Lattice QCD and Flavour Anomalies

Chris Bouchard
University of Glasgow

Flavour Anomalies, IPPP
09 April 2020

How can lattice QCD help reduce theory uncertainty?

Standard methods

- better precision for hadronic matrix elements
  - improved lattices
  - no EFT matching
  - isospin and QED corrections

- extended q^2 range

New methods

- multiple hadrons in initial and/or final state

- inclusive decays

speculative

Standard methods

Continued improvement of "gold plated" quantities:

At most one hadron in initial and final state.

All hadrons are stable in QCD.

Factorizable non-QCD contributions.

Gold plated quantities include:

$$\langle 0|J_\mu^5|B\rangle \propto f_B$$

$$\langle \bar{B}|Q_i|B\rangle \propto f_B^2 B_B^{(i)}$$

$$\langle \pi(p')|V_\mu|B(p)\rangle = f_+^{B\pi}(q^2)(\cdots)_\mu + f_0^{B\pi}(q^2)(\cdots)_\mu$$

[not $B \to K^*(\to K\pi)\ell\bar{\ell}$, $B \to K\ell\bar{\ell}$ near resonances, …]

More mature calculations with better understood systematics — we know what to do to reduce errors, the standard methods …

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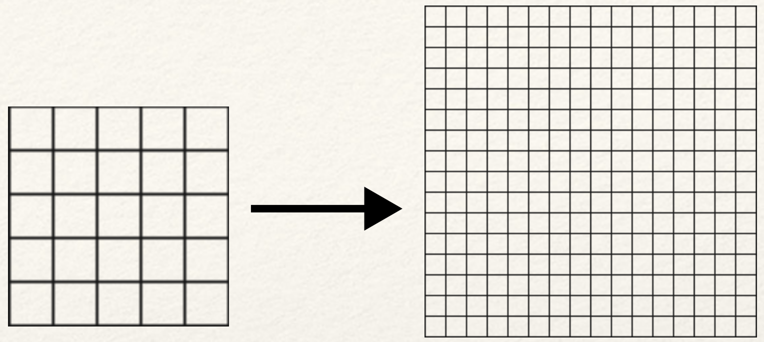
Standard methods: better precision for hadronic matrix elements

Improved lattices



- Larger volumes and finer lattices

  MILC, Phys. Rev. D87 (2013) 054505
  + continued additions from (MILC, CalLAT, …)

  - Improved HISQ action for sea quarks

    a: 0.03, 0.045, 0.06, 0.09, 0.12, 0.15 fm
    L up to ~ 6 fm

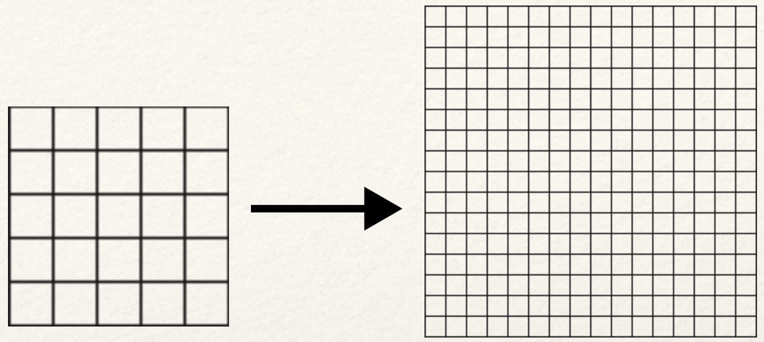
  - hitting a wall at 0.03 fm "critical slowing down"

$$S_{\text{Wilson}}(a) = S_{\text{QCD}} + \mathcal{O}(a^2)$$

$$\delta_{\text{FV}}(L) \sim e^{-M_\pi L}$$

Standard methods: better precision for hadronic matrix elements

Improved lattices



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    L up to ~ 6 fm

  - hitting a wall at 0.03 fm "critical slowing down"

not standard

- interesting possible way forward via "master field" simulations:

  Lüscher 1707.09758 Giusti, Lüscher 1812.02062 Francis, Fritzsch, Lüscher, Rago 1911.04533

  QCD mass gap means distant regions weakly correlated.

  Instead of many lattices, one very large lattice with many measurements from distant regions.

Standard methods: better precision for hadronic matrix elements

Improved lattices

- Vacuum polarisation effects u/d, s, c:

$$n_f = 2 + 1 + 1$$

- Large statistics

$$\langle 0|X|0\rangle = \int dG\ X[G,\ldots]\,e^{-S[G,\ldots]} = \frac{1}{N}\sum_{n=1}^{N} X[G_n,\ldots] + \mathcal{O}\left(\frac{1}{\sqrt{N}}\right)$$

Typically *N* ~ O(10,000)

allows for better characterisation of systematic effects

Standard methods: better precision for hadronic matrix elements

Improved lattices

- Physical (and/or a range of) light quark masses

  Eliminate (or anchor) chiral extrapolation

  Comes with costs:

  - expensive
  - harder to isolate ground state
  - enhanced finite volume effects

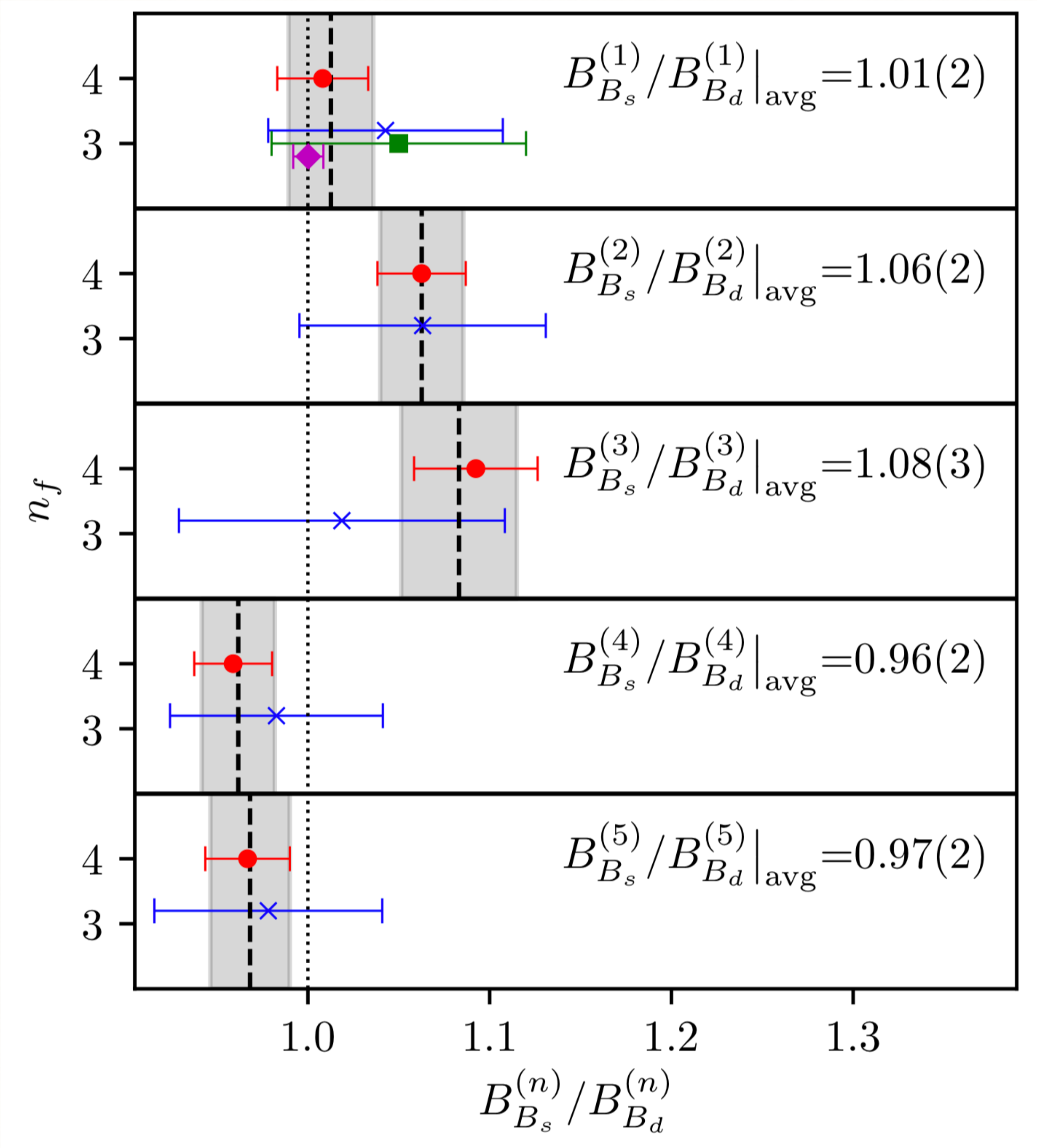
$n_f = 2+1+1$ HISQ lattices

$\approx a$ (fm)	$(L/a)^3 \times (T/a)$	L (fm)	M_π (MeV)	$M_\pi L$	$N_{\rm conf}$
0.15	$16^3 \times 48$	2.45	305	3.8	1020
0.15	$24^3 \times 48$	3.67	214	4.0	1000
▶ 0.15	$32^3 \times 48$	4.89	131	3.3	1000
0.12	$24^3 \times 64$	2.93	305	4.5	1040
0.12	$24^3 \times 64$	2.93	304	4.5	1020
0.12	$24^3 \times 64$	2.93	218	3.2	1020
0.12	$32^3 \times 64$	3.91	217	4.3	1000
0.12	$40^3 \times 64$	4.89	216	5.4	1028
0.12	$24^3 \times 64$	2.93	337	5.0	1020
0.12	$32^3 \times 64$	3.91	215	4.3	1020
0.12	$32^3 \times 64$	3.91	214	4.2	1020
0.12	$32^3 \times 64$	3.91	214	4.2	1020
0.12	$32^3 \times 64$	3.91	213	4.2	1020
0.12	$32^3 \times 64$	3.91	282	5.6	1020
▶ 0.12	$48^3 \times 64$	5.87	132	3.9	999
0.09	$32^3 \times 96$	2.81	316	4.5	1005
0.09	$48^3 \times 96$	4.22	221	4.7	999
▶ 0.09	$64^3 \times 96$	5.62	129	3.7	484
0.06	$48^3 \times 144$	2.72	329	4.5	1016
0.06	$64^3 \times 144$	3.62	234	4.3	572
▶ 0.06	$96^3 \times 192$	5.44	135	3.7	842
0.042	$64^3 \times 192$	2.73	315	4.3	1167
▶ 0.042	$144^3 \times 288$	6.13	134	4.2	420
0.03	$96^3 \times 288$	3.09	309	4.8	724

MILC, Phys. Rev. D98 (2018) 074512

Standard methods: better precision for hadronic matrix elements

Improved lattices



Dowdall et al (HPQCD), PRD100 (2019) 094508

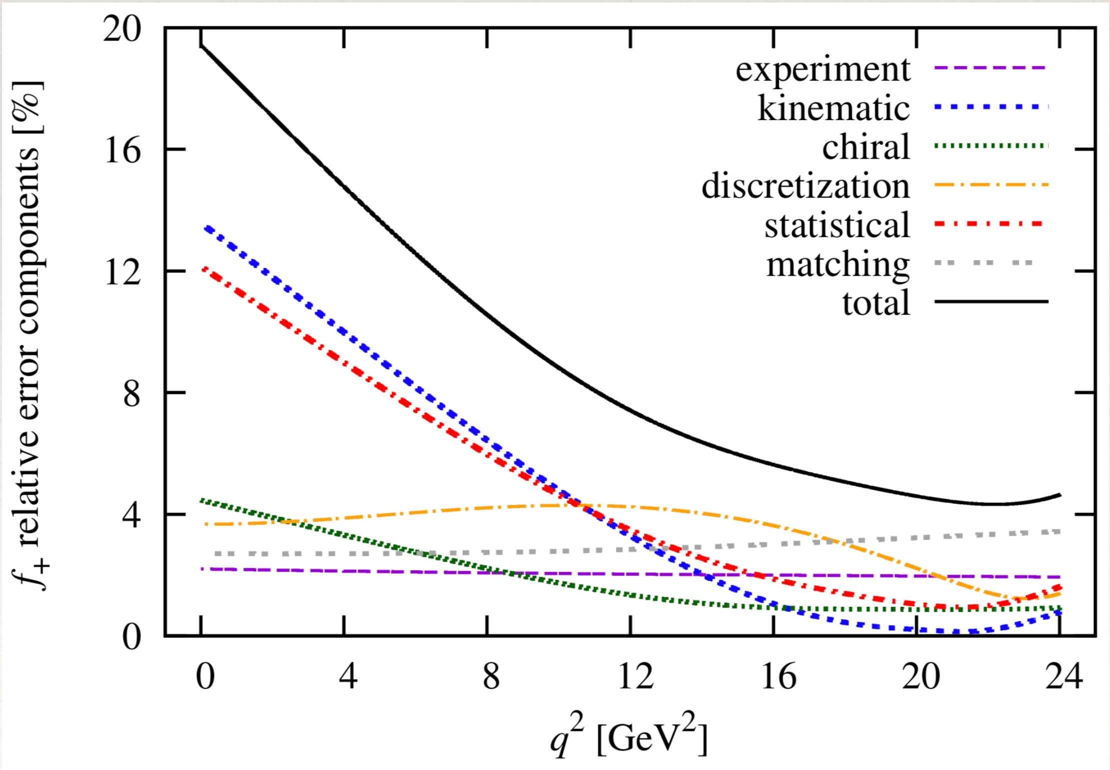
- $B_{(s)} - \bar{B}_{(s)}$ with NRQCD b quark
- ● used NRQCD b quark and $n_f = 2+1+1$ HISQ lattices
- ✕ used FNAL (HQET) b quark and $n_f = 2+1$ asqtad lattices
- NRQCD ≠ FNAL (HQET), so this is a bit naive, but an obvious difference is the use of improved lattices in ●

Standard methods: better precision for hadronic matrix elements

<u>No EFT matching</u>

- am_c and am_b discretization effects too large for pre ~2007 fermion actions

- EFT approaches were developed
  - NRQCD (HPQCD) Lepage et al, PRD46 (1992) 4052
  - HQET (FNAL, RBC-UKQCD) El-Khadra et al, PRD55 (1997) 3933; Christ et al, PRD76 (2007) 074505



CMB et al, PRD90 (2014) 054506

- $B_s \to K\ell\nu$ with NRQCD b quark
- lattice simulation: $17\ \mathrm{GeV}^2 \lesssim q^2 \lesssim 24\ \mathrm{GeV}^2$
- matching is dominant error in simulation region

Standard methods: better precision for hadronic matrix elements

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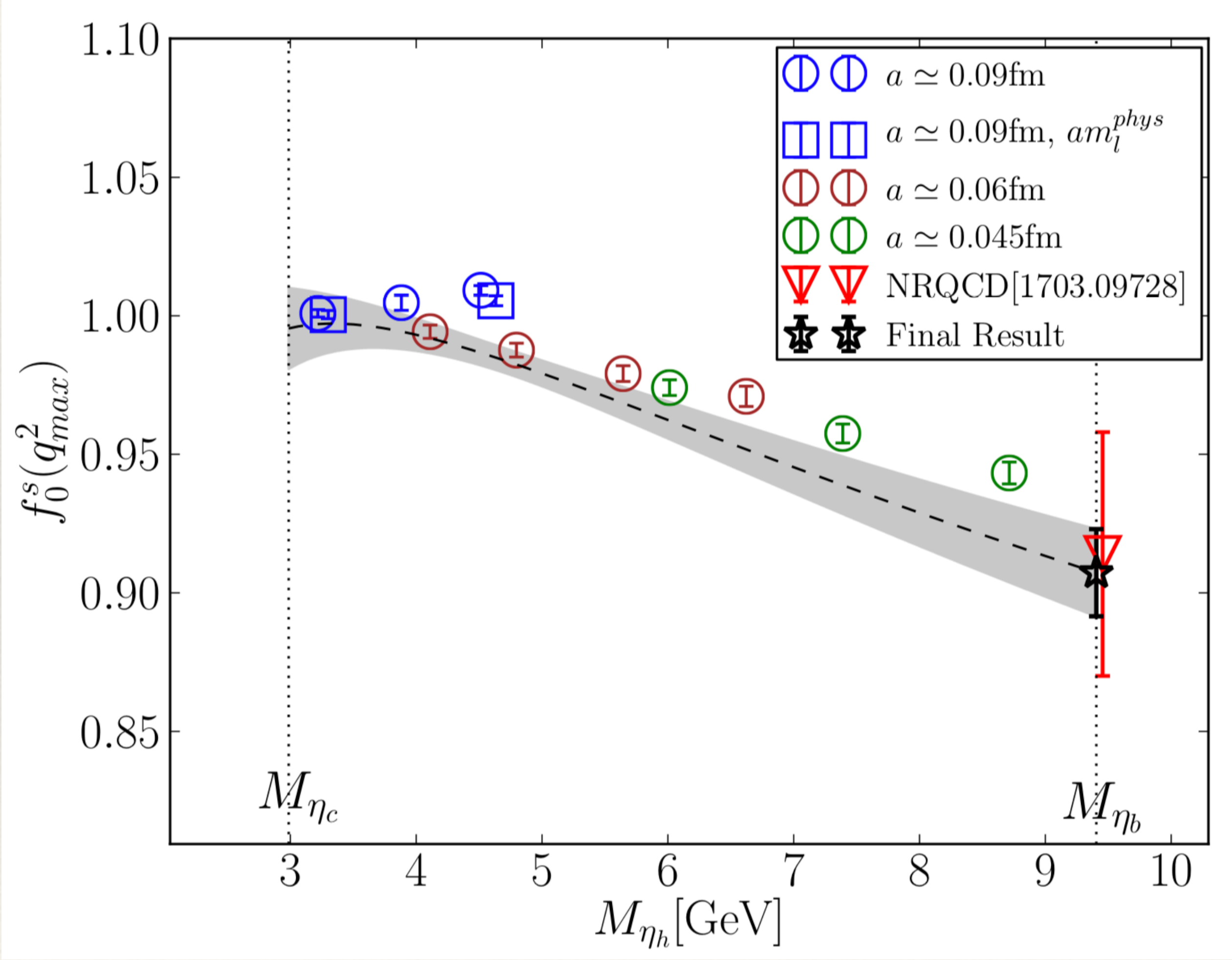
- HISQ (highly improved staggered quark) action Follana et al, PRD75 (2007) 054502
  - reduces am effects sufficiently to allow use to $am_b \sim 0.8$
  - fully relativistic so no EFT used and no matching
  - mild extrapolation in m_h based on HQET and guided by range $m_c \lesssim m_h \lesssim m_b$

Standard methods: better precision for hadronic matrix elements

No EFT matching

McLean et al (HPQCD), 1906.00701



- $B_s \to D_s \ell \nu$ with HISQ b quark
- Mild HQET extrapolation to m_b
- Consistent with NRQCD b quark result, but with improved precision

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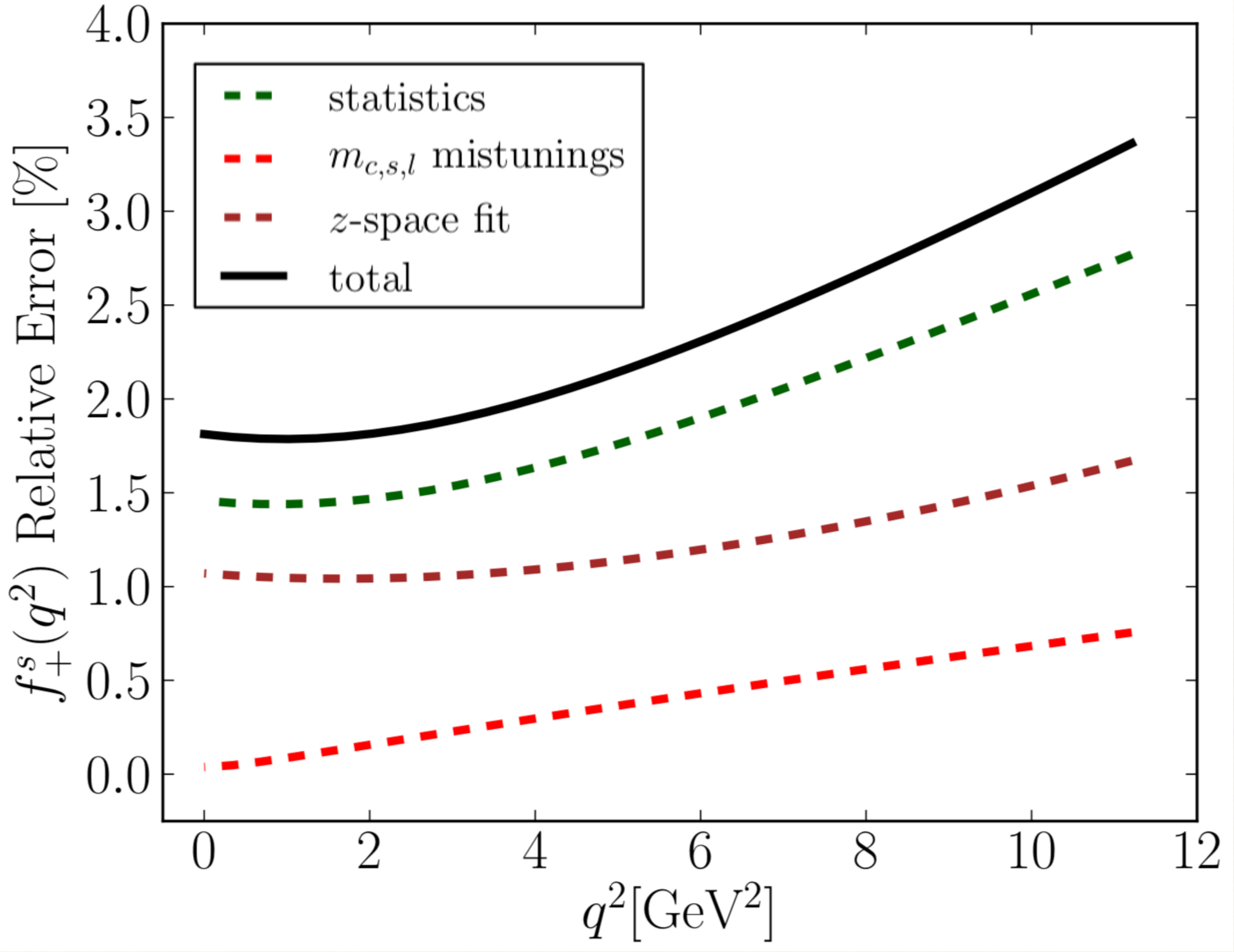


- $B_s \to D_s \ell \nu$ with HISQ b quark
- Mild HQET extrapolation to m_b
- Consistent with NRQCD b quark result, but with improved precision
- No EFT matching error for b quark - dominant error is statistics

Standard methods: better precision for hadronic matrix elements

No EFT matching

- HPQCD has calculated several $B_{(c,s)}$ semileptonic decays with HISQ b quarks:

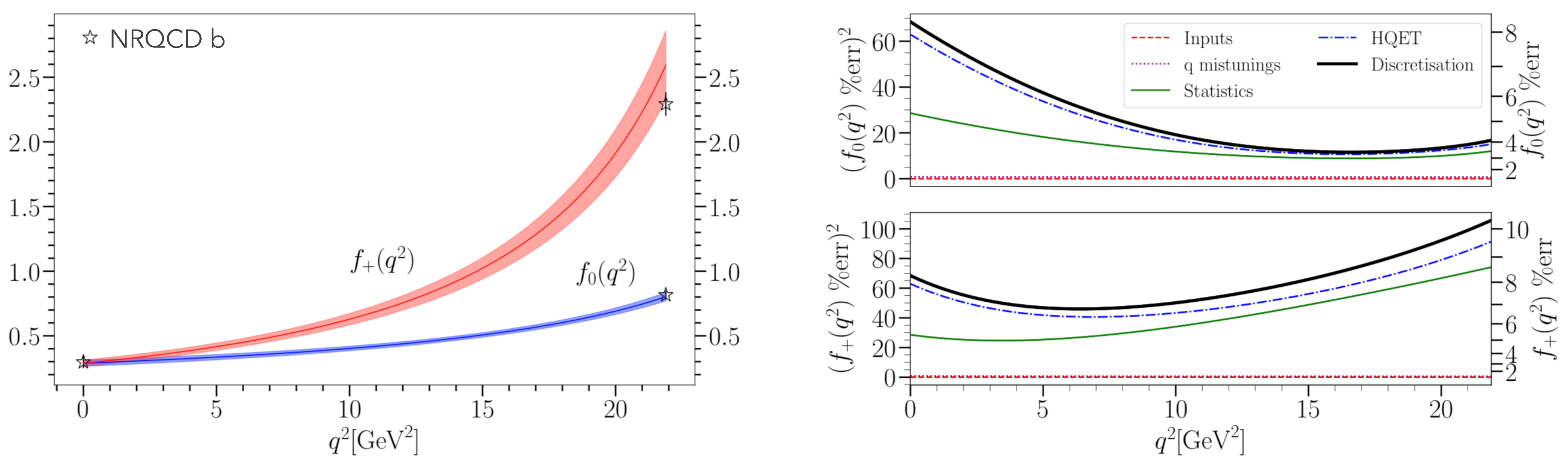
$B_c \to J/\Psi \ell\nu$ and $B_c \to \eta_c \ell\nu$ Colquhoun et al, 1611.01987

$B_s \to D_s^* \ell\nu$ (zero recoil) McLean et al, PRD99 (2019) 114512

$B_s \to D_s \ell\nu$ McLean et al, 1906.00701

$B_c \to B_{(s)} \ell\nu$ Cooper et al, 2003.00914

- Extending towards $b \to u/d$ via fictitious $B_s \to \eta_s \ell\nu$



Standard methods: better precision for hadronic matrix elements

Isospin and QED

- Current lattice simulations (typically) assume:
  - Strong isospin, $m_u = m_d$
  - QED effects are negligible

- Combined QED + isospin breaking effect estimated to be < 1%

- Using HISQ 2+1+1 lattices and HISQ b quarks, most precise B/D decay constant calculation confirms this Bazavov et al (Fermilab-MILC), PRD98 (2018) 074512

$$f_{D^0} = 211.6(0.3)_{\rm stat}(0.5)_{\rm syst}(0.2)_{f_\pi,{\rm PDG}}[0.2]_{\rm EM\ scheme}\ {\rm MeV}$$
$$f_{D^+} = 212.7(0.3)_{\rm stat}(0.4)_{\rm syst}(0.2)_{f_\pi,{\rm PDG}}[0.2]_{\rm EM\ scheme}\ {\rm MeV}$$

$$f_{B^+} = 189.4(0.8)_{\rm stat}(1.1)_{\rm syst}(0.3)_{f_\pi,{\rm PDG}}[0.1]_{\rm EM\ scheme}\ {\rm MeV}$$
$$f_{B^0} = 190.5(0.8)_{\rm stat}(1.0)_{\rm syst}(0.3)_{f_\pi,{\rm PDG}}[0.1]_{\rm EM\ scheme}\ {\rm MeV}$$

Standard methods: better precision for hadronic matrix elements

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- Ability to vary $m_{u/d}$ in analysis gives handle on strong isospin

- QED effects can be explicitly accounted for by adding U(1) gauge fields to QCD Duncan et al, PRL76 (1996) 3894; Blum et al, PRD76 (2007) 114508

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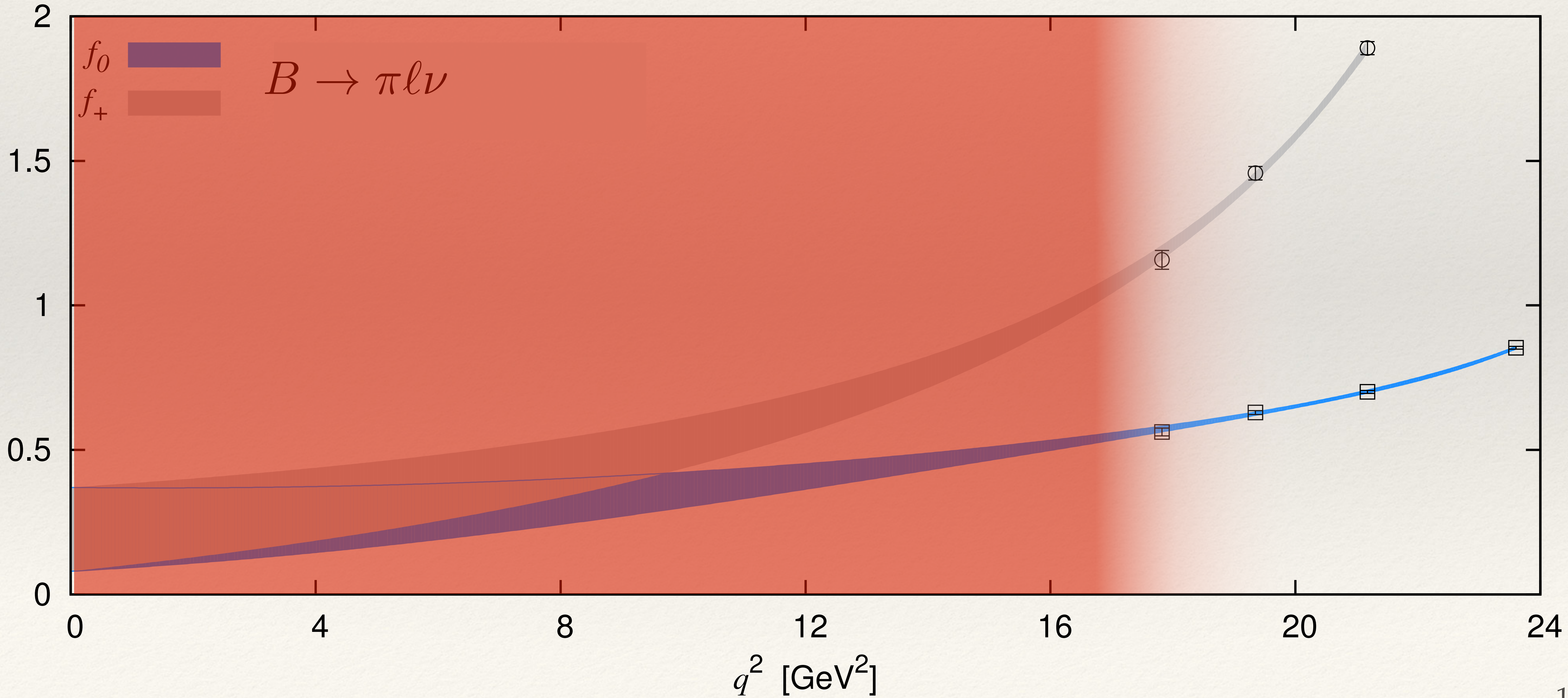
Standard methods: extended q^2 range

Consider $B \to \pi \ell \nu$

- Chiral Perturbation Theory valid only for $q^2 \gtrsim 17\ \mathrm{GeV}^2$
- kinematics not a problem (z-expansion)



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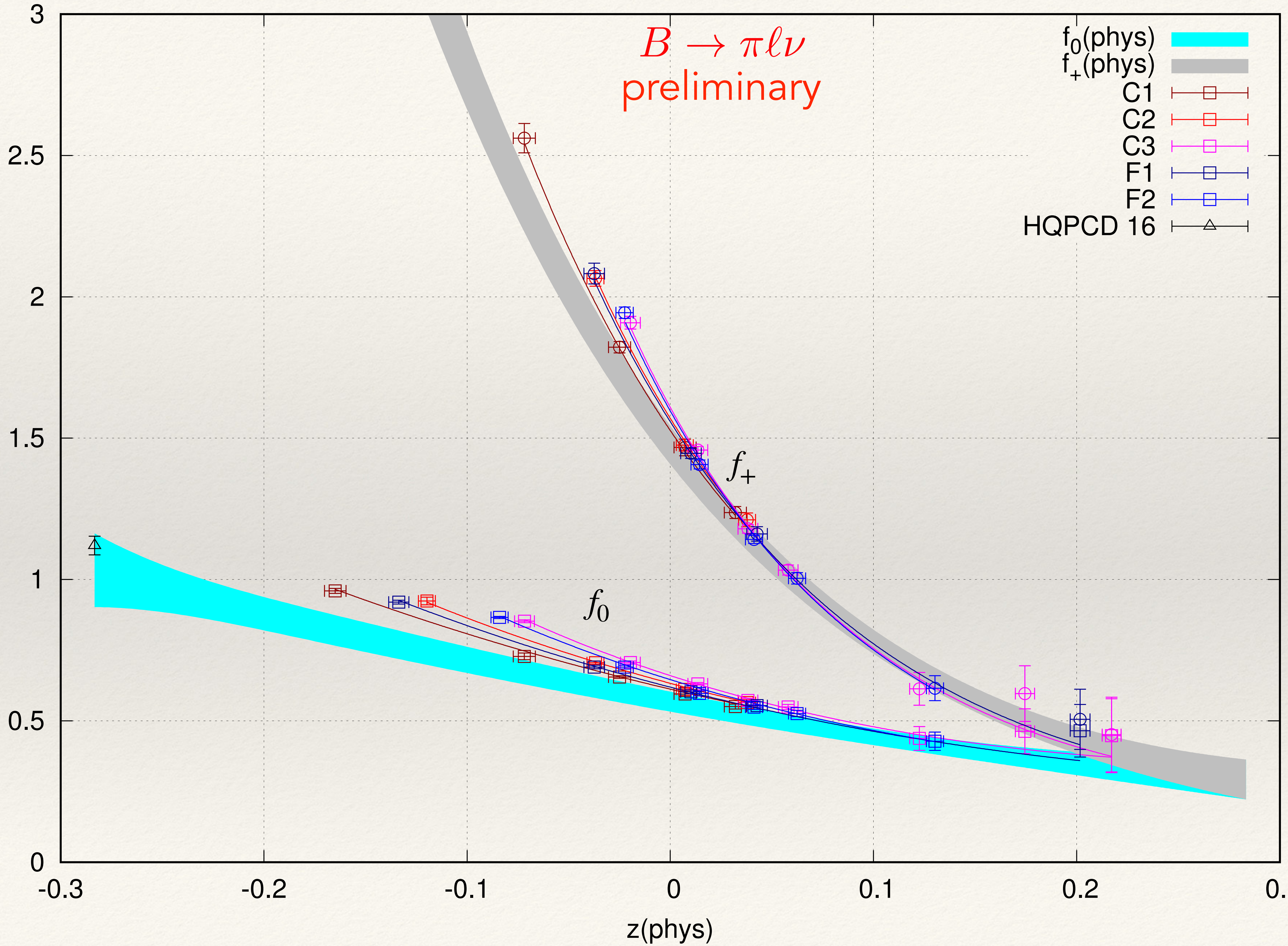
Solution:

- *Hard Pion* Chiral Perturbation Theory Bijnens and Jemos, NPB 846 (2011) 145
- chiral physics and kinematics factorize $f(z) = (1 + \mathrm{logs}])\mathcal{F}(z)$
- HPChPT with z-expansion $f(z) = (1 + \mathrm{logs}]) \sum_n c_n(a, m) z^n$

CMB et al, PRD90 (2014) 054506 (2014)

Standard methods: extended q^2 range



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New methods: multiple hadrons in initial and/or final state

$$B \to K^* \ell\bar{\ell} \to K\pi\ell\bar{\ell}\ , \quad B \to \rho\ell\nu \to \pi\pi\ell\nu \quad \ldots$$

- Involve 2 hadrons in the final state
- Multiple hadrons in finite volume involves all particle combinations consistent with kinematics and strong interaction selection rules
- Luscher and Lellouch developed relationship between FV multiparticle state energies and infinite volume scattering matrix

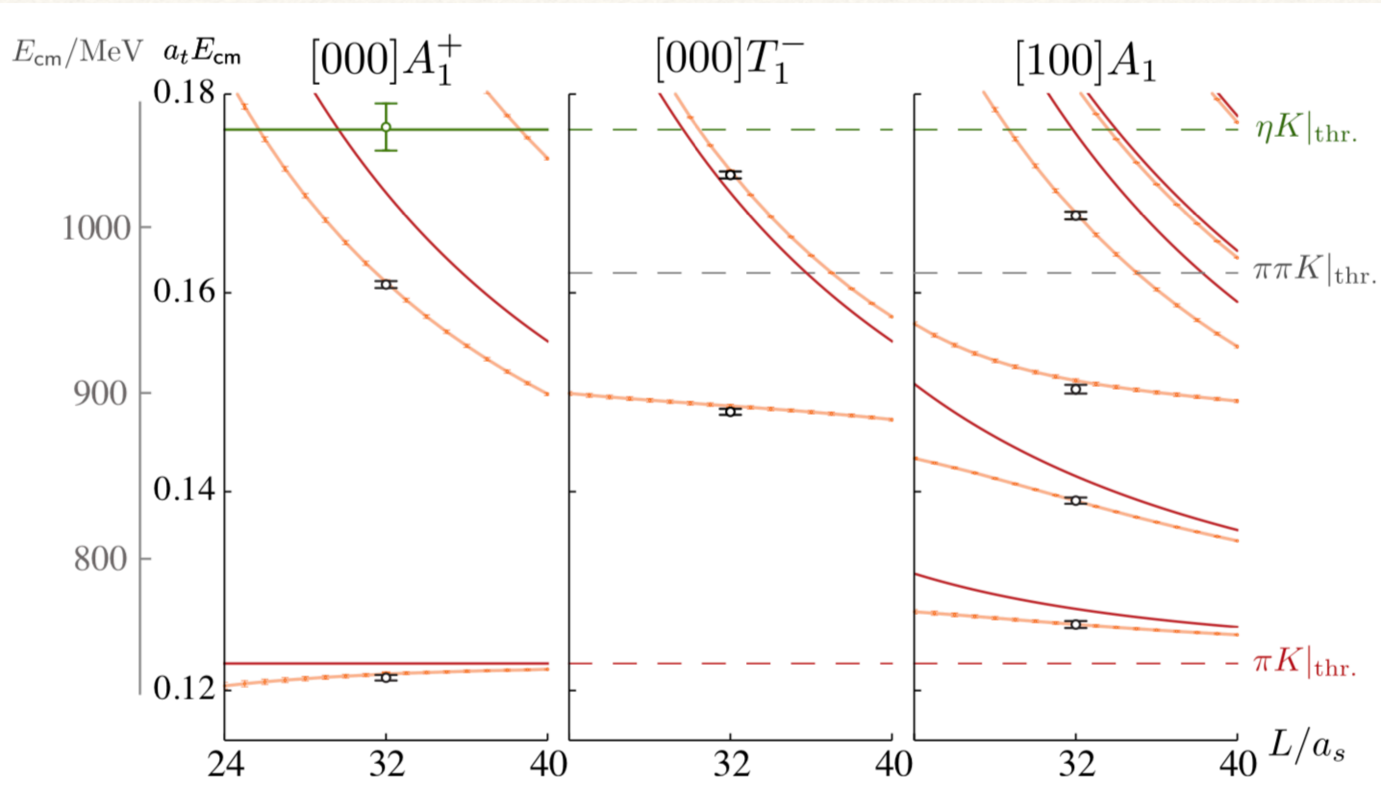
  Luscher, Comm. Math. Phys. 105 (1986) 153; NPB364 (1991) 237; Lellouch and Luscher, Comm. Math. Phys. 219 (2001) 31

$$\{E_n(L)\} \leftrightarrow S\ \mathrm{matrix}(E)$$

- Active area, extending to 2 and 3 particles in/and or out

  Briceno, Dudek, Young, RMP90 (2018) 025001

New methods: multiple hadrons in initial and/or final state



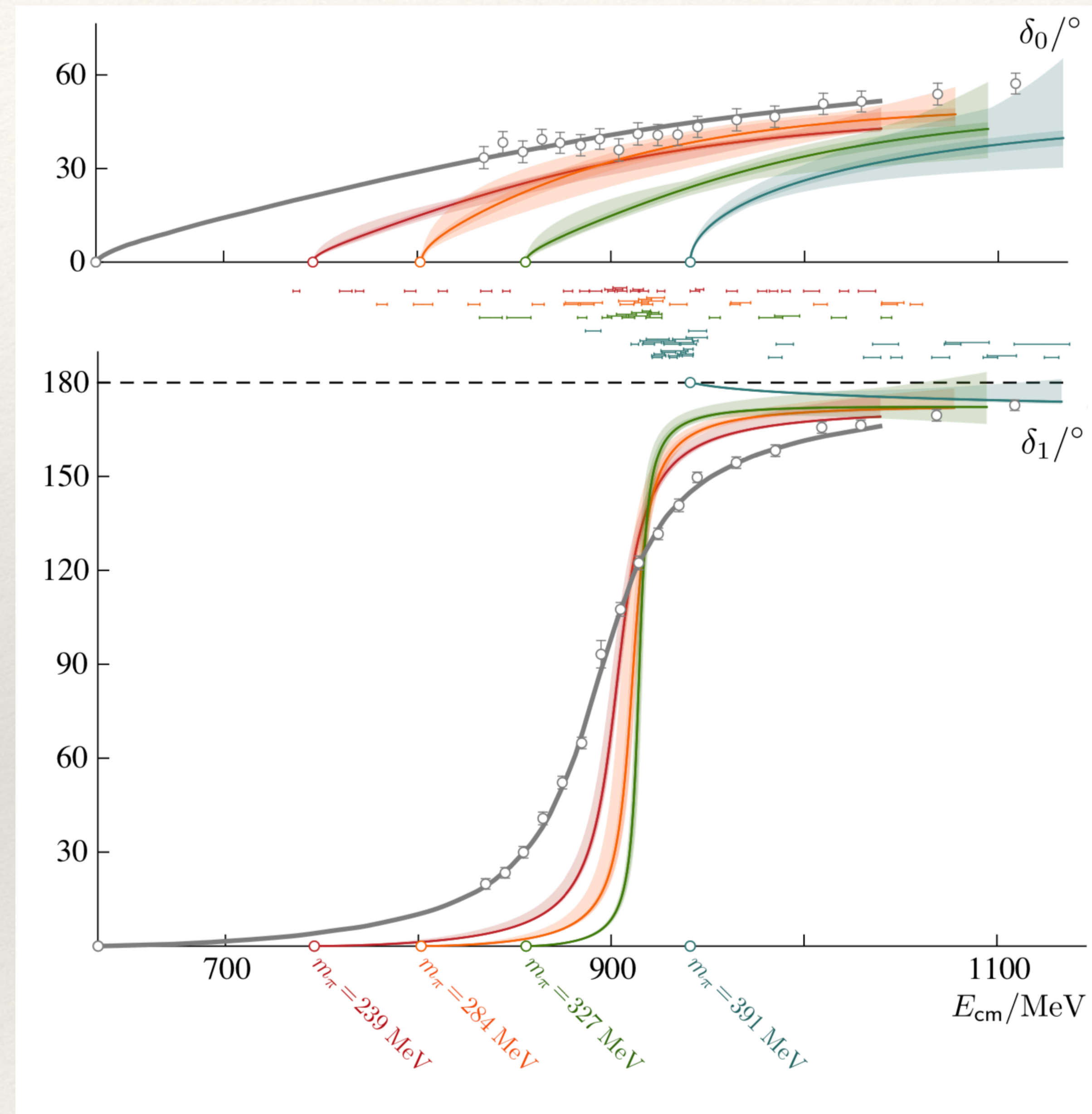


FIG. 2. *S*-wave (top) and *P*-wave (bottom) phase shifts. The

Wilson et al (HadSpec), PRL123 (2019) 042002

- πK elastic S- and P-wave scattering
- (Left) FV spectra for cubic irreps are measured in lattice QCD, $\{E_n(L)\}$
- (Right) These energies are converted into scattering phase shifts

New methods: multiple hadrons in initial and/or final state

- A lot of activity in both developing theory and application
- Still early days (extrapolation to continuum and physical masses not happening yet)
- Limitation: intermediate scattering states must be individually accounted for — appears to be prohibitive for heavy mesons.
  - DD scattering would involve many intermediate state πs
  - Idea to get around this by studying **inclusive** quantities on the lattice

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New methods: inclusive decays

- Start with 4pt correlation function on the lattice

$$\langle D|\mathcal{H}_W(\tau,\vec{x})\mathcal{H}_W(0)|D\rangle$$

- An inverse Laplace transform (imaginary time $\tau = it$) would allow the decay rate to be calculated

$$\Gamma \propto \sum_n |\langle n|\mathcal{H}_W|D\rangle|^2 \rho(E)$$

- At stage of feasibility studies

Hansen, Meyer, Robaina, PRD96 (2017) 094513

Liu, 1603.07352

Liang et at, 1710.11145; 1906.05312

Hashimoto, 1703.01881

Bailas et al (JLQCD), 2001.11678

- Convenient overlap with finite temperature and nucleon structure efforts

Summary

- The current generation of lattices (e.g. HISQ nf = 2 + 1 + 1) offer physical light quarks, large volumes, and lattices fine enough to allow fully relativistic b quarks.
- Based on work (largely HPQCD), these improvements should allow significant improvement in B semileptonic decay form factors over full range of q^2. A few percent seems reasonable for $B_{(s,c)}$.
- Promising early results for rigorous approach to scattering of light mesons, but phase space too large for B (and D?) decays
- Quite speculate possibility of calculating inclusive rates on the lattice

Discussion

- Bs and Bc are easier, faster, and more precise than B in lattice QCD: I second Alex's comment on an $\Upsilon(5S)$ run at Belle-II
- Is there useful overlap between long distance $b \to c\bar{c}s$ resonances and the lattice QCD calculation of long distance effects in $K \to \pi$ (Gino et al)?
- What are the interesting (probably D or K) decays to unstable states with limited phase space (say up to 3 πs)?