Top Quark Physics (≡ Mass Determination)

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This is not an overview, I will focus on one main point:

**top mass determinations from measurements of invariant mass of decay products have a systematic theoretical error** $\delta m_t \simeq \Gamma_t \simeq 1.5$ GeV not accounted for

setting the stage  ● issues in/ overview of top quark physics

problem in $m_t$ determinations  ● what is $m_t$ ??
● scheme dependence

$t\bar{t}$ production  ● theory overview
● required theory improvements

single top production  ● theory overview
● other mass measurements
Outline

- $e_Q; T_3; \text{spin; } SU(N_c)$
  - Test indirect constraints
  - Not main motivation

- $m_t \text{ (what mass?)}$
  - Input for (EW) precision
  - The measurement

- Yukawa coupling $y_t$
  - Direct test of Higgs mech.
  - Important

- CKM element $V_{tb}$
  - (Only) direct measurement
  - Nice

- Width $\Gamma_t$
  - SM theory accurate at 1%
  - (Would be) really nice

- Anom. coupl; BSM
  - We are desperate for it
  - No comment

- $t \to Wb$
  - $pp \to tt\gamma$

- $tt\gamma$ production
  - Other possibilities?

- $pp \to ttH$
  - Single top production

- Only at ILC ??
  - Spin correlations, rare decays, single top ...
one-page summary

- **width** known at $\alpha_s^2$ and one-loop electroweak $\Rightarrow$ theoretical uncertainty $\sim 1\%$ [Czarnecki, Melnikov; Chetyrkin et.al; Denner, Sack; Eilam et.al.]

- $m_{t,pole}/\bar{m}_t(m_t)$ known at $\alpha_s^3$ [Chetyrkin, Steinhauser]

- top quark pair production known at $\sim$ one-loop $\Rightarrow$ see later
  included in MC@NLO [Frixione, Webber]

- single top production known at $\sim$ one-loop $\Rightarrow$ see later
  s- and t-channel included in MC@NLO [Frixione, Laenen, Motylinski, Webber]

- $pp \rightarrow t\bar{t}H$ known at $\sim$ one-loop [Beenakker et. al; Reina et.al.]

- $pp \rightarrow t\bar{t}j$ known at $\sim$ one-loop [Dittmaier, Uwer, Weinzierl]
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Top quarks are basically treated in narrow-width approximation for most (but not all !!) applications this is sufficient
• most precise $m_t$ determinations from measurement of invariant mass of decay products
• you measure $m_t$ really precisely, but you haven’t got a clue what $m_t$ is
• the following equation is not correct

$$\text{WORLD} = \text{HERWIG} \lor \text{PYTHIA} \lor \text{SHERPA} \lor \text{MC@NLO} \lor \text{ANY OTHER MC}$$

• what you determined (and call $m_t$) is not even defined

A reliable determination of the top quark mass with an error $\delta m_t \simeq \Gamma_t \simeq 1.5$ GeV at a hadron collider requires theoretical input which is not (yet) available
• $m_t$ has no meaning, unless you precisely specify what you mean by it
• quark mass definition is not unique, it is simply a theoretical parameter
• different definitions (schemes) are possible and widely used e.g. $m_{\text{pole}}, \overline{m}, m_{\text{PS}}, m_{1S}, m_{\text{DR}} \ldots$
• for each (acceptable) scheme $s_1$ the mass $m_{s_1}$ can be related to the bare mass $m_0$ by divergent relations to any order in perturbation theory

\[ m_{s_1}^{(i)} = m_0 \left( 1 + \alpha_s d_{s_1}^{(1)} + \alpha_s^2 d_{s_1}^{(2)} + \ldots + \alpha_s^i d_{s_1}^{(i)} \right) \]

• the masses in two (acceptable) schemes $s_1$ and $s_2$ are related by finite relations

\[ m_{s_1}^{(i)} = m_{s_2}^{(i)} \left( 1 + \alpha_s f_{s_1,s_2}^{(1)} + \alpha_s^2 f_{s_1,s_2}^{(2)} + \ldots + \alpha_s^i f_{s_1,s_2}^{(i)} \right) \]

• at tree level, all mass definitions are equal, but the higher-order coefficients can be numerically large, e.g. relating $m_{\text{pole}}^{(3)}$ to $\overline{m}^{(3)}$:

\[ 172.5 \text{ GeV} \simeq (162.0 + 8.0 + 1.9 + 0.6) \text{ GeV} \]
observable $O$, mass scheme $s_1$

\[ O_{\text{exp}} = O_{s_1}^{(0)}(m_{s_1} \ldots) + \alpha_s O_{s_1}^{(1)}(m_{s_1} \ldots) + \alpha_s^2 O_{s_1}^{(2)}(m_{s_1} \ldots) + \ldots \]

- working at order $\alpha_s^n$, the determinations of $m_{s_2}$ by
  - using mass scheme $s_2$ directly in determination above
  - using mass scheme $s_1$ as above and then converting $m_{s_1}$ to $m_{s_2}$

are different at order $\alpha_s^{n+1}$

- but $m_{\text{pole}} - \bar{m} \simeq 10\ \text{GeV}$, thus if we are working at LO there is a theoretical error of
  \[ \delta m_t \sim 10\ \text{GeV} \]?

  fortunately it is not quite that bad ⇒ see later
Theory status (top not decaying)

- NLO QCD corrections to top pair production [Dawson et.al.; Beenakker et.al. . . .]
- resummation (in threshold region $\beta \rightarrow 0$) [not for arbitrary distributions] [Bonciani, Catani, Mangano, Nason . . . .]

$$\hat{\sigma}_{t\bar{t}}^{(1)} = \hat{\sigma}_{t\bar{t}}^{(0)} \left( 1 + \alpha_s \left[ \sim \frac{1}{\beta} + \sim \log^2 \beta + \sim \log \beta + c \right] \right)$$

resummation of logs considerably improves the scale dependence of the cross section

- one-loop electroweak corrections known [Beenakker et.al., Kao, Wackeroth, Bernreuther et.al; Kühn, Scharf, Uwer]
small for total cross section, can be important for differential distributions

- NLO QED available [Hollik, Kollar]
- NNLO QCD on its way [Czakon, Moch, Mitov]
- MSSM/ Susy QCD effects [Ross, Wiebusch; Berge et.al.]
Theory status (top decaying) have to consider the decay for experimental cuts

- spin correlations known at NLO [Bernreuther, Brandenburg, Si, Uwer]
- off-shell and off-resonance effects studied [Kauer, Zeppenfeld] but generally not included

\[ p^2 = m_t^2 \Rightarrow \text{singularity} \Rightarrow \text{include width} \Rightarrow \text{gauge invariance issues} \]

Importance of these effects crucially depends on final state cuts

- non-factorizable corrections studied \( \sim \alpha_s \Gamma_t / m_t \) [Beenakker,Berends, Chapovsky] but generally not included
- colour reconnection effects studied [Skands, Wicke] but generally not included
- no program available including all these effects but they can be important at the \( \delta m_t \sim \Gamma_t \sim 1.5 \text{ GeV} \) level
$m_t$ measurements from invariant mass of top decay products (which mass ??)
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corrections to production and decay of on-shell top are included factorizable corrections
$m_t$ measurements from invariant mass of top decay products (which mass ??)

non-factorizable corrections not included

- usual argument: they are suppressed by $\alpha_s \Gamma_t/m_t$, since top are not on-shell any longer;

$$\frac{1}{p^2 - m_t^2 + i m_t \Gamma_t} \quad \text{vs.} \quad \frac{1}{(p + k)^2 - m_t^2 + i m_t \Gamma_t}$$

not true for soft gluons $E \sim \Gamma_t \quad \Rightarrow \quad$ impact on $m_t$ measurement: shift in peak (!!)
without (!!) additional cuts $\delta m_t \sim 100$ MeV [Beenakker,Berends, Chapovsky]
$m_t$ measurements from invariant mass of top decay products (which mass ??)

interconnection effects not included

- non-perturbative interconnection between top quarks (similar to $W$ mass measurement at LEP, but here strong interaction) and with beam remnants:
  impact on $m_t$ measurement: $\delta m_t \sim 0.5 \text{ GeV} - 1 \text{ GeV}$ [Skands, Wicke]
scaremonger summary

• all theoretical descriptions used in the determination of invariant mass distributions are essentially tree-level descriptions
• at this order all mass definitions are equivalent
• after extraction of $m_t$ we do not know whether this is $m_{\text{pole}}$ or $\bar{m}$ or ..., thus there is a "theoretical systematic error" of $\delta m_t \approx 10 \text{ GeV}$
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luckily, this is not the full truth...

- use a “good” scheme, i.e. one where corrections are small
- work at “high” orders in perturbation theory
the statement “all schemes are equally acceptable” is correct in principle, but in practice a scheme with small corrections is better

\[ m_{s_1} = m_{s_1}^{(0)} (1 + c_{s_1}^{(1)} \alpha_s + c_{s_1}^{(2)} \alpha_s^2 + \ldots) \]

the tree level result \( m_{s_1}^{(0)} \) is closer to the true value of the mass \( m_{s_1} \) if the coefficients \( c_{s_1}^{(i)} \) (and in particular \( c_{s_1}^{(1)} \)) are small

this is not the case in the \( \overline{MS} \) scheme since the propagator peaks at \( p^2 = m_{\text{pole}}^2 \)

\[
\frac{1}{p^2 - m_{\text{pole}}^2 - \Sigma(p^2)} \quad \text{\( p^2 \to m_{\text{pole}}^2 \)} \quad \frac{1}{i m_{\text{pole}} \Gamma_t} \quad \frac{1}{p^2 - \overline{m}^2 - \Sigma(p^2)} \quad \text{\( p^2 \to \overline{m} \)} \quad \frac{1}{\Delta m^2 + i m_{\text{pole}} \Gamma_t}
\]

in the \( \overline{MS} \) there are large corrections (e.g. from self-energy insertions) thus a tree-level extracted mass is much closer to the pole mass.
\[ \delta m_t \equiv m_{\text{pole}} - \bar m \simeq 10 \text{ GeV} \] is clearly overestimating the theoretical uncertainty.

- however, the NLO (and higher-order) effects not included in theory prediction do lead to a shift in the measured value of \( m_t \) i.e.

\[ m_{\text{pole}} \neq m_{\text{whatever MC scheme}} \]

- naive estimate \( \delta m_t \simeq \Gamma_t \)

- would be very useful to have a (fixed-order) general purpose MC for \( pp \rightarrow W^+ \bar{b}W^- b \) with
  - beyond narrow width
  - resummation of \( \log \beta \)
  - resummation of Coulomb \( 1/\beta \)
  - non-factorizable corrections
  - modelling of interconnection effects
  - combined with parton shower

- there is still the issue that \( m_{\text{pole}} \) has an intrinsic uncertainty \( \sim \Lambda_{\text{QCD}} \simeq 0.25 \text{ GeV} \)
small branching ratio, but clean signal
- determine $m_t$ from $M_{J/\Psi \ell}$
- initial claims $\delta m_t \lesssim 1 \text{ GeV}$ [Kharchilava]
- updated analysis $\delta m_t \sim 1.5 \text{ GeV}$, theory dominated [Chierici, Dierlamm]

- theory error due to higher orders $\sim 0.7 \text{ GeV}$ from scale variation in PYTHIA (???)
- theory error due to fragmentation function $\sim 0.5 \text{ GeV}$ from variation of Peterson fragmentation function parameter (???)
- using directly moments:

$$\int dM_{b\ell} M_{b\ell}^{n} \frac{d\sigma}{dM_{b\ell}}$$

claim $\delta m_t \sim 0.5 \text{ GeV}$ (???) [Nekrasov]
• \( m_t \) from cross section
  theoretical uncertainty (mainly scale and PDF's)  \( \delta \sigma_{th} = 10\% \quad \Rightarrow \quad \delta m_t \simeq 4 \text{ GeV} \)
  LHC:  \( \delta \sigma_{th} = 5\% \quad \Rightarrow \quad \delta m_t \simeq 2 \text{ GeV} \)

• ratios of cross sections ??
a smart ratio might decrease the dependence on the PDF’s or at least serve as a cross check e.g. \( \sigma_{ttj}/\sigma_{tt} \) or \( \sigma_{tt\gamma}/\sigma_{tt} \)

• \( m_t \) from single top production ??
in particular associated production \( pp \rightarrow tW \) would be affected by “different” non-factorizable corrections (no cross talk between two decaying top quarks)
Theory status

- NLO QCD corrections, production and hadronic decay for t-, s-channel and $Wt$ known [..., Harris et.al (plots below); Campbell, Ellis, Tramontano (MCMF)]
- note at NLO $tW$ mixes with $t\bar{t}$ through inclusion of real radiation diagrams

- the last diagram is the same as $t\bar{t}$ production with (one) subsequent $t$ decay
- disentangle:
  - subtract contribution from resonant diagram [Tait]
  - make cut on invariant mass $M_{Wb}$ to prevent top from becoming resonant [Belayev, Boos, Dudko]
  - the use $p_t$ of $b$ quarks as discriminating variable is preferable [Campbell, Tramontano]
• initial state $b$ quarks from “collinear” splitting of gluons
• resum these contributions, up to a certain factorization scale $\mu_F$ via PDF
• must choose $\mu_F$ small enough such that collinear splitting is a reasonable approximation $\mu_F \sim (m_W + m_t)/4 \sim 65$ GeV
• veto $b$ jets with $p_t > \mu_F$ [Campbell, Tramontano]
mass parameters in MC do not precisely correspond to the pole mass and are not defined beyond leading order

remember: people take whatever number you quote at face value and plug it into whatever they do !!! the theory error has to be taken into account

it is better to have a larger reliable error than a small error that cannot be trusted, in particular given the “strain” in precision tests caused by small values of $m_t$

for $\delta m_t \lesssim 2 \text{ GeV}$ many “small” effects require further work or at least proper inclusion in the error analysis

alternative top-mass measurements are very useful as cross checks, even if they are not competitive