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Top Quark Physics ($\equiv$ Mass Determination)

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This is not an overview, I will focus on one main point:

top mass determinations from measurements of invariant mass of decay products have a systematic theoretical error $\delta m_t \simeq \Gamma_t \simeq 1.5 \text{ GeV}$ not accounted for

setting the stage	• issues in/ overview of top quark physics
problem in m_t determinations	• what is m_t ??
	• scheme dependence
$t\bar{t}$ production	• theory overview
	• required theory improvements
single top production	• theory overview
	• other mass measurements

$e_Q; T_3;$ spin; $SU(N_c)$	test indirect constraints not main motivation	$t \to Wb; \quad pp \to t\bar{t}\gamma$
m_t (what mass?)	input for (EW) precision THE measurement	$t\bar{t}$ production other possibilities?
Yukawa coupling y_t	direct test of Higgs mech. important	$pp \to t\bar{t}H$
CKM element V_{tb}	(only) direct measurement nice	single top production
width Γ_t	SM theory accurate at 1% (would be) really nice	only at ILC ??
anom. coupl; BSM	we are desperate for it no comment	spin correlations, rare decays, single top ...

one-page summary

- width known at α_s^2 and one-loop electroweak $\Rightarrow$ theoretical uncertainty $\sim 1\%$ [Czarnecki, Melnikov; Chetyrkin et.al; Denner, Sack; Eilam et.al.]
- $m_{t,\text{pole}}/\overline{m_t}(\overline{m_t})$ known at α_s^3 [Chetyrkin, Steinhauser]
- top quark pair production known at $\sim$ one-loop $\Rightarrow$ see later
  included in MC@NLO [Frixione, Webber]
- single top production known at $\sim$ one-loop $\Rightarrow$ see later
  s- and t-channel included in MC@NLO [Frixione, Laenen, Motylinski, Webber]
- $pp \to t\bar{t}H$ known at $\sim$ one-loop [Beenakker et. al; Reina et.al.]
- $pp \to t\bar{t}j$ known at $\sim$ one-loop [Dittmaier, Uwer, Weinzierl]

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top quarks are basically treated in narrow-width approximation
for most (but not all !!) applications this is sufficient

- most precise m_t determinations from measurement of invariant mass of decay products
- you measure m_t really precisely, but you haven't got a clue what m_t is
- the following equation is not correct

$$\mathrm{WORLD} = \mathrm{HERWIG} \vee \mathrm{PYTHIA} \vee \mathrm{SHERPA} \vee \mathrm{MC@NLO} \vee \mathrm{ANY\ OTHER\ MC}$$

- what you determined (and call m_t) is not even defined

a reliable determination of the top quark mass with an error $\delta m_t \simeq \Gamma_t \simeq 1.5\ \mathrm{GeV}$ at a hadron collider requires theoretical input which is not (yet) available

- m_t has no meaning, unless you precisely specify what you mean by it
- quark mass definition is not unique, it is simply a theoretical parameter
- different definitions (schemes) are possible and widely used e.g. $m_{\text{pole}}, \overline{m}, m_{\text{PS}}, m_{\text{1S}}, \overline{m_{\text{DR}}} \ldots$
- for each (acceptable) scheme s_1 the mass m_{s_1} can be related to the bare mass m_0 by divergent relations to any order in perturbation theory

$$m_{s_1}^{(i)} = m_0\,(1 + \alpha_s\, d_{s_1}^{(1)} + \alpha_s^2\, d_{s_1}^{(2)} + \ldots + \alpha_s^i\, d_{s_1}^{(i)})$$

- the masses in two (acceptable) schemes s_1 and s_2 are related by finite relations

$$m_{s_1}^{(i)} = m_{s_2}^{(i)}\,(1 + \alpha_s\, f_{s_1,s_2}^{(1)} + \alpha_s^2\, f_{s_1,s_2}^{(2)} + \ldots + \alpha_s^i\, f_{s_1,s_2}^{(i)})$$

- at tree level, all mass definitions are equal, but the higher-order coefficients can be numerically large, e.g. relating $m_{\text{pole}}^{(3)}$ to $\overline{m}^{(3)}$:

$$172.5\ \text{GeV} \simeq (162.0 + 8.0 + 1.9 + 0.6)\ \text{GeV}$$

observable O, mass scheme s_1

$$\underbrace{O_{\rm exp} = O^{(0)}_{s_1}(m_{s_1}\dots)}_{\text{determination of } m^{(0)}_{s_1}} + \alpha_s\, O^{(1)}_{s_1}(m_{s_1}\dots) + \alpha_s^2\, O^{(2)}_{s_1}(m_{s_1}\dots) + \dots$$

$$\underbrace{O_{\rm exp} = O^{(0)}_{s_1}(m_{s_1}\dots) + \alpha_s\, O^{(1)}_{s_1}(m_{s_1}\dots)}_{\text{determination of } m^{(1)}_{s_1} = m^{(0)}_{s_1}(1 + c^{(1)}_{s1}\alpha_s)}$$

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- working at order α_s^n, the determinations of m_{s_2} by
  - using mass scheme s_2 directly in determination above
  - using mass scheme s_1 as above and then converting m_{s_1} to m_{s_2}

  are different at order α_s^{n+1}
- but $m_{\rm pole} - \overline{m} \simeq 10\ \mathrm{GeV}$, thus if we are working at LO there is a theoretical error of $\delta m_t \sim 10\ \mathrm{GeV}$?!?!? fortunately it is not quite that bad $\Rightarrow$ see later

Theory status (top not decaying)

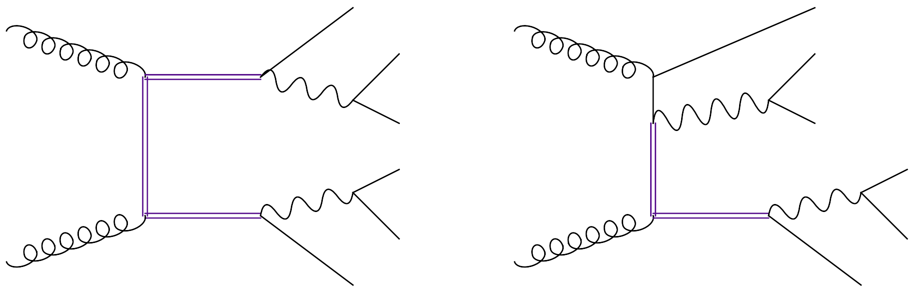
- NLO QCD corrections to top pair production [Dawson et.al.; Beenakker et.al. . . .]
- resummation (in threshold region $\beta \to 0$) [not for arbitrary distributions] [Bonciani, Catani, Mangano, Nason . . . . . .]

$$\hat{\sigma}^{(1)}_{t\bar{t}} = \hat{\sigma}^{(0)}_{t\bar{t}} \left(1 + \alpha_s \left[\underbrace{\sim \frac{1}{\beta}}_{\text{not res.}} + \underbrace{\sim \log^2 \beta + \sim \log \beta}_{\text{resummed}} + c \right] \right)$$

  resummation of logs considerably improves the scale dependence of the cross section
- one-loop electroweak corrections known [Beenakker et.al., Kao, Wackeroth, Bernreuther et.al; Kühn, Scharf, Uwer]
  small for total cross section, can be important for differential distributions
- NLO QED available [Hollik, Kollar]
- NNLO QCD on its way [Czakon, Moch, Mitov]
- MSSM/ Susy QCD effects [Ross, Wiebusch; Berge et.al.]

Theory status (top decaying) have to consider the decay for experimental cuts

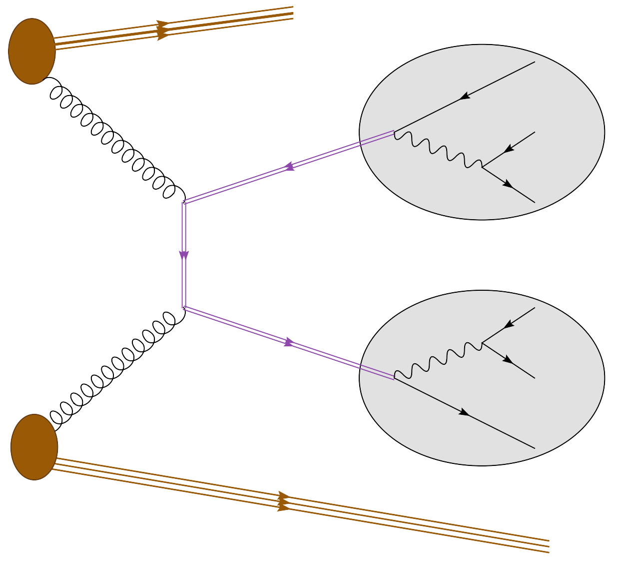
- spin correlations known at NLO [Bernreuther, Brandenburg, Si, Uwer]
- off-shell and off-resonance effects studied [Kauer, Zeppenfeld] but generally not included



in general: $p^2 = m_t^2 \Rightarrow$ singularity $\Rightarrow$ include width $\Rightarrow$ gauge invariance issues
importance of these effects crucially depends on final state cuts

- non-factorizable corrections studied $\sim \alpha_s \Gamma_t / m_t$ [Beenakker,Berends, Chapovsky] but generally not included
- colour reconnection effects studied [Skands, Wicke] but generally not included
- no program avaliable including all these effects but they can be important at the $\delta m_t \sim \Gamma_t \sim 1.5\ \mathrm{GeV}$ level

m_t measurements from invariant mass of top decay products (which mass ??)



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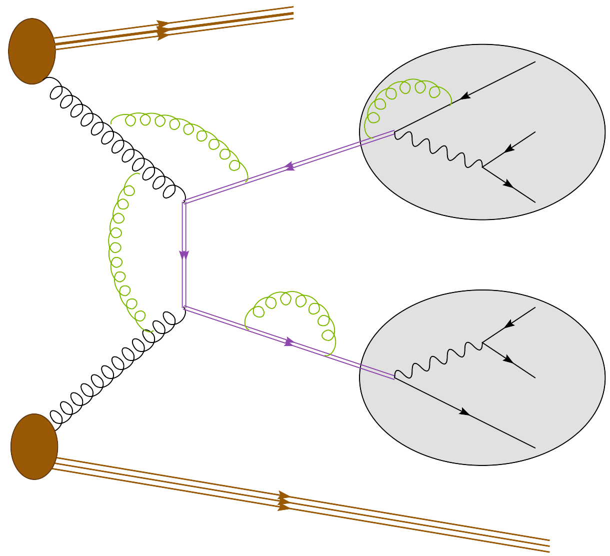
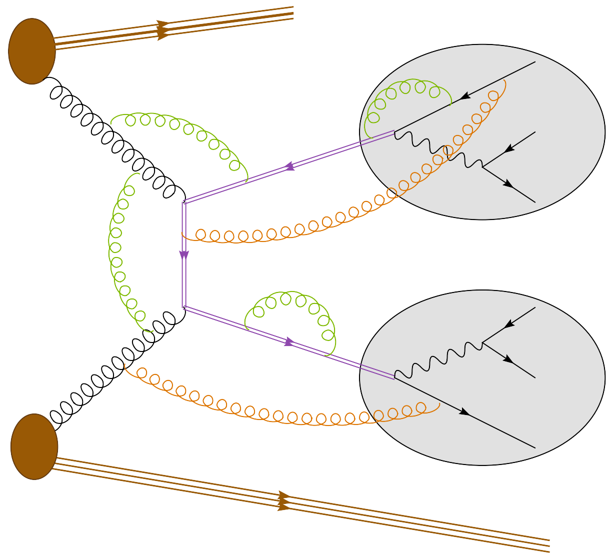


corrections to production and decay of on-shell top are included
factorizable corrections

m_t measurements from invariant mass of top decay products (which mass ??)



non-factorizable corrections not included

- usual argument: they are suppressed by $\alpha_s \Gamma_t / m_t$, since top are not on-shell any longer;

$$\frac{1}{p^2 - m_t^2 + i m_t \Gamma_t} \quad \text{vs.} \quad \frac{1}{(p+k)^2 - m_t^2 + i m_t \Gamma_t}$$

not true for soft gluons $E \sim \Gamma_t$ $\Rightarrow$ impact on m_t measurement: shift in peak (!!) without (!!) additional cuts $\delta m_t \sim 100 \text{ MeV}$ [Beenakker,Berends, Chapovsky]

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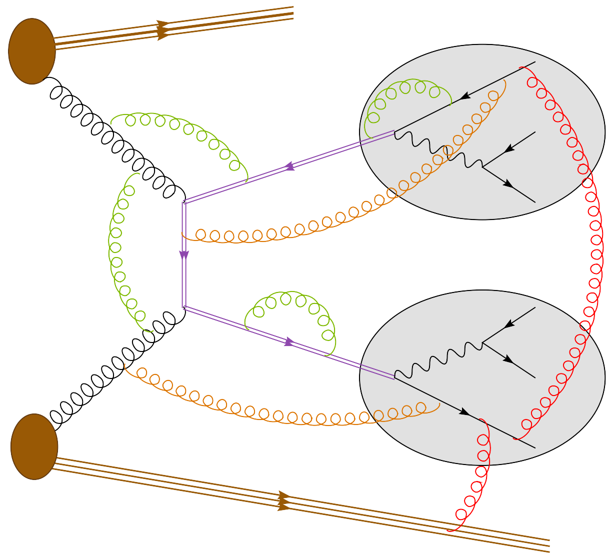

interconnection effects not included

- non-perturbative interconnection between top quarks (similar to W mass measurement at LEP, but here strong interaction) and with beam remnants:
  impact on m_t measurement: $\delta m_t \sim 0.5\ \mathrm{GeV} - 1\ \mathrm{GeV}$ [Skands, Wicke]

scaremonger summary

- all theoretical descriptions used in the determination of invariant mass distributions are essentially tree-level descriptions
- at this order all mass definitions are equivalent
- after extraction of m_t we do not know whether this is $m_{\rm pole}$ or $\overline{m}$ or ... , thus there is a "theoretical systematic error" of $\delta m_t \simeq 10\ \mathrm{GeV}$

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luckily, this is not the full truth...

- use a "good" scheme, i.e. one where corrections are small
- work at "high" orders in perturbation theory

- the statement "all schemes are equally acceptable" is correct in principle, but in practice a scheme with small corrections is better

$$m_{s_1} = m_{s_1}^{(0)}(1 + c_{s1}^{(1)}\alpha_s + c_{s1}^{(2)}\alpha_s^2 + \ldots)$$

- the tree level result $m_{s_1}^{(0)}$ is closer to the true value of the mass m_{s_1} if the coefficients $c_{s1}^{(i)}$ (and in particular $c_{s1}^{(1)}$) are small
- this is not the case in the $\overline{MS}$ scheme since the propagator peaks at $p^2 = m^2_{\text{pole}}$

$$\frac{1}{p^2 - m^2_{\text{pole}} - \Sigma(p^2)} \quad \overset{p^2 \to m^2_{\text{pole}}}{\longrightarrow} \quad \frac{1}{i m_{\text{pole}}\Gamma_t}$$

$$\frac{1}{p^2 - \overline{m}^2 - \Sigma(p^2)} \quad \overset{p^2 \to \overline{m}}{\longrightarrow} \quad \frac{1}{\Delta m^2 + i m_{\text{pole}}\Gamma_t}$$

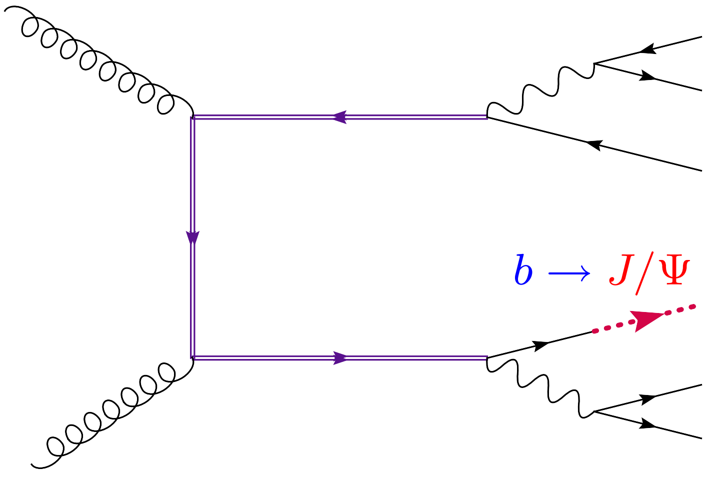
- in the $\overline{MS}$ there are large corrections (e.g. from self-energy insertions) thus a tree-level extracted mass is much closer to the pole mass.

- $\delta m_t \equiv m_{\rm pole} - \overline{m} \simeq 10 \text{ GeV}$ is clearly overestimating the theoretical uncertainty.
- however, the NLO (and higher-order) effects not included in theory prediction do lead to a shift in the measured value of m_t i.e.

$$m_{\rm pole} \neq m_{\text{whatever MC scheme}}$$

- naive estimate $\delta m_t \simeq \Gamma_t$
- would be very useful to have a (fixed-order) general purpose MC for $pp \to W^+ \bar{b} W^- b$ with
  - beyond narrow width
  - resummation of $\log \beta$
  - resummation of Coulomb $1/\beta$
  - non-factorizable corrections
  - modelling of interconnection effects
  - combined with parton shower
- there is still the issue that $m_{\rm pole}$ has an intrinsic uncertainty $\sim \Lambda_{\rm QCD} \simeq 0.25 \text{ GeV}$

top mass from $t \to \ell\nu J/\Psi X$



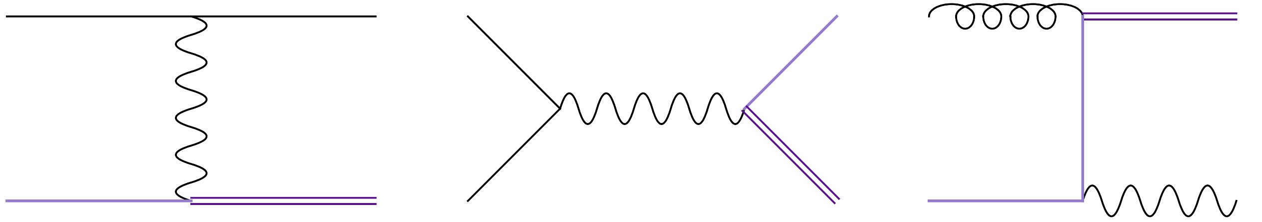
- small branching ratio, but clean signal
- determine m_t from $M_{J/\Psi\,\ell}$
- initial claims $\delta m_t \lesssim 1$ GeV [Kharchilava]
- updated analysis $\delta m_t \sim 1.5$ GeV, theory dominated [Chierici, Dierlamm]

- theory error due to higher orders ~ 0.7 GeV from scale variation in PYTHIA (??)
- theory error due to fragmentation function ~ 0.5 GeV from variation of Peterson fragmentation function parameter (??)
- using directly moments :

$$\int dM_{b\ell}\, M_{b\ell}^n \frac{d\sigma}{dM_{b\ell}}$$

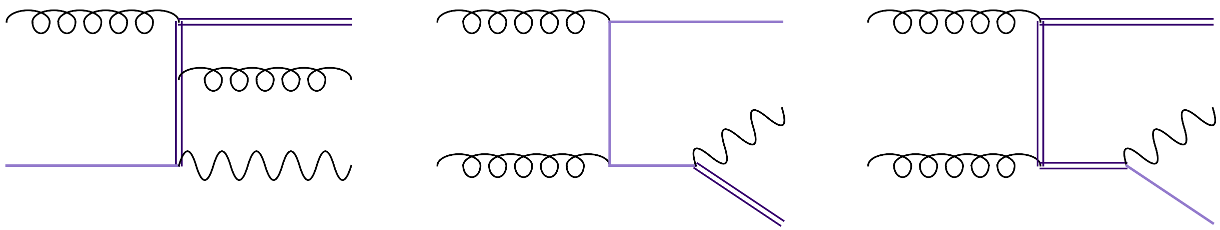
claim $\delta m_t \sim 0.5$ GeV (???) [Nekrasov]

- m_t from cross section
  theoretical uncertainty (mainly scale and PDF's) $\delta\sigma_{\rm th} = 10\%$ $\Rightarrow$ $\delta m_t \simeq 4$ GeV
  LHC: $\delta\sigma_{\rm th} = 5\%$ $\Rightarrow$ $\delta m_t \simeq 2$ GeV
- ratios of cross sections ??
  a smart ratio might decrease the dependence on the PDF's or at least serve as a cross check e.g. σ_{ttj}/σ_{tt} or $\sigma_{tt\gamma}/\sigma_{tt}$
- m_t from single top production ??
  in particular associated production $pp \to tW$ would be affected by "different" non-factorizble corrections (no cross talk between two decaying top quarks)



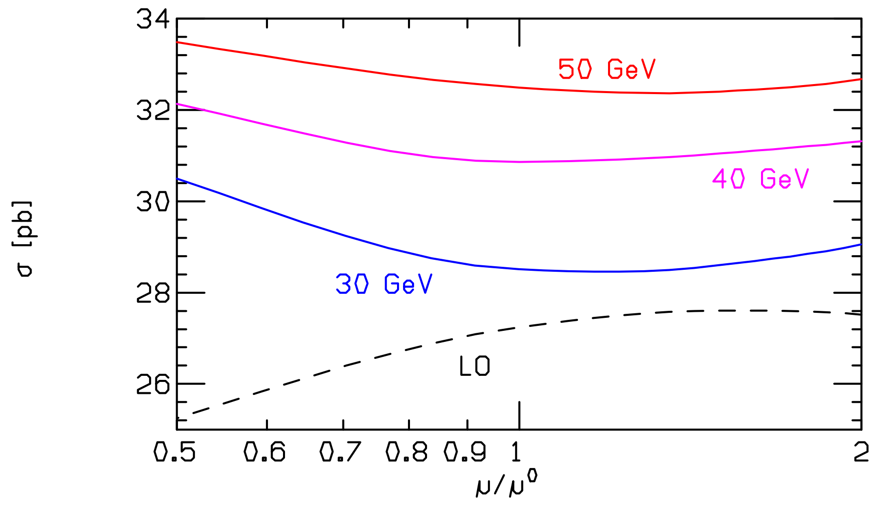
Theory status

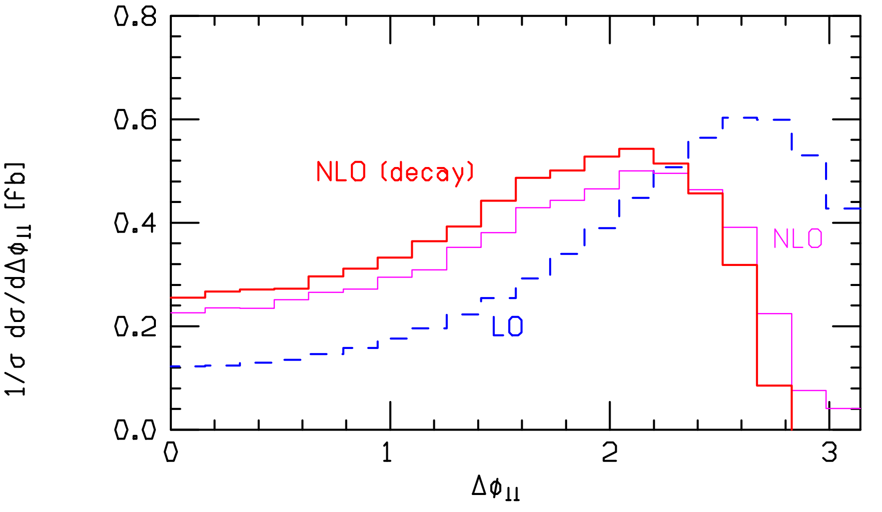
- NLO QCD corrections, production and hadronic decay for t–, s–channel and Wt known [. . ., Harris et.al (plots below); Campbell, Ellis, Tramontano (MCMF)]
- note at NLO tW mixes with $t\bar{t}$ through inclusion of real radiation diagrams



- the last diagram is the same as $t\bar{t}$ production with (one) subsequent t decay
- disentangle:
  - subtract contribution from resonant diagram [Tait]
  - make cut on invariant mass M_{Wb} to prevent top from becoming resonant [Belayev, Boos,Dudko]
  - the use p_t of b quarks as discriminating variable is preferable [Campbell, Tramontano]

- initial state b quarks from "collinear" splitting of gluons
- resum these contributions, up to a certain factorization scale μ_F via PDF
- must choose μ_F small enough such that collinear splitting is a reasonable approximation
  $\mu_F \sim (m_W + m_t)/4 \sim 65$ GeV
- veto b jets with $p_t > \mu_F$ [Campbell, Tramontano]





- mass parameters in MC do not precisely correspond to the pole mass and are not defined beyond leading order
- remember: people take whatever number you quote at face value and plug it into whatever they do !!! the theory error has to be taken into account
- it is better to have a larger reliable error than a small error that cannot be trusted, in particular given the “strain” in precision tests caused by small values of m_t
- for $\delta m_t \lesssim 2\ \mathrm{GeV}$ many “small” effects require further work or at least proper inclusion in the error analysis
- alternative top-mass measurements are very useful as cross checks, even if they are not competitive