Technische Universität München

A path toward establishing or ruling out NP in C_9

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based on prelim. work with N. Gubernari and J. Virto
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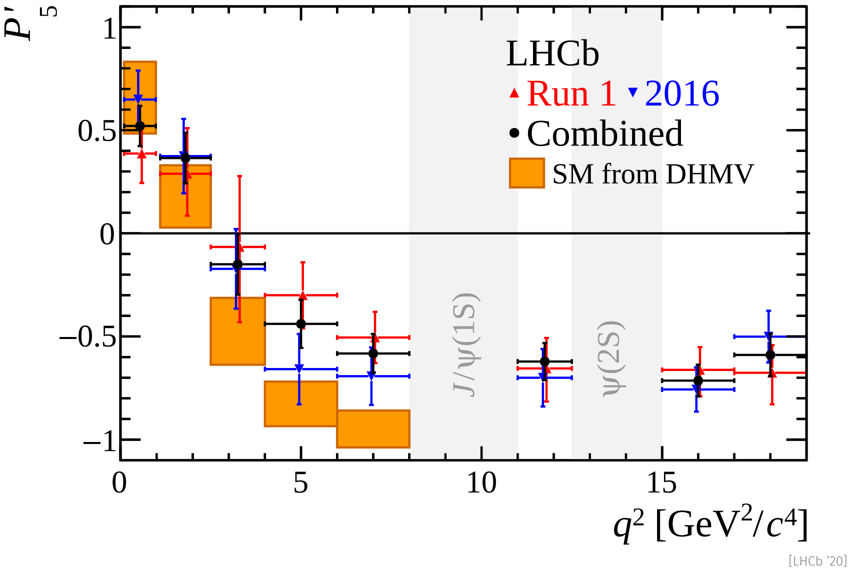
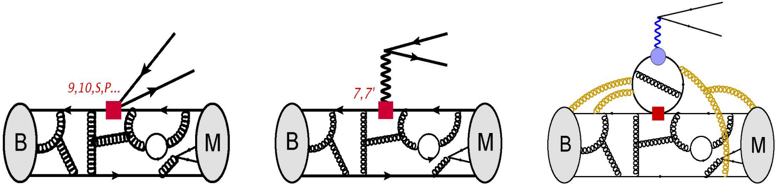
Status Quo


P'_5
1
0.5
0
−0.5
−1
0
5
10
15
q^2 [GeV^2/c^4]
LHCb
Run 1
2016
Combined
SM from DHMV
$J/\psi(1S)$
$\psi(2S)$
[LHCb '20]

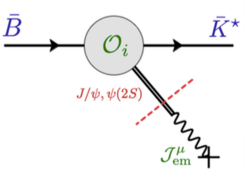
Exclusive $b \to s\ell^+\ell^-$ Amplitudes



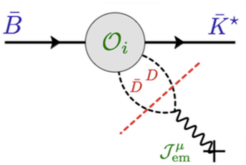
$$\mathcal{A}_\lambda = \mathcal{N}_\lambda \left\{ (C_9 \mp C_{10})\mathcal{F}_\lambda(q^2) + \frac{2m_b M_B}{q^2}\left[C_7 \mathcal{F}_\lambda^T(q^2) - 16\pi^2 \frac{M_B}{m_b}\mathcal{H}_\lambda(q^2)\right]\right\}$$

- non-local : $\mathcal{H}_\lambda(q^2) = i\,\mathcal{P}_\mu^\lambda \int d^4x\, e^{iq\cdot x}\, \langle \bar{K}(k)|\, T\{\mathcal{J}^\mu_{\rm em}(x), \mathcal{C}_i\mathcal{O}_i(0)\}\, |\bar{B}(q+k)\rangle$
  - major source of systematic uncertainty
  - several approaches dependent on the phase space region (i.e.: q^2)
  - what are the properties of $\mathcal{H}_\lambda$ as a function of q^2?

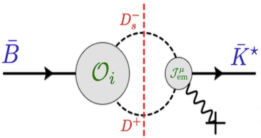
Analytic structure of the $\mathcal{H}$: dynamical singularities

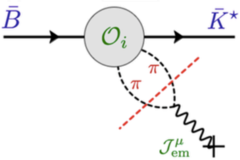


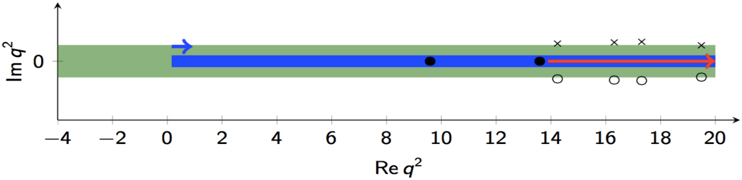
(a)



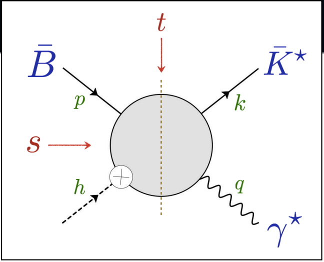
(b)







Understanding the p^2 cut



Trick: add spurious momentum h to $\mathcal{O}_i$
recover physical kinematics as $h \to 0$

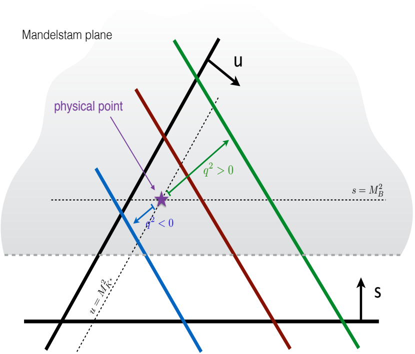
- consider Mandelstam variables

$$s \equiv (p+h)^2 \longrightarrow M_B^2$$

$$u \equiv (k-h)^2 \longrightarrow M_{K*}^2$$

$$t \equiv (q-h)^2 \xrightarrow[\text{physical point}]{} q^2$$

- s independent of t
  - cut in $s \sim p^2$ does not translate into cut in $t \sim q^2$



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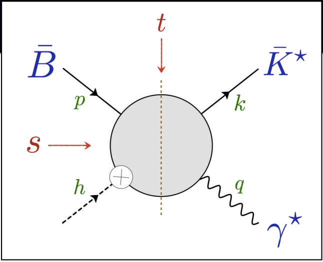

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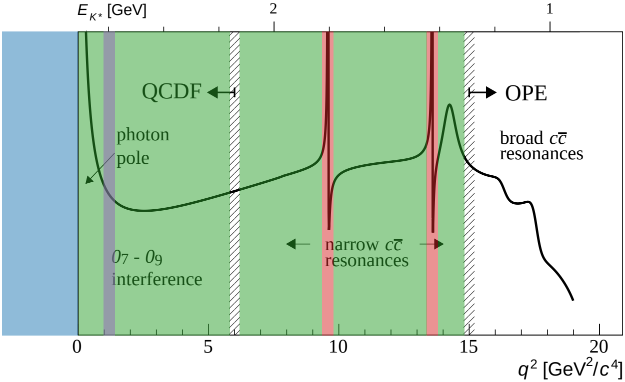
- s independent of t
  - cut in $s \sim p^2$ does not translate into cut in $t \sim q^2$

- p^2 cut does not induce singularities in q^2 as long as k^2 is fixed
- two correlators:

$$\mathcal{H}_\lambda(q^2) \to \mathcal{H}_\lambda^{\text{real}}(q^2) + i\,\mathcal{H}_\lambda^{\text{imag}}(q^2)$$

- the same dispersion relation governs their q^2-dependence

Light-hadron cut



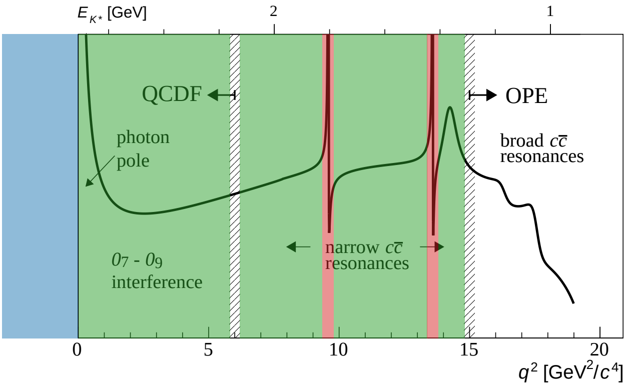
[sketch from Blake, Gershon, Hiller 1501.03309]

We do not consider the light-hadron cut here or in the following!

- peaks only locally around $\sim 1\,\mathrm{GeV}^2$
- perturbatively small ($\mathcal{O}(\alpha_s)$)
- empirically small ($\Gamma(J/\psi)$)

▶ will need to be considered once very precise data become available

Strategy



[sketch from Blake, Gershon, Hiller 1501.03309]

Parametrize the local matrix elements in the complex half plane with $\mathrm{Re}\, q^2 < 4M_D^2$ [Bobeth.Chrzaszcz,DvD,Virto '17]

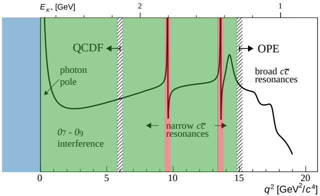
- fit to experimental data on narrow charmonium resonances
- fit to theory predictions in Light-Cone OPE region ($\mathcal{O}(\alpha_s)$)
- use in semileptonic region

Basics of the parametrization

$$q^2 < 0$$

$$\cap \quad 0 \leq q^2 \leq 4M_D^2$$

$$\cap \quad q^2 \in \{M_{J/\psi}^2, M_{\psi'}^2\}$$

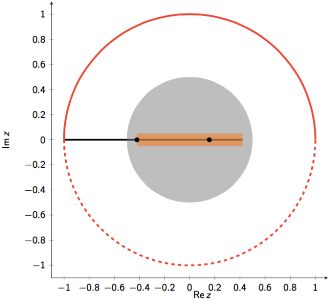


ansatz in z valid below the $D\overline{D}$ threshold

[Bobeth, Chrzaszcz, van Dyk, Virto 2017]

motivated by "z-parametrization" of form factors

[Boyd et al 1994, Bourelly et al 2008]



1. Extract the poles:
   $\hat{\mathcal{H}}_\lambda(q^2) = (q^2 - M_{J/\psi}^2)(q^2 - M_{\psi(2S)}^2)\,\mathcal{H}_\lambda(q^2)$
2. $\hat{\mathcal{H}}_\lambda(q^2)$ still has $D\overline{D}$ cut.
3. perform conformal mapping $q^2 \mapsto z(q^2)$.
4. branch cut in q^2 is mapped onto the unit circle in z.
5. $\hat{\mathcal{H}}_\lambda(z)$ analytic within unit circle.
6. power expand $\hat{\mathcal{H}}_\lambda(z)$ around $z = 0$.

Three open questions

Light-Cone OPE *What is the size of next-to-leading power terms in the light-cone OPE?*

First LCSR calculation of the hadronic matrix elements was incomplete. [Khodjamirian et al. '10]
Missing terms can have numerically significant impact. [Kokulu,Gubernari,DvD '18]

Singularities *Are there further singularities?*

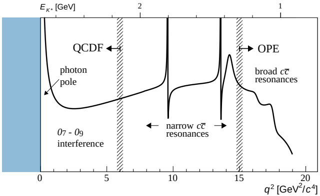
In a previous analysis we only discussed singularities of dynamical origin. [Bobeth,Chrzaszcz,DvD,Virto '17]

Bounds *Is there an upper bound on the size of the z-expansion coefficients?*

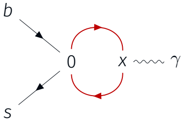
For form factors we can derive an upper bound through an integral representation of vacuum matrix elements. [e.g. Boyd,Grinstein,Lebed '95]

Light-Cone OPE and Matrix Elements

$$4m_c^2 - q^2 \gg \Lambda_{\text{hadr.}}^2$$

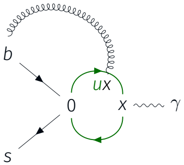


- expansion in operators at light-like distances $x^2 \simeq 0$ [Khodjamirian, Mannel, Pivovarov, Wang 2010]
- employing light-cone expansion of charm propagator [Balitsky, Braun 1989]



$$\xrightarrow{q^2 \ll 4m_c^2} \underbrace{\left(\frac{C_1}{3} + C_2\right) g(m_c^2, q^2)}_{\text{coeff \#1}} [\bar{s}\,\Gamma^\mu\, b] + \mathcal{O}(\alpha_s)$$

$$+\, \mathcal{I}^{\mu\alpha\beta\gamma}(q^2)\, [\bar{s}_L \gamma_\alpha \tilde{G}_{\beta\gamma} b_L] + \mathcal{O}(\alpha_s)$$

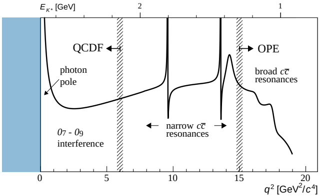


- leading coefficients now known analytically to $\mathcal{O}(\alpha_s)$ [de Boer '17; Asatrian,Greub,Virto '19]

$0 \leq u \leq 1$

Calculation: Light-Cone OPE

$$4m_c^2 - q^2 \gg \Lambda^2_{\text{hadr.}}$$



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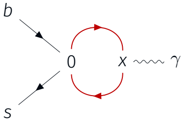

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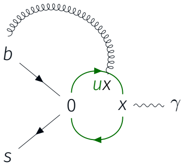
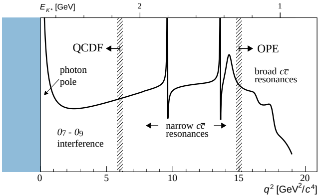


$0 \le u \le 1$

- subleading coefficient $\mathcal{I}$ known [Khodjamirian,Mannel,Pivovarov,Wang '10]
- we *confirm* result for $\mathcal{I}$: LC expansion apparently breaks Ward identity [Gubernari,DvD,Virto w.i.p]

Calculation: Matrix Elements

$$4m_c^2 - q^2 \gg \Lambda^2_{\text{hadr.}}$$



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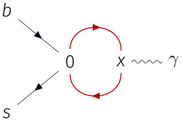
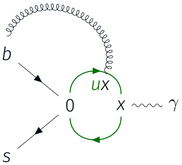




$$0 \leq u \leq 1$$

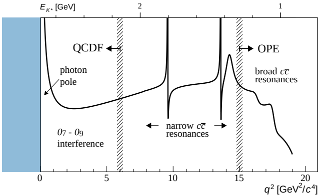
matrix elements schematically:

$$\mathcal{H}_\lambda = g(q^2) \times \mathcal{F}_\lambda + \mathcal{H}_\lambda^{\text{spect.}}$$
$$+ \mathcal{I} \times \mathcal{F}_\lambda^{\text{soft}}$$

- ▶ leading part identical to QCD Fact. results [Beneke, Feldmann, Seidel '01&'04]
- ▶ subleading matrix element $\mathcal{F}_\lambda^{\text{soft}}$ can be inferred from B-LCSRs [Khodjamirian, Mannel, Pivovarov, Wang 2010]

Calculation: Matrix Elements

$$4m_c^2 - q^2 \gg \Lambda_{\text{hadr.}}^2$$



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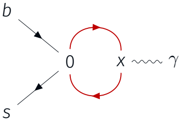
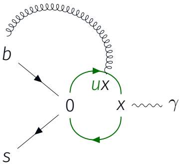




$0 \leq u \leq 1$

- Updating *B*-LCSR calculation of the matrix element of $\mathcal{F}_\lambda^{\text{soft}}$ [Gubernari,DvD,Virto w.i.p.]
- Ward identity restored in LCSR! [Gubernari,DvD,Virto w.i.p.]
- PRELIMINARY at $q^2 = 1\,\text{GeV}^2$ we find suppression by a factor of 10×20
  - 10 from updated numerical inputs
  - 20 from cancellations between “new” and “old” LCDAs; indep. of inputs

Singularities

Ambiguity in definition of matrix elements

- previous work only considered singularities of dynamical origin:
  - poles from bound states
  - branch cuts due to two-body thresholds
- another type of singularity is of kinematical origin.

will discuss these singularities at hand of a simpler example

Ambiguity in defining the $B \to K$ vector form factor

Consider two decompositions of the same matrix element

$$\langle K(k)|\, \bar{s}\gamma^\mu b\, |B(p)\rangle \equiv f_+(q^2)(p+k)^\mu + \dots$$
$$\equiv \left[\frac{\sqrt{\lambda(M_B^2, M_K^2, q^2)}}{4M_B^2}\right]^{-3} \tilde{f}_+(q^2)(p+k)^\mu + \dots$$

- left-hand side stays the same
- $\sqrt{\lambda(\dots)}$ term has singularity of kinematical origin if $\lambda < 0$
- f_+ and $\tilde{f}_+$ have different properties
  - which is the correct one?

Dispersive bound

Define an two-point correlation function $\Pi_{1^-}(q^2)$

$$\Pi_{1^-}(q^2) \equiv P_{\mu\nu} \int e^{iq\cdot x} \langle 0| \mathcal{T}\{\bar{s}\gamma^\mu b(x), \bar{b}\gamma^\nu s(0)\} ket0$$

- for suitable $q^2 \ll (m_b + m_s)^2$ one can compute a certain derivative of Π_{1^-}:

$$\chi_{1^-}(q^2)\Big|_{\text{OPE}} = \left[-\frac{d}{dq^2}\right]^2 q^2\, \Pi_{1^-}(q^2)\Big|_{\text{OPE}} \propto \int ds\, \frac{s\ \text{Im}\,\Pi_{1^-}(s)}{[s-q^2]^4}$$

- for any $q^2 < (M_B + M_K)^2$ one can express Π_{1^-} as a dispersive integral:

$$\chi_{1^-}(q^2)\Big|_{\text{OPE}} \propto \int ds\, \frac{s\, \sqrt{\lambda(s)}^3\, |f_+(s)|^2}{[s-q^2]^4}$$

Integral representation

$$\chi_{1^-}(q^2)\Big|_{\mathrm{OPE}} \propto \int ds\, \frac{s\,\sqrt{\lambda(s)}^{\,3}\,|f_+(s)|^2}{[s-q^2]^4}$$

- λ enters as combination of two-particle phase space and angular momentum factor
- further terms depend on the type of the current and the number of subtractions in the dispersive integral
- the integrand is supposed to be free of singularities
- absorb both types of terms into definition of $f_+ \to \mathcal{F}_+$, rendering the integrand analytic
- the power of $[s-q^2]$ and the power of $\sqrt{\lambda(s)}$ are fixed:

$[s-q^2]$	by the perturbative calculation
$\sqrt{\lambda(s)}$	by phase space and angular momentum conservation

Resolving the ambiguity

Neither of the previous suggestions was the correct one:

$$\langle K(k)|\,\bar{s}\gamma^\mu b\,|B(p)\rangle \equiv f_+(q^2)(p+k)^\mu + \ldots$$

$$\equiv \left[\frac{\sqrt{\lambda(M_B^2, M_K^2, q^2)}}{4M_B^2}\right]^{-3} \tilde{f}_+(q^2)(p+k)^\mu + \ldots$$

Instead, require

$$\langle K(k)|\,\bar{s}\gamma^\mu b\,|B(p)\rangle \equiv \left[\frac{\sqrt{\lambda(M_B^2, M_K^2, q^2)}}{4M_B^2}\right]^{-3/2} \mathcal{F}_+(q^2)$$

tasks for the non-local matrix elements:

- need to come up with a dispersive representation of a suitable correlation function!
- determine powers of poles and angular momentum branch cuts

Bounds

Correlation Function

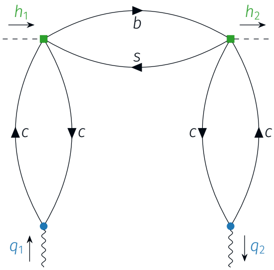
We investigate a four-point correlation function of the type

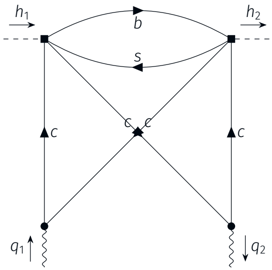
$$\Pi(q^2) \propto \int d^4x\, d^4y\, d^4z\, e^{-iq_1\cdot x}\, e^{+iq_2\cdot y}\, e^{-ih_1\cdot z}$$

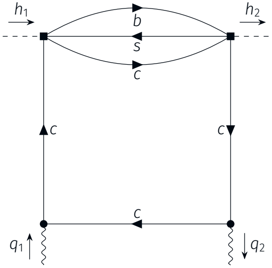
$$\langle 0|\, \mathcal{T}\left\{J^\mu(x), J^\nu(y), O(z), O^\dagger(0)\right\}|0\rangle\; g_{\mu\nu}$$

- incoming momenta q_1 of the e.m. current and h_1 artifically inserted into the four quark operator O
- outgoing momenta q_2 and h_2
- impose $h_1^2 = 0 = h_2^2$
- physical limit is $h_{1,2} \to 0$

Diagrammatically







$$s \equiv (q_1 + h_1)^2 \quad t \equiv (q_1 - q_2)^2 \quad u \equiv (q_1 - h_2)^2$$

- cuts in s are $b\bar{s}$ cuts $\Rightarrow$ dispersion relation in s
- cuts in t are flavour-less
- cuts in u are $b\bar{s}$, but not independent of s or t cuts

Preliminary results (1)

$$\chi(q^2) \propto \left[\frac{d}{dq^2}\right]^n \Pi(q^2)\Big|_{q^2\to -m_b^2}$$

- after renormalizing Π, $1/\varepsilon$ divergence is analytic in s
- OPE requires n = 3 subtractions to render χ finite

results obtained by applying large-q^2 OPE to the time-ordered product

- results dominated by two insertions of dim-3 operators imaginary part of Π informs the kinematical singularities

- $B \to K$ and $B \to K^*$ matrix elements enter with different powers of λ, as expected from total angular momentum
  - $B \to K$ needs additional angular momentum to create P wave $\to \mathrm{Im}\,\Pi(s) \propto \lambda(s)^{3/2}$
  - $B \to K^*$ needs no additional angular momentum to create P wave $\to \mathrm{Im}\,\Pi(s) \propto \lambda(s)^{1/2}$
- previous analysis did not remove kinematical singularities

Ende

Summary / Outlook

- previous parametrization is only the first step toward toward taming systematic uncertainties due to non-local effects
  - light-hadron cut explicitly *not yet* included
  - singularities due to charmonia and open-charm dynamics included
  - kinematical singularities work in progress
- reanalysis of theory predictions at negative q^2 ongoing
  - form factors updated already
  - large cancellations in $\mathcal{F}_\lambda^{\text{soft}}$ due to terms missing in original work
  - only few terms remains to be cross checked
- calculating dispersive bounds is a necessary step to understand the parametrization
  - allows to remove the kinematical singularities
  - gives insight into rate of convergence of the parametrization
  - provides parametric handle on systematic uncertainties