

As is customary, this formula is then written in terms of typical values for the quantities in question. The one tricky conversion is the halo density, which must be written in units eV[4] using the conversion that 1 cm[3] is $1.31 \times 10^{14}\,\mathrm{eV}^{-3}$

$$P = 1.52 \times 10^{-21}\,\mathrm{W}\,\mathrm{f_{nlm}} \left(\frac{B}{7.6\,T}\right)^2 \left(\frac{V}{220\,\mathrm{litres}}\right) \left(\frac{g_\gamma}{0.97}\right)^2 \times \left(\frac{\rho_a}{0.45\,\mathrm{GeV\,cm^{-3}}}\right) \left(\frac{f}{750\,\mathrm{MHz}}\right) \left(\frac{Q}{70,000}\right).$$

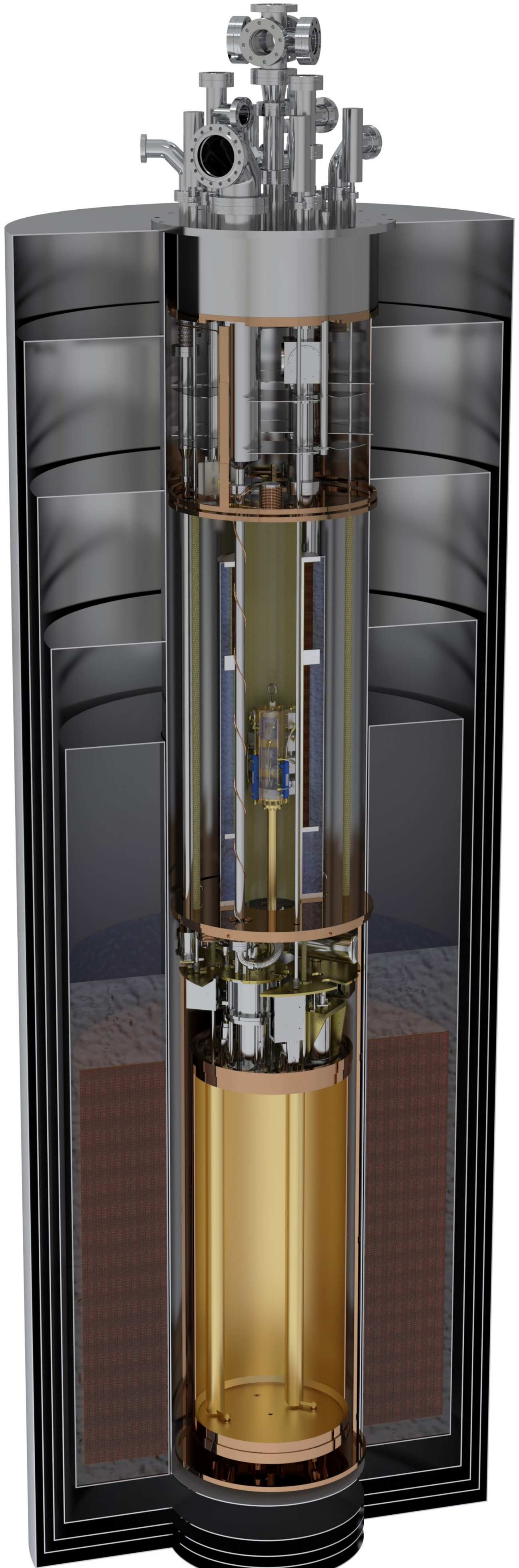
Notice the prefactor! The power levels we expect are order of a thousandth of an attowatt!

Pesky factors of 2

Annoyingly it gets worse. A theorem about impedance matching is that the best you can hope to extract from a source using an amplifier is half the energy, so we lose half this power to the walls of the cavity. Also, the loading effect of the antenna that couples the signal out degrades the cavity Q by a factor of 2; no this isn't the same factor of 2 - it's another one, so you only get 1/4 of the power in to your amplifier. Rats.

Summary of Seminar 1

1. **We introduced axions as a by-product of Peccei and Quinn's theoretical mechanism to explain CP conservation in quantum chromodynamics**
2. **We figured out how the axion terms in the Lagrangian for QCD cause modifications to Maxwell's equations that can be used to cause axions to convert into electromagnetic field oscillations.**
3. **Resonant cavity detectors enhance the size of these signals by a factor of the cavity resonance quality factor.**
4. **We then derived the power from axion to photon conversion in a cavity, discovering why we need resonant enhancement of the signal - even with the cavity Q, the signal is of order 1/1000 of an atoWatt.**
5. **This assumes the whole halo is axions, and that the local halo density reflects that in a slightly flattened (2:3 aspect ratio) halo with no clumping or voids. Probably the wrong institute to state these assumptions….oh well…**
6. **This only matters if there is a noise source that spoils the party. And there is! It's thermal noise in the walls of the cavity, and more thermal noise from the electronics used to amplify the signal. More about that in the next seminar.**



Sikivie-style resonant axion haloscopes - 2

QSFP School 2020

Institute for Particle Physics Phenomenology

Durham University

Ed Daw, The University of Sheffield

ADMX detector rendering
Cavity (gold) is 1m high, 50cm in diameter.
Superconducting magnet coils surrounding the cavity are immersed in liquid helium

Noise background

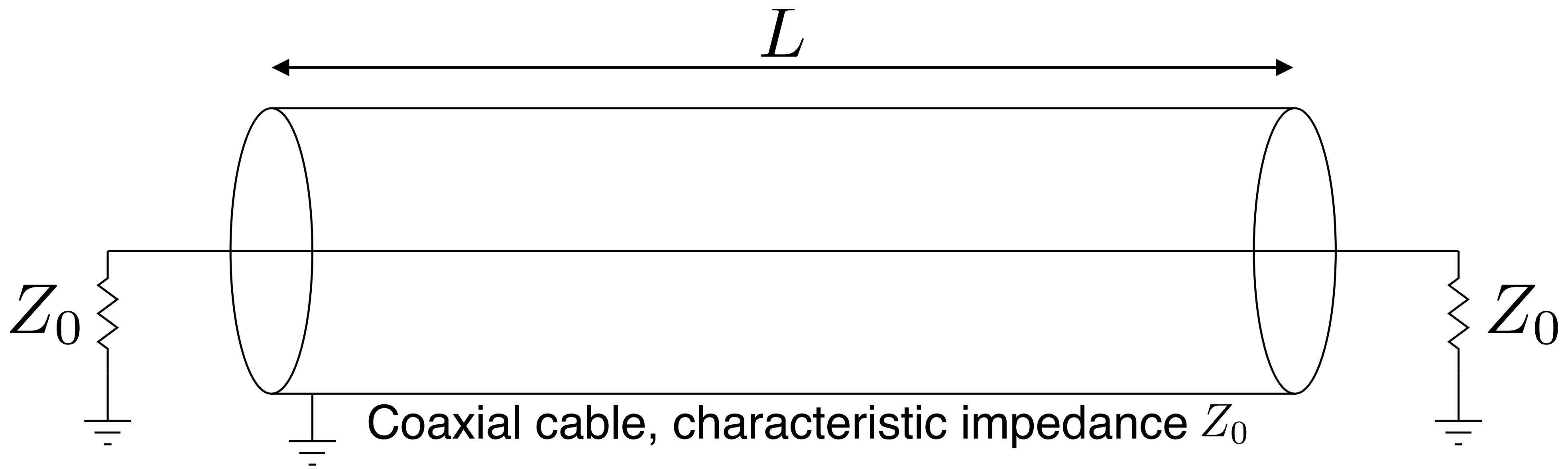
An advantage of resonant cavity axion detectors is that the active volume is surrounded in many layers of conductive metal shielding. This provides natural shielding against many sources of outside noise. Also note the signals are persistent, so there are no backgrounds from radioactive decays, and no need for an underground lab.

The electromagnetic shielding does not protect from Johnson noise produced *inside* the apparatus. The cavity walls, and the amplifier electronics, are at a non-zero temperature. They contain charged particles in random thermal motion, hence they are emitting electromagnetic waves, also random. In turn, these waves exert forces on the charges, so that the radiation reaches thermodynamic equilibrium with the charges, at a characteristic temperature T.

T is either the *physical temperature* of the walls of the cavity, or
it is the *noise temperature* of the electronics, or, in reality, both added together.

We will now work out the power emitted by a body at temperature T emitting Johnson noise using a simple classical derivation. This treatment follows that of many textbooks on thermal physics [5]

Johnson noise in a matched line



Consider a coaxial transmission line of characteristic impedance Z_0 terminated at each end by matched impedances. Any radiation that is incident down the cable on the terminations will be absorbed without reflection. Similarly, any radiation emitted by the resistors will travel into the cable without reflection. If the resistors and the cable are all at temperature T, then the radiation must be in thermodynamic equilibrium; the radiation flux out of the cable into the resistors must equal the radiation flux out of the resistors and into the cable.

However, we know how to calculate the radiation flux out of the cable. It's just the radiation density in the cable times the velocity of the radiation. So, let's do that calculation. The electromagnetic modes of oscillation of the cable have wavelengths

$$\lambda_n = \frac{2L}{n} \quad \text{where } n \text{ is a positive integer.}$$

How many cable modes?

$$\lambda_n = \frac{2L}{n}$$ corresponds to a wavenumber of $$k_n = \frac{2\pi}{\lambda_n} = \frac{n\pi}{L}$$

The wavenumber interval occupied by a single mode is $$\delta k = \frac{\pi}{L}$$

The frequency interval and the wavelength interval occupied by a mode are related using $$\omega = ck \qquad 2\pi\nu = ck$$

$$\frac{\delta\nu}{\nu} = \frac{\delta k}{k} = \frac{\pi}{kL} = \frac{\pi\lambda}{2\pi L} = \frac{\lambda}{2L}$$

so that $$\delta\nu = \frac{\nu\lambda}{2L} = \frac{c}{2L}$$ This is the frequency interval occupied by a single cable mode.

Now consider a frequency bandwidth B in the cable. This bandwidth contains $$n = \frac{B}{\delta\nu} = \frac{2LB}{c}$$ modes.

Equipartition of energy

Each mode of the cable obeys a harmonic oscillator equation, and so possesses 2 degrees of freedom. By classical equipartition, at temperature T the average energy per oscillator mode is $k_B T$. So the average energy in the modes in a bandwidth B of cable is

$$\bar{E} = \frac{2LB}{c} k_B T = \frac{2L k_B T B}{c}$$

The average energy density (energy per unit length) in the cable in this same bandwidth is

$$\bar{u} = \frac{\bar{E}}{L} = \frac{2 k_B T B}{c}$$

The energy flux, or just the power emitted by the cable is just the energy density times c, but note that it gets shared between the two terminating resistors. Hence the power emitted by the cable towards each resistor in a bandwidth B is

$$\bar{P} = \frac{\bar{u} c}{2} = k_B T B$$

Since the resistors and the cable are in thermodynamic equilibrium, that same power must be emitted by the resistors. This is the desired result. Note this is an *average* power; from moment to moment the power emitted will fluctuate. We will discuss these fluctuations presently.

Thermal noise in resonant cavities

A cavity cooled to 150mK searches for $4\,\mu\mathrm{eV}$ axions. What is the noise level in the signal bandwidth?

The bandwidth of this signal was 290Hz. At resonance, the effective impedance of the cavity mode matches the characteristic impedance of an antenna critically coupled (see later) to the mode. So, the thermal noise emitted by the cavity is

$$\bar{P} = k_B T B = 1.4 \times 10^{-23}\,\mathrm{J\,K^{-1}} \times 0.15\,\mathrm{K} \times 290\mathrm{Hz} = 6 \times 10^{-22}\,W$$

The signal power to compare this with is about $1.5 \times 10^{-21}\,W$ times a cavity form factor of order 0.5, times 0.5 because half the power is dissipated in the cavity walls, so the signal level is about $P_{\mathrm{S}} \sim 4 \times 10^{-22}\,W$

The ratio of the signal power to the raw noise power is of order 1 even for noise from the cavity walls only and axion signal levels at the more optimistic levels predicted by the KSVZ axion model. It is worse in practice; we want also to look for more faintly coupled DFSZ axions, and we have a potential additional noise source in the noise temperature of the electronics.

Effects on the design of axion halo scopes

- The signal power is inherently weak. Make it as strong as possible using a large magnetic field and volume. Figure of merit for detectors is B^2V.
- V of a single cavity is dictated by the wavelength of the target photons. For large volumes at high frequencies, you will end up power-combining multiple cavities. This is technically difficult, and each cavity must be tuned in sync.
- Physical temperature of cavity as low as is justified by the noise temperature of the electronics.
- Considerable progress has been made on electronics noise temperatures. 20 years ago, 2K - currently 150mK - but devices at and below the standard quantum limit around 10mK noise temperatures are now a possibility. More about electronics later.

Axion **D**ark **M**atter e**X**periment - 1

1990s configuration - Lawrence Livermore National Laboratory, U.S.A.

- A resonant search for halo axions - first science data in 1995

- 8 T superconducting NbTi winding magnet
- Early amplifiers were HFETS - first model 4K noise temperature; later models 2K. Made by the US NRAO lab in Richard Bradleys group.
- Early electronics was a double heterodyne receiver with IF and AF frequencies of 10.7MHz and 50kHz.
- Cryogenics was pumped liquid helium, 2K cavity temperature

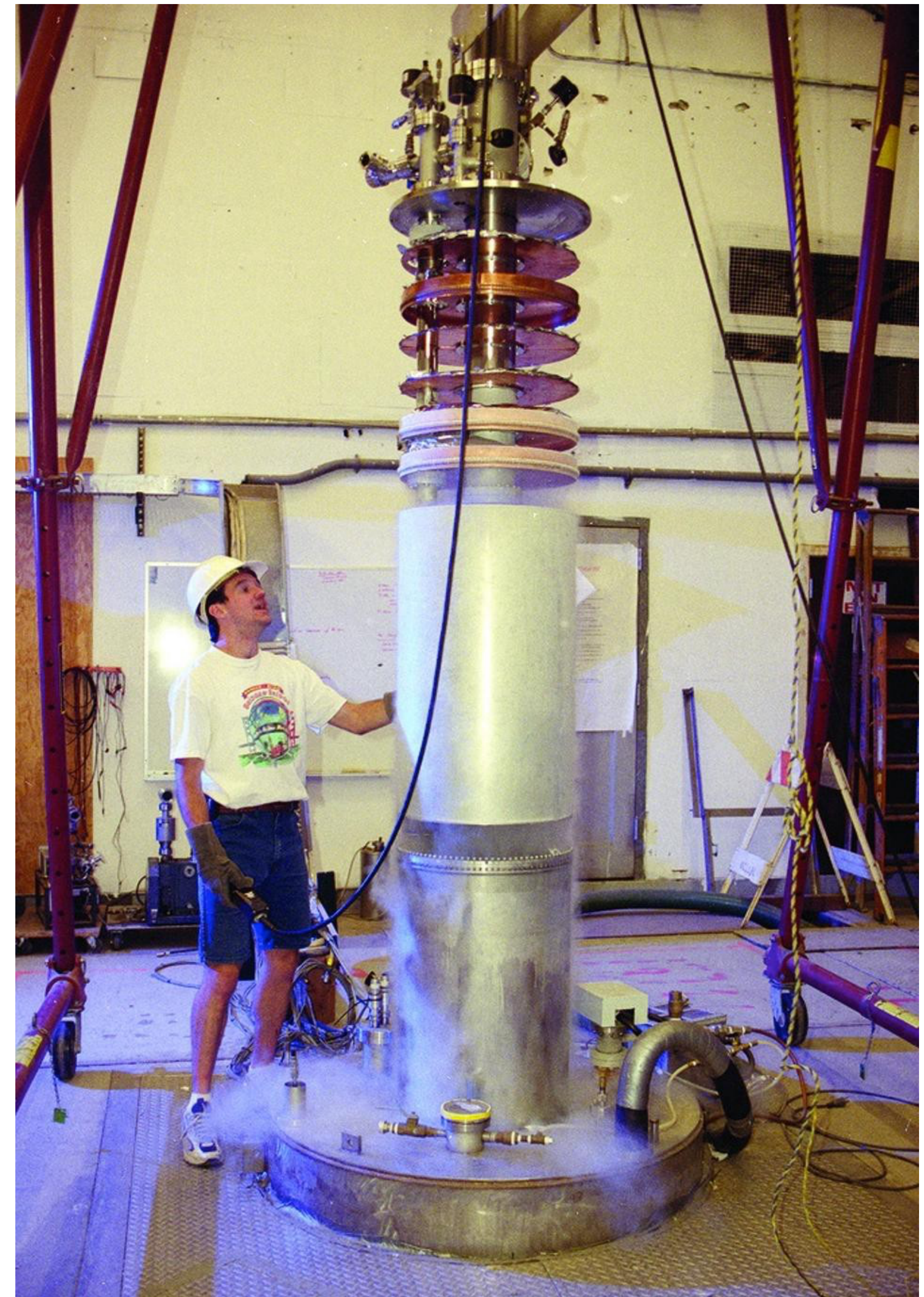
STEPPER MOTOR TOWER

THERMAL BAFFLES

HELIUM RESERVOIR

CAVITY HOUSING

MAGNET (4m HIGH IN FLOOR PIT)



Darren Kinion - built the DAQ system used in the early LLNL phase.

Crash course in microwave electronics

Gains in dB

$$G[\mathrm{dB}] = 10\log_{10}\left(\frac{P_{\mathrm{OUT}}}{P_{\mathrm{IN}}}\right)$$

Powers in dBm

$$P[\mathrm{dBm}] = 10\log_{10}\left(\frac{P\,[\mathrm{W}]}{10^{-3}\,\mathrm{W}}\right)$$

Useful rules of thumb:

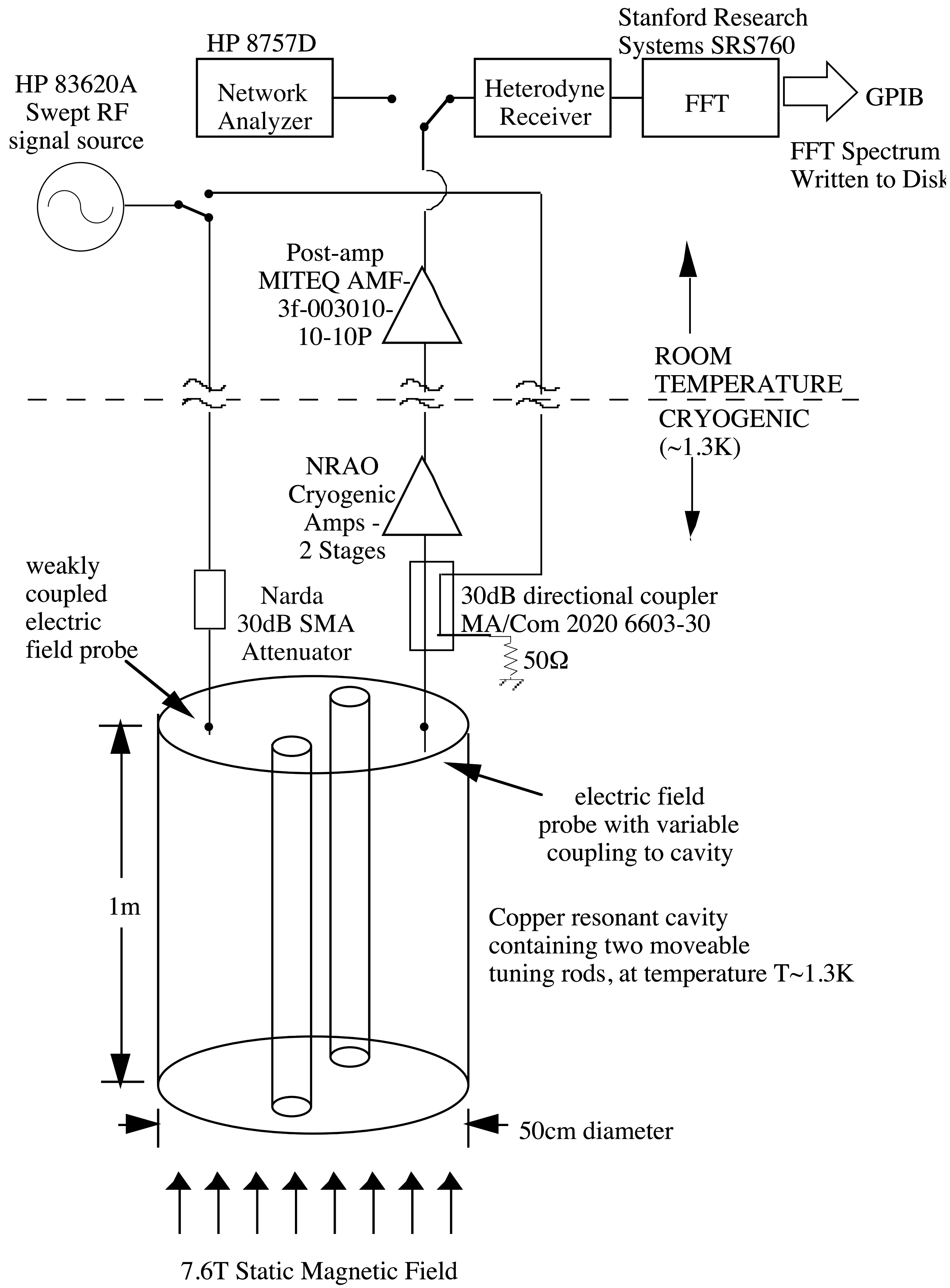
- 3dB is a factor of 2 in power.
- -3dB is a factor of 1/2 in power.
- 10dB is a factor of 10 in power.
- 0dBm is a power of 1mW.
- output power (dBm) is input power (dBm) + gain (dB). Negative gains mean attenuation (signal gets smaller).

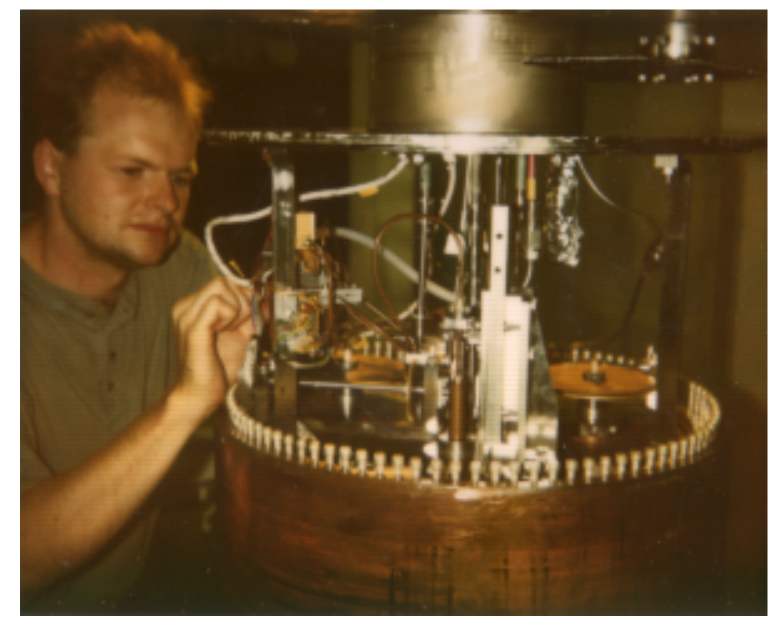
- Between 500MHz and 20GHz, we can usually use high quality coaxial cable to move these signals.
- In a vacuum 1GHz = 30cm wavelength.

Key components.

1. Amplifiers.
2. Attenuators.
3. Mixers - multiply a signal by a sine wave at a different frequency. A way of moving a signal from a higher frequency to a lower one.
4. Network analyser. Sends a swept sine wave towards one port of a component, and at each frequency measures the frequency response at one or more device ports.
5. Spectrum analyser. Measures power vs. frequency in a signal.
6. Directional coupler. A 3 port device - the third port injects a signal onto the line between ports 1 and 2, such that the signal is only travelling in one direction.
7. Circulator. A 3-port device. Inject a signal into port 1 and it comes out through port 2, but not port 3, and cyclic permutations. Useful for matching. Usually contains ferrites.

Early ADMX Electronics - HEMT amplifiers





Double heterodyne receivers

Double heterodyne receiver. Cavity resonance only about 10kHz wide, so a final data sampling rate of 200kHz is ample. Cavity search between 700 and 800 MHz, for a mass range between 2.9 and 3.3 micro-eV.

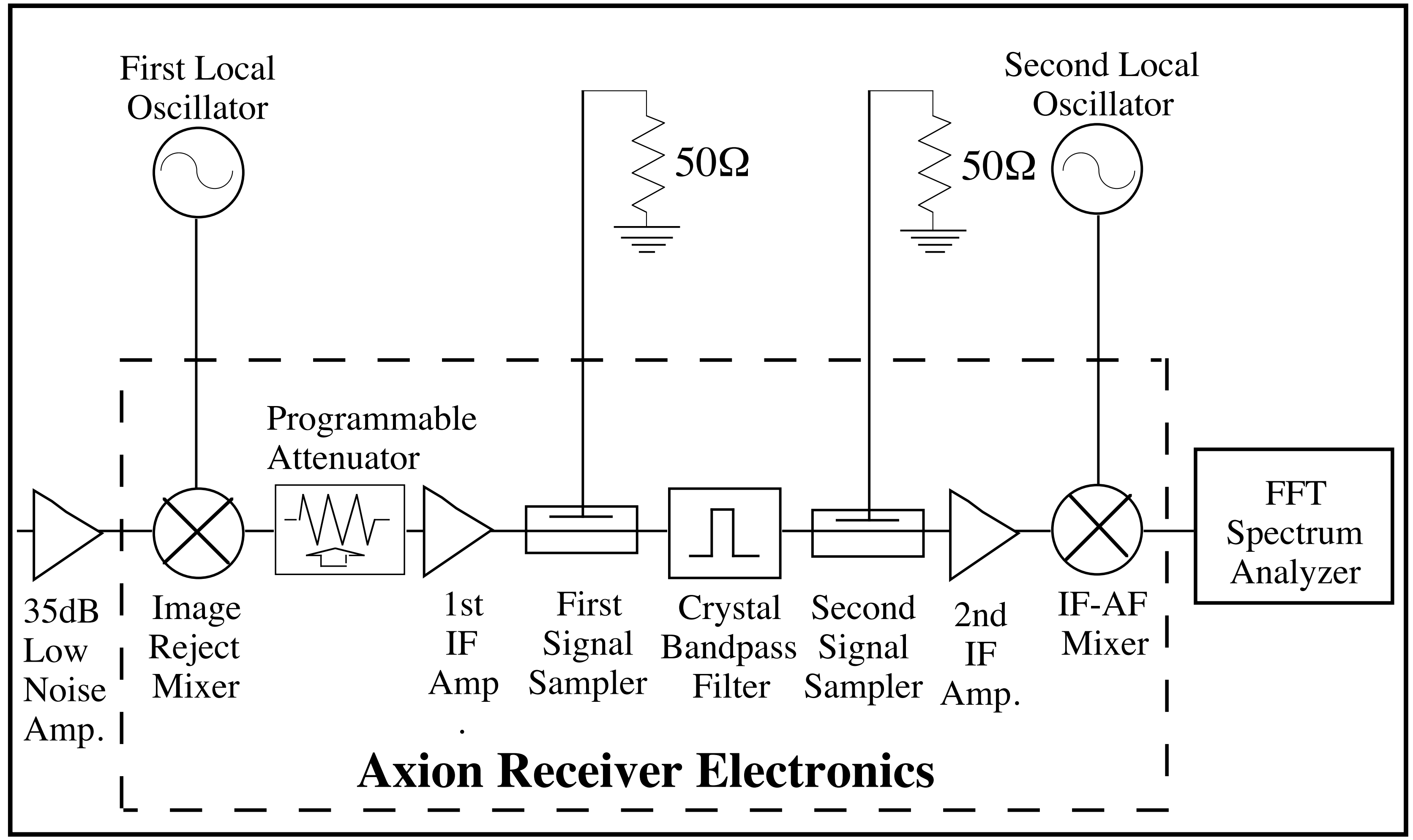
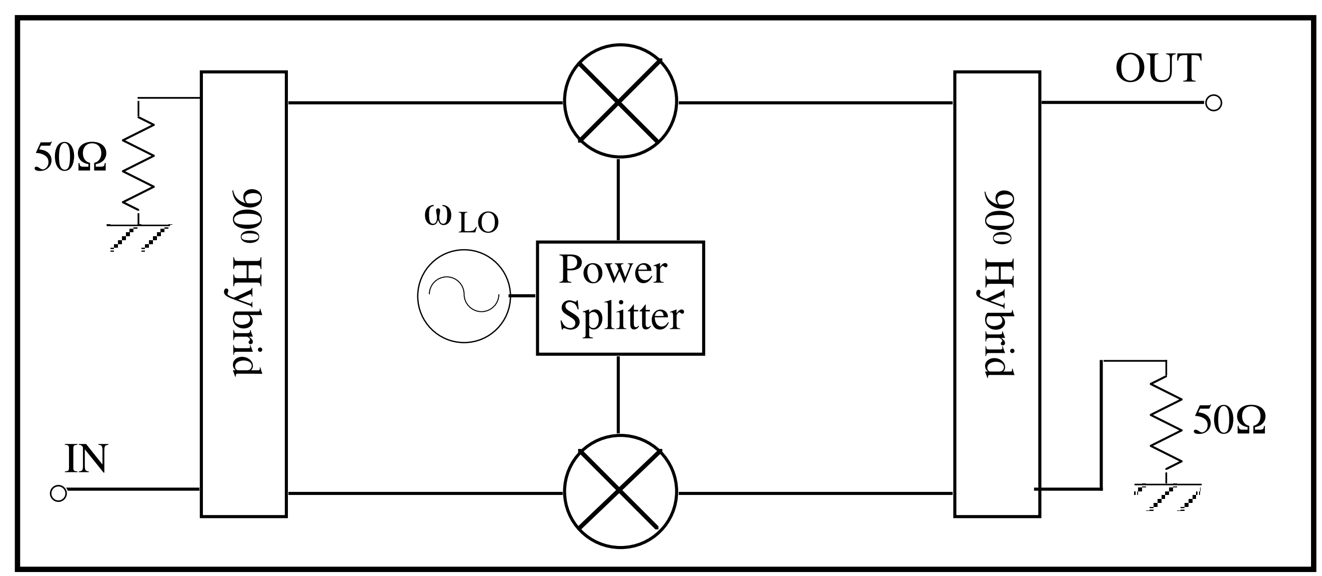


IMAGE-REJECT MIXER



Early ADMX Electronics - Power Budget

Component gains

Noise contribution from electronics chain in a 125Hz bandwidth

Know power over full bandwidth and don't let it rise above about 0dBm

Component	Gain (dB)	Power per 125Hz at Output (dBm)	Total Output Power (dBm)
Cavity	-	-170*	-101*
Cryogenic Amplifiers	34	-136	-67
Room Temperature 'Post Amplifier'	35	-101	-32
Flexible Cable to Analysis Hut	-6	-107	-38
Image Reject Mixer	-7	-114	-45
1st IF Amplifier	30	-84	-15
Crystal Filter	-3	-87	-60
Second IF Amplifier	30	-57	-30
IF - AF Mixer	-7	-64	-37

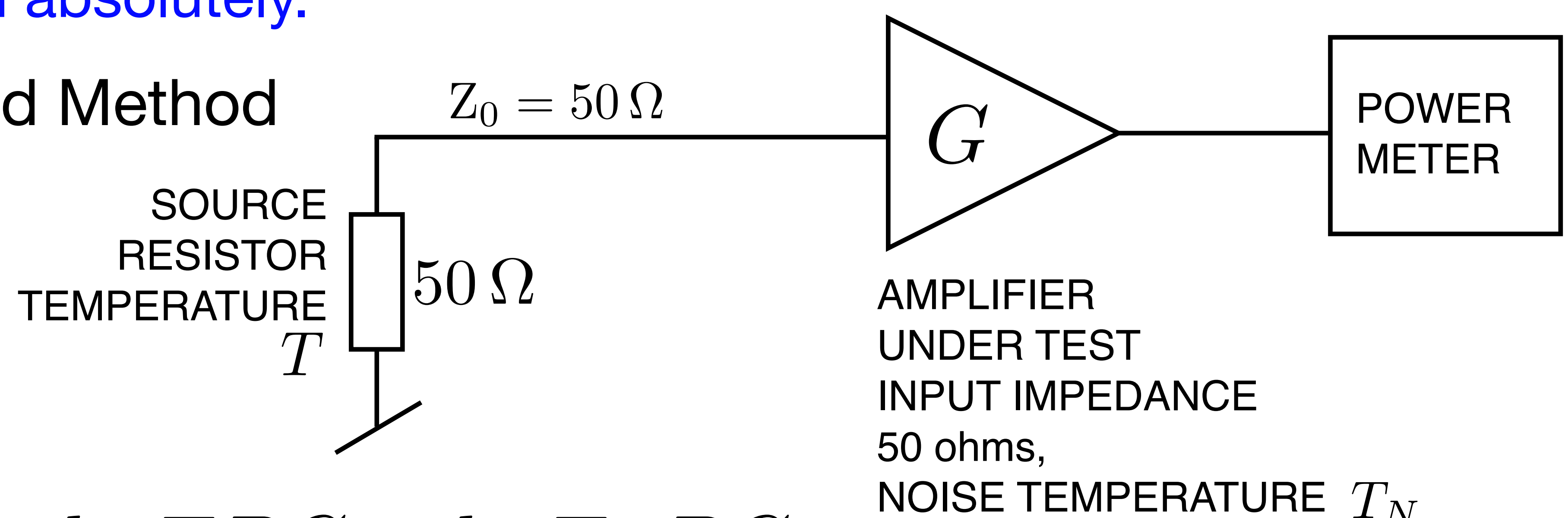
*Assumes that the 1st cryogenic amplifier has a bandwidth of 1GHz and that the sum of the cavity and amplifier noise temperatures is 6K.

Amplifier noise temperature measurements

The problem of measuring noise of amplifiers

Amplifiers contribute noise, but how much? Measuring the noise contribution of an amplifier isn't simple. For a start, absolute measurements of power are very hard. Secondly, amplifiers behave badly when their inputs are not connected to anything, but if you connect something it will also contribute noise. It is not so easy to separate amplifier and source noise. Furthermore, the bandwidth of a power meter is rarely known absolutely.

Hot-Cold Load Method



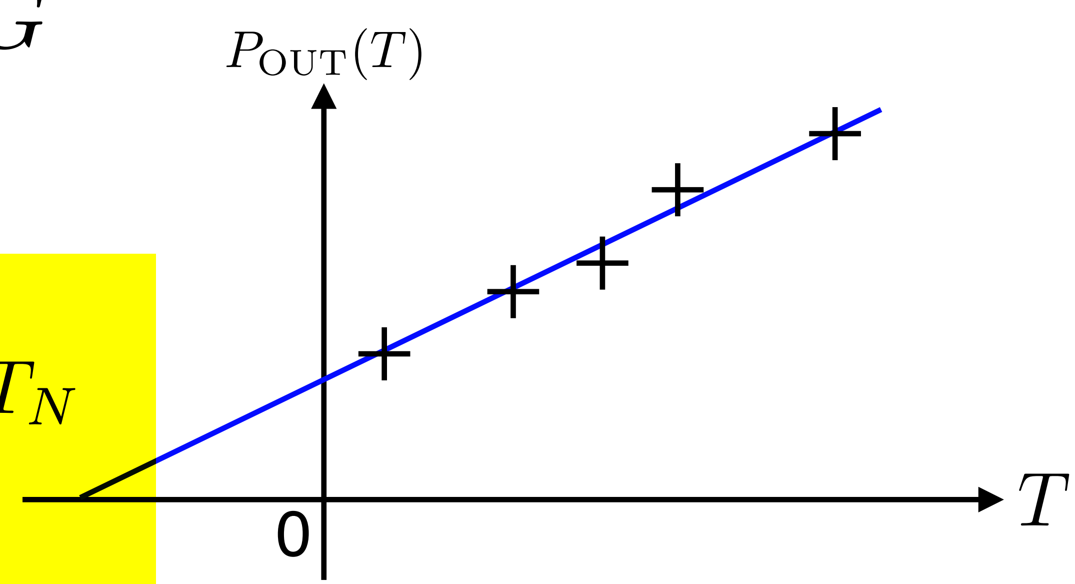
$$P_{\mathrm{OUT}}(T) = k_B T B G + k_B T_N B G$$

Slope is $k_B B G$

Vertical axis intercept is $k_B T_N B G$

Ratio of vertical axis intercept to slope is T_N

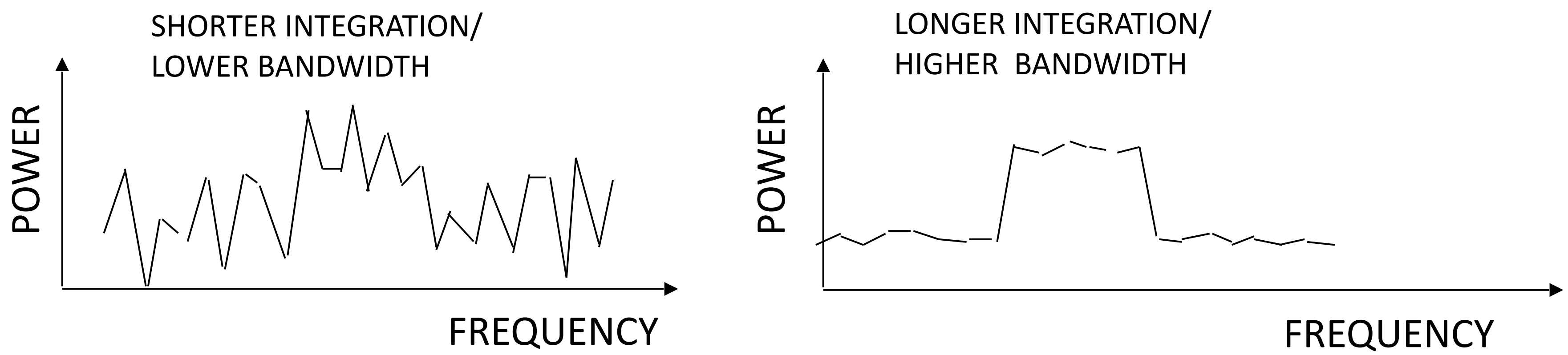
Horizontal axis intercept is $-T_N$



How to handle signal power to noise power ratios of order 1.

ADMX already achieves sensitivity to KSVZ and DFSZ axions, albeit with very slow coverage of the mass space. How are they doing this? The answer is by exploiting the signals persistence in time and power, and stable frequency.

I’ve carefully avoided using the term signal to noise ratio, because the ratio of the signal power to the raw noise power in the signal bandwidth is NOT the signal to noise ratio. To appreciate this consider the sketch below.



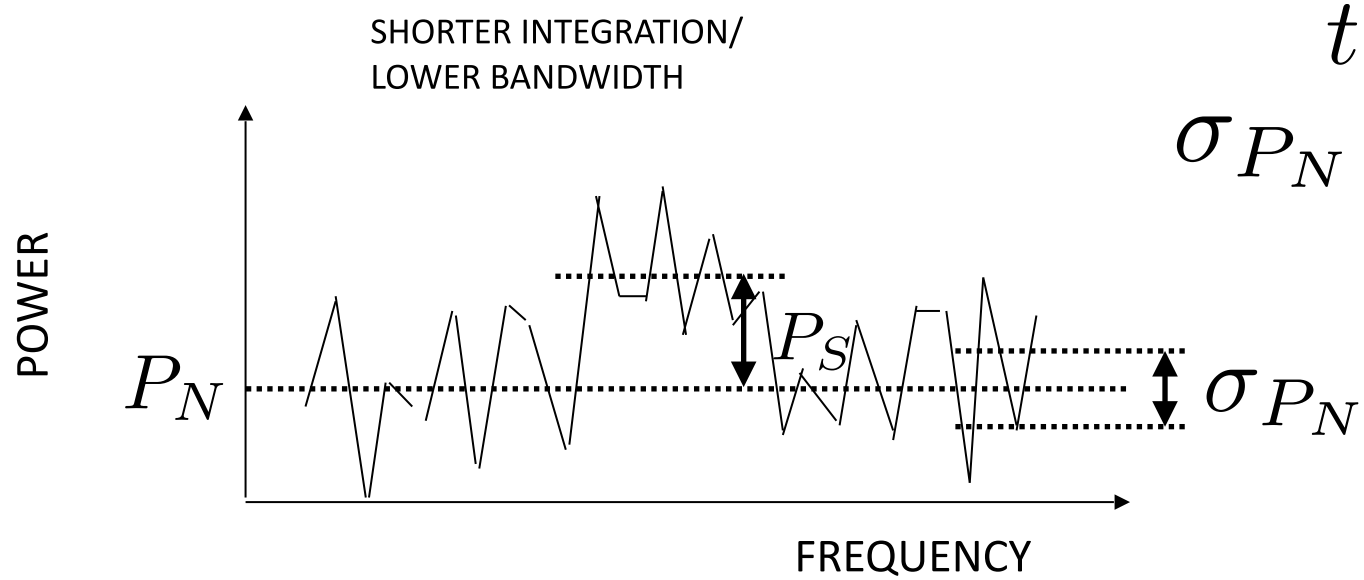
The figure shows power versus frequency in the vicinity of a toy axion signal, taken after short and longer integration times. As the integration time increases, the bin-to-bin fluctuations in the noise drop, leaving the signal visible on top of the noise that appears at all frequencies.

This intuitive fact is formalised in the *radiometer equation*, a key result that we shall spend some time looking in to.

The Radiometer Equation.

$$\mathrm{SNR} = \frac{P_S}{\sigma_{P_N}} = \frac{P_S}{P_N}\sqrt{Bt}$$

B : bandwidth
P_S : signal power
P_N : noise power
t : integration time
σ_{P_N} : r.m.s. of bin-to-bin fluctuations in noise



The radiometer equation is useful here because the signal is at a static frequency, and the noise at surrounding frequencies is relatively flat (because the cavity resonance is much wider band than the signal peak). Thus the signal appears as *excess power* in its bandwidth on top of the noise power that is in every bin.

Whether the signal is discernible or not depends not depends on whether the bin-to-bin fluctuations in the noise swamp the signal. The radiometer equation tells you how long you have to integrate for to discern the signal against the background of these fluctuations.

Proof of the radiometer equation - 1

The probability density of a classical system having an energy E is given by the Maxwell Bolzmann distribution.

$$p(E) = f(\Theta)e^{-\frac{E}{\Theta}}$$

where $\Theta = k_B T$ and $f(\Theta)$ is some function of temperature. This normalises to unity, integrated over all energies, so

$$\int p(E)\, dE = \int dE f(\Theta) e^{-\frac{E}{\Theta}} = 1$$

Differentiate this with respect to Θ

$$\int dE \left(\frac{df}{d\Theta} e^{-\frac{E}{\Theta}} + f(\Theta) \frac{E}{\Theta^2} e^{-\frac{E}{\Theta}} \right) = 0$$

Rearranging

$$\frac{1}{f(\Theta)} \frac{df}{d\Theta} \int dE\, f(\Theta) e^{-\frac{E}{\Theta}} + \frac{1}{\Theta^2} \int dE f(\Theta) E e^{-\frac{E}{\Theta}} = 0$$

The expectation of any function of energy is given by

$$\overline{h(E)} = \int dE h(E) f(\Theta) e^{-\frac{E}{\Theta}}$$

Proof of the radiometer equation - 2

$$\frac{1}{f(\Theta)}\frac{df}{d\Theta}\int dE\, f(\Theta)e^{-\frac{E}{\Theta}} + \frac{1}{\Theta^2}\int dE f(\Theta)Ee^{-\frac{E}{\Theta}} = 0$$

Simplifies to $\frac{1}{f(\Theta)}\frac{df}{d\Theta} + \frac{\bar{E}}{\Theta^2} = 0$ or $\frac{df}{d\Theta} = -\frac{\bar{E}f(\Theta)}{\Theta^2}$

Next start with the expectation value of the energy

$$\overline{E} = \int dE\, E f(\Theta)e^{-\frac{E}{\Theta}}$$

Differentiating with respect to Θ

$$\frac{d\overline{E}}{d\Theta} = \int dE\, E\frac{df}{d\Theta}e^{-\frac{E}{\Theta}} + \int dE\, E f(\Theta)\frac{E}{\Theta^2}e^{-\frac{E}{\Theta}}$$

Substituting in for $df/d\Theta$ and rearranging

$$\frac{d\overline{E}}{d\Theta} = -\frac{\bar{E}}{\Theta^2}\int dE\, E\, f(\Theta)e^{-\frac{E}{\Theta}} + \frac{1}{\Theta^2}\int dE\, E^2 f(\Theta)e^{-\frac{E}{\Theta}}$$

or

$$\Theta^2\frac{d\bar{E}}{d\Theta} = \overline{E^2} - \bar{E}^2 = \sigma_E^2$$

or, in terms of T:

$$\boxed{\sigma_E^2 = k_B T^2\frac{d\bar{E}}{dT}}$$

Proof of the radiometer equation - 3

This result is quite general, applicable to any expression where an average energy of a classical system is expressed in terms of its temperature. We apply it to a Johnson noise emitter.

$$\bar{P} = k_B T B$$

$$\bar{E} = k_B T B t \qquad \frac{d\bar{E}}{dT} = k_B B t$$

$$\sigma_E^2 = k_B T^2 \frac{d\bar{E}}{dT} = k_B^2 T^2 B t$$

$$\sigma_E = k_B T \sqrt{Bt} \qquad \frac{\sigma_E}{\bar{E}} = \frac{1}{\sqrt{Bt}}$$

Divide top and bottom on the left by time to go from energies and powers, and recognise that both the average and spread of energies are of noise.

$$\frac{\sigma_{P_N}}{P_N} = \frac{1}{\sqrt{Bt}}$$

Rearranging and multiplying by the signal power,

$$\boxed{\frac{P_S}{\sigma_{P_N}} = \frac{P_S}{P_N}\sqrt{Bt}}$$

which is the radiometer equation.

Proof of the radiometer equation - 3

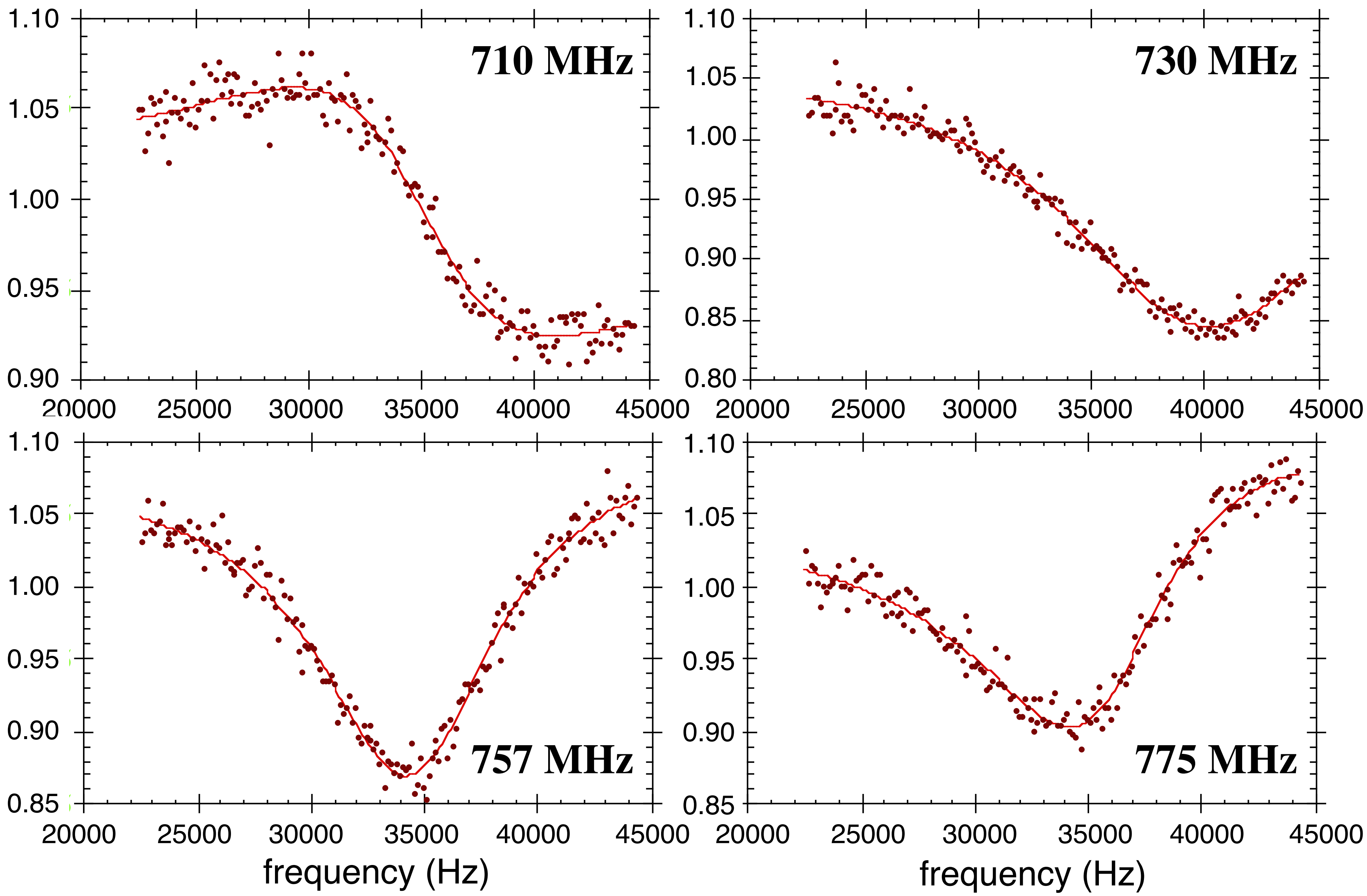
Search in practice

- Move tuning rods to a new position, corresponding to a new frequency of the TM_{010} mode resonance.
- Use a network analyser to measure the transmitted power between two ports (one weakly coupled, the other critically coupled) of the cavity.
- Identify the right mode, and measure its frequency and quality factor.
- Set the local oscillator frequency for the first mixer to heterodyne this mode frequency down to the intermediate frequency (IF).
- Bandpass filter in the IF - either with a hardware crystal filter or using a digital filter.
- Either heterodyne down again to a lower frequency then digitise, or digitise at the IF and then decimate before taking a Fourier transform.
- Write the power spectrum over the cavity resonance to disk
- Rinse and repeat….

- Now open the data off-line and try and make sense of it all.

What raw data looks like

In practice, the power spectrum of a cavity coupled to an amplifier has structure over the bandwidth of the cavity resonance. However, this structure is frequency dependent, and reflects the characteristics of the circuit connecting the amplifier to the cavity, and the input components of the amplifier itself. Here are some examples:



Background shape removal

Technique 1: use an average of nearby spectra

Neighbouring spectra have similar shapes, so an average of many accumulated power spectra can be used to normalise spectra. eg, use an average of n spectra, in which the rms noise is reduced by a factor of $\sqrt{n}$

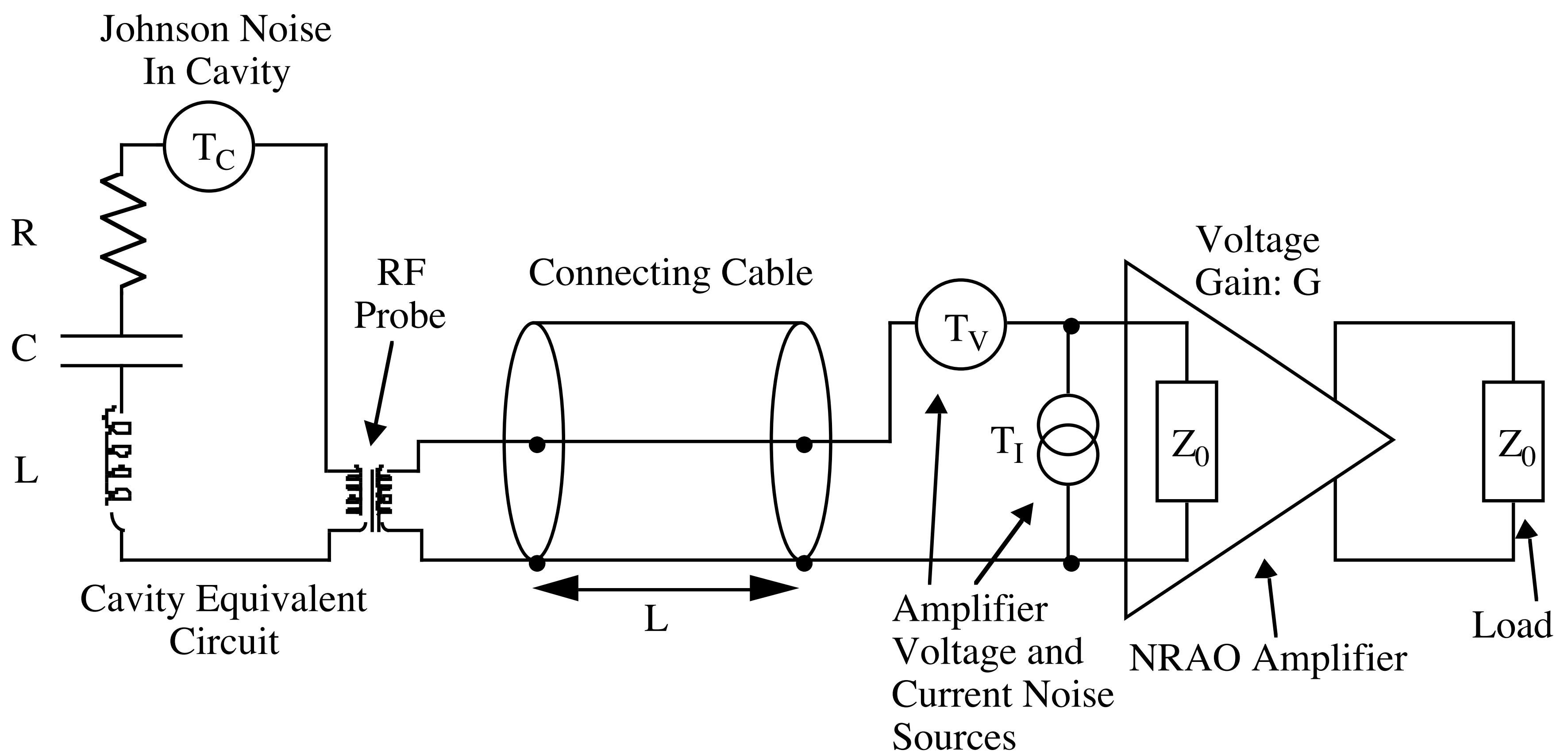
Normalising to an average adds noise to the signal, but it turns out that this isn't such a large effect. If the fluctuations about the mean in a single spectrum are a fraction p of the power, then the difference between a single spectrum and the average of n spectra has an error

$$p\sqrt{1+\frac{1}{n}}$$

Where n=10 this is an additional 5% error, which is manageable.

Disadvantage of this method is that any large excursions appearing in multiple spectra cause confusing nests of signals in the normalised data.

Technique 2: model and fit the background shape using a mathematical model derived from an equivalent circuit.



$$P(\Delta f) = \frac{k_B B G^2}{1 + \frac{4(\Delta f)^2}{\Gamma^2}} \left((T_C + T_V + T_I) + \frac{4(\Delta f)}{\Gamma} (T_I - T_V) \sin(2kL) + \frac{8(\Delta f)^2}{\Gamma^2} \left((T_V + T_I) + (T_I - T_V) \cos(2kL) \right) \right)$$

This complicated function is used to construct an empirical fit having 5 free parameters

Technique 2: Performance of the fit.

$$P(\Delta) = \frac{a_1 + 8a_3\left(\frac{\Delta - a_5}{a_2}\right)^2 + 4a_4\left(\frac{\Delta - a_5}{a_2}\right)}{1 + 4\left(\frac{\Delta - a_5}{a_2}\right)^2}$$

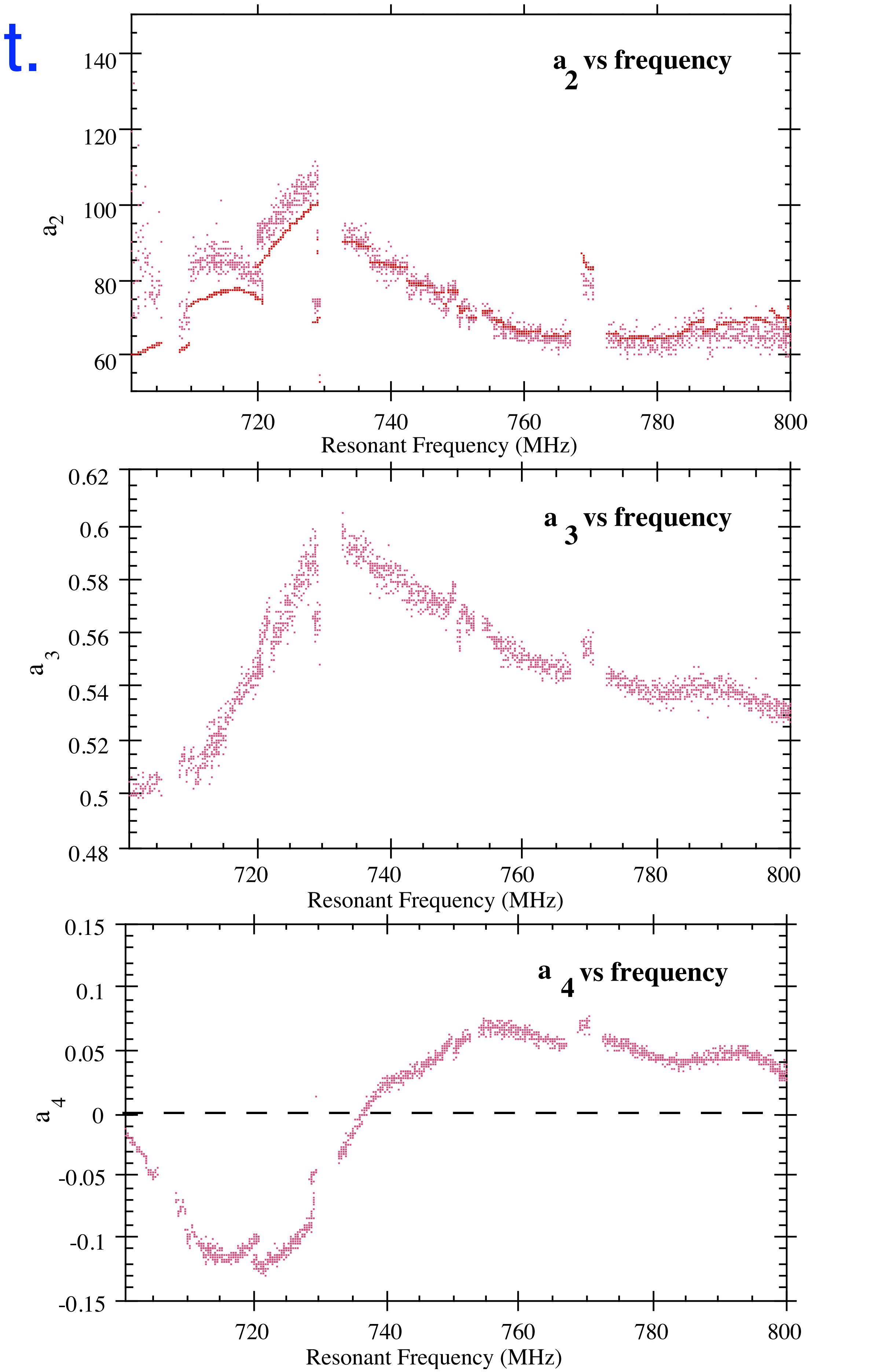
$$a_1 = T_C + T_I + T_V$$

$$a_2 = \frac{f_0}{Q}$$

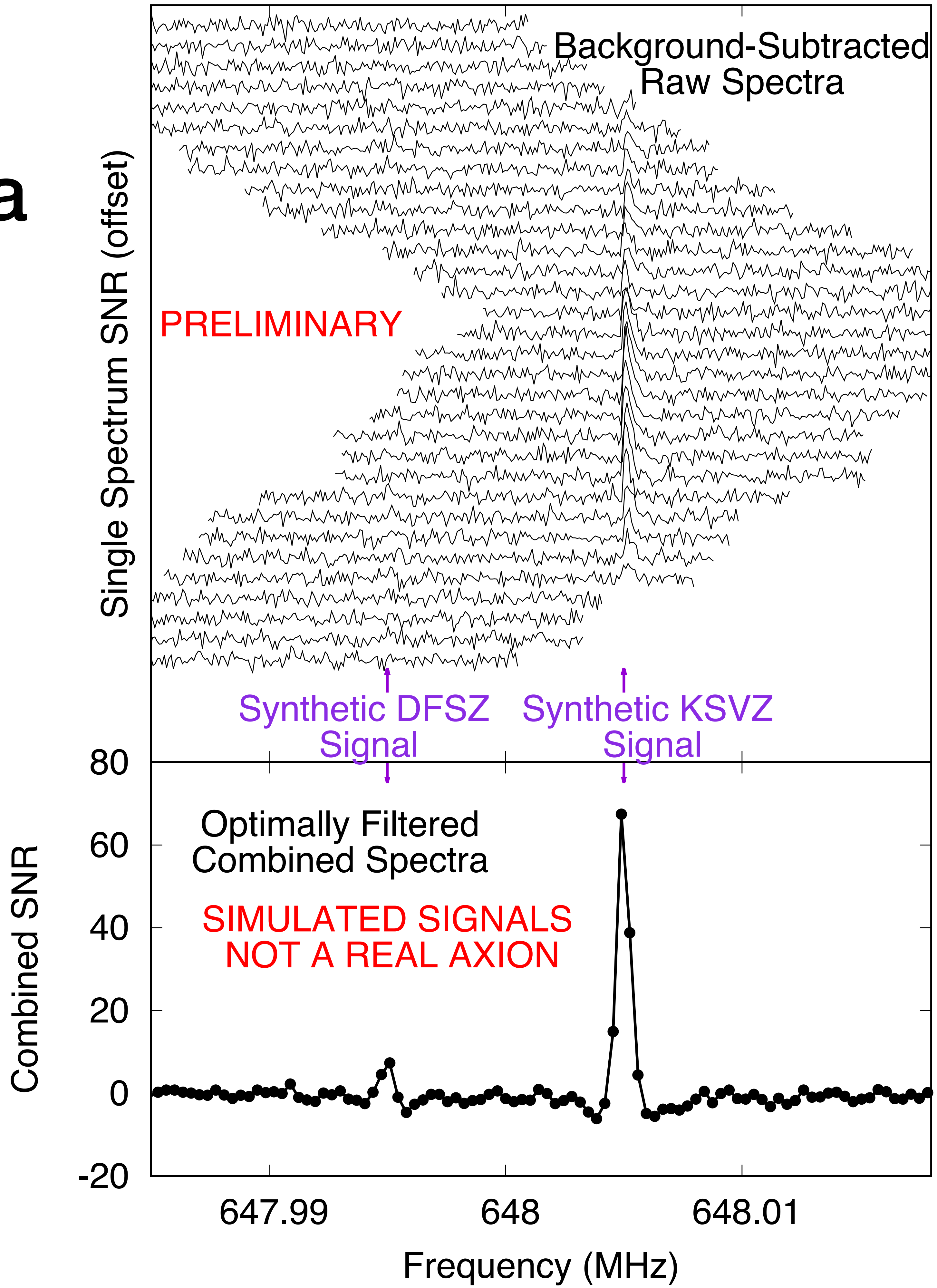
$$a_3 = (T_I + T_V + (T_I - T_V)\cos 2kL)$$

$$a_4 = (T_I - T_V)\sin 2kL$$

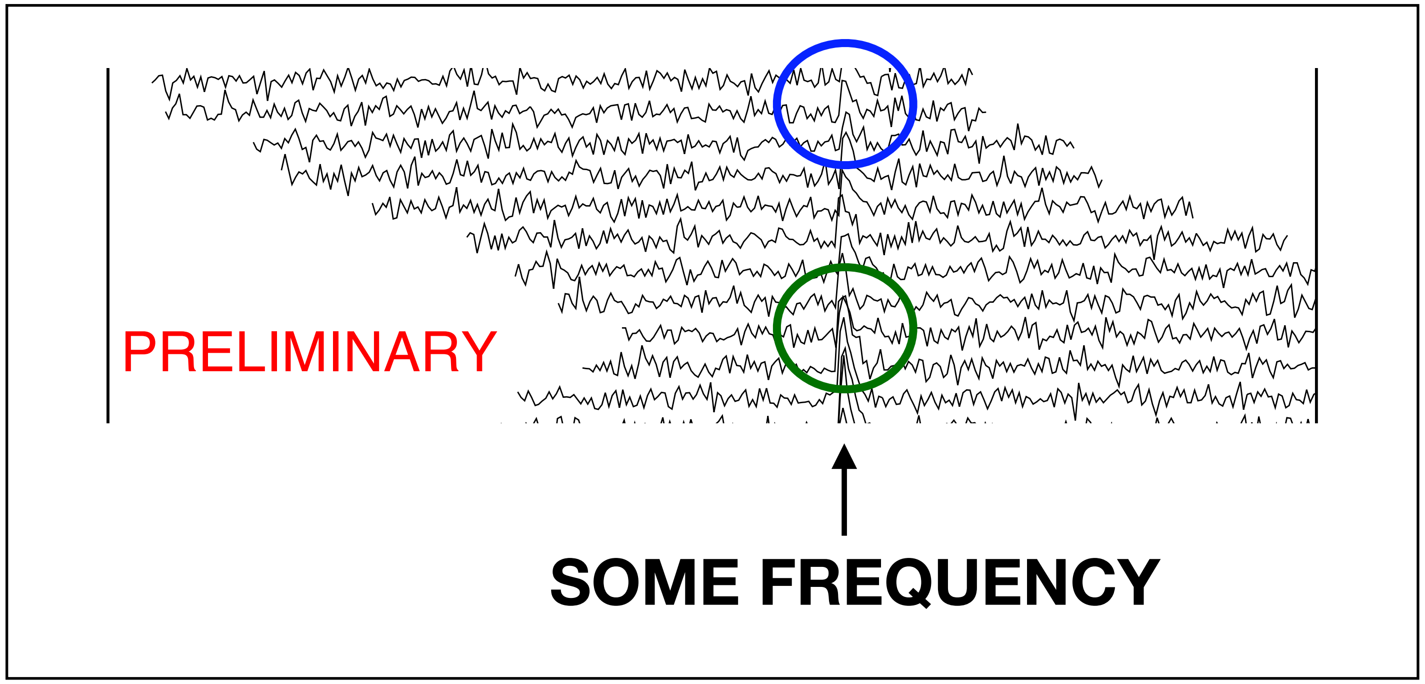
a_5 is an offset of the centre of the resonance from the centre of the spectrum.



Combining raw spectra



Optimal linear combination of power spectra



Consider two component spectra in which the following quantities take different values:

δ_1 δ_2 measured power in bin

σ_1 σ_2 standard deviation of power across spectrum

h_1 h_2 height of cavity resonance relative to peak height

P_1 P_2 signal power at that frequency for an assumed axion model

The optimal combination turns out to be

$$\delta_{\mathrm{WS}} = \frac{\frac{h_1 P_1}{\sigma_1^2}\delta_1 + \frac{h_2 P_2}{\sigma_2^2}\delta_2}{\frac{h_1 P_1}{\sigma_1^2} + \frac{h_2 P_2}{\sigma_2^2}}$$

Details of the ‘optimal’ data combining algorithm

Crop off the 1st 100 and last 125 bins of each raw spectrum. Divide spectrum by IF transferred power. Fit the IF normalized spectrum to a five parameter equivalent circuit model Divide the spectrum by the fit function and subtract 1.

For each trace normalized read/measure:

1) Γ, the width of the TM010 resonance.
2) T_N, the system noise temperature.
3) N, the number of averaged spectra per trace.
4) $\sigma(\delta)$, the rms noise level in the normalized trace.

Multiply the fluctuations $\sigma(\delta)$ in each raw trace by $k_B B$.

Make 4 combined data streams for each 125Hz bin accross the frequency range scanned

1. Number of Raw Spectra Contributing
2. $\sum_i \frac{h_i P_i \delta_i}{\sigma_i^{w2}}$
3. $\sum_i \frac{h_i^2 P_i^2}{\sigma_i^{w2}}$
4. $\sum_i \frac{h_i P_i}{\sigma_i^{w2}}$

data array for peak search

$$\frac{\sum_i \frac{h_i P_i \delta_i}{\sigma_i^{w2}}}{\sqrt{\sum_i \frac{h_i^2 P_i^2}{\sigma_i^{w2}}}}$$

Signal-to-Noise Ratio (SNR) in jth bin of combined data

$$SNR = \frac{P_0}{k_B}\sqrt{\sum_i \frac{h_i^2 P_i^2}{\sigma_i^{w2}}} = 25.3\sqrt{\sum_i \frac{h_i^2 P_i^2}{\sigma_i^{w2}}}$$

deviation from mean power (W)

$$k_B \frac{\sum_i \frac{h_i P_i \delta_i}{\sigma_i^{w2}}}{\sum_i \frac{h_i P_i}{\sigma_i^{w2}}}$$

rms noise level (W/125Hz)

$$k_B \frac{\sqrt{\sum_i \frac{h_i^2 P_i^2}{\sigma_i^{w2}}}}{\sum_i \frac{h_i P_i}{\sigma_i^{w2}}}$$

power deposited in amplifier by KSVZ axions (halo density $=7.5.10^{-25}$ gcm^{-3})

$$3.5 \times 10^{-22} W \frac{\sum_i \left(\frac{h_i P_i}{\sigma_i^{w}}\right)^2}{\sum_i \frac{h_i P_i}{\sigma_i^{w2}}}$$

Post-data combining steps and search sensitivity

- Expected signal is a few hundred Hz wide.
- Signal MAY be shaped like a Maxwell-Bolzmann distribution.
- If your binning has the width of the signal, the signal may be about evenly split between neighbouring bins, and your search sensitivity is then degraded by yet another factor of 2 in single bin searches.
- A better strategy is to make the bins significantly narrower than the anticipated signal width.
- In ADMX 1, we had 125Hz bins for an expected signal width of about 600Hz. We then co-added (boxcar averaged) overlapping neighbouring sets of 6 bins.
- This makes the output data of the co-adding algorithm bin-to-bin correlated.
- This means that the sensitivity of your search can no longer be deduced directly from knowledge of the Gaussian distribution; the data is still Gaussian, but it is correlated.
- As in most experiments, you end up running a simulation to determine your sensitivity.

Simulation strategy

1. Randomly generate n frequencies in the search range of your data. Ensure they don't overlap (only expect 1 axion signal).
2. Loop over your raw data set. For each spectrum, determine whether one or more signal frequencies fall within the spectrum.
3. For each raw (uncombined) power spectrum in your measured data, create a surrogate with gaussian noise replacing the physical noise in the spectrum, having the same RMS width. Add signals from your chosen model partitioned between appropriate bins of each raw spectrum, using the physically measured position of the Lorentzian resonant peak to modulate the signal power.
4. Apply your standard data combining, bin coadding, and thresholding algorithm to the synthetic data. Measure the fraction of signals identified by your algorithm.
5. Repeat these steps for a range of signal powers. The signal power leading to detection of 90% of your signals is the signal power you have excluded at 90% confidence, as long as you haven't found anything that you can't eliminate in your actual measured data.

What if you DO see something in your data?

Here cavity axion searches have an edge over experiments looking for dark matter particles that interact via discrete particle collisions, such as direct WIMP searches.

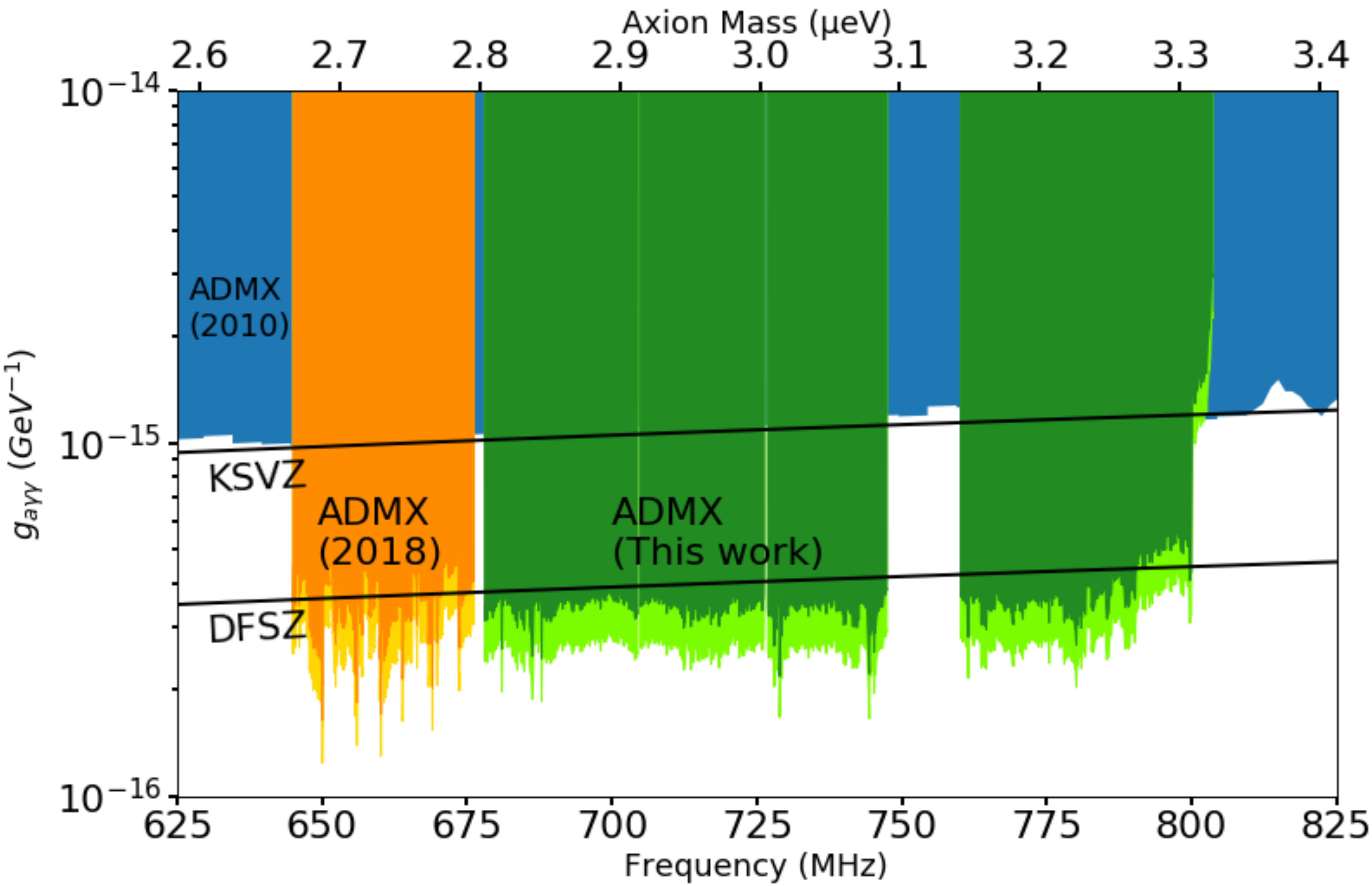
The axion signal should be (1) persistent at a given frequency and
(2) only present if there is a static magnetic field, furthermore
(3) proportional in signal power to the square of that magnetic field.

Therefore there are several strategies that you can use to 'check' signals that you may have seen in your data.

* Go back to the same frequency and see if the signal is still there. If it's gone, then that eliminates at least some axion models.
* Detach your receiver from the heavily shielded insert and measure the same frequency in free space. Background fields may be much bigger in free space; axion fields should vanish.
* Ramp down the magnetic field and see if signal power is proportional to the square of the magnetic field.

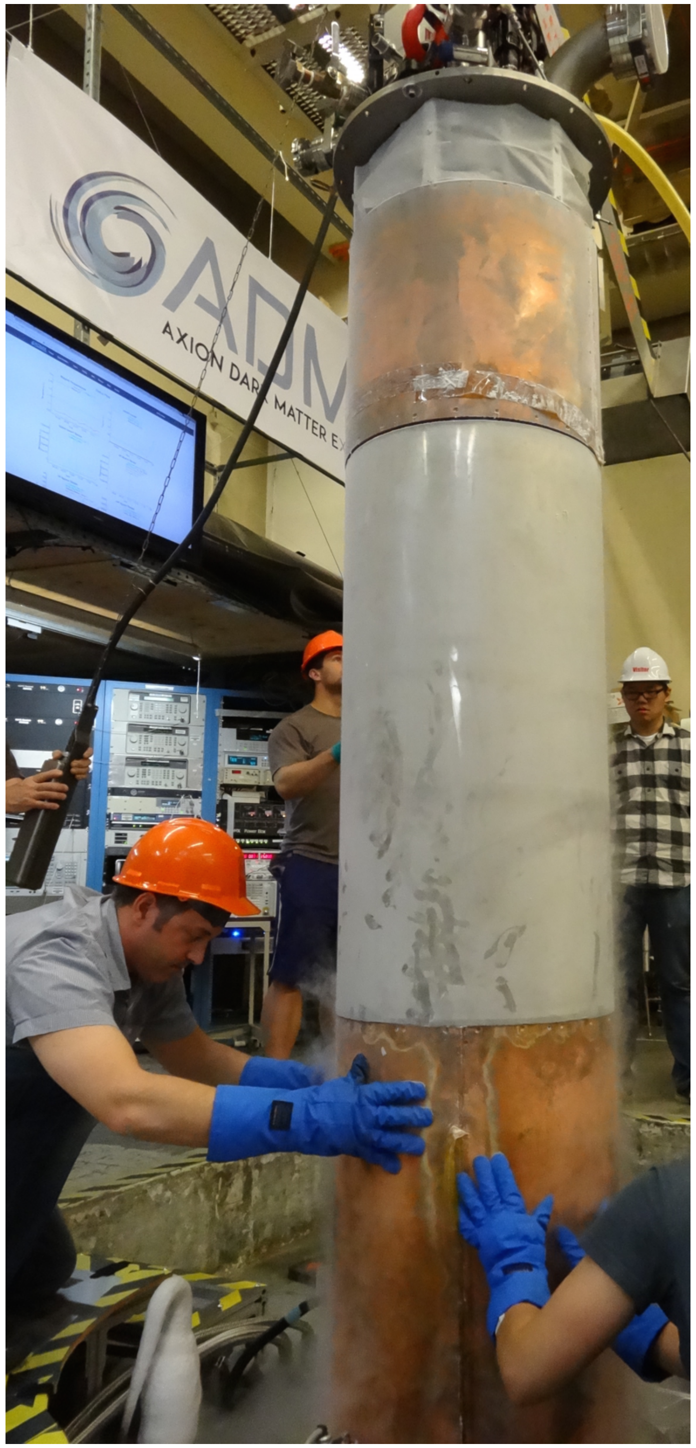
Recent ADMX results

ADMX hasn't yet detected a signal that has passed all of these tests.



https://arxiv.org/abs/1910.08638 [under review at PRL]

Sikivie-style resonant axion haloscopes - 3



QSFP School 2020

Institute for Particle Physics Phenomenology

Durham University

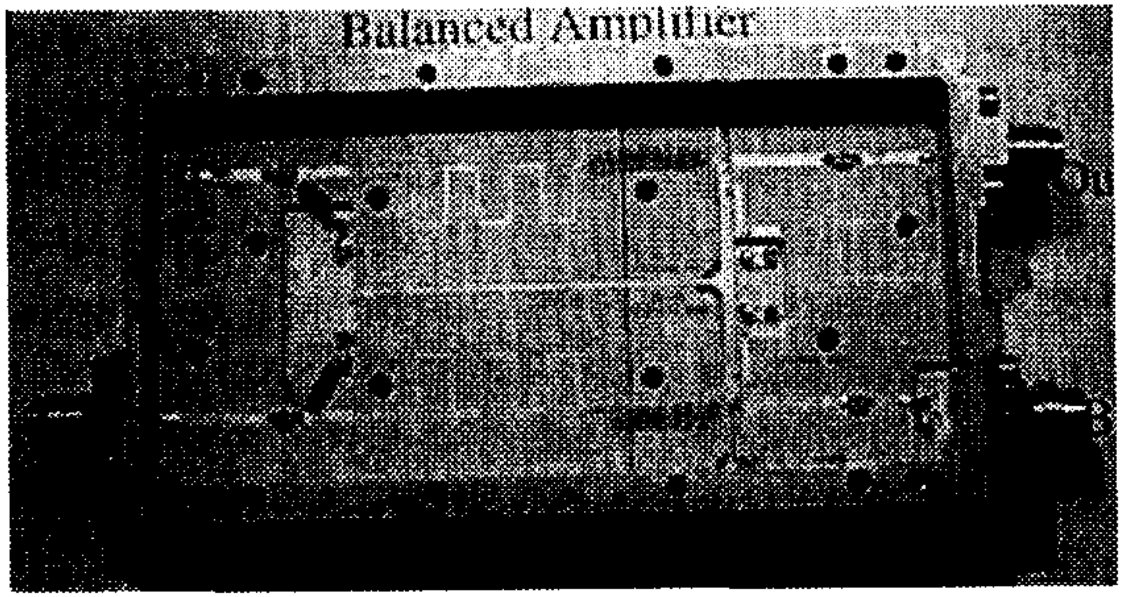
Ed Daw, The University of Sheffield

Low noise amplifier techniques - 1

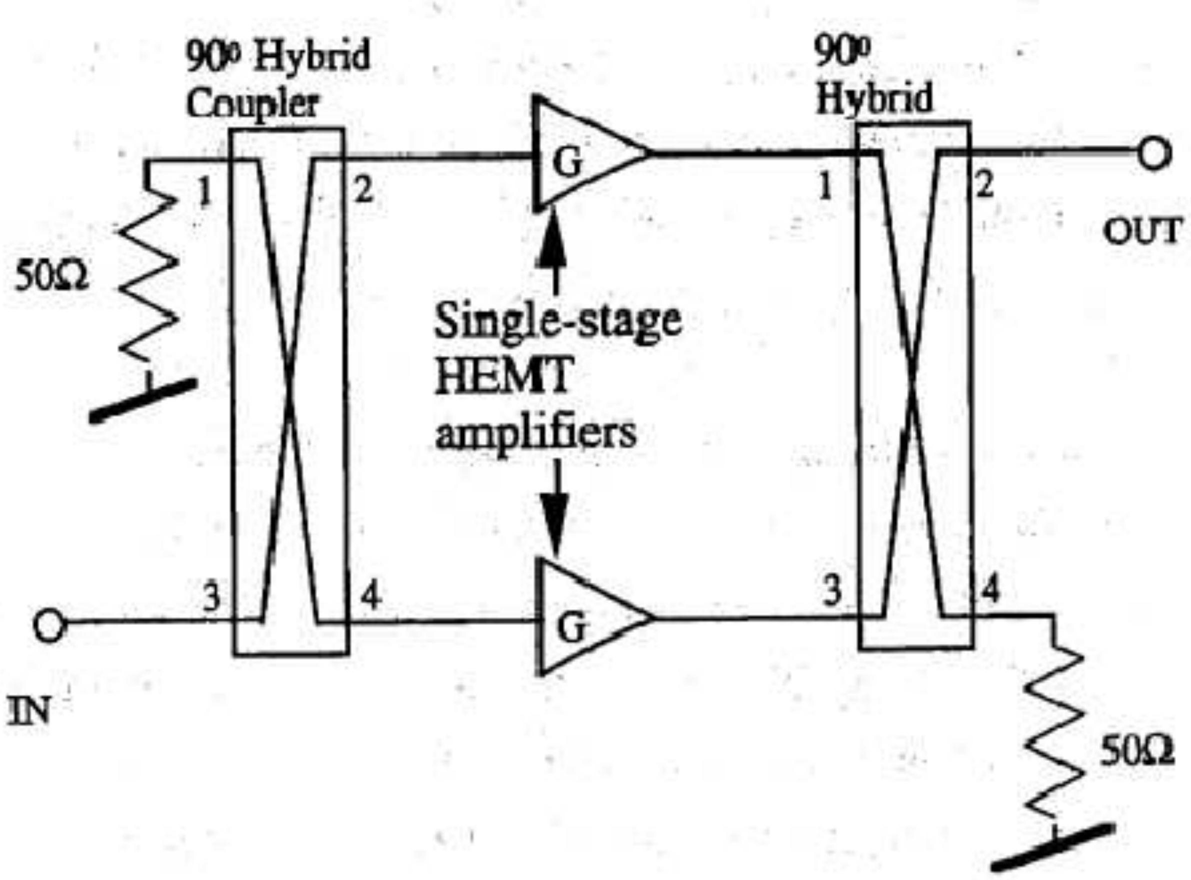
HFET amplifiers (commercial name of HFETs is HEMTs - high electron mobility transistors

Modern HFETs can achieve 1K noise temperatures (www.lownoisefactory.com).

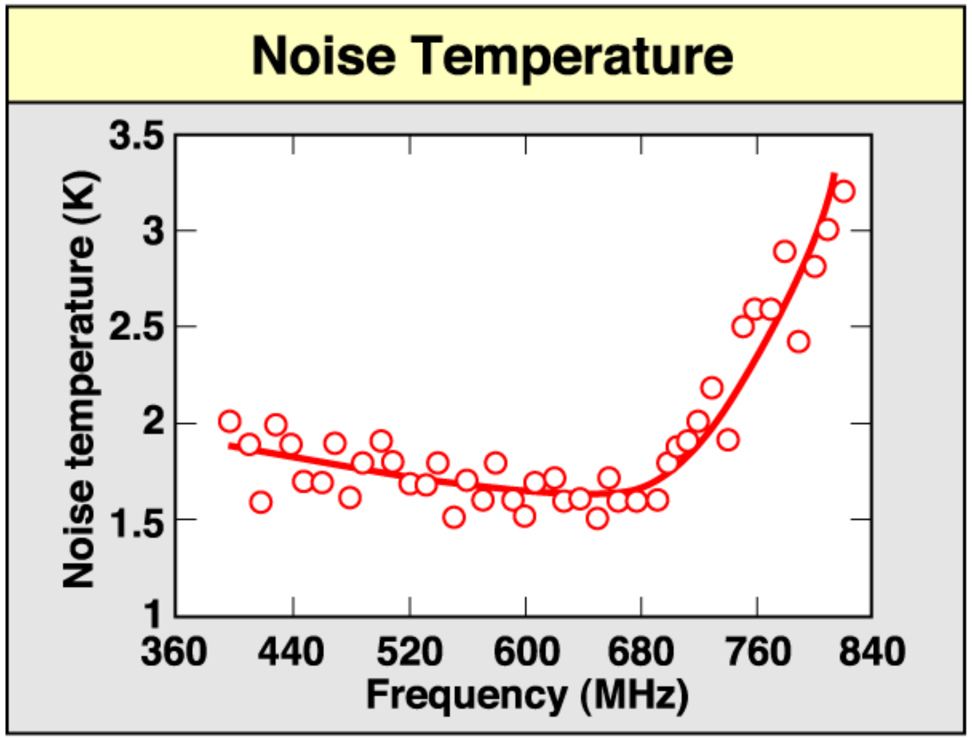
Matching is an issue! No ferrites unless you have a field-free region in your detector. Use a balanced design.

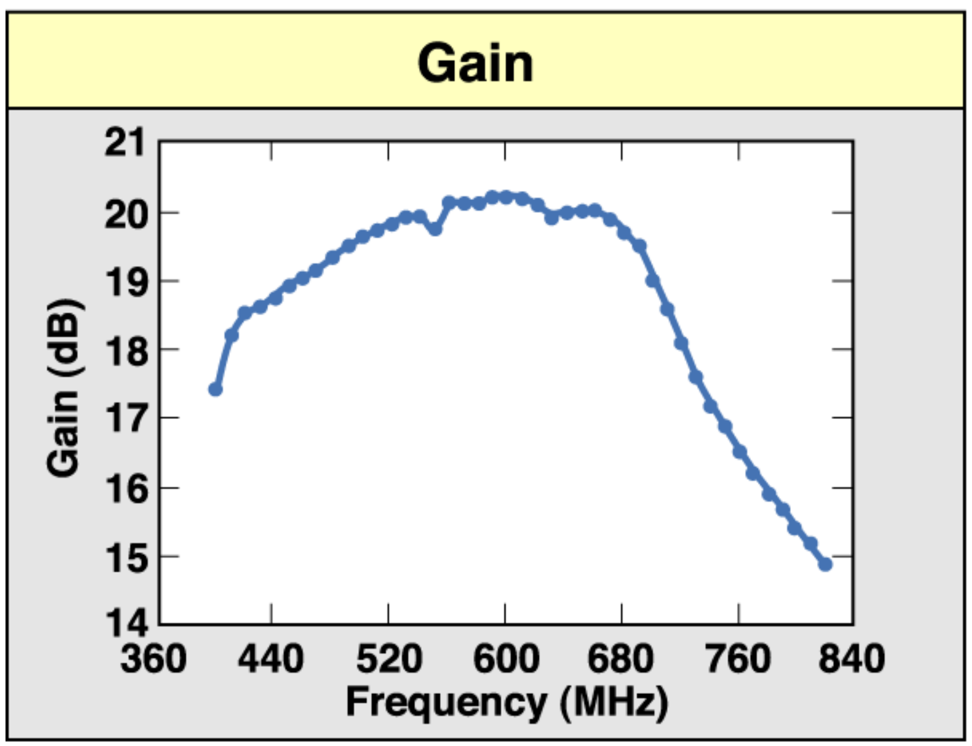


Balanced design by NRAO - Bradley
Nucl. Phys. B Suppl. **72** (1999) 137-144



HFET performance





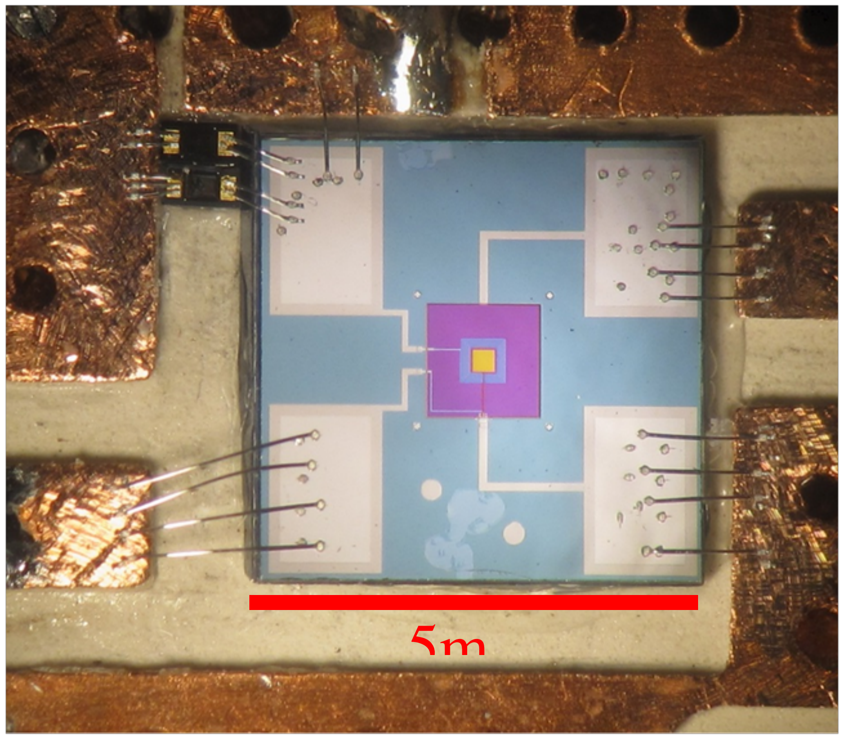
- **Currently HFET amplifiers (Heterojunction Field-Effect Transistor)**
  - **A.k.a. HEMT™ (High Electron Mobility Transistor)**
  - **Workhorse of radio astronomy, military communications, etc.**

Cooling HFET amplifiers below a physical temperature of about 1K does not result in a commensurate decrease in noise temperature. Mechanism thought to be trapping of electrons in the two dimensional electron gas contained in the HFET gate region.

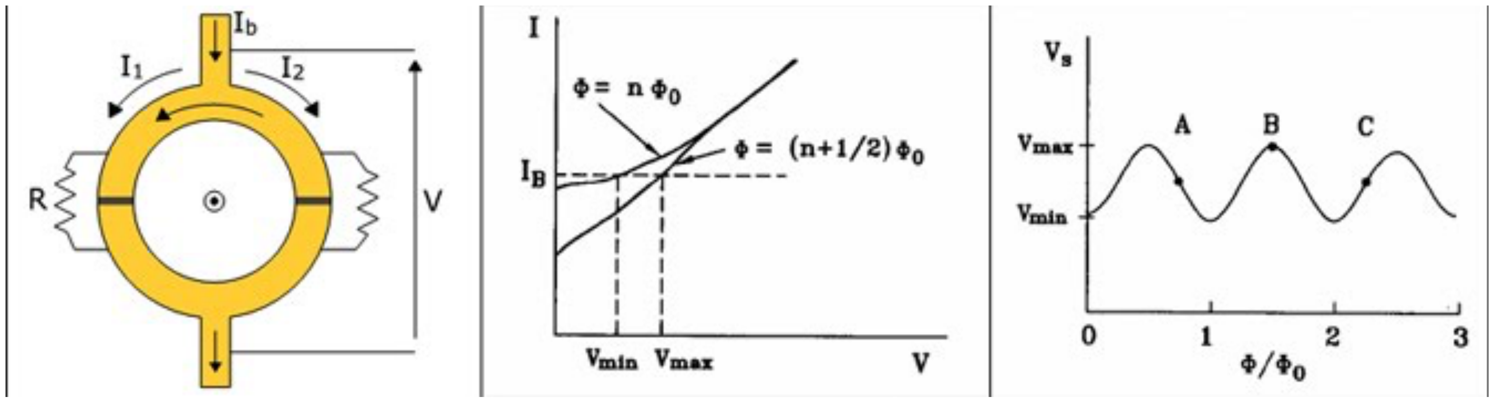
Further improvements in noise temperature have proceeded using a competitor technology, BUT HFET devices still play a critical role as low cost low-noise post amplifiers to quantum devices stages.

Squids with varactor matching

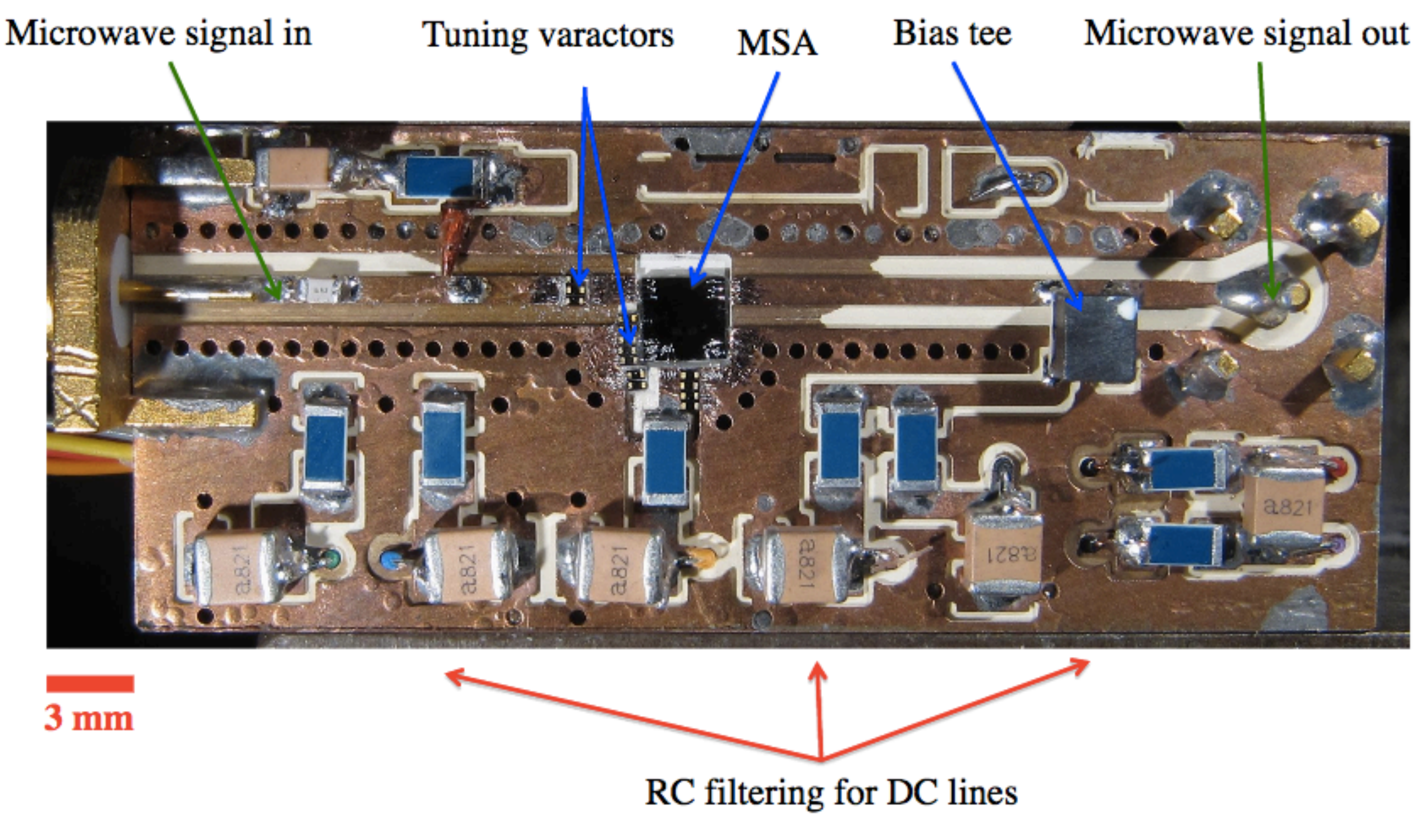
Clarke group, UC Berkeley



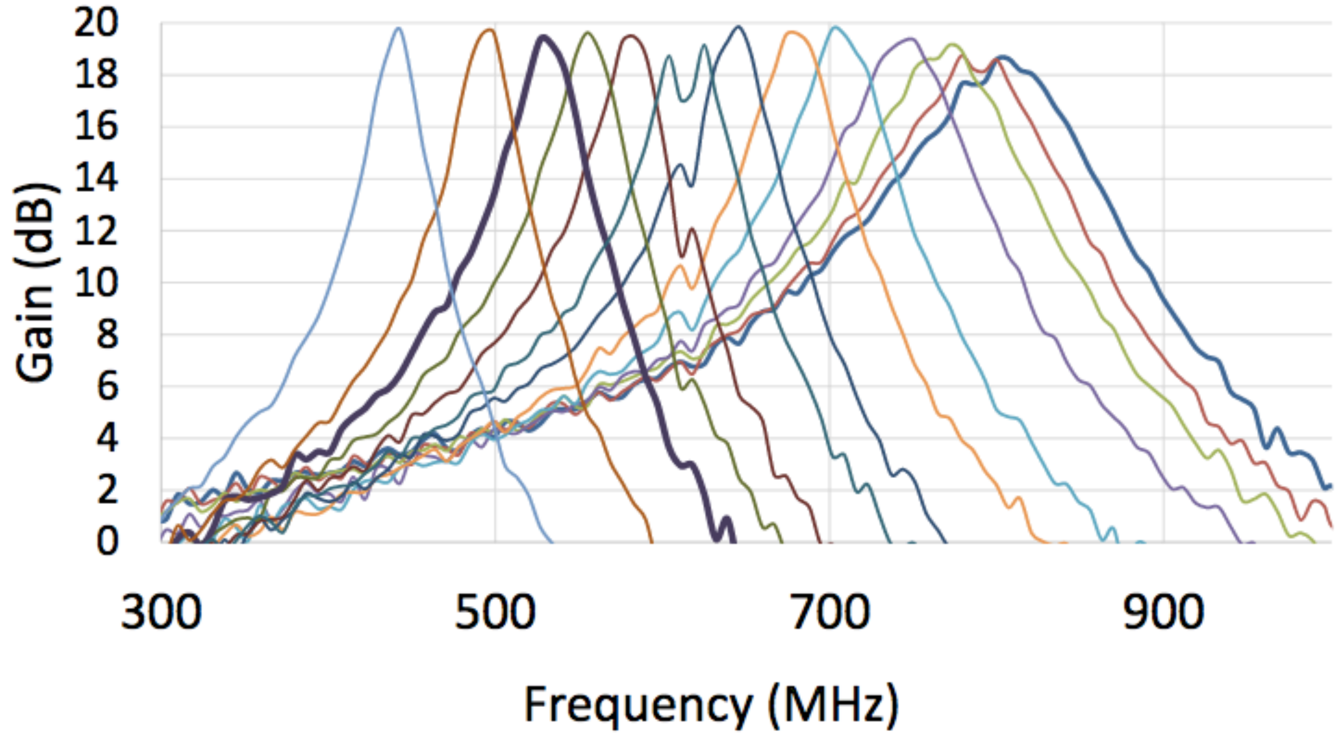
https://sites.google.com/site/squiddevices/home

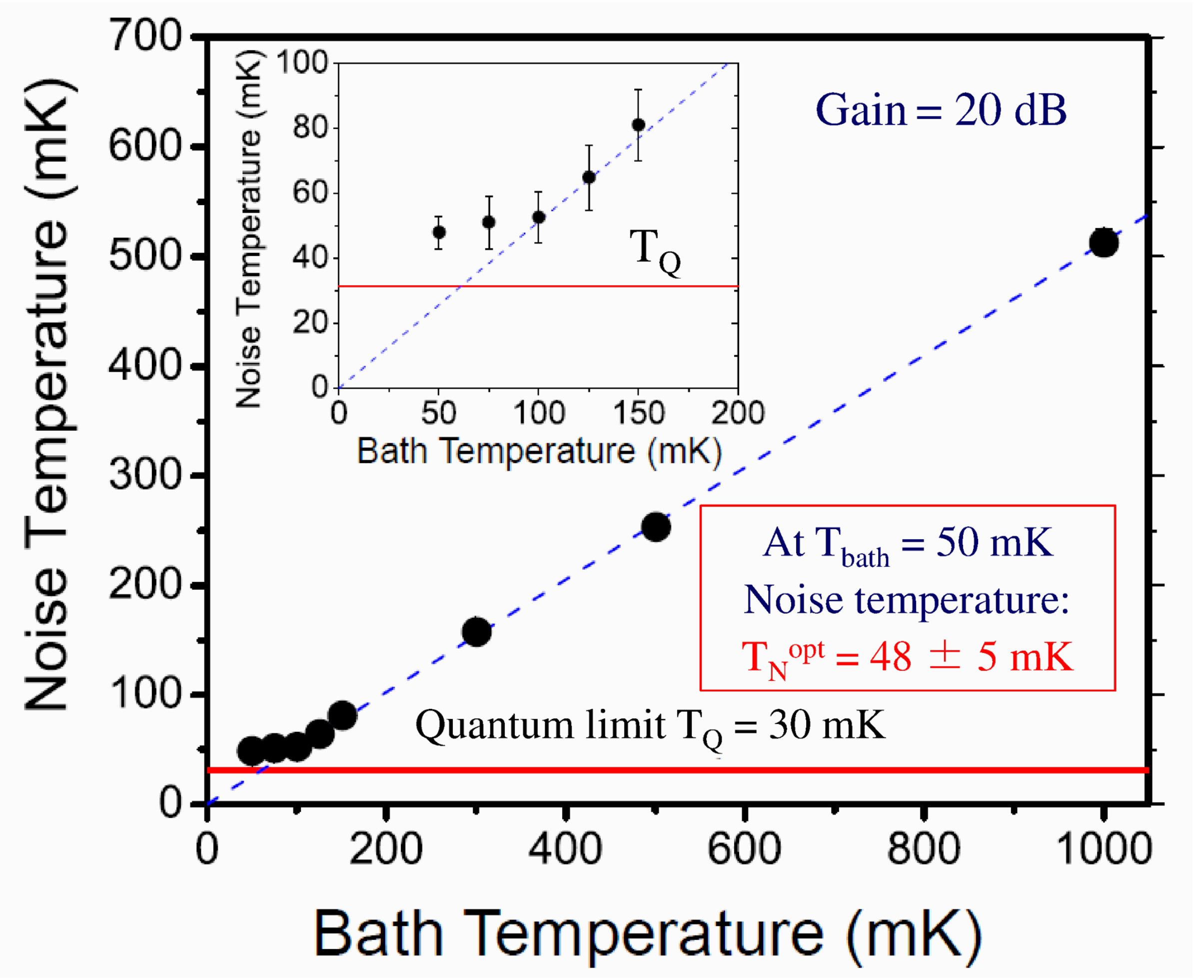


Institute of Applied Sciences & Intelligent Systems "E. Caianiello"



Squids measured noise vs. frequency



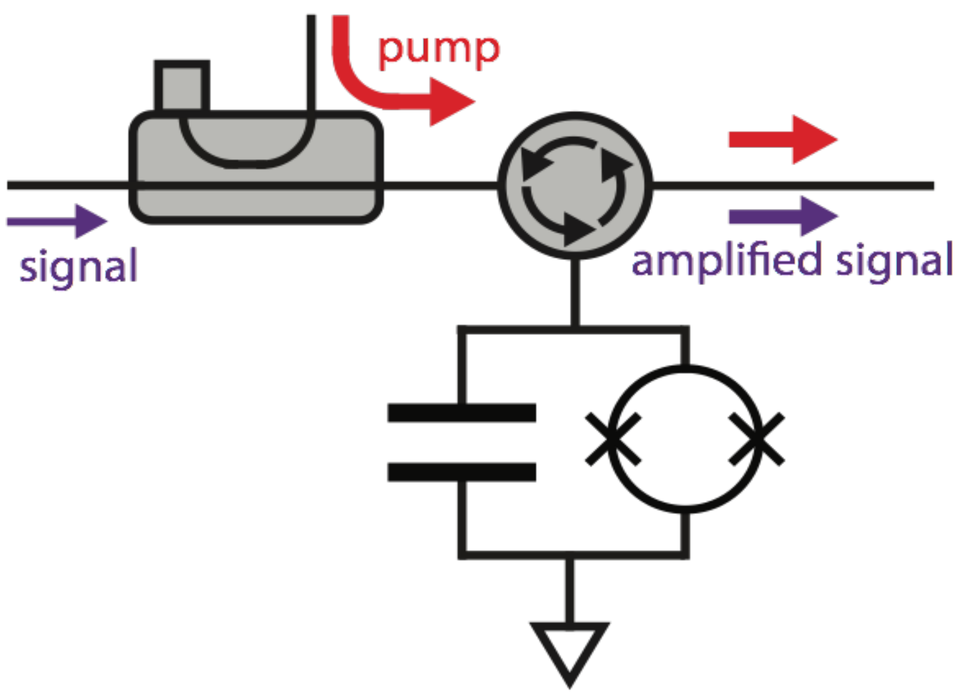


Josephson Parametric Amplifiers

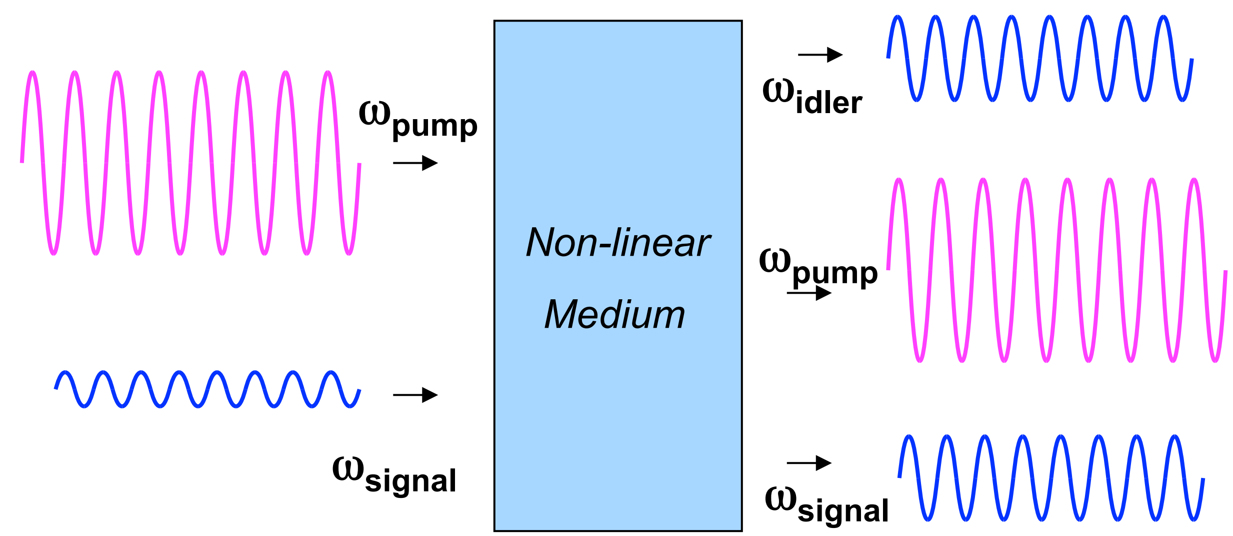
Yanjie Qiu (Jack), Allison Dove, Andrew Eddins, Ifran Siddiqi

Workshop on Microwave Cavities and Detectors for Axion Research
Livermore Valley Open Campus
January 12, 2017

JPA Operation



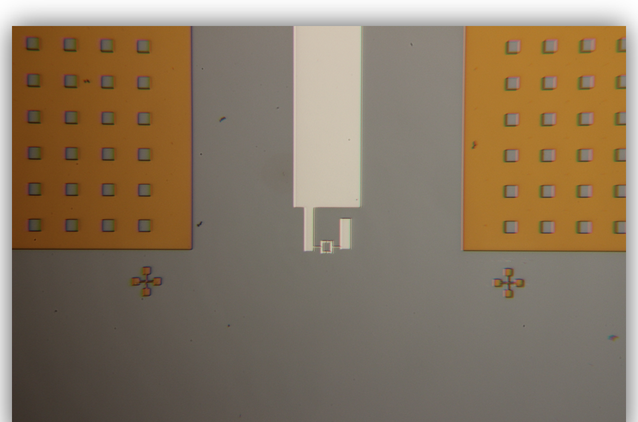
Wave Mixing Processes



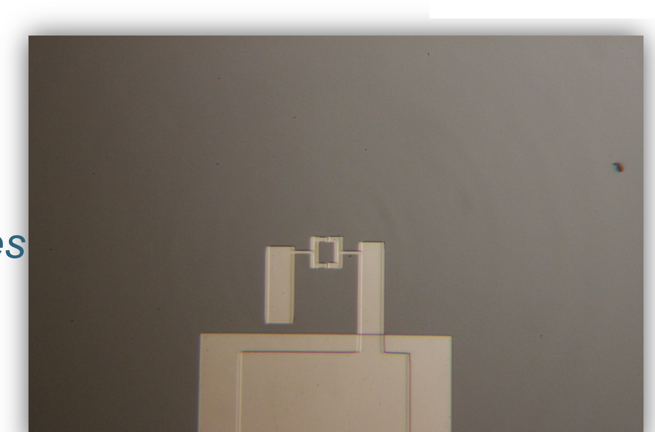
Three types:

1. $\omega_{pump} = \omega_{signal} + \omega_{idler}$ → non-degenerate
2. $2\omega_{pump} = \omega_{signal}$ → degenerate
3. $2\omega_{pump}$ → doubly degenerate

4GHz - 9GHz JPA

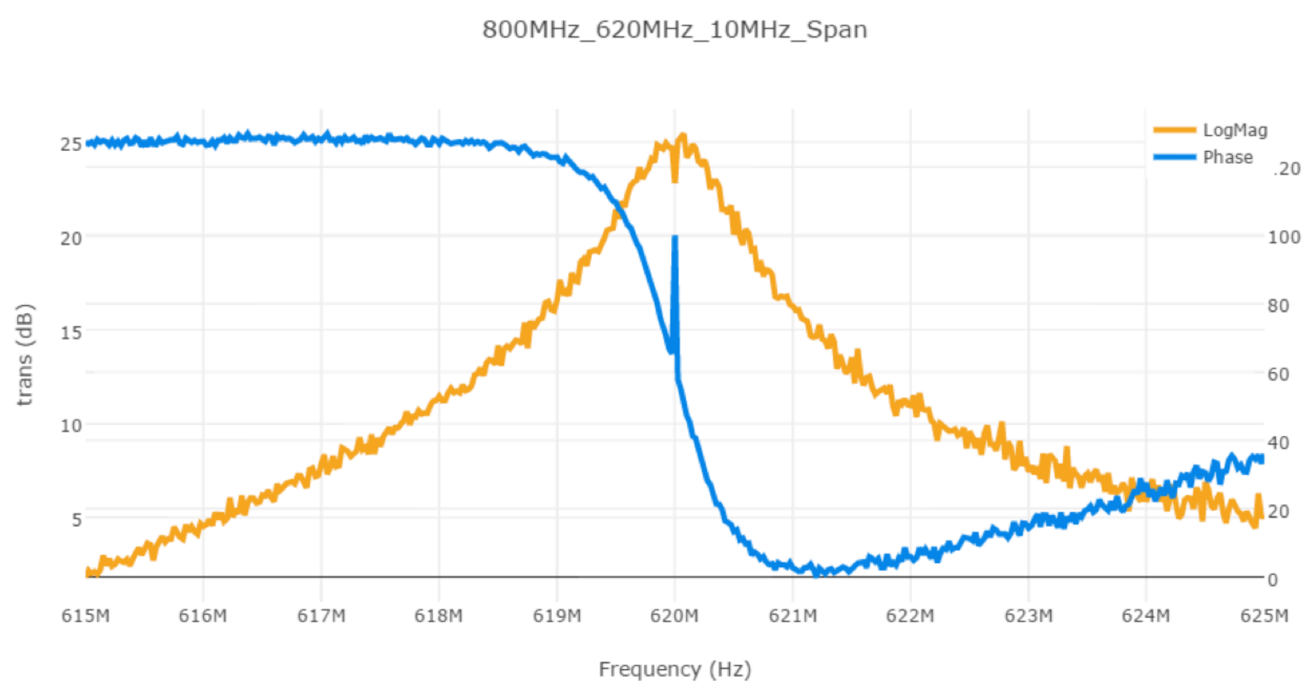


Junction Images



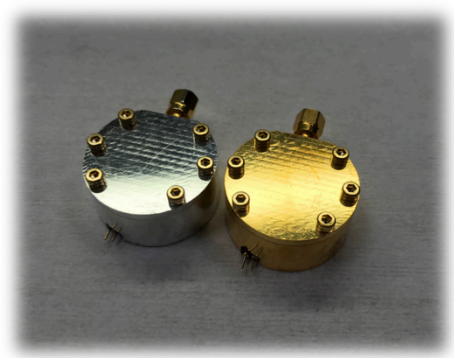
Gain performance

Yanjie Qiu (Jack), Allison Dove, Andrew Eddins, Ifran Siddiqi



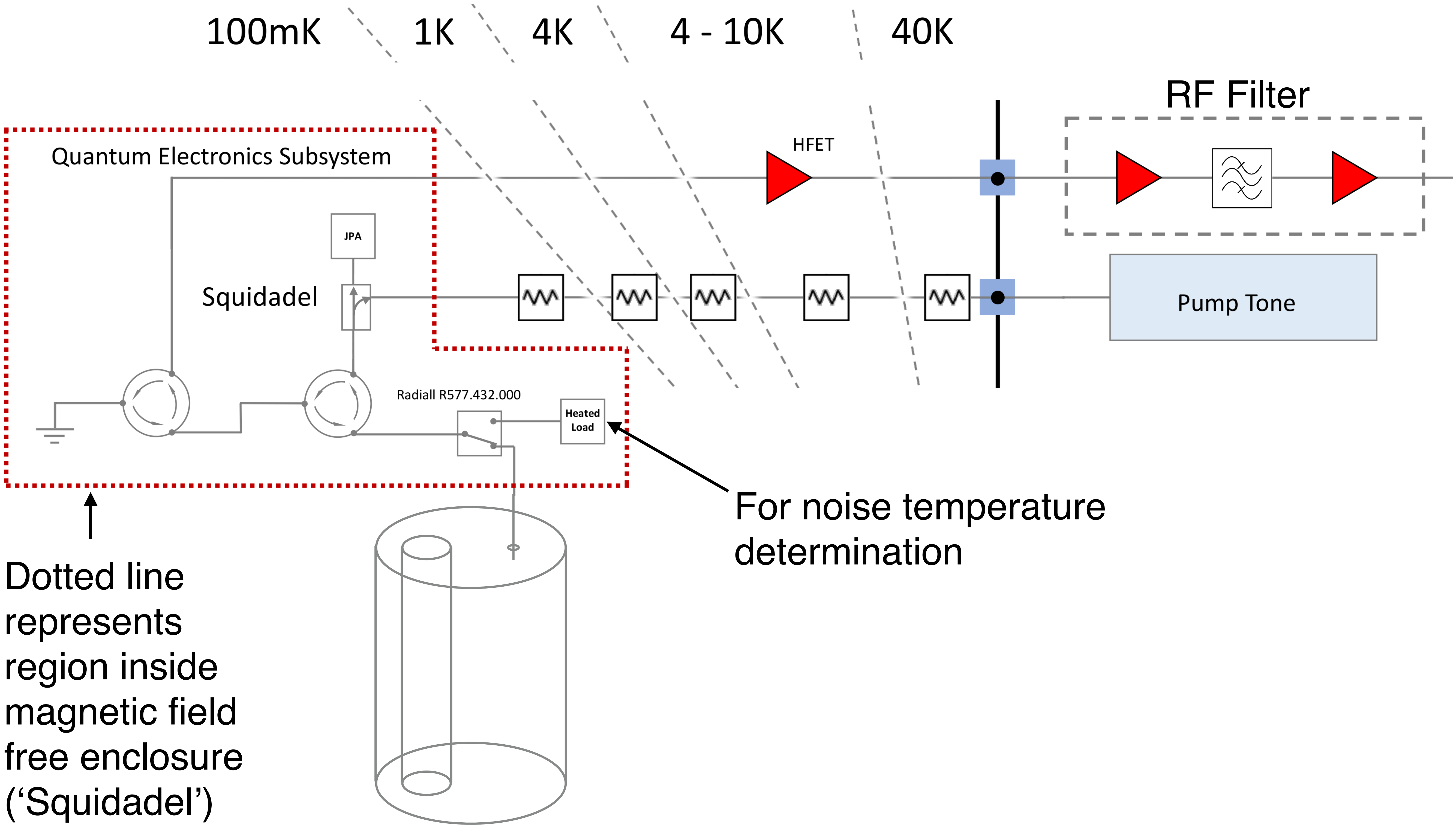
- Gain: 25dB peak
- Bandwidth: 2.5 MHz
- Pump power: -104.17dBm
- Coil current: -7.484mA

- No hanging thermalization strap

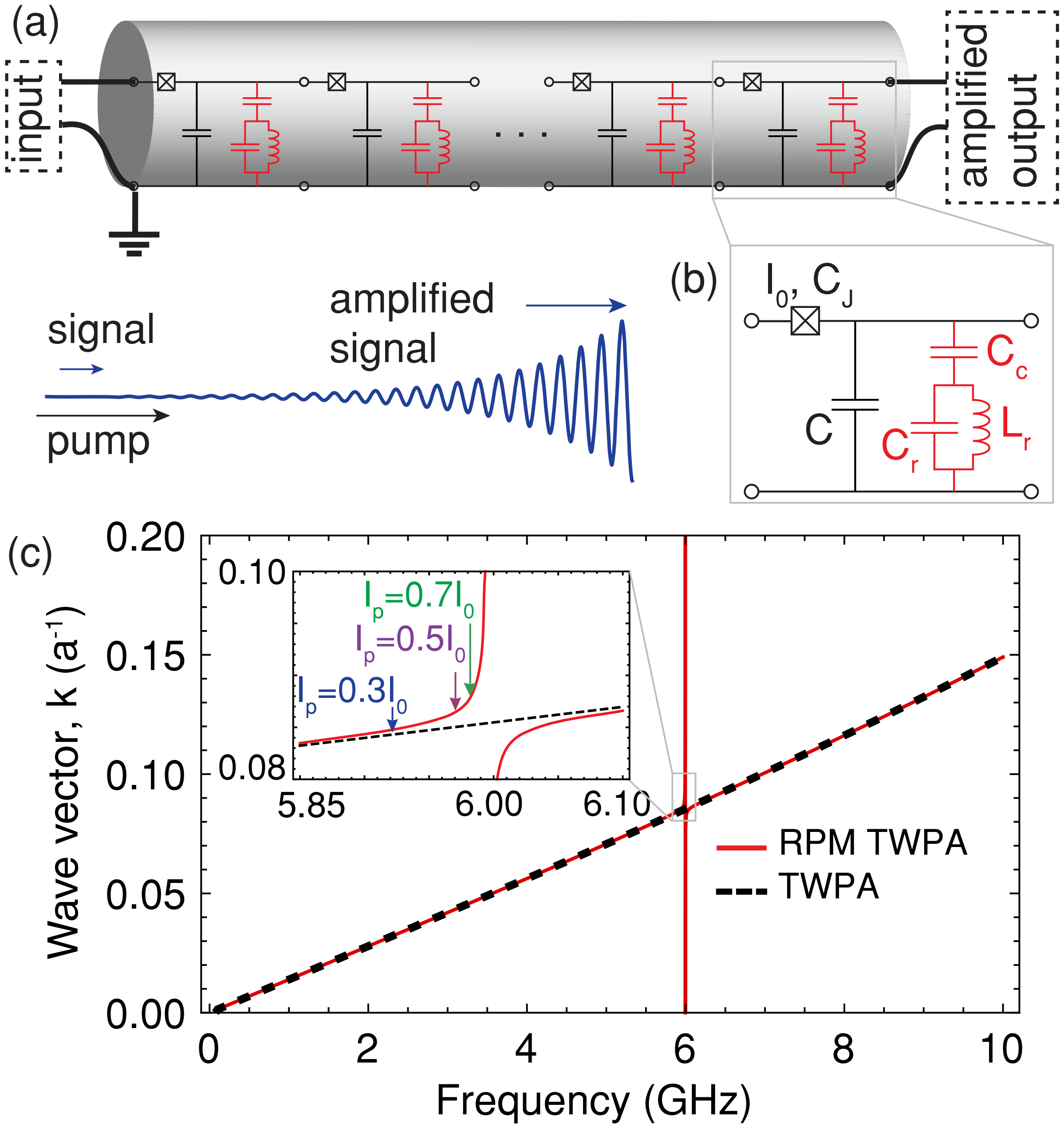


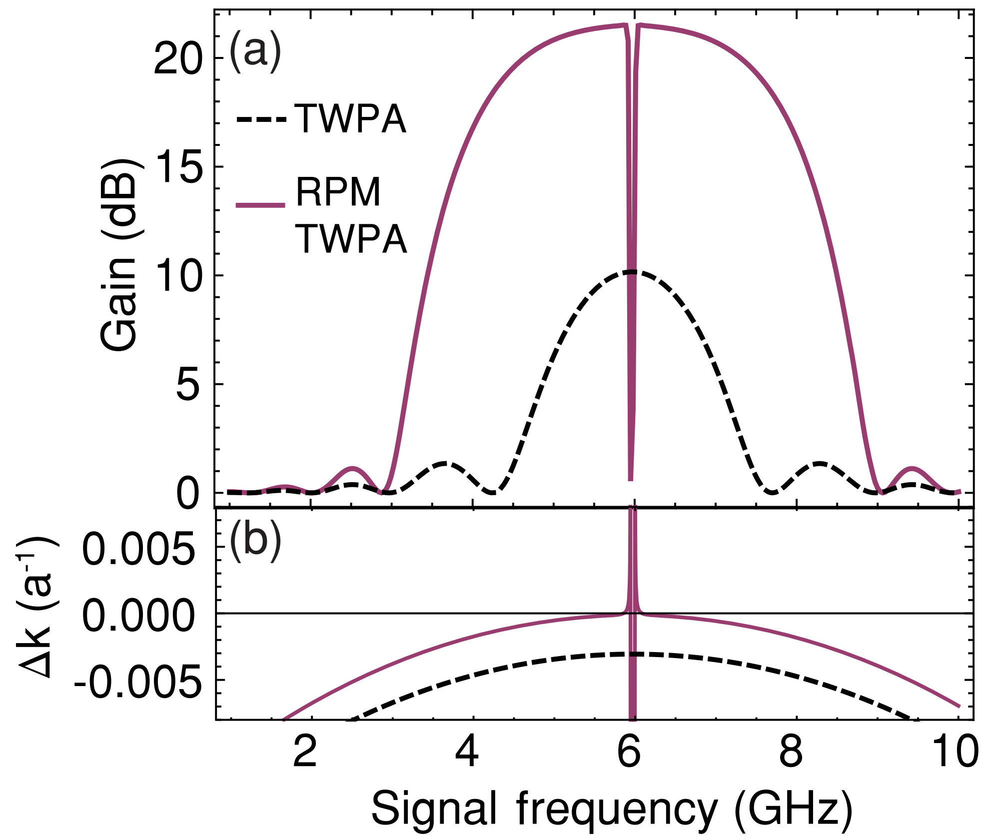
- Built-in dc connections reinforced with Stycast

Using a JPA with ADMX



Travelling wave parametric amplifiers





PRL **113,** 157001 (2014)

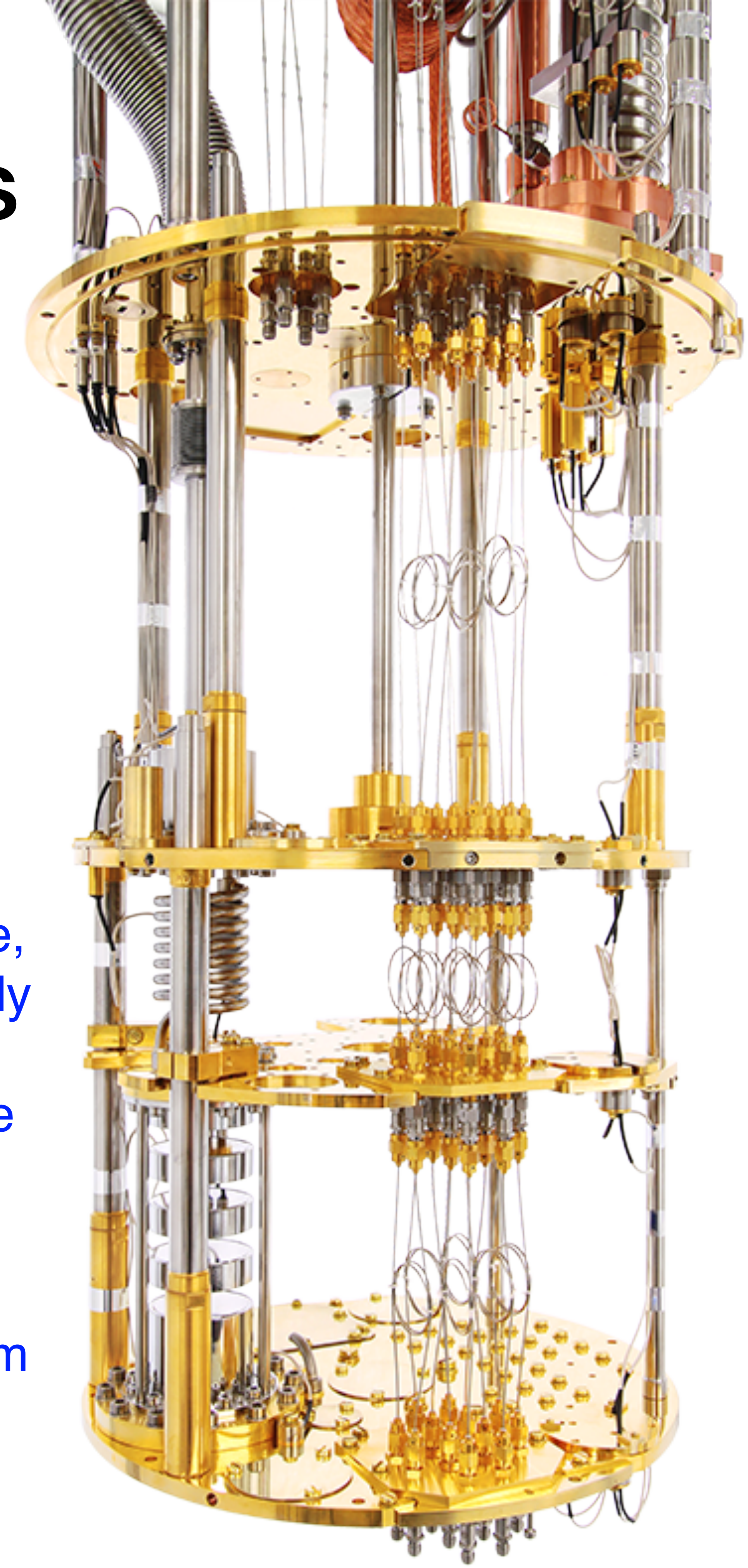
Kevin O'Brien,[1] Chris Macklin,[2] Irfan Siddiqi,[2] and Xiang Zhang[1,3,*]

Larger high field magnets, cold almost-dry cryogenics

Mechanically cooled (small LHe reservoir for quench protection), 8T, 90cm bore magnet.
Tesla Engineering (UK!)

10mk base temperature, mechanically pre-cooled closed cycle dilution refrigerator.

bluefors.com

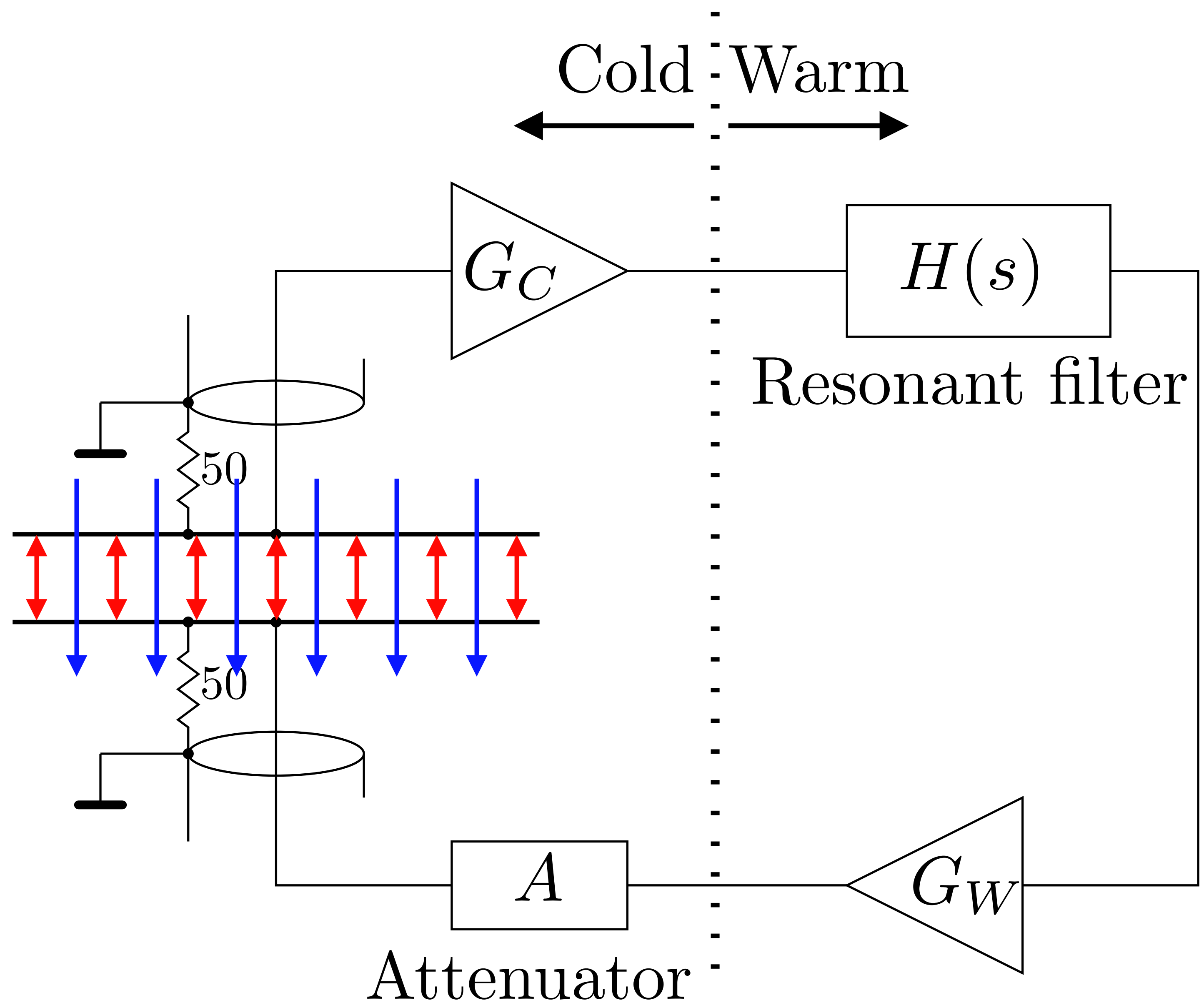


The University Of Sheffield.

A practical resonant feedback circuit

Courtesy of Holger Notzel, www.kometamps.com

Feedback resonance

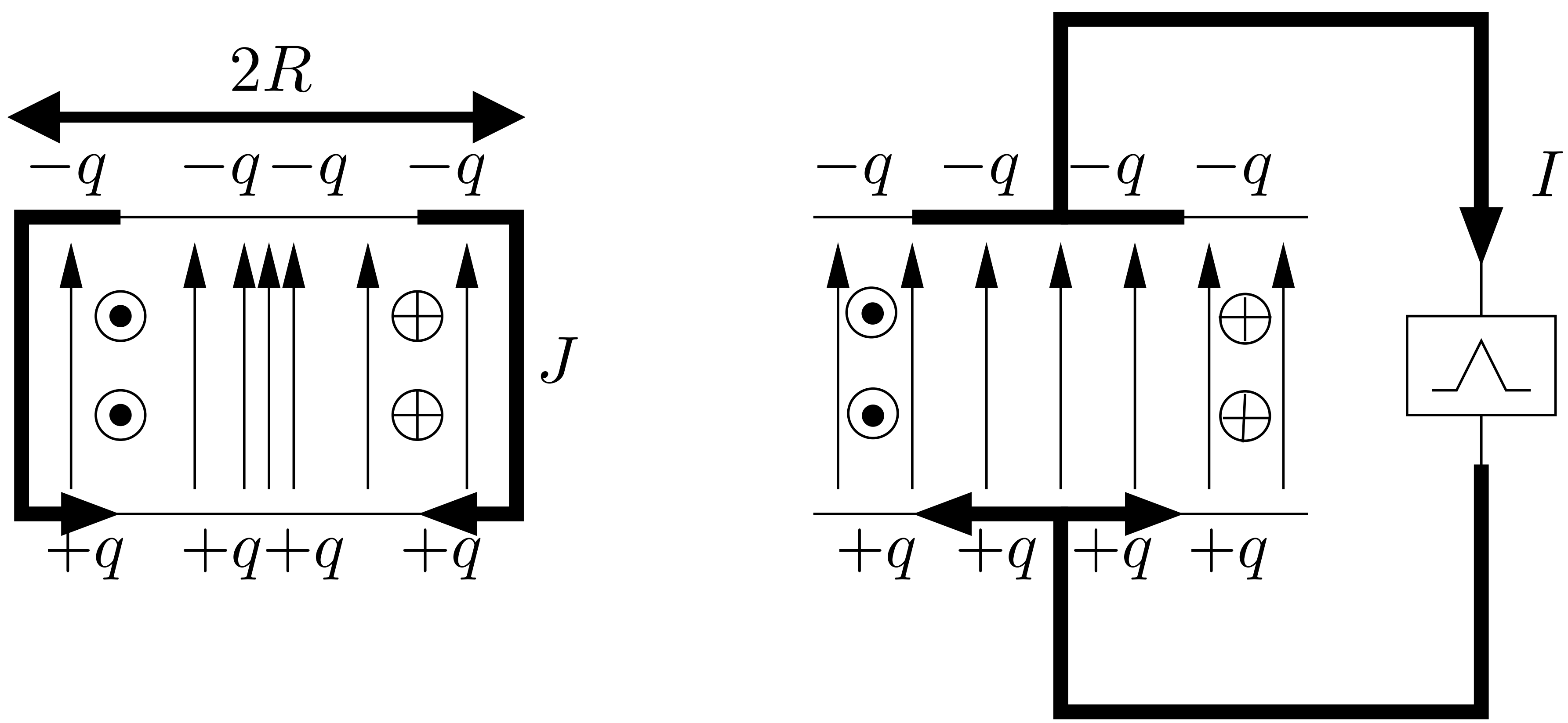


Maintain open-loop gain at <1 so the circuit doesn't start to oscillate by using a suitably large attenuator.

https://arxiv.org/abs/1805.11523

Is resonant feedback through a circuit equivalent to a cavity resonance ?

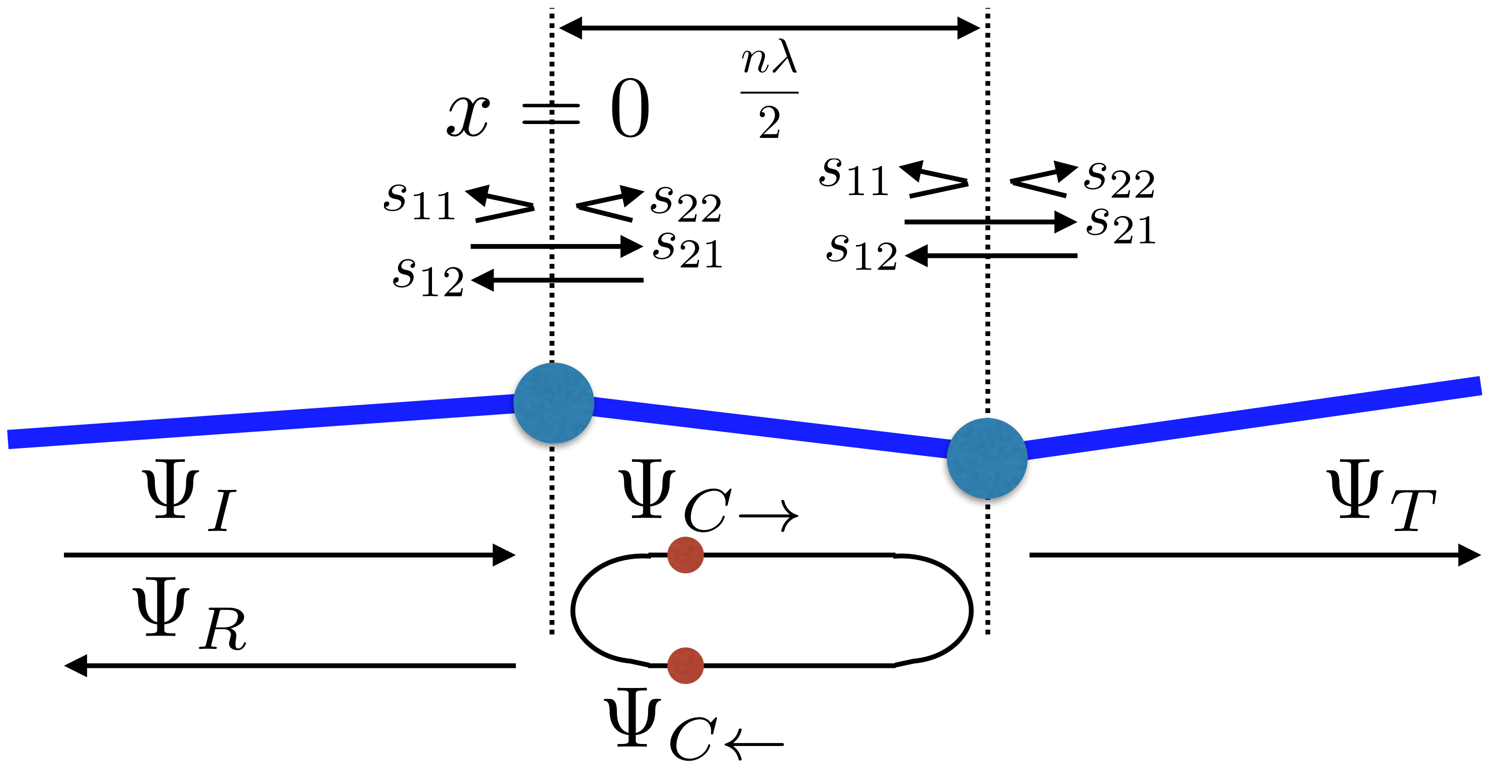
The answer is, not quite - but close enough for practical purposes.



Cavity TM_{010} mode   Resonant feedback

Cavity Mode: currents in walls complete resonant circuit.
Capacitor+feedback: feedback loop completes circuit.

What is resonance?



1.) Resonance results when the circulating field, in this case between the masses, interferes constructively with itself around on a round-trip.

2.) For an incident field to drive the resonance to high amplitude, it must be coherent over multiple round trips of the circulating field, and losses around the loop must be small.

Cavity mode circulating field decomposition

Electric fields in the cavity TM_{010} mode are usually written

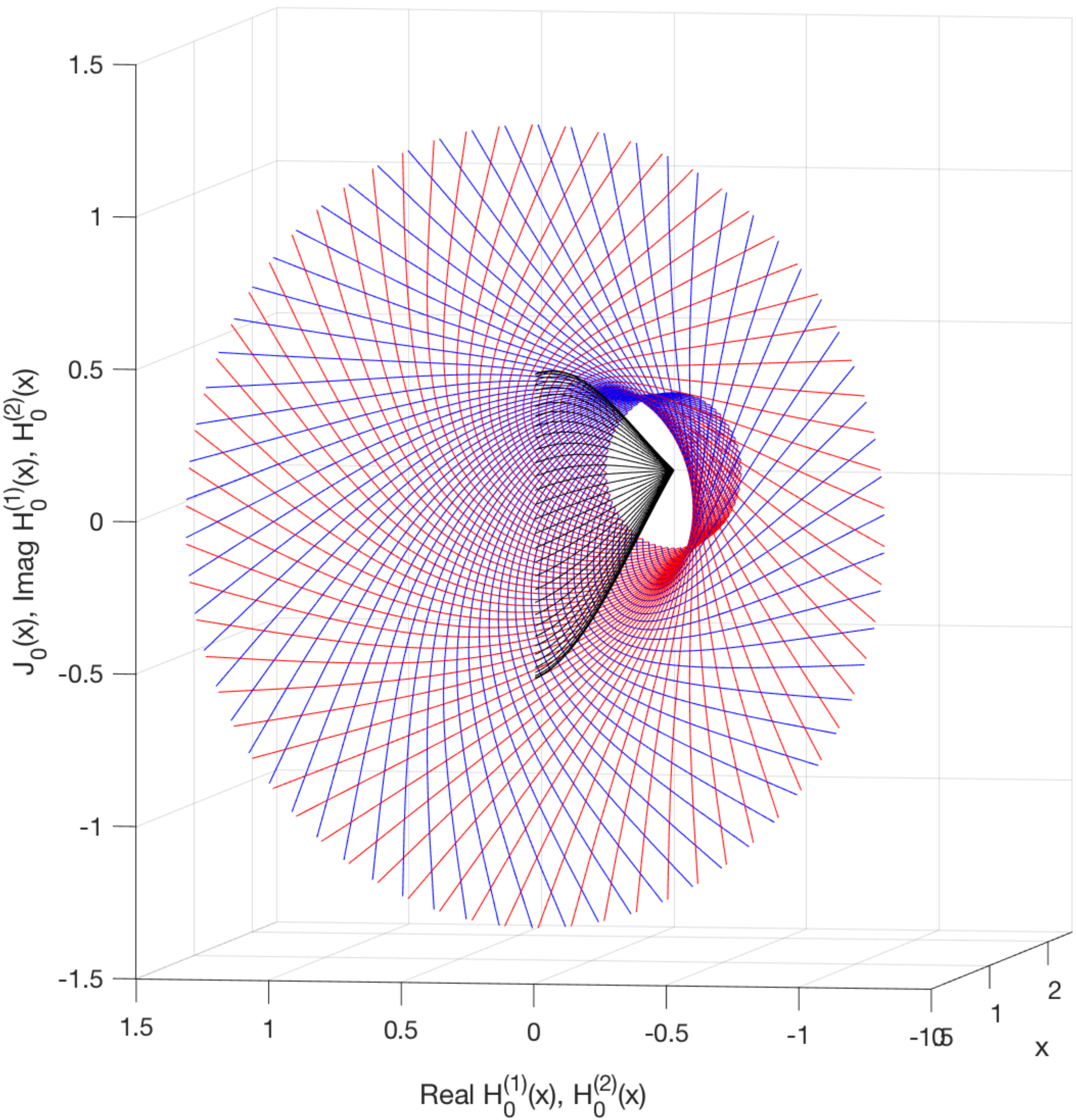
$$E(\rho, t) = E_0 J_0 \left(\frac{2.405\rho}{R} \right) \cos(\omega t)$$

This 'drum mode' standing wave can be re-cast in terms of counter-propagating travelling waves that move radial-cylindrically inwards and outwards, through the central axis, then bounce off the circular cavity wall, back through the axis, bounce off the opposite wall, and return again to the axis.

$$E(\rho, t) = \frac{E_0}{2} \Re \left\{ \left[H_0^{(1)} \left(\frac{2.405\rho}{R} \right) + H_0^{(2)} \left(\frac{2.405\rho}{R} \right) \right] e^{-i\omega t} \right\}$$

Propagating: **radially-outwards** **radially-inwards**

The University Of Sheffield.



Axion signal driving a cavity resonance

De Broglie wavelength of halo axions (assuming 1.8 microelectronvolts), $\lambda = \frac{2\pi\hbar c}{\beta m c^2} \simeq 830\,\mathrm{m}$

so the coherence time is $\tau_{\mathrm{coh}} = \frac{\lambda}{v_0} \simeq 3.4\,\mathrm{ms}$

Round trip time of a circulating field around the loop is 4R/c, which is 3.4ns.

Therefore within the coherence time the circulating fields from axions can make a million round trips.
The fundamental upper limit on resonant enhancement of the cavity signal in this cavity mode is

axion intrinsic Q= $\pi N_{\mathrm{cav}} = 3.1 \times 10^6$

In practice, losses in the copper walls on reflection dominate, and the cavity mode Q is around 35,000.

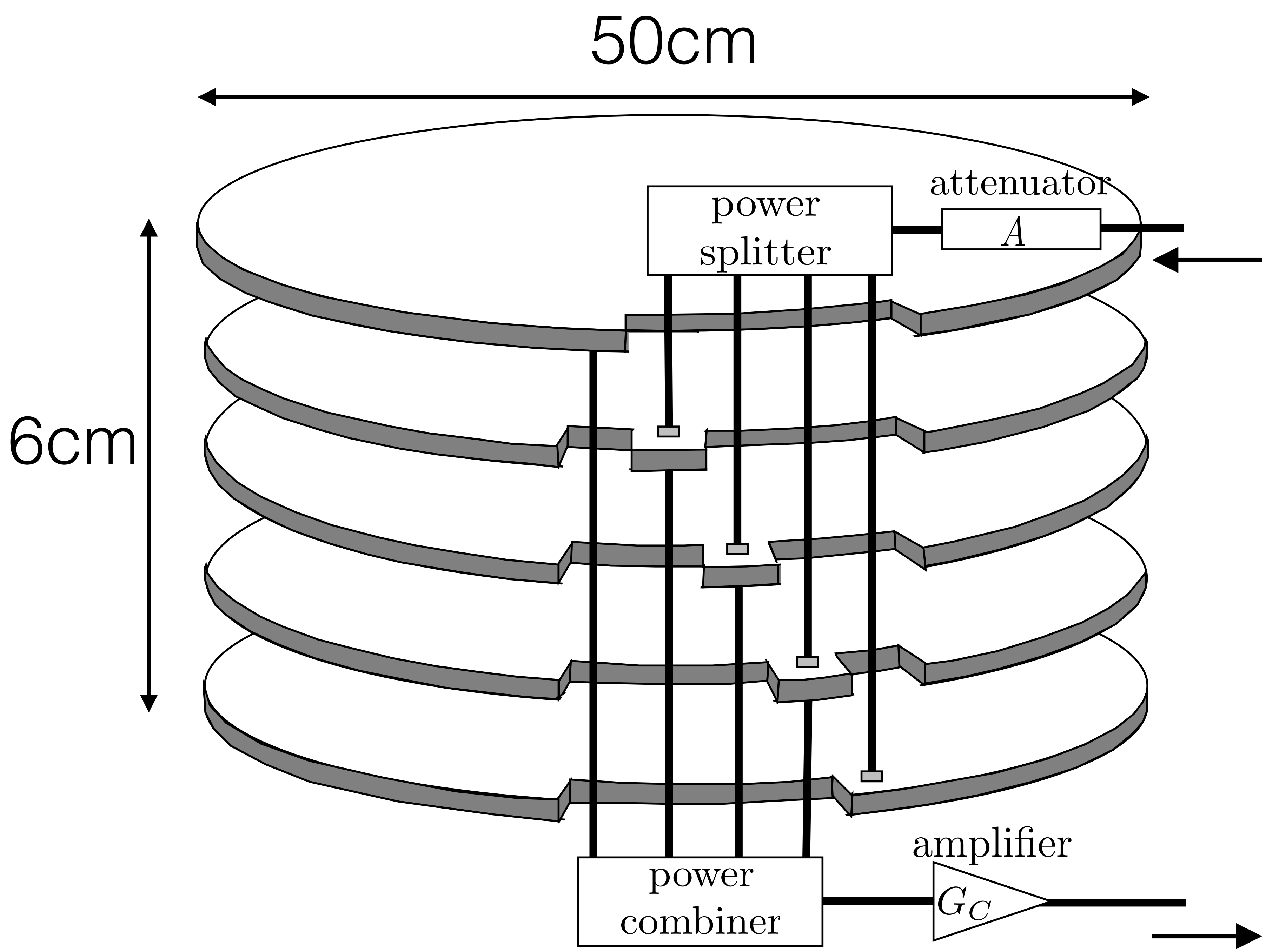
Coherence time and Q of axion-driven feedback circuit

Assume feedback loop involves 20m of RG401 coax and a 250MHz ADC/DAC pair, the total delay time round the loop is 103 ns (dominated by the cable). This means the 34,000 cycles around the feedback loop is 1 e-folding of coherence, equivalent to a Q of $\pi N_{\mathrm{coh}} = 107,000$

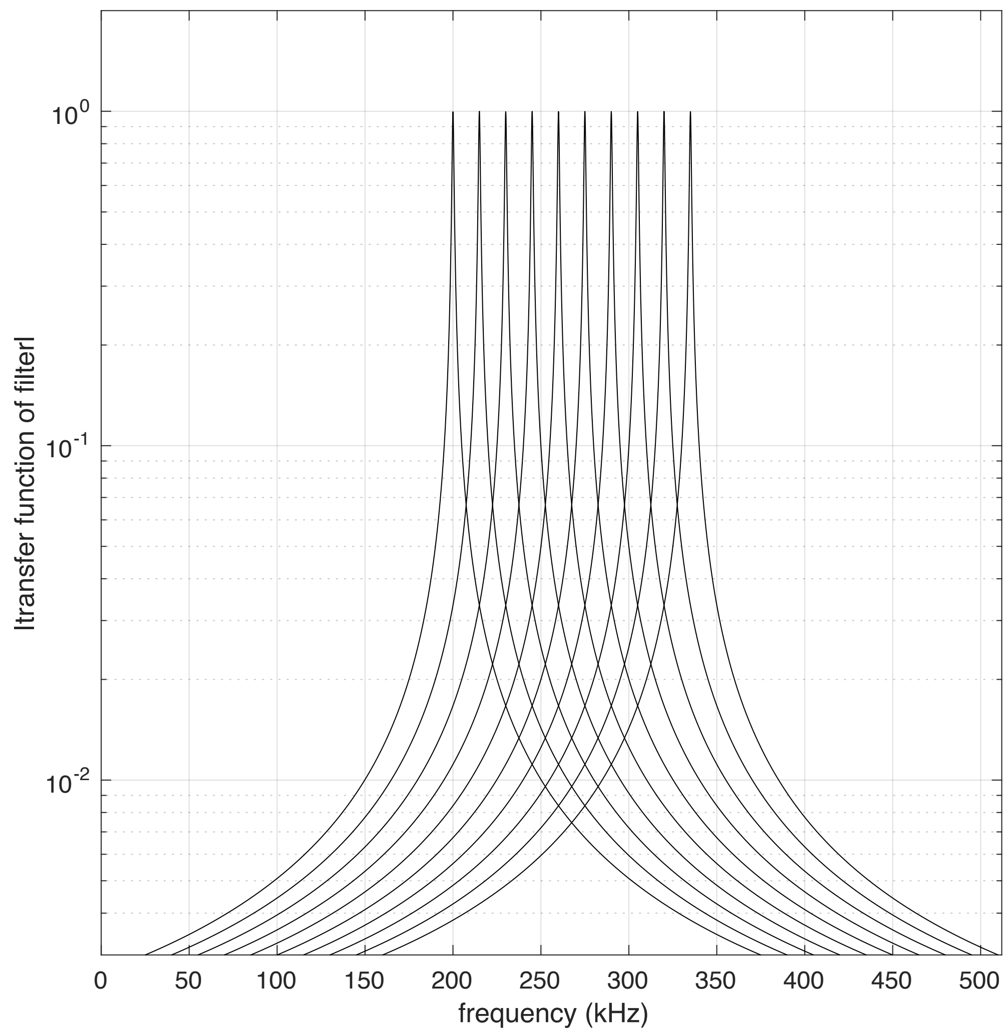
This Q is very competitive with that inherent in normal conducting cavity modes.

The equivalent here of the cavity wall losses is the Q of the external circuit, controlled by the parameters of the feedback filter. Experimental tests will inform the question of possible internal losses, but there should be no skin-depth losses at the end walls as electric fields terminate on surface charges.

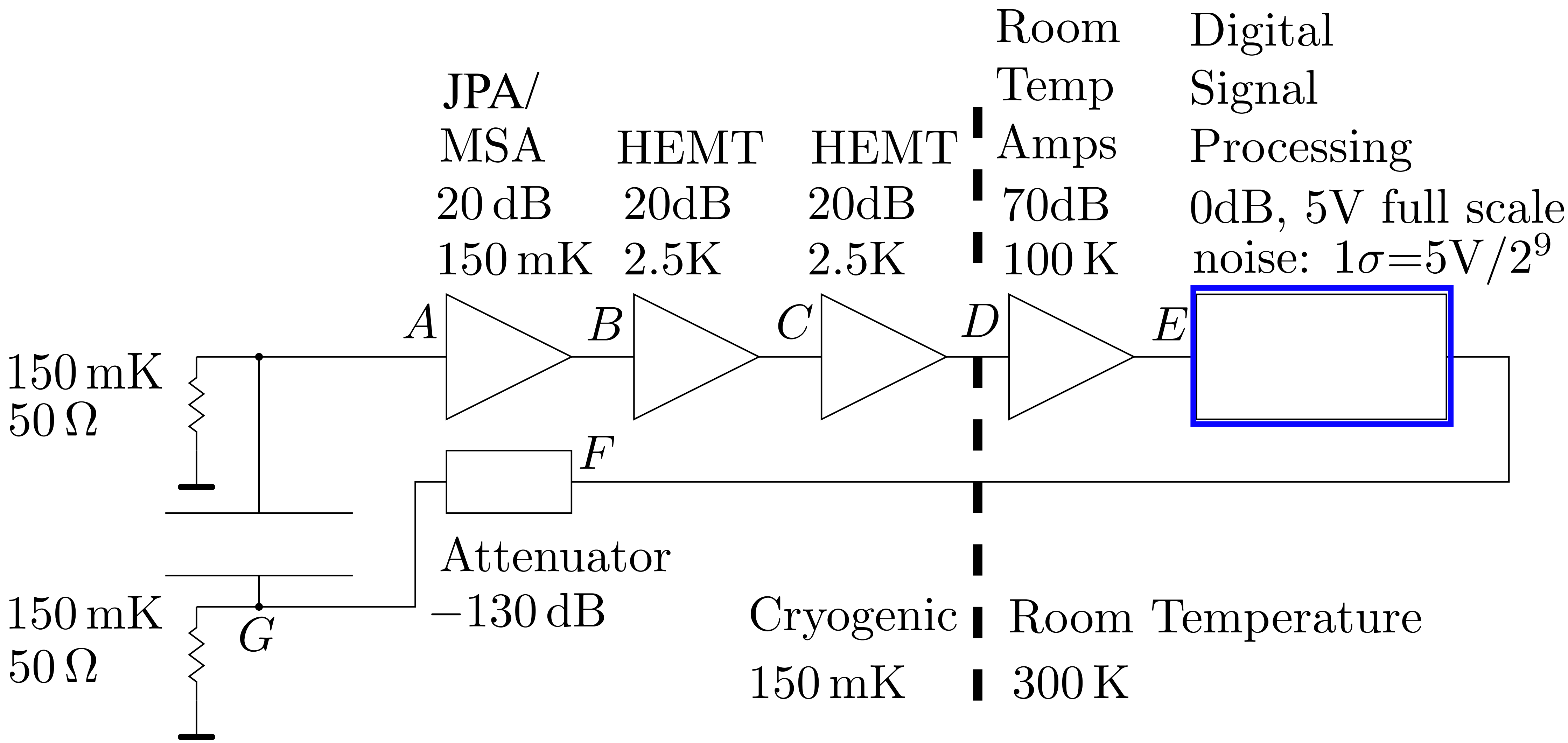
Capacitors in parallel - a prototype 4-capacitor model



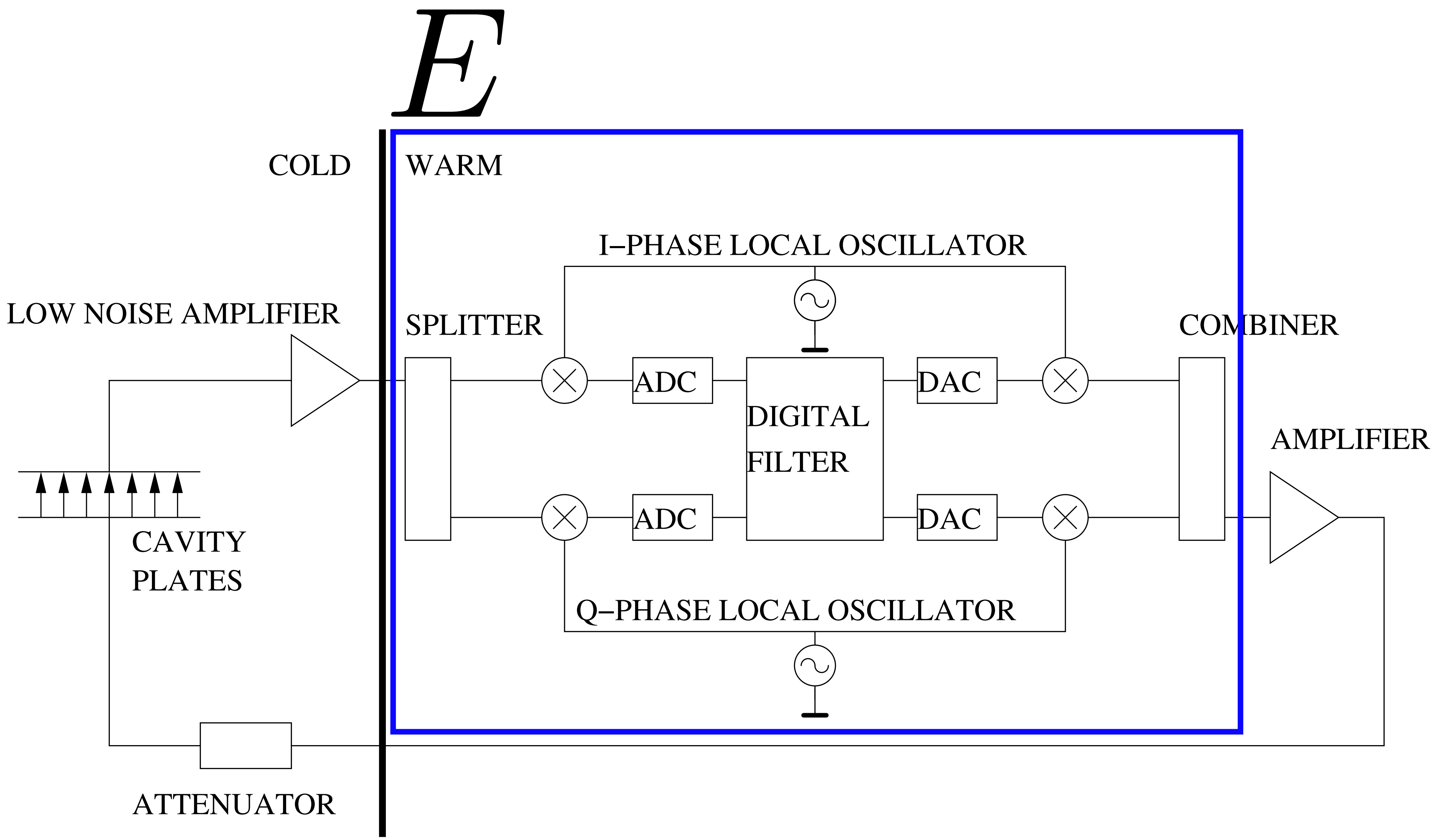
Resonances in parallel



Electronics in practice



Resonant electronics



Total noise power

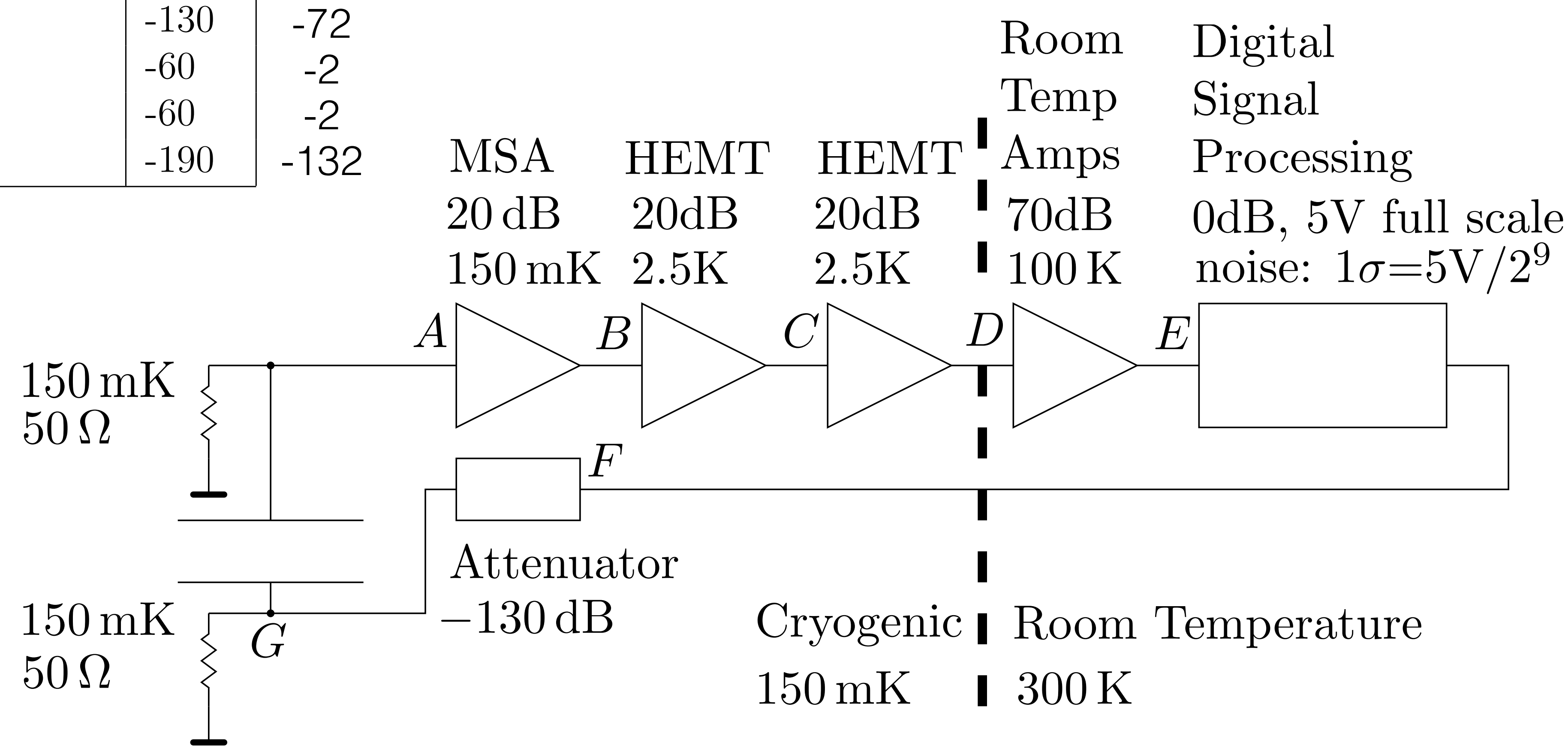
100 15kHz wide resonances, separated by 150kHz.
Q per resonance of approx 1GHz/15kHz=67000.
Total bandwidth into digital electronics 15MHz.
Noise in 15MHz band assuming system noise temperature of 300mK, -132dBm.

Total integration time for DFSZ

Assume an axion signal bandwidth of 750Hz, 300mK system noise, hence a signal-to-noise ratio of (10^{-22}W/ 3.1×10^{-21}W).DFSZ sensitivity requires an integration time of 1120s, during which we cover 1.5MHz.
2-40 micro eV corresponds to 4.34GHz bandwidth, so that the total integration time is 1120s x 4340/1.5 which is 37.5 days. This assumes a form factor of 0.4!

Noise budget

Location	Total summed noise into 750Hz bandwidth [dBm]	Noise from local component into 750Hz bandwidth [dBm]	Signal power [dBm]	**Noise in 15MHz bandwidth [dBm]**
A	-175	-178	-190	-132
B	-155	-166	-170	-112
C	-135	-166	-150	-92
D	-115	-150	-130	-72
E	-45	-76	-60	-2
F	-45	-178	-60	-2
G	-175	-178	-190	-132



Future axion searches

- Cavity searches should continue to dominate. No other method has sufficient sensitivity.
- Practical improvements in technology are critical. Best established of these is quantum electronics for low noise amplification.
- Advent of practical high field, large magnets with minimal liquid helium requirements make these experiments far more practical.
- Possibility of replacing the hard-wall resonant cavity detector with something having induced resonances in a comb. Great potential for improving scan rate.
- Still a very difficult job to search what is a large parameter space.
- Like LIGO, maybe we will get lucky.