Form Factors and High-Mass Moments in $B \to K\pi\ell\ell$

Javier Virto

Universitat de Barcelona

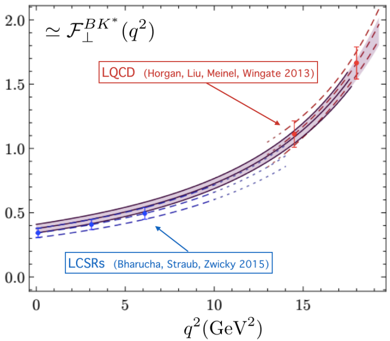
arXiv:1908.02267 [hep-ph] in collaboration with S. Descotes-Genon, A. Khodjamirian

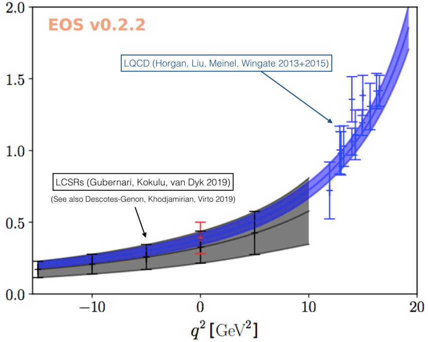
Beyond the Flavor Anomalies – Durham – April 3rd, 2020

Local $B \to V$ Form Factors

1. **Definition:** $\mathcal{F}_i(q^2) \sim \langle V(k)|\bar{q}\Gamma_i b|B(q+k)\rangle$
2. Necessary for:
   - Semileptonic decays: $B \to \rho\ell\nu$, $B_s \to K^*\ell\nu$, ...
   - Non-Leptonic decays: $B \to K^*\pi$, ...
   - "Rare" FCNC decays: $B \to K^*\bar{\nu}\nu$, $B \to K^*\ell^+\ell^-$

Local $B \to K^*$ Form Factors





- Two main approaches: (1) Lattice QCD (large q^2) (2) LCSRs (low q^2)
- Two approaches to LCSRs, in terms of (Left) K^* LCDAs (Right) B LCDAs
- q^2 dependence can be parametrized model-independently

Subject of this talk

However:

- $\rho, K^*, \ldots$ are not stable in QCD (e.g. $K^* \to K\pi$ strong decay)
- Form factor calculations done in the narrow-width limit

This talk:

$$B \to K^* X \quad - - - - \longrightarrow \quad B \to K\pi X$$

Naively, corrections from finite width are

$$\mathcal{W} \sim 1 + \textit{coeff.} \times \frac{\Gamma}{M} + \cdots$$

Target precision: $\sim 10\%$ $\Gamma/M \sim 20\%(\rho), 6\%(K^*), 0.5\%(\phi)$

But there are also "non-resonant" effects (higher resonances, S, D-waves, ...)

$B \to K\pi$ Form factors

Definition of Lorentz-Invariant Form Factors:

$$
\begin{aligned}
i\langle K^-(k_1)\pi^+(k_2)|\bar{s}\gamma^\mu b|\bar{B}^0(q+k)\rangle &= F_\perp\, k_\perp^\mu \\
-i\langle K^-(k_1)\pi^+(k_2)|\bar{s}\gamma^\mu\gamma_5 b|\bar{B}^0(q+k)\rangle &= F_t\, k_t^\mu + F_0\, k_0^\mu + F_\parallel\, k_\parallel^\mu \\
\langle K^-(k_1)\pi^+(k_2)|\bar{s}\sigma^{\mu\nu}q_\nu b|\bar{B}^0(q+k)\rangle &= F_\perp^T\, k_\perp^\mu \\
\langle K^-(k_1)\pi^+(k_2)|\bar{s}\sigma^{\mu\nu}q_\nu\gamma_5 b|\bar{B}^0(q+k)\rangle &= F_0^T\, k_0^\mu + F_\parallel^T\, k_\parallel^\mu
\end{aligned}
$$

Functions $F_i^{(T)}(k^2, q^2, q\cdot\bar{k})$. Partial-wave expansion:

$$
\begin{aligned}
F_{0,t}(k^2, q^2, q\cdot\bar{k}) &= \sum_{\ell=0}^{\infty}\sqrt{2\ell+1}\, F_{0,t}^{(\ell)}(k^2,q^2)\, P_\ell^{(0)}(\cos\theta_K) \\
F_{\perp,\parallel}(k^2, q^2, q\cdot\bar{k}) &= \sum_{\ell=1}^{\infty}\sqrt{2\ell+1}\, F_{\perp,\parallel}^{(\ell)}(k^2,q^2)\, \frac{P_\ell^{(1)}(\cos\theta_K)}{\sin\theta_K}
\end{aligned}
$$

A toy model with a new (light) scalar

Consider a new scalar particle Φ that couples to the pseudoscalar current $\bar{s}\gamma_5 b$:

$$\mathcal{L}_{sb\Phi} = -g\,\bar{s}\gamma_5 b\,\Phi + h.c.\ .$$

The $B \to \Phi K^- \pi^+$ amplitude to leading order in g is

$$i\mathcal{A}(B \to \Phi K^- \pi^+) = -\frac{g\sqrt{q^2}}{m_b + m_s} F_t(k^2, m_\Phi^2, \theta_K)\ ,$$

and the differential decay rate is given by

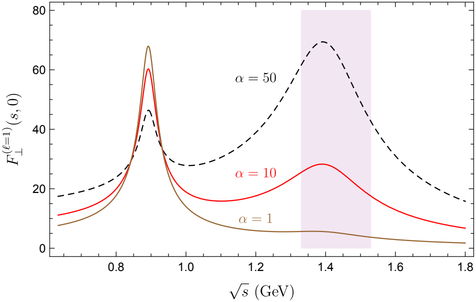
$$\frac{d\Gamma}{dk^2\, d\cos\theta_K} = \frac{1}{(2\pi)^3 32 m_B^3} \frac{\sqrt{\lambda\lambda_{K\pi}}}{2k^2} |\mathcal{A}|^2\ .$$

Integrating over the angle θ_K and using orthogonality of Legendre polynomials:

$$\frac{d\Gamma}{dk^2} = \frac{1}{(2\pi)^3 32 m_B^3} \frac{g^2 q^2 \sqrt{\lambda\lambda_{K\pi}}}{(m_b + m_s)^2 k^2} \sum_{\ell=0}^{\infty} |F_t^{(\ell)}(k^2, m_\Phi^2)|^2\ .$$

A toy model with a new (light) scalar

$$\frac{d\Gamma}{dk^2} = \frac{1}{(2\pi)^3 32 m_B^3} \frac{g^2 q^2 \sqrt{\lambda \lambda_{K\pi}}}{(m_b + m_s)^2 k^2} \sum_{\ell=0}^{\infty} |F_t^{(\ell)}(k^2, m_\Phi^2)|^2 \ .$$



The K^* contribution : take only the $\ell = 1$ term, with only one resonance:

$$|F_t^{(\ell=1)}|^2 = \frac{32\pi^2 s\lambda}{3q^2 \lambda_{K\pi}^{1/2}} |\mathcal{F}_{K^*,t}(m_\Phi^2)|^2 \, \Delta(s, m_{K^*}) \ ; \quad \Delta(s, m_{K^*}) \equiv \frac{1}{\pi} \frac{\sqrt{s}\,\Gamma_{K^*}(s)}{(m_{K^*}^2 - s)^2 + s\Gamma_{K^*}(s)} \ .$$

A toy model with a new (light) scalar

The K^* contribution :

$$|F_t^{(\ell=1)}|^2 = \frac{32\pi^2 s\lambda}{3q^2\lambda_{K\pi}^{1/2}}\,|\mathcal{F}_{K^*,t}(m_\Phi^2)|^2\,\Delta(s,m_{K^*})\,;\quad \Delta(s,m_{K^*}) \equiv \underbrace{\frac{1}{\pi}\frac{\sqrt{s}\,\Gamma_{K^*}(s)}{(m_{K^*}^2-s)^2+s\Gamma_{K^*}(s)}}_{\longrightarrow\,\delta(s-m_{K^*})}\ .$$

Notice $\int ds\,\Delta(s,m_{K^*}) = 1$. Integrating around a window containing the K^*:

$$\Gamma(B\to\Phi K^*[\to K^-\pi^+]) = \frac{g^2\lambda^{3/2}(m_K^*)}{24\pi m_B^3(m_b+m_s)^2}|\mathcal{F}_{K^*,t}(m_\Phi^2)|^2\ .$$

We need to compare this result with its narrow-width approximation $B\to K^*\Phi$:

$$\Gamma(B\to\Phi K^*) = \frac{g^2\lambda^{3/2}(m_K^*)}{16\pi m_B^3(m_b+m_s)^2}|A_0^{BK^*}(m_\Phi^2)|^2\ .$$

Since the $\mathcal{B}(K^*\to K^-\pi^+) = 2/3$, **both rates coincide if $\mathcal{F}_{K^*,t} = A_0^{BK^*}$.**

But beyond the NWL $\mathcal{F}_{K^*,t}$ has a different value!

The case of $B \to K\pi\ell\ell$

$$\frac{d\Gamma}{dq^2 dk^2 d\cos\theta_\ell d\cos\theta_K d\phi} = \frac{9}{32\pi} I(q^2, k^2, \theta_\ell, \theta_K, \phi) = \frac{9}{32\pi} \sum_i I_i(q^2, k^2) \Omega_i(\theta_\ell, \theta_K, \phi)$$

- ☐ In $B \to K^*\ell\ell$: $\quad \bar{I}(q^2, m_{K^*}^2)$ are the usual functions of $A_i^{L,R}(q^2)$.
- ☐ In $B \to K\pi\ell\ell$ we must do: $A_i^{L,R}(q^2) \to \widehat{A}_i^{L,R}(q^2, k^2)$

with

$$\widehat{A}_\perp^{L,R} = \frac{\sqrt{\lambda_{K\pi}}}{k^2} \mathcal{A}_\perp^{L,R(1)} \,, \qquad \widehat{A}_\parallel^{L,R} = \frac{\sqrt{\lambda_{K\pi}}}{k^2} \mathcal{A}_\parallel^{L,R(1)} \,,$$

$$\widehat{A}_0^{L,R} = -\mathcal{A}_0^{L,R(1)}/\sqrt{2} \,, \qquad \widehat{A}_t = -\mathcal{A}_t^{(1)}/\sqrt{2} \,.$$

and

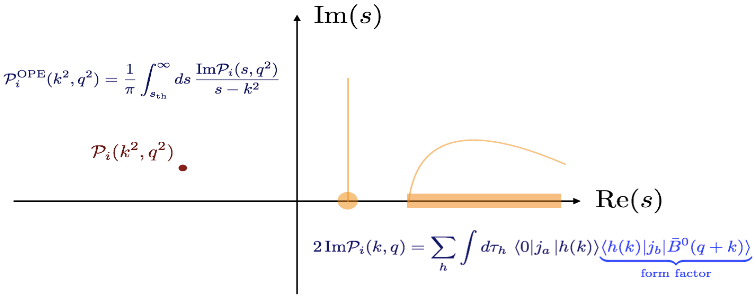
$$\mathcal{A}_i^{L,R}(k^2, q^2, \theta_K) = \mathcal{N}\left[(C_9 \mp C_{10}) F_i + \frac{2m_b}{q^2} \left\{ C_7 F_i^T - i \frac{16\pi^2}{m_b} \mathcal{H}_i \right\} \right] , \quad i = \{\perp, \parallel, 0, t\}$$

with partial-wave expansion giving $\mathcal{A}_i^{L,R(\ell)}(k^2, q^2)$.

Khodjamirian, Mannel, Offen 2006

[Analyticity+Unitarity+Duality]

Consider a correlation function: $\mathcal{P}_{ab}(k,q) = i\int d^4x\, e^{ik\cdot x}\langle 0|\mathrm{T}\{j_a(x), j_b(0)\}|\bar{B}^0(q+k)\rangle$

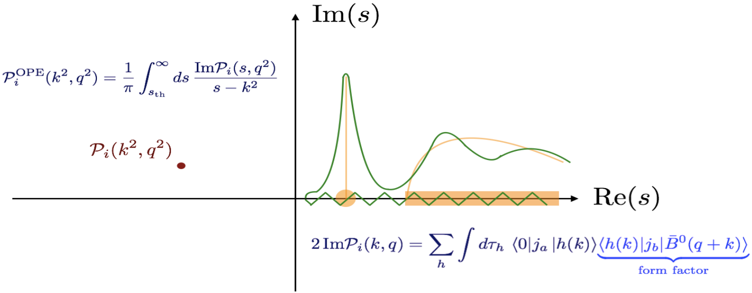


- Traditionally, $h(k) = K^* + \mathit{continuum} \;\Rightarrow\; 2\,\mathrm{Im}\mathcal{P}_{ab}(k,q) \sim f_K^*\, F^{BK*}\,\delta(k^2 - m_{K^*}) + \cdots$

Light-Cone Sum Rules with B-meson LCDAs

Khodjamirian, Mannel, Offen 2006 [Analyticity+Unitarity+Duality]

Consider a correlation function: $\mathcal{P}_{ab}(k,q) = i \int d^4x\, e^{ik\cdot x} \langle 0|\mathrm{T}\{j_a(x), j_b(0)\}|\bar{B}^0(q+k)\rangle$



- Traditionally, $h(k) = K^* + continuum \;\Rightarrow\; 2\,\mathrm{Im}\mathcal{P}_{ab}(k,q) \sim f_K^* F^{BK*}\delta(k^2 - m_{K^*}) + \cdots$
- Generalization for unstable mesons Cheng, Khodjamirian, Virto 2017 : $h(k) = K\pi + \cdots$

LCSRs with B-meson DAs, natural for this generalization.

Light-Cone Sum Rules for P-wave $B \to K\pi$ Form Factors

$$\boxed{\int_{s_{\rm th}}^{s_0} ds\; e^{-s/M^2}\;\; \omega_i(s,q^2)\;\; f_+^\star(s)\;\; F_i^{(T)(\ell=1)}(s,q^2) = \mathcal{P}_i^{(T),{\rm OPE}}(q^2,\sigma_0,M^2)}$$

- s_0 – Effective threshold
- $\omega_i(s,q^2)$ – (known) kinematic factors
- $\langle K^-(k_1)\pi^+(k_2)|\bar{s}\gamma_\mu d|0\rangle = f_+(k^2)\;\bar{k}_\mu + \frac{m_K^2-m_\pi^2}{k^2} f_0(k^2)\; k_\mu$
- $\mathcal{P}_i^{(T),{\rm OPE}}$ – OPE result for the correlation function

What have we done? arXiv:1908.02267 [hep-ph]

$$\boxed{\int_{s_{\rm th}}^{s_0} ds\, e^{-s/M^2}\ \ \omega_i(s,q^2)\ \ f_+^\star(s)\ \ F_i^{(T)(\ell=1)}(s,q^2) = \mathcal{P}_i^{(T),\rm OPE}(q^2,\sigma_0,M^2)}$$

- Generalize LCSRs in Khodjamirian, Mannel, Offen 2006 beyond the K^* case, including LCSRs for A_0, $T_{2,3}$
- Recalculate $\mathcal{P}_i^{(T),\rm OPE}$ including 3-particle contributions, and extended consistently to twist-4 accuracy. Full (numerical) agreement with Gubernari,Kokulu,van Dyk 2018 (not input parameters)
- Revisit s_0 $\Rightarrow$ significantly lower value!! – f_{K^*} is derived quantity
- Study of Narrow-width limit, Finite-Width effects, and effects beyond the K^*
- Applications to $B \to K\pi\ell\ell$

$K\pi$ form factor $f_+(s)$ from $\tau \to K\pi\nu_\tau$

Differential decay rate of $\tau \to K\pi\nu_\tau$:

$$\frac{d\Gamma}{ds} = \frac{N_\tau}{s^3}\left(1-\frac{s}{m_\tau^2}\right)^2\left(1+2\frac{s}{m_\tau^2}\right)\lambda_{K\pi}^{3/2}\,|\widetilde{f}_+(s)|^2\left\{1+\frac{3(\Delta m^2)^2}{(1+2s/m_\tau^2)\,\lambda_{K\pi}}\,|\widetilde{f}_0(s)|^2\right\}$$

with the normalization [Total BR will give $|f_+(0)|^2 = 0.99$, consistent with $f_+^{LQCD}(0) = 0.97$]

$$N_\tau = \frac{G_F^2\,|V_{us}|^2\,|f_+(0)|^2\,m_\tau^3}{1536\pi^3}S_{EW}^{\rm had}$$

Belle fits to models: [This gives $f_{K*} \simeq 205$ MeV, compared to $f_{K*} = 217(5)$ MeV (NWL)]

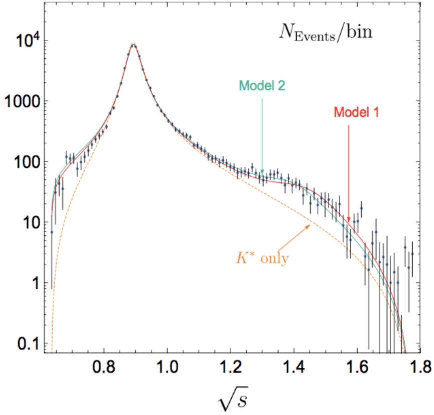
$$\widetilde{f}_+(s) = \sum_R \frac{\xi_R\,m_R^2}{m_R^2 - s - i\sqrt{s}\,\Gamma_R(s)}\,,\quad f_0(s) = f_+(0)\cdot\sum_{R_0}\frac{\xi_{R_0}\,s}{m_{R_0}^2 - s - i\sqrt{s}\,\Gamma_{R_0}(s)}\,,$$

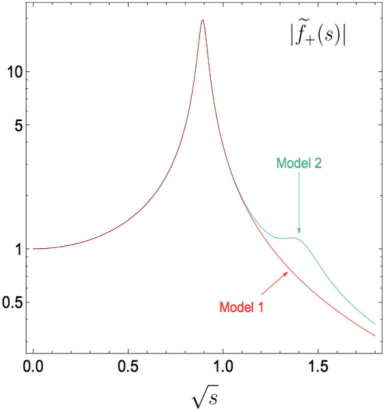
Model 1 : $\quad \xi_{K^*(892)} = 1\,,\ \xi_{K_0^*(800)} = 1.27\,,\ \xi_{K_0^*(1430)} = 0.954\,e^{i\,0.62}$

Model 2 : $\quad \xi_{K^*(892)} = 0.988\,e^{-i\,0.07}\,,\ \xi_{K^*(1410)} = 0.074\,e^{i\,1.37}\,,\ \xi_{K_0^*(800)} = 1.57$

$K\pi$ form factor $f_+(s)$ from $\tau \to K\pi\nu_\tau$

Data from Belle, arXiv:0706.2231 [hep-ex]





Effective threshold: 2-point SVZ sum rule

Knowing $|f_+(s)|$ we can extract s_0 from a QCD sum rule:

$$\begin{aligned}\Pi_{\mu\nu}(k) &= i\int d^4x e^{ikx}\langle 0|\mathrm{T}\{\bar{d}(x)\gamma_\mu s(x), \bar{s}(0)\gamma_\nu d(0)|0\rangle \\ &= (k_\mu k_\nu - k^2 g_{\mu\nu})\,\Pi(k^2) + k_\mu k_\nu\,\widetilde{\Pi}(k^2)\end{aligned}$$

$$\Pi(M^2, s_0) \equiv \frac{1}{\pi}\int_{s_{\rm th}}^{s_0} ds\, e^{-s/M^2}\,\mathrm{Im}\Pi(s) = \int_{s_{\rm th}}^{s_0} ds\, e^{-s/M^2}\,\frac{\lambda_{K\pi}^{3/2}(s)}{32\pi^2 s^3}\,|f_+(s)|^2$$

$$\begin{aligned}\Pi^{\rm OPE}(M^2, s_0) &= \frac{1}{8\pi^2}\int_{m_s^2}^{s_0} ds\, e^{-s/M^2}\,\frac{(s-m_s^2)^2(2s+m_s^2)}{s^3} \\ &\quad + \frac{\alpha_s(M)}{\pi}\,\frac{M^2}{4\pi^2}\left(1 - e^{-s_0/M^2}\right) + \frac{v_4}{M^2} + \frac{v_6}{2M^4}\end{aligned}$$

Effective threshold: 2-point SVZ sum rule

Borel parameter M^2	Effective threshold s_0	
$1.00\ \mathrm{GeV}^2$	$1.28 \pm 0.18\ \mathrm{GeV}^2$ (Model 1) $1.25 \pm 0.18\ \mathrm{GeV}^2$ (Model 2)	$1.26 \pm 0.18\ \mathrm{GeV}^2$ (Average)
$1.25\ \mathrm{GeV}^2$	$1.33 \pm 0.12\ \mathrm{GeV}^2$ (Model 1) $1.31 \pm 0.12\ \mathrm{GeV}^2$ (Model 2)	$1.31 \pm 0.12\ \mathrm{GeV}^2$ (Average)
$1.50\ \mathrm{GeV}^2$	$1.36 \pm 0.09\ \mathrm{GeV}^2$ (Model 1) $1.34 \pm 0.09\ \mathrm{GeV}^2$ (Model 2)	$1.35 \pm 0.09\ \mathrm{GeV}^2$ (Average)

Table 3: *Values for the effective threshold s_0 extracted from the SVZ sum rules.*

Significantly low value compared to the usual $s_0^{K^*} \simeq 1.7\ \mathrm{GeV}^2 \sim (\sqrt{s_0^\rho} + m_s)^2$

Models for $B \to K\pi$ form factors

Assume that the P-wave $K\pi$ state couples to its interpolating current $\bar{s}\Gamma d$ resonantly, through a set of Breit-Wigner-type vector resonances:

$$\langle K(k_1)\pi(k_2)|\bar{s}\gamma^\mu d|X\rangle = \sum_{R,\eta} BW_R(k^2)\langle K(k_1)\pi(k_2)|R(k,\eta)\rangle\langle R(k,\eta)|\bar{s}\gamma^\mu d|X\rangle$$

$$f_+(s) = -\sum_R \frac{m_R\, f_R\, g_{RK\pi}\; e^{i\phi_R(s)}}{m_R^2 - s - i\sqrt{s}\,\Gamma_R(s)}$$

$$F_i^{(T),(\ell=1)}(s,q^2) = \sum_R \frac{Y_{R,i}^{(T)}(s,q^2)\, g_{RK\pi}\, \mathcal{F}_{R,i}^{(T)}(q^2)\, e^{i\phi_R(s)}}{m_R^2 - s - i\sqrt{s}\,\Gamma_R(s)}$$

This model is totally equivalent to the model fitted by Belle for $f_+(s)$.

Light-Cone Sum Rule + BW model

$$\boxed{\sum_R \mathcal{F}^{(T)}_{R,i}(q^2)\, d^{(T)}_{R,i}\; I_R(s_0, M^2) = \mathcal{P}_i^{(T),\mathrm{OPE}}(q^2, \sigma_0, M^2)}$$

with

$$I_R(s_0, M^2) = \frac{m_R}{16\,\pi^2} \int_{s_{\rm th}}^{s_0} ds\; e^{-s/M^2}\; \frac{g_{RK\pi}\,\lambda_{K\pi}^{3/2}(s)\,|f_+(s)|}{s^{5/2}\sqrt{(m_R^2 - s)^2 + s\,\Gamma_R^2(s)}}$$

and

$$d_{R,\perp} = -d_{R,-} = (m_B + m_R)^{-1}\,, \quad d_{R,\|} = \frac{(m_B + m_R)}{2}\,, \quad d_{R,t} = -m_R\,,$$

$$d^T_{R,\perp} = -d^T_{R,-} = 1\,, \quad d^T_{R,\|} = \frac{(m_B^2 - m_R^2)}{2}\,.$$

Narrow-width limit

Consider the sum rule with a single resonance R:

$$\boxed{\mathcal{F}_{R,i}^{(T)}(q^2)\, d_{R,i}^{(T)}\, I_R(s_0, M^2) = \mathcal{P}_i^{(T),\mathrm{OPE}}(q^2, \sigma_0, M^2)}$$

$$\begin{aligned} I_R(s_0, M^2) \quad &= \quad 3\, m_R\, f_R\, \mathcal{B}(R \to K^+\pi^-) \int_{s_{\rm th}}^{s_0} ds\, e^{-s/M^2}\, \frac{m_R}{\sqrt{s}} \left[\frac{1}{\pi} \frac{\sqrt{s}\,\Gamma_R(s)}{(m_R^2 - s)^2 + s\,\Gamma_R^2(s)}\right] \\ &\xrightarrow{\Gamma_R^{\rm tot} \to 0} \; 3\, m_R\, f_R\, \mathcal{B}(R \to K^+\pi^-)\, e^{-m_R^2/M^2} \end{aligned}$$

$$\Rightarrow \quad 3\, m_R\, f_R\, d_{R,i}^{(T)}\, \mathcal{F}_{R,i}^{(T)}(q^2)\, e^{-m_R^2/M^2}\, \mathcal{B}(R \to K^+\pi^-) = \mathcal{P}_i^{(T),\mathrm{OPE}}(q^2, \sigma_0, M^2)$$

This agrees with Khodjamirian, Mannel, Offen 2006

Finite-width effects

Consider the sum rule with a single K^*:

$$\boxed{\mathcal{F}^{(T)}_{K^*,i}(q^2)\, d^{(T)}_{K^*,i}\, I_{K^*}(s_0, M^2) = \mathcal{P}_i^{(T),\mathrm{OPE}}(q^2, \sigma_0, M^2)}$$

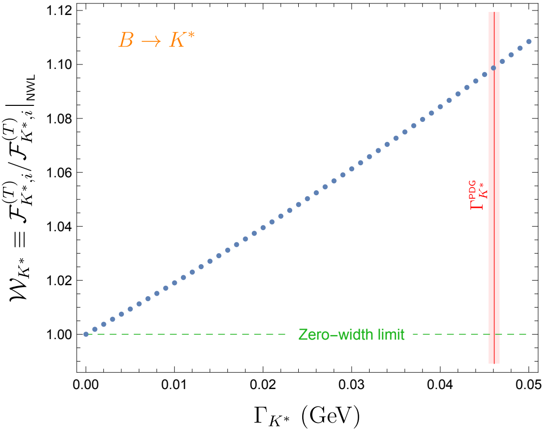
Define the "Width ratio" $\mathcal{W}_{K^*}$:

$$\mathcal{W}_{K^*} \equiv \frac{\mathcal{F}^{(T)}_{K^*,i}(q^2)}{\mathcal{F}^{(T)}_{K^*,i}(q^2)_{\mathrm{NWL}}} = \frac{I_{K^*}(s_0, M^2)\big|_{\Gamma_{K^*}\to 0}}{I_{K^*}(s_0, M^2)} = \frac{2 m_{K^*} f_{K^*} e^{-m^2_{K^*}/M^2}}{I_{K^*}(s_0, M^2)}$$

- $\mathcal{W}_{K^*}$ is independent of the form factor type
- $\mathcal{W}_{K^*}$ is independent of q^2

$\Rightarrow$ BRs are corrected by $|\mathcal{W}_{K^*}|^2$, ratios are uncorrected! + true in q^2 bins.

Finite-width effects



$$\mathcal{W}_{K^*} \simeq 1 + 1.9\, \frac{\Gamma_{K^*}}{m_{K^*}}$$

$$\mathcal{W}_{K^*} = 1.09 \pm 0.01$$

$\Rightarrow$ BRs are corrected by a factor $|\mathcal{W}_{K^*}|^2 \simeq 1.2$

Beyond the $K^*(892)$

Consider the sum rule with $R = \{K^*(892), K^*(1410)\}$:

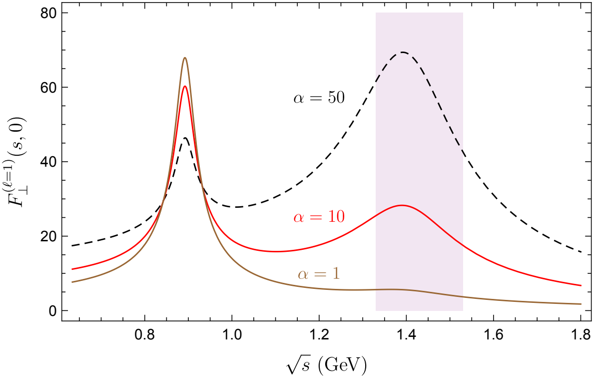
$$\boxed{\sum_R \mathcal{F}_{R,i}^{(T)}(q^2)\, d_{R,i}^{(T)}\, I_R(s_0, M^2) = \mathcal{P}_i^{(T),\mathrm{OPE}}(q^2, \sigma_0, M^2)}$$

		$M^2 = 1.00\,\mathrm{GeV}^2$	$M^2 = 1.25\,\mathrm{GeV}^2$	$M^2 = 1.50\,\mathrm{GeV}^2$
Model 1	$I_{K^*(892)}$	0.1506(23)	0.1781(16)	0.1992(13)
	$I_{K^*(1410)}$	0.0050(07)	0.0062(07)	0.0072(06)
Model 2	$I_{K^*(892)}$	0.1491(22)	0.1766(20)	0.1975(16)
	$I_{K^*(1410)}$	0.0048(07)	0.0061(06)	0.0070(06)

Table 8: *Values for the quantities I_R for $R = \{K^*(892), K^*(1410)\}$ for the different values of the Borel parameter M^2 and for the two models for the $K\pi$ form factor. The $K^*(1410)$ contribution is very suppressed in the sum rules, with $I_{K^*(1410)}/I_{K^*(892)} \simeq 0.03$ in all cases.*

Beyond the $K^*(892)$

Set $\mathcal{F}_{K^*(1410)} = \alpha\, \mathcal{F}_{K^*(892)}$ with α a floating parameter



$\alpha = 1: \ \mathcal{F}_{K^*,\perp}(0) = 0.28$; $\alpha = 10: \ \mathcal{F}_{K^*,\perp}(0) = 0.22$; $\alpha = 50: \ \mathcal{F}_{K^*,\perp}(0) = 0.11$.

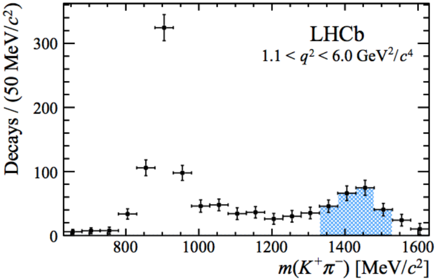
High $K\pi$-Mass Moments in $B \to K\pi\ell\ell$

Differential decay rate including S,P,D waves – – [$d\Omega = d\cos\theta_\ell \, d\cos\theta_K \, d\phi$]

$$\frac{d\Gamma}{dq^2 dk^2 d\Omega} = \frac{1}{4\pi} \sum_{i=1}^{41} f_i(\Omega) \, \tilde{\Gamma}_i(q^2, k^2)$$

The 41 moments $\tilde{\Gamma}_i(q^2, k^2)$ have been measured by LHCb (arXiv:1609.04736) in the bins

$$\sqrt{k^2} \in [1.33, 1.53]\text{GeV} \,, \quad q^2 \in [1.1, 6]\text{GeV}^2$$



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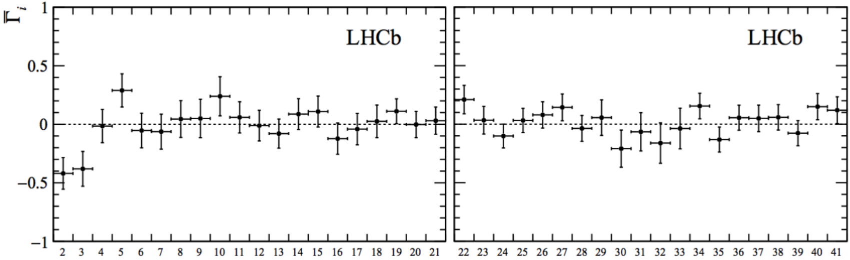
The 41 moments $\tilde{\Gamma}_i(q^2, k^2)$ have been measured by LHCb (arXiv:1609.04736) in the bins

$$\sqrt{k^2} \in [1.33, 1.53]\mathrm{GeV}\ , \quad q^2 \in [1.1, 6]\mathrm{GeV}^2$$



High $K\pi$-Mass Moments in $B \to K\pi\ell\ell$

Differential decay rate including S,P,D waves – – [$d\Omega = d\cos\theta_\ell\, d\cos\theta_K\, d\phi$]

$$\frac{d\Gamma}{dq^2 dk^2 d\Omega} = \frac{1}{4\pi} \sum_{i=1}^{41} f_i(\Omega)\, \tilde{\Gamma}_i(q^2, k^2)$$

The 41 moments $\tilde{\Gamma}_i(q^2, k^2)$ depend on S, P, D-wave amplitudes:

i	$f_i(\Omega)$	$\Gamma_i^{L,\mathrm{tr}}(q^2)/\mathbf{k}q^2$	$\eta_i^{L\to R}$
1	$P_0^0 Y_0^0$	$\Big[\lvert H_0^L\rvert^2 + \lvert H_\parallel^L\rvert^2 + \lvert H_\perp^L\rvert^2 + \lvert S^L\rvert^2 + \lvert D_0^L\rvert^2 + \lvert D_\parallel^L\rvert^2 + \lvert D_\perp^L\rvert^2\Big]$	$+1$
2	$P_1^0 Y_0^0$	$2\Big[\frac{2}{\sqrt5} Re(H_0^L D_0^{L*}) + Re(S^L H_0^{L*}) + \sqrt{\frac35}\, Re(H_\parallel^L D_\parallel^{L*} + H_\perp^L D_\perp^{L*})\Big]$	$+1$
3	$P_2^0 Y_0^0$	$\frac{\sqrt5}{7}\,(\lvert D_\parallel^L\rvert^2 + \lvert D_\perp^L\rvert^2)$ - $\frac{1}{\sqrt5}\,(\lvert H_\parallel^L\rvert^2 + \lvert H_\perp^L\rvert^2) + \frac{2}{\sqrt5}\,\lvert H_0^L\rvert^2 + \frac{10}{7\sqrt5}\,\lvert D_0^L\rvert^2 + 2\, Re(S^L D_0^{L*})$	$+1$
4	$P_3^0 Y_0^0$	$\frac{6}{\sqrt{35}}\Big[- Re(H_\parallel^L D_\parallel^{L*} + H_\perp^L D_\perp^{L*}) + \sqrt3 Re(H_0^L D_0^{L*})\Big]$	$+1$
5	$P_4^0 Y_0^0$	$\frac27\Big[-2(\lvert D_\parallel^L\rvert^2 + \lvert D_\perp^L\rvert^2) + 3\lvert D_0^L\rvert^2\Big]$	$+1$
6	$P_0^0 Y_2^0$	$\frac{1}{2\sqrt5}\Big[(\lvert D_\parallel^L\rvert^2 + \lvert D_\perp^L\rvert^2) + (\lvert H_\parallel^L\rvert^2 + \lvert H_\perp^L\rvert^2) - 2\lvert S^L\rvert^2 - 2\lvert D_0^L\rvert^2 - 2\lvert H_0^L\rvert^2\Big]$	$+1$
7	$P_1^0 Y_2^0$	$\Big[\frac{\sqrt3}{5}\, Re(H_\parallel^L D_\parallel^{L*} + H_\perp^L D_\perp^{L*}) - \frac{2}{\sqrt5}\, Re(S^L H_0^{L*}) - \frac45\, Re(H_0^L D_0^{L*})\Big]$	$+1$
8	$P_2^0 Y_2^0$	$\Big[\frac{1}{14}(\lvert D_\parallel^L\rvert^2 + \lvert D_\perp^L\rvert^2) - \frac27\lvert D_0^L\rvert^2 - \frac{1}{10}(\lvert H_\parallel^L\rvert^2 + \lvert H_\perp^L\rvert^2) - \frac25\lvert H_0^L\rvert^2 - \frac{2}{\sqrt5} Re(S^L D_0^{L*})\Big]$	$+1$
9	$P_3^0 Y_2^0$	$-\frac{3}{5\sqrt7}\Big[Re(H_\parallel^L D_\parallel^{L*} + H_\perp^L D_\perp^{L*}) + 2\sqrt3\, Re(H_0^L D_0^{L*})\Big]$	$+1$
10	$P_4^0 Y_2^0$	$-\frac{2}{7\sqrt5}\Big[\lvert D_\parallel^L\rvert^2 + \lvert D_\perp^L\rvert^2 + 3\lvert D_0^L\rvert^2\Big]$	$+1$
11	$P_1^1 \sqrt2\, Re(Y_2^1)$	$-\frac{3}{\sqrt{10}}\Big[\sqrt{\frac23}\, Re(H_\parallel^L S^{L*}) - \sqrt{\frac{2}{15}}\, Re(H_\parallel^L D_0^{L*}) + \sqrt{\frac25}\, Re(D_\parallel^L H_0^{L*})\Big]$	$+1$
12	$P_2^1 \sqrt2\, Re(Y_2^1)$	$-\frac{3}{[illegible]}\Big[Re(H_\parallel^L H_0^{L*}) + \sqrt{\frac{5}{[illegible]}}\, Re(D_\parallel^L S^{L*}) + \frac{5}{[illegible]}\, Re(D_\parallel^L D_0^{L*})\Big]$	$+1$

High $K\pi$-Mass Moments in $B \to K\pi\ell\ell$

Combinations of moments depending only on P-wave:

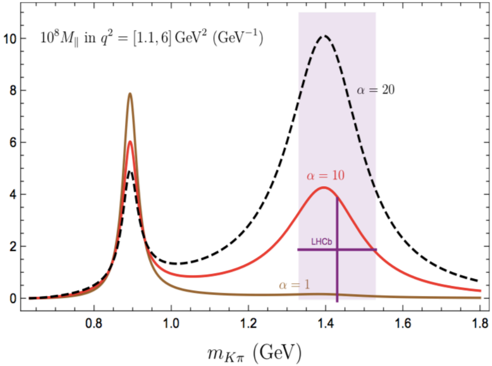
$$
\begin{aligned}
|\widehat{A}_{\|}^{L}|^2 + |\widehat{A}_{\|}^{R}|^2 &= \frac{1}{36}(5\tilde{\Gamma}_1 - 7\sqrt{5}\tilde{\Gamma}_3 + 5\sqrt{5}\tilde{\Gamma}_6 - 35\tilde{\Gamma}_8 - 5\sqrt{15}\tilde{\Gamma}_{19} + 35\sqrt{3}\tilde{\Gamma}_{21}) \\
|\widehat{A}_{\perp}^{L}|^2 + |\widehat{A}_{\perp}^{R}|^2 &= \frac{1}{36}(5\tilde{\Gamma}_1 - 7\sqrt{5}\tilde{\Gamma}_3 + 5\sqrt{5}\tilde{\Gamma}_6 - 35\tilde{\Gamma}_8 + 5\sqrt{15}\tilde{\Gamma}_{19} - 35\sqrt{3}\tilde{\Gamma}_{21}) \\
\mathrm{Im}(\widehat{A}_{\perp}^{L}\widehat{A}_{\|}^{L*} + \widehat{A}_{\perp}^{R}\widehat{A}_{\|}^{R*}) &= \frac{5}{36}(\sqrt{15}\tilde{\Gamma}_{24} - 7\sqrt{3}\tilde{\Gamma}_{26}) \\
\mathrm{Re}(\widehat{A}_{\perp}^{L}\widehat{A}_{\|}^{L*} - \widehat{A}_{\perp}^{R}\widehat{A}_{\|}^{R*}) &= \frac{1}{36}(-5\sqrt{3}\tilde{\Gamma}_{29} + 7\sqrt{15}\tilde{\Gamma}_{31})
\end{aligned}
$$

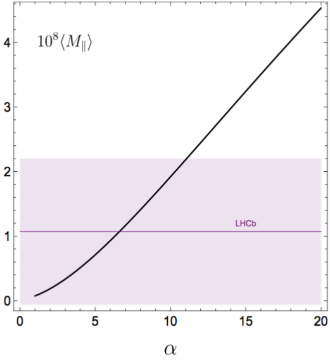
Binned LHCb results (arXiv: 1609.04736) imply:

$$
\begin{aligned}
\tau_B\,\langle|\widehat{A}_{\|}^{L}|^2 + |\widehat{A}_{\|}^{R}|^2\rangle &\equiv \langle M_{\|}\rangle = (1.07 \pm 1.13) \times 10^{-8} \\
\tau_B\,\langle|\widehat{A}_{\perp}^{L}|^2 + |\widehat{A}_{\perp}^{R}|^2\rangle &\equiv \langle M_{\perp}\rangle = (0.94 \pm 1.06) \times 10^{-8} \\
\tau_B\,\langle\mathrm{Im}(\widehat{A}_{\perp}^{L}\widehat{A}_{\|}^{L*} + \widehat{A}_{\perp}^{R}\widehat{A}_{\|}^{R*})\rangle &\equiv \langle M_{\mathrm{im}}\rangle = (-0.75 \pm 0.79) \times 10^{-8} \\
\tau_B\,\langle\mathrm{Re}(\widehat{A}_{\perp}^{L}\widehat{A}_{\|}^{L*} - \widehat{A}_{\perp}^{R}\widehat{A}_{\|}^{R*})\rangle &\equiv \langle M_{\mathrm{re}}\rangle = (0.27 \pm 0.50) \times 10^{-8}
\end{aligned}
$$

High $K\pi$-Mass Moments in $B \to K\pi\ell\ell$

Example: $\langle M_{\|} \rangle$:



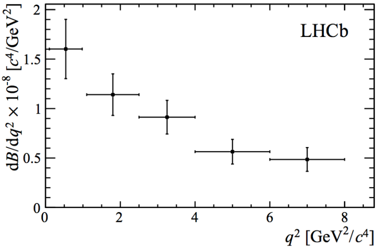


Bounds: From $\langle M_{\|} \rangle$: $\alpha \lesssim 11$; From $\langle M_{\perp} \rangle$: $\alpha \lesssim 17$; From $\langle M_{\rm re} \rangle$: $\alpha \lesssim 18$.

High $K\pi$-Mass Moments in $B \to K\pi\ell\ell$

Upper bounds on P-wave from differential BR:

$$\frac{d\Gamma}{dq^2 dk^2} = \tilde{\Gamma}_1 = |\widehat{A}^L_{\|}|^2 + |\widehat{A}^R_{\|}|^2 + |\widehat{A}^L_{\perp}|^2 + |\widehat{A}^R_{\perp}|^2 + |\widehat{A}^L_0|^2 + |\widehat{A}^R_0|^2 + \ldots$$



$$\begin{aligned}
10^8 \cdot \langle \mathcal{B} \rangle_{[0.10,0.98]} &= 1.41 \pm 0.27 \to \alpha \lesssim 5 \\
10^8 \cdot \langle \mathcal{B} \rangle_{[1.10,2.50]} &= 1.60 \pm 0.29 \to \alpha \lesssim 6 \\
10^8 \cdot \langle \mathcal{B} \rangle_{[2.50,4.00]} &= 1.37 \pm 0.26 \to \alpha \lesssim 5 \\
10^8 \cdot \langle \mathcal{B} \rangle_{[4.00,6.00]} &= 1.12 \pm 0.26 \to \alpha \lesssim 4 \\
10^8 \cdot \langle \mathcal{B} \rangle_{[6.00,8.00]} &= 0.98 \pm 0.23 \to \alpha \lesssim 3
\end{aligned}$$

Bounds are easily improved with some info on S-wave form factors.

Summary

- Absolutely no excuse to do the transition $K^* \to K\pi$ in your life
- Recalculation of all $B \to K^*$ form factors from LCSRs with B-DAs with twist-4 accuracy
- Finite-Width effects are 20% at the level of BRs, universal and q^2-independent
$\Rightarrow$ Global factor 1.2 in BRs; but ratios (e.g. P_5' unaffected.)
- Higher resonance effects can have dramatic effect on $B \to K^*$.
High mass BRs and Moments very efficient to bound this possibility
- Measurements of angular moments in bins across the q^2 and k^2 spectra $\Rightarrow$ **very useful**

Thank You

Extra

Form Factor	This work	Ref. [12]	Ref. [24]	Ref. [15]	Ref. [17]
$\mathcal{F}_{K^*,\perp}(0) = V^{BK^*}(0)$	0.26(15)	0.39(11)	0.36(18)	0.32(11)	0.34(4)
$\mathcal{F}_{K^*,\parallel}(0) = A_1^{BK^*}(0)$	0.20(12)	0.30(8)	0.25(13)	0.26(8)	0.27(3)
$\mathcal{F}_{K^*,-}(0) = A_2^{BK^*}(0)$	0.14(13)	0.26(8)	0.23(15)	0.24(9)	0.23(5)
$\mathcal{F}_{K^*,t}(0) = A_0^{BK^*}(0)$	0.30(7)	–	0.29(8)	0.31(7)	0.36(5)
$\mathcal{F}^T_{K^*,\perp}(0) = T_1^{BK^*}(0)$	0.22(13)	0.33(10)	0.31(14)	0.29(10)	0.28(3)
$\mathcal{F}^T_{K^*,\parallel}(0) = T_2^{BK^*}(0)$	0.22(13)	0.33(10)	0.31(14)	0.29(10)	0.28(3)
$\mathcal{F}^T_{K^*,-}(0) = T_3^{BK^*}(0)$	0.13(12)	–	0.22(14)	0.20(8)	0.18(3)

Table 6: *Results for the form factors at $q^2 = 0$ in the narrow-width limit,compared to corresponding results in the literature. The approach in Ref. [17] is a completely different LCSR approach, in terms of K^* DAs.*

$\mathcal{F}^{BK^*}(q^2=0)$	V^{BK^*}	$A_1^{BK^*}$	$A_2^{BK^*}$	$A_0^{BK^*}$	$T_{1,2}^{BK^*}$	$T_3^{BK^*}$
Ref. [12]	0.39	0.30	0.26	–	0.33	–
Inputs [12], no g_+	0.38	0.29	0.26	0.31	0.33	0.25
Inputs [12], with g_+	0.27	0.21	0.14	0.31	0.24	0.14
Our inputs, but $s_0 = 1.7\,\mathrm{GeV}^2$	0.33	0.26	0.17	0.38	0.29	0.17
Our inputs, our s_0, no g_+	0.36	0.28	0.25	0.30	0.31	0.23
Our inputs, our s_0, with g_+	0.26	0.20	0.14	0.30	0.22	0.13

Table 7: *Deconstruction of the different effects explaining the difference between our results for the form factors at $q^2 = 0$ and those in Ref. [12]. The difference stems mainly from the inclusion of the twist-four two-particle contributions. See the text for more details.*

Light-Cone Sum Rules with B-meson LCDAs

Khodjamirian, Mannel, Offen 2006

Consider a correlation function of the type:

$$\mathcal{P}_{ab}(k,q) = i\int d^4x\, e^{ik\cdot x}\langle 0|\mathrm{T}\{j_a(x), j_b(0)\}|\bar{B}^0(q+k)\rangle$$

which obeys a dispersion relation:

$$\mathcal{P}_{ab}^{\rm OPE}(k^2,q^2) = \frac{1}{\pi}\int_{s_{\rm th}}^{\infty} ds\, \frac{\mathrm{Im}\mathcal{P}_{ab}(s,q^2)}{s-k^2}$$

Duality + Borel transformation:

$$\frac{1}{\pi}\int_{s_{\rm th}}^{s_0} ds\, e^{-s/M^2}\,\mathrm{Im}\mathcal{P}_{ab}(s,q^2) = \mathcal{P}_{ab}^{\rm OPE}(q^2,\sigma_0,M^2)\,,$$

Light-Cone Sum Rules with B-meson LCDAs

Khodjamirian, Mannel, Offen 2006

From Unitarity:

$$2\,\mathrm{Im}\mathcal{P}_{ab}(k,q) = \sum_h \int d\tau_h\ \langle 0|j_a\,|h(k)\rangle \underbrace{\langle h(k)|j_b|\bar{B}^0(q+k)\rangle}_{\text{form factor}}$$

- Traditionally,

$$h(k) = K^* + continuum \quad \Rightarrow \quad 2\,\mathrm{Im}\mathcal{P}_{ab}(k,q) \sim f_K^*\,F^{BK*}\delta(k^2 - m_{K^*}) + \cdots$$

- Generalization for unstable mesons Cheng, Khodjamirian, Virto 2017

$$h(k) = K\pi + \cdots$$

LCSRs with B-meson DAs, natural for this generalization.