Gravity Simulators (a.k.a. Analogue Gravity), Part I

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Extreme Gravitational Fields

- black holes (real data!) [Event horizon telescope, LIGO, PLANCK]
  - Hawking radiation (black hole evaporation) — horizon
  - super-radiance $\leftrightarrow$ rotation of black hole (or cylinder) — ergo-region
  - quasi-normal modes — horizon
- expanding Universe
  - cosmological particle creation
  - horizon crossing $\rightarrow$ freezing $\rightarrow$ squeezing — horizon
  - Gibbons-Hawking effect — horizon

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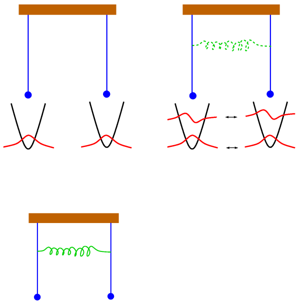
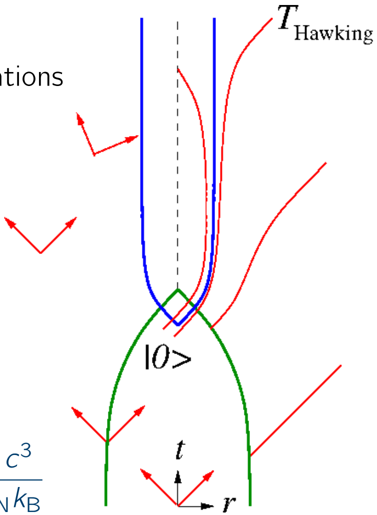
Hawking Radiation

Tunneling??

Particle–anti-particle pairs?

Tearing apart of quantum vacuum fluctuations





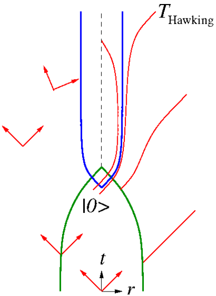
$$T_{\text{Hawking}} = \frac{1}{8\pi M} \frac{\hbar c^3}{G_N k_B}$$

Note: regularity at horizon is vital...

Black Hole Evaporation

Formula for Hawking temperature

$$T_{\text{Hawking}} = \frac{1}{8\pi M}\frac{\hbar\, c^3}{G_{\text{N}}\, k_{\text{B}}}$$



Combines four (apparently) different areas of physics

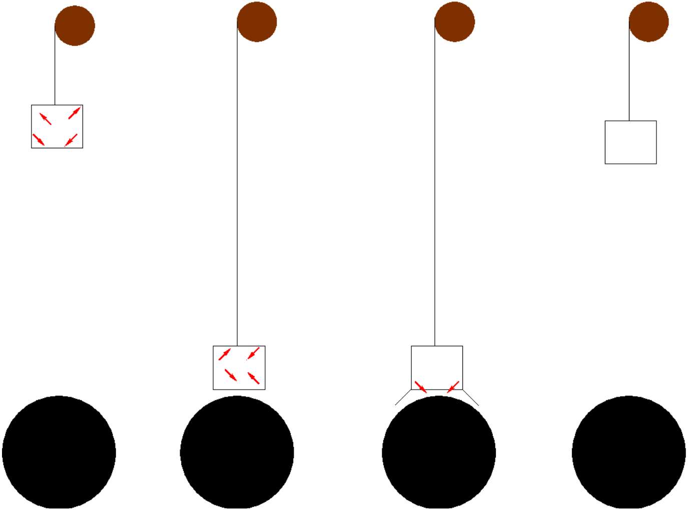
- $\hbar$ quantum theory
- c relativity
- G_{N} gravity
- k_{B} thermodynamics

Is nature trying to give us a hint?

$$S_{\text{BH}} = k_{\text{B}}\frac{A}{4\ell_{\text{P}}^2} = k_{\text{B}}\frac{A}{4\hbar G_{\text{N}}/c^3}$$

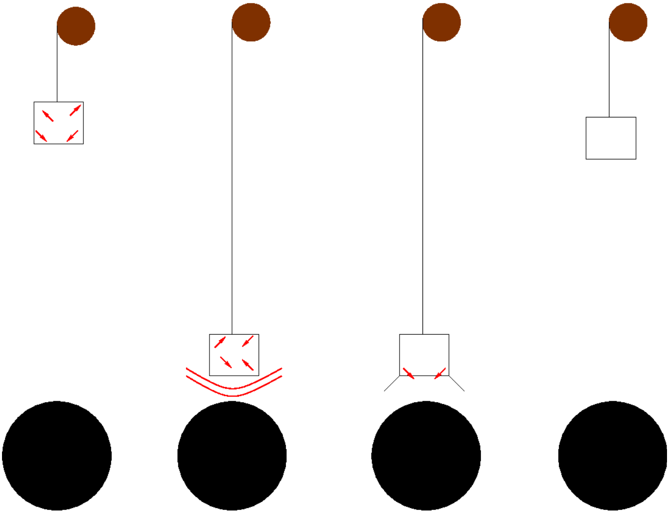
$\rightarrow$ black hole entropy $\propto$ area etc.

Black-Hole Heat Engine?



[Unruh & Wald, PRD 1982]

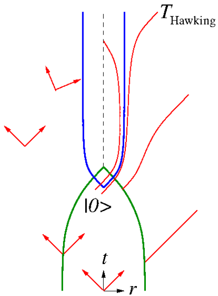
Resolution: Quantum Effects



[Unruh & Wald, PRD 1982]

Problems & Open Questions

- observability
  $M_{\rm BH} = 30 M_{\rm sun} \leadsto T_{\rm Hawking} \approx 2\rm nK \dots$
- trans-Plankian origin
- interacting fields
- back-reaction (e.g., final stage)
- robustness
- partners & entanglement
- information puzzle
- microscopic origin of $S_{\rm BH}$
- and many more...



Quasi-Normal Modes

Schwarzschild metric with horizon at $r = 2M$

$$ds^2 = \left(1 - \frac{2M}{r}\right) dt^2 - \frac{dr^2}{1 - \frac{2M}{r}} - r^2 d\vartheta^2 - r^2 \sin^2\vartheta \, d\varphi^2$$

Separation $\Phi_{\omega,\ell,m}(t, r, \vartheta, \varphi) = \exp\{-i\omega t\}\, \phi_{\omega,\ell,m}(r) Y_{\ell,m}(\vartheta, \varphi)$

Ordinary 2nd-order differential equation for $\phi_{\omega,\ell,m}(r)$

Resonances with complex ω (analogous for metric perturbations)

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Super-Radiance

Kerr metric with $\Delta = r^2 - 2Mr + a^2$ and $\Sigma = r^2 + a^2\cos^2\vartheta$

$$
\begin{aligned}
ds^2 \;=\; & \left(1 - \frac{2Mr}{\Sigma}\right) dt^2 + \frac{4Mar\sin^2\vartheta}{\Sigma}\, dt\, d\varphi - \frac{\Sigma}{\Delta}\, dr^2 - \Sigma\, d\vartheta^2 - \\
& - \left(r^2 + a^2 + \frac{2Ma^2 r\sin^2\vartheta}{\Sigma}\right) \sin^2\vartheta\, d\varphi^2
\end{aligned}
$$

Horizons ($\Delta = 0$) at $r_\pm = M \pm \sqrt{M^2 - a^2}$ and ergo-region $g_{00} < 0$

Separation $\Phi_{\omega,\ell,m}(t,r,\vartheta,\varphi) = \exp\{-i\omega t + im\varphi\}\, \phi_{\omega,\ell,m}(r) S_{\omega,\ell,m}(\vartheta)$

Ordinary 2nd-order differential equation for $\phi_{\omega,\ell,m}(r) \to$ Wronskian

$$
1 - |\mathcal{R}_{\omega m}|^2 = \frac{\omega - m\Omega_{\mathrm{h}}}{\omega} |\mathcal{T}_{\omega m}|^2
$$

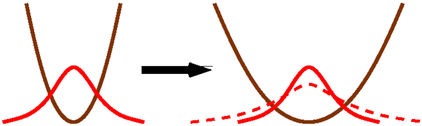
Amplification for $\omega < m\Omega_{\mathrm{h}}$

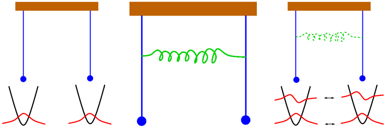
Quantum Amplifiers

Amplification of signal $q \to e^{\xi} q$ as quantum gate?

- phase-sensitive amplifier – single-mode squeezing (bosons)
  $\hat{q} \to e^{+\xi}\hat{q}$ and $\hat{p} \to e^{-\xi}\hat{p}$ with $\hat{U} = \exp\{\xi\hat{a}^2/2 - \text{h.c.}\}$
- phase-insensitive amplifier – two-mode squeezing (entanglement)
  $\hat{q} \to \hat{q}\cosh\xi + \hat{Q}\sinh\xi$ and $\hat{p} \to \hat{p}\cosh\xi - \hat{P}\sinh\xi$

Send in vacuum state → creation of particle pairs (→ partners)





[Unruh]

Cosmological Particle Creation

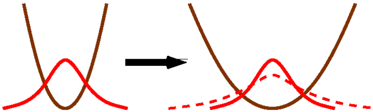
Friedmann-Robertson-Walker metric in 1+1 dimensions

$$ds^2 = a^2(t)\left[dt^2 - dx^2\right]$$

Massive scalar field

$$(\Box + m^2)\Phi = 0 \rightsquigarrow \left(\frac{d^2}{dt^2} + k^2 + a^2(t)m^2\right)\phi_k(t) = 0$$

Harmonic oscillators with time-dependent potentials $\rightarrow$ squeezing



Horizon Crossing and Freezing

Friedmann-Robertson-Walker metric in 3+1 dimensions (proper time)

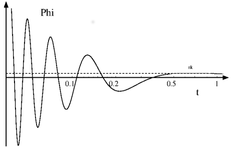
$$ds^2 = d\tau^2 - a^2(\tau)d\vec{r}^{\,2}$$

Massless scalar field in de Sitter metric with $a(\tau) \propto \exp\{H\tau\}$

$$\Box\Phi = 0 \rightsquigarrow \left(\frac{d^2}{d\tau^2} + 3H\frac{d}{d\tau} + \frac{k^2}{a^2(\tau)}\right)\phi_k(t) = 0$$

Damped harmonic oscillators with decaying spring stiffness

- oscillation (under-damped)
- horizon-crossing (critical)
- freezing (over-damped)



$\to$ amplification

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Gibbons-Hawking Effect

Friedmann-Robertson-Walker metric in 3+1 dimensions

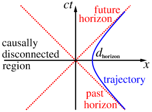
$$ds^2 = a^2(t)\left[dt^2 - d\vec{r}^{\,2}\right] = d\tau^2 - a^2(\tau)d\vec{r}^{\,2}$$

Electromagnetic field: conformal invariance $\to$ no particle creation

$$\langle \hat{A}(t,\vec{r})\hat{A}(t',\vec{r}\,')\rangle \propto \frac{1}{(t-t')^2 - (\vec{r}-\vec{r}\,')^2}$$

However, particle detector "ticks" with proper time τ

de Sitter metric $a(\tau) \propto \exp\{H\tau\} \to$ thermal response with temperature



$$T_{\rm GH} = \frac{H}{2\pi}$$

.

Analogy to Unruh effect. . .

Gravity Simulators, Part II

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Sonic/Acoustic Analogy

[Unruh, PRL 1981]

Sound waves in flowing fluids (velocity $\vec{v}$ and density ϱ)

- ideal fluid without viscosity or friction
- irrotational flow $\vec{v} = \nabla\phi$
- conservative forces $\to$ potential V
- barotropic equation of state $p = p(\varrho)$

Euler equation $\to$ Bernoulli equation with specific enthalpy $h(\varrho)$

$$\frac{d\vec{v}}{dt} = \left(\frac{\partial}{\partial t} + \vec{v}\cdot\nabla\right)\vec{v} = -\frac{\nabla p}{\varrho} - \nabla V \leadsto \dot{\phi} + V + \frac{(\nabla\phi)^2}{2} + h(\varrho) = 0$$

Equation of continuity

$$\frac{\partial\varrho}{\partial t} + \nabla\cdot(\varrho\vec{v}) = 0 \leadsto \frac{\partial\varrho}{\partial t} + \nabla\cdot(\varrho\nabla\phi) = 0$$

Linearise...

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Sound Waves

Lagrangian with $d\mu/d\varrho = h(\varrho)$

$$\mathcal{L} = -\varrho\left(\dot{\phi} + V + \frac{(\nabla\phi)^2}{2}\right) - \mu(\varrho)$$

Linearise $\varrho = \varrho_0 + \delta\varrho$, $\phi = \phi_0 + \delta\phi \rightsquigarrow \vec{v} = \vec{v}_0 + \delta\vec{v}$ and eliminate $\delta\varrho$

$$\delta^2\mathcal{A} = \frac{1}{2}\int dt\, d^D r \left(\frac{(\delta\dot{\phi} + \vec{v}_0 \cdot \nabla\delta\phi)^2}{\mu''(\varrho_0)} - \varrho_0\,\frac{(\nabla\delta\phi)^2}{2}\right) ,$$

Painlevé-Gullstrand-Lemaître metric with $c_s^2 = \varrho\mu'' = \varrho h' = dp/d\varrho$

$$g_{\mu\nu}^{\text{eff}} = \left(\frac{\varrho_0}{c}\right)^{2/(D-1)} \begin{pmatrix} c^2 - \vec{v}_0^2 & \vec{v}_0 \\ \vec{v}_0 & -\mathbf{1} \end{pmatrix}$$

Phonons in fluid behave as scalar field in curved space-time!

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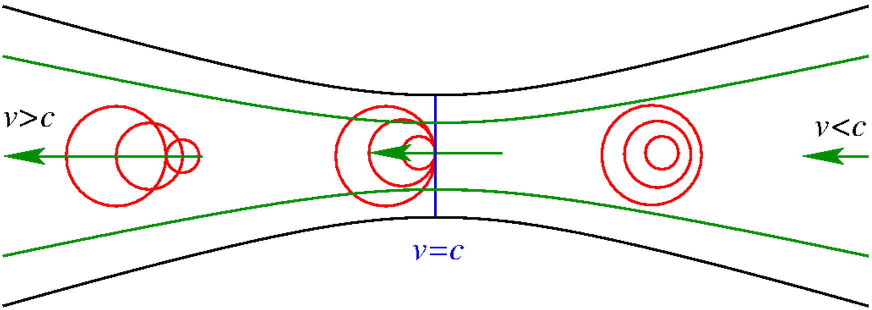
Black-Hole Analogues



"The same equations have the same solutions."

$$T_{\text{Hawking}} = \frac{\hbar}{2\pi\, k_{\text{B}}} \left| \frac{\partial}{\partial r} (v - c) \right|$$

$\rightarrow$ trans-Plankian origin?

Dispersion Relation

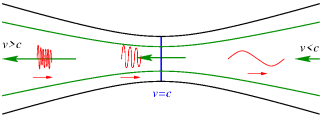
WKB approach: $(\omega + \vec{v}_0 \cdot \vec{k})^2 = f^2(\vec{k}) = c_s^2 \vec{k}^2 + \dots$

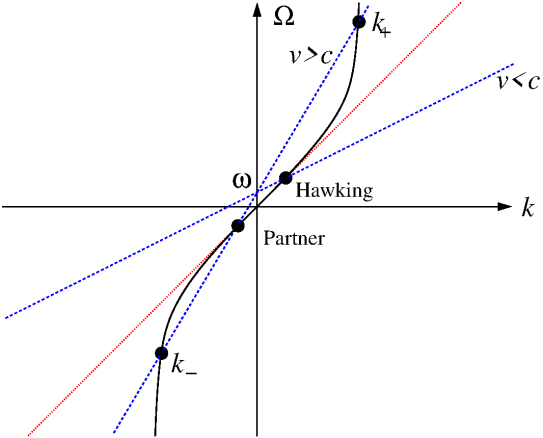
Super-“luminal” case

Effectively 1D

Upstream modes

Breakdown of WKB
at horizon

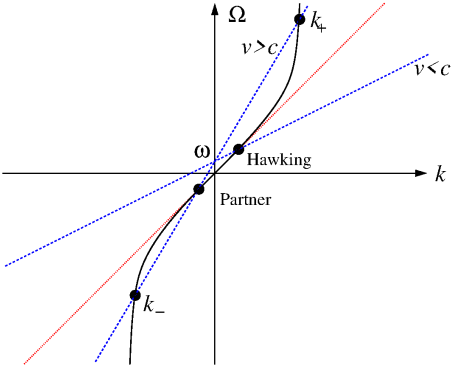


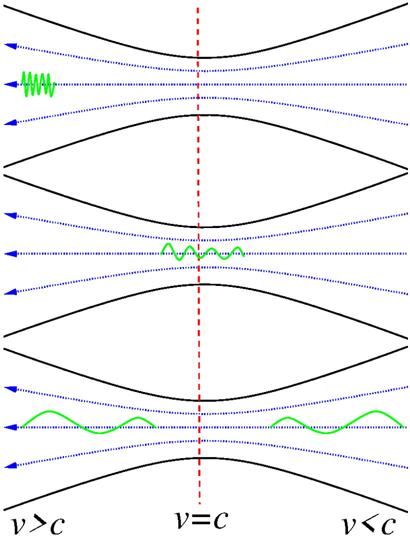


[Unruh, Jacobson, Corley, RS, Leonhardt, Parentani,...]

Origin of Hawking Radiation

Tearing apart of initial wave
(from the interior)





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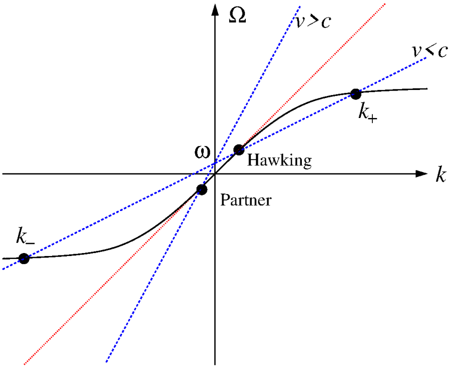
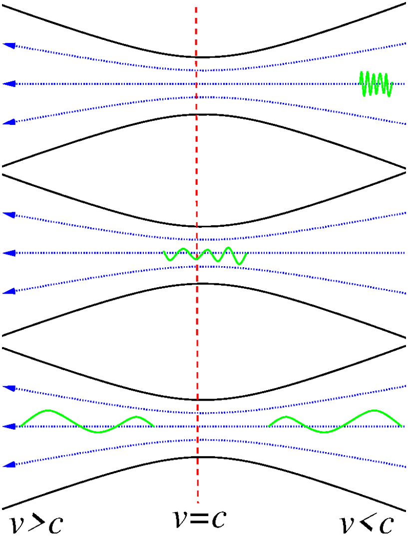
Sub-“Luminal” Case

Origin: exterior region





Lessons for Hawking Radiation

- trans-Planckian origin resolved
- universality
- robustness (within limits)

$$T_{\text{Hawking}}(\omega) = \frac{v_{\text{group}}(\omega)v_{\text{phase}}(\omega)}{8\pi M}$$

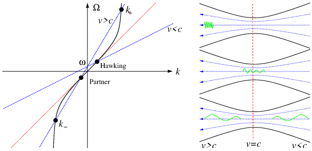
more than $\omega^2 \sim k^4$ is problematic

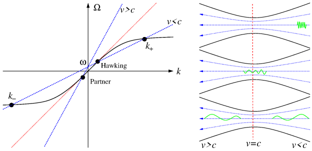
[RS+Unruh, PRD (R) 2008]

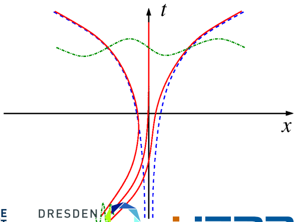
- breakdown of WKB
  in t, x coordinates
- near-horizon metric with Kruskal
  coordinate [RS+Unruh, PRD 2013]

$$ds^2 = 2e^{\kappa t}\,dt\,dU - e^{2\kappa t}\,dU^2$$

analogy to cosmic expansion
$\rightarrow$ tearing apart of waves







Super-Radiance

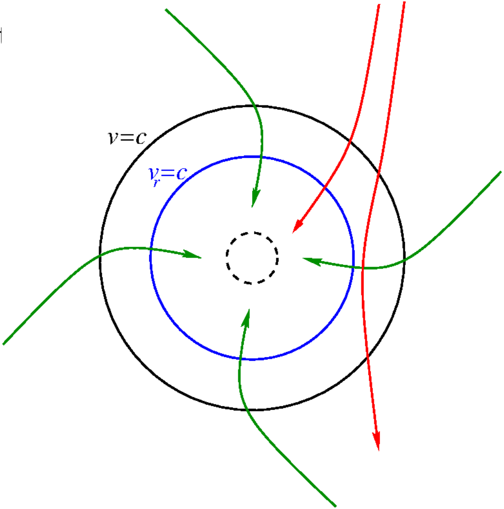
Inward radial $v_r < 0$ plus azimut

Ergo-region $\vec{v}^2 > c_s^2$
$\rightarrow g_{00}^{\text{eff}} < 0$

Horizon $v_r^2 > c_s^2$

Singularity $\sim$ drain

Phonons may extract
rotational energy. . .



Also: quasi-normal modes etc.

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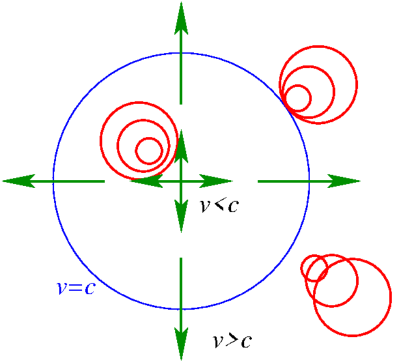
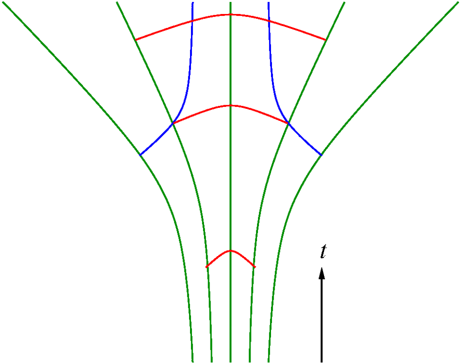
Analogue of Cosmic Expansion I

Option I: homogeneously expanding fluid → analogue of cosmic horizon

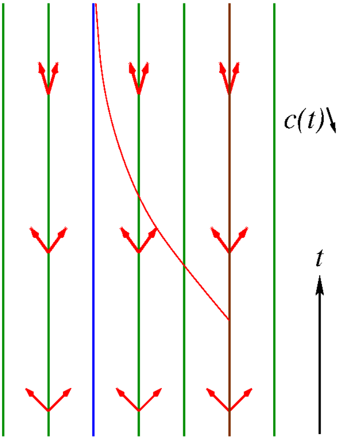


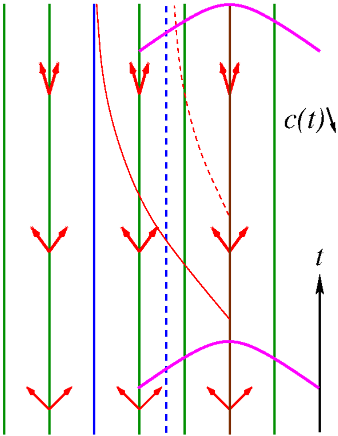


Oscillation → horizon crossing → freezing → squeezing (amplification)

Analogue of Cosmic Expansion II

Option I: fluid at rest with decaying speed of sound





Oscillation → horizon crossing → freezing → squeezing (amplification)
Or combination of both I and II

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Gravity Simulators, Part III

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Bose-Einstein Condensates

[Anglin, Barceló, Busch, Carusotto, Cirac, Coutant, de Nova, Fabbri, Fedichev, Fischer, Finazzi, Garay, Guéry-Odelin, Jacobson, Jain, Liberati, Macher, Michel, Parentani, Pavloff, Prain, Robertson, Sols, Unruh, Visser, Volovik, Weinfurtner, Zapata, Zoller,...]

Gross-Pitaevskiĭ equation for condensate wave-function

$$i\dot{\psi} = \left(-\frac{\nabla^2}{2m} + V_{\rm ext} + g|\psi|^2\right)\psi$$

Madelung split $\psi = \sqrt{\varrho}e^{iS}$ → Hamilton-Jacobi (Bernoulli) equation

$$\dot{S} + V_{\rm ext} + g\varrho + \frac{(\nabla S)^2}{2m} = \frac{1}{2m}\frac{\nabla^2\sqrt{\varrho}}{\sqrt{\varrho}}\,, \quad \dot{\varrho} + \nabla\cdot(\varrho\vec{v}) = 0$$

Long wavelengths → neglect "quantum pressure term"

- low temperatures (✓), super-fluid → "no" viscosity/vorticity (✓)
- well understood (✓), controllable (✓), super-"luminal" dispersion
- small size (?), three-body losses (?), measure single phonons (?)

in situ [RS, PRL 2006] correlations [Balbinot, Fabbri, Fagnocchi, Recati, Carusotto]

time-of-flight [Westbrook]

Quantum Back-Reaction

Naive "calculation" of cosmological constant

$$\langle \hat{T}_{\mu\nu} \rangle \sim g_{\mu\nu} \int d^3k \, \frac{|\vec{k}|}{2} \sim g_{\mu\nu} \, k_{\rm cut}^4 \sim g_{\mu\nu} \, \ell_{\rm Planck}^{-4}$$

Analogously for zero-point pressure from quantum Bernoulli equation

$$\dot{S} + V_{\rm ext} + g\varrho + \frac{(\nabla S)^2}{2m} = 0 \leadsto p_{\rm zero} = -\frac{\langle (\nabla \hat{S})^2 \rangle}{2m} \sim k_{\rm cut}^4$$

But: additional contribution from "quantum pressure term"

$$\dot{S} + V_{\rm ext} + g\varrho + \frac{(\nabla S)^2}{2m} = \frac{1}{2m} \frac{\nabla^2 \sqrt{\varrho}}{\sqrt{\varrho}} \leadsto p_{\rm zero} = -\frac{\langle (\nabla \hat{S})^2 \rangle}{2m} + \frac{\langle (\nabla \delta \hat{\varrho})^2 \rangle}{8m\varrho^2}$$

Naively $p_{\rm zero} \sim k_{\rm cut}^6$, with dispersion relation $p_{\rm zero} \sim k_{\rm cut}^3$ (cancellation!)

$$i \frac{\partial}{\partial t} \hat{\Psi} = \left(-\frac{\nabla^2}{2m} + V_{\rm ext} + g \, \hat{\Psi}^\dagger \hat{\Psi} \right) \hat{\Psi}$$

Full theory: $p_{\rm zero}$ from $\langle \hat{\chi}^\dagger \hat{\chi} \rangle \sim k_{\rm cut}^0$ and $\langle \hat{\chi}^2 \rangle \sim k_{\rm cut}^1 \neq 1/\xi_{\rm healing}$

[RS, PoSQG-Ph 2007] [Balbinot et al, PRD 2005] [Volovik]

Gravity Simulators, Part IV

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Electromagnetic Analogues

[RS+Plunien+Soff, PRL 2002;. . .]

Electromagnetism in media with constant dielectric permittivity ε

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{\varepsilon - 1}{2} F_{\mu\nu} u^{\nu} F^{\mu\lambda} u_{\lambda} = -\frac{1}{4} F_{\mu\nu}\, g_{\text{eff}}^{\mu\rho}\, g_{\text{eff}}^{\nu\sigma}\, F_{\rho\sigma}$$

Gordon metric → horizon, ergo-region ($g_{00}^{\text{eff}} = 0 \leftrightarrow \beta^2 = 1/\varepsilon$)

$$g_{\text{eff}}^{\mu\nu} = g_{\text{Minkowski}}^{\mu\nu} + (\varepsilon - 1) u^{\mu} u^{\nu} \rightsquigarrow g_{\mu\nu}^{\text{eff}} = g_{\mu\nu}^{\text{Minkowski}} - \frac{\varepsilon - 1}{\varepsilon} u^{\mu} u^{\nu}$$

Problem: speed of light (in medium) is typically too large. . .

- slow light

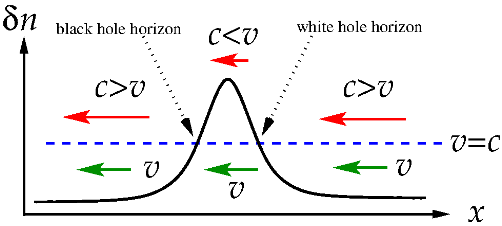
  [Leonhardt+Piwnicki, PRL 2000]

  [Comment by Visser, Reply]

  [Unruh+RS, PRD 2003]

- moving pulse/front

  [RS+Unruh, PRL 2005;. . .]



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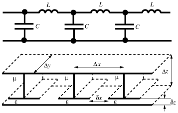
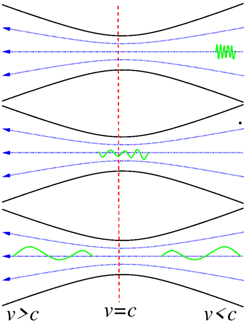
Optical and Electromagnetic Experiments

- Wave-guides & meta-materials [RS+Unruh, PRL 2005]



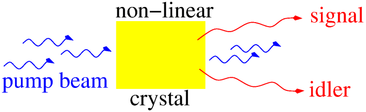


Dynamical Casimir Effect (quantum!)

[Wilson *et al*, Nature 2011; Lahteenmaki *et al*, PNAS 2013]

see also [Tian, Jing, Dragan, Nation, Blencowe, Rimberg, Buks, RS, Unruh,. . .]

- Non-linear (Kerr) media: fibres or bulk



Classical mode conversion (✓)

Quantum effects: Hawking radiation (?)

[Belgiorno, Brevik, De Lorenci, Faccio, Jacquet, Koenig, Leonhardt, Liberati, Novello, Philbin, Prain, Thompson, Unruh, Visser]

- Outlook: “photon fluids”

Gravity Simulators, Part V

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Water (Gravity) Waves

Surface waves on ideal flowing liquid with gravity [RS+Unruh, PRD 2002]

- incompressible flow (✓)
- irrotational flow without friction and viscosity (?)
- shallow water waves $\lambda \gg h$
- small (linear) waves $\delta h \ll h$

Generalised Painlevé-Gullstrand-Lemaître metric

$$g_{\rm eff}^{\mu\nu} = \frac{1}{h_0^2} \begin{pmatrix} 1 & v_0^i \\ v_0^j & v_0^i v_0^j - \mathfrak{g}_{\rm eff}^{\perp} g^{ij} \end{pmatrix}$$

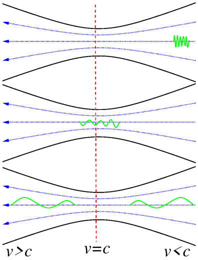
More flexibility due to possibly curved bottom $\to g^{ij}$

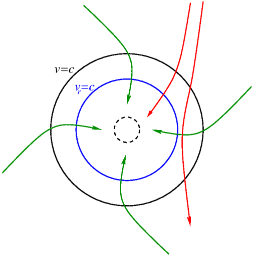
- dispersion relation:
  deep water → sub-“luminal” or surface tension → super-“luminal”
- easy to manipulate and measure (✓)
- quantum effects are out of reach
  → super-fluid films (?) [Skyba]

Water Wave Experiments

[Rousseaux *et al*, Weinfurtner *et al*]

- classical mode conversion (stimulated)
  $\rightarrow$ "thermal" Bogoliubov coefficients
- rotating black holes: super-radiance
- quasi-normal modes





Gravity Simulators, Part VI

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9th January 2020

Ion Trap Analogues

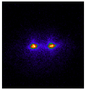
[Alsing, Dowling, Milburn, Cirac, Fey, Schätz, R.S.,...]

Friedmann-Lemaître-Robertson-Walker metric $ds^2 = a^2(t)[dt^2 - dx^2]$

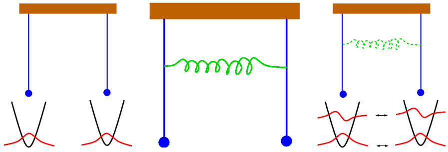
$$(\Box + m^2)\Phi = 0 \quad \rightsquigarrow \quad \ddot{\phi}_k + \left[k^2 + a^2(t)m^2\right]\phi_k = 0$$

$$\delta\ddot{q}_{1,2} + \omega^2\delta q_{1,2} = \gamma\delta q_{2,1} \quad \rightsquigarrow \quad \delta\ddot{q}_- + \left[\gamma + \omega^2(t)\right]\delta q_- = 0$$

scalar field $\Phi(t, x)$	ion motion $\delta q_{1,2}(t)$
Fourier mode ϕ_k	rocking mode δq_-
cosmic expansion $a^2(t)$	trap potential $\omega^2(t)$
internal dynamics k^2	Coulomb interaction γ
spatial entanglement	entangled ions



Increase distance $\gamma\downarrow$ or tighten potential $\omega\uparrow$ $\to$ tear apart fluctuations



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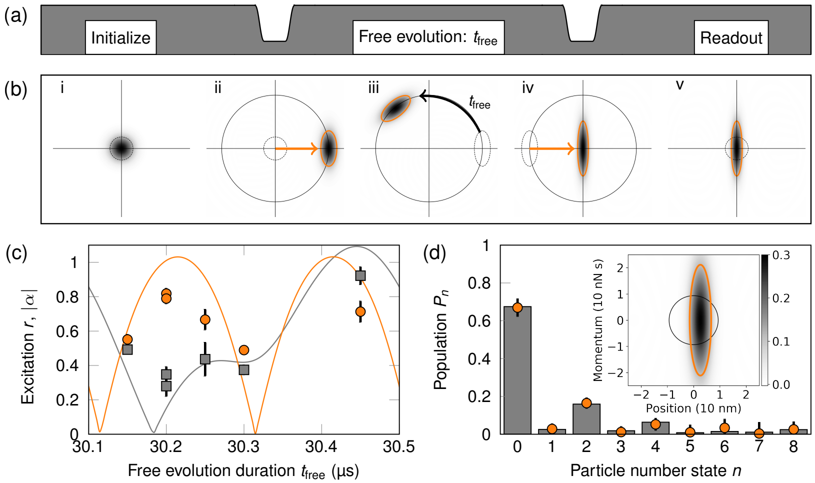
Experiment

[Wittemer, Hakelberg, Kiefer, Schröder, Fey, R.S., Warring, Schaetz, PRL 2019]



→ squeezing $\xi \approx 0.8$, coherent $\alpha \approx 0.3$, thermal $n_{\text{th}} \approx 0.03$

→ entanglement $E_F \approx 0.4$

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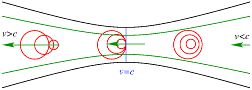
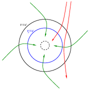
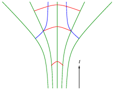
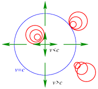
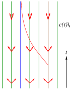
Summary and Outlook

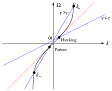
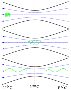
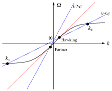
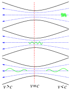
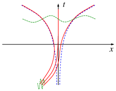
Effects: black (and white) holes and expanding Universe

Analogues: phonons, condensates, photons, water waves, ion traps



Lessons: trans-Planckian origin, robustness, back-reaction

Outlook: fermionic fields

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Experiments in Bose-Einstein Condensates

- black-hole analogue
  Hawking radiation from density correlations
  quantum nature – entanglement (!?)

  [Steinhauer] ↔ [Leonhardt] [Jacobson] [Parentani] . . .

- expanding Bose-Einstein condensates

  [Eckel, Kumar, Jacobson, Spielman, Campbell, PRX 2018]

  creation of phonons with opposite momenta

  [Jaskula, Partridge, Bonneau, Lopes, Ruaudel, Boiron, Westbrook, PRL 2012]

- changing speed of sound

  [Donley, Claussen, Cornish, Roberts, Cornell, Wieman, Nature 2001]

  signature change → "Bose Nova"

  [Calzetta, Hu, Weinfurtner, White, Visser,. . .]