NNLO QCD corrections to differential top-quark pair production with the $\overline{\mathrm{MS}}$ mass

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Outline

- Relation between pole mass and $\overline{\text{MS}}$ mass
  some general features
- Top-quark production at the LHC
  unequal role of pole mass and $\overline{\text{MS}}$ mass
- Cross sections for $t\bar{t}$ on-shell production
  from pole mass and $\overline{\text{MS}}$ mass
- QCD results with $\overline{\text{MS}}$ mass
  total and single-differential cross sections up to NNLO
- Effects due to the $\overline{\text{MS}}$ running mass
  a first study: invariant-mass distribution of $t\bar{t}$ pair
- Summary

POLE vs. $\overline{\text{MS}}$ MASS

- top-quark mass: fundamental parameter of SM to be properly defined by renormalization of related UV divergences

  - pole mass M_t : pole of renormalized propagator ("customary" mass for physical particle)

  - $\overline{\text{MS}}$ mass $m_t(\mu_m)$: "subtract" UV divergences in dimensional regularization (more abstract definition)

- different renormalization schemes are perturbatively related:

$$M_t = m_t(\mu_m)\, d(m_t(\mu_m), \mu_m) = m_t(\mu_m) \left(1 + \sum_{k=1}^{\infty} \left(\frac{\alpha_S(\mu_m)}{\pi} \right)^k d^{(k)}(\mu_m) \right)$$

  - we specifically use mass relation at NNLO ($k \leq 3$)

coefficients $d^{(k)}$ known for $k \leq 4$

- $\overline{\text{MS}}$ mass depends on arbitrary renormalization scale μ_m (similarly to QCD coupling $\alpha_S(\mu_R)$) and scale dependence is perturbatively computable [Renormalization Group (RG) evolution]

$$\frac{d \ln m_t(\mu_m)}{d \ln \mu_m^2} = - \sum_{k=0}^{\infty} \gamma_k \left(\frac{\alpha_S(\mu_m)}{\pi} \right)^{k+1}$$

coefficients γ_k known for $k \leq 4$

  - we specifically use RG evolution at NNLO ($k \leq 2$)

Note: scale dependence of $\overline{\text{MS}}$ mass much slower than α_S

$$\frac{d \ln m_t(\mu)}{d \ln \mu} \sim \frac{1}{2} \frac{d \ln \alpha_S(\mu)}{d \ln \mu} \quad \text{at LO}$$

- $\overline{\text{MS}}$ mass $m_t(\mu_m)$ can be specified by: its value at a reference scale + RG evolution

  customary reference scale: $\bar{m}_t$ (no special physical meaning; somehow analogous to reference scale M_Z for $\alpha_S(\mu_R)$)

  a scale of the order of the mass itself ("intrinsic" definition)

  $$m_t(\bar{m}_t) = \bar{m}_t$$

  - typical values at NNLO ($\mathcal{O}$(GeV) variations w.r.t. LO, NLO)

    $M_t = 173$ GeV $\longleftrightarrow$ $\bar{m}_t = 164$ GeV (~ 10 GeV difference)

  [Note: at scale $\mu_m = \bar{m}_t/2$ ➜ $m_t(\mu_m) = M_t + \mathcal{O}(1\ \text{GeV})$, simply because to $d^{(1)} \sim 0$]

- Two main consequences of scale dependence of $\overline{\text{MS}}$ mass

  - perturbative QCD predictions unavoidably depend on μ_m (in addition to renormalization scale μ_R from $\alpha_S(\mu_R)$ and factorization scale μ_F from PDFs)

  - μ_m can possibly be set to a scale very different from $M_t \sim \bar{m}_t$ to embody ("resum") higher-order corrections ➜ running mass effects

TOP QUARK at the LHC

- indirect studies/sensitivity :

top quark enters as virtual (highly off-shell) particle
[e.g., Higgs boson production by gluon-gluon fusion through top-quark loop]

pole and $\overline{\mathrm{MS}}$ masses can be introduced on equal footing

TOP QUARK at the LHC

- direct studies/sensitivity:

top quark (its decay products) is (are) directly observed in the final state

- based on definite physical picture

top quark is "physical", though unstable, particle
with definite pole mass M_t ($\sim$ 173 GeV) and small decay width Γ_t ($\sim$ 1.4 GeV)

then

data on top-quark production extracted from quasi-resonant behavior (around pole mass) of its decay products

- no data without the concept of pole mass
- pole mass has primary role [$\overline{\text{MS}}$ mass has (somehow) an auxiliary role]
  * difference pole vs. $\overline{\text{MS}}$ mass can be much larger than width Γ_t

- theory counterpart:

after integration over top-quark decay products and in narrow-width limit

→ compute cross section for production of on-shell top quark with pole mass M_t

[M_t is not only a parameter of the Lagrangian but also a key kinematical parameter of the phase space (of the underlying physical picture)]

ON-SHELL CROSS SECTION for $t\bar{t}$ PRODUCTION: from pole to $\overline{\text{MS}}$ mass

- Start from on-shell cross section $\sigma(M_t, X)$ with pole mass M_t (total σ or differential $d\sigma/dX$)

  e.g. up to NNLO $$\sigma_{\text{NNLO}}(\alpha_{\text{S}}(\mu_R), \mu_R, \mu_F; M_t; X) = \sum_{i=0}^{2} \left(\frac{\alpha_{\text{S}}(\mu_R)}{\pi}\right)^{i+2} \sigma^{(i)}(M_t; \mu_R, \mu_F; X)$$

- Perform all-order replacement $M_t \to m_t(\mu_m)$ and define $\overline{\text{MS}}$ scheme cross section $\bar{\sigma}$ by ALL-ORDER (formal) EXACT EQUALITY

$$\bar{\sigma}(\alpha_{\text{S}}(\mu_R), \mu_R, \mu_F; \mu_m, m_t(\mu_m); X) = \sigma(\alpha_{\text{S}}(\mu_R), \mu_R, \mu_F; M_t = m_t(\mu_m)\, d(m_t(\mu_m), \mu_m); X)$$

$\overline{\text{MS}}$ scheme Pole scheme

Note: mass and kinematic variable(s) X are treated as independent variables

- Express M_t in terms of $\{ m_t(\mu_m)$ and $\alpha_{\text{S}}(\mu_R) \}$ and expand σ in α_{S} at fixed $m_t(\mu_m)$

  e.g. up to NNLO

$$\bar{\sigma}_{\text{NNLO}}(\alpha_{\text{S}}(\mu_R), \mu_R, \mu_F; \mu_m, m_t(\mu_m); X) = \sum_{i=0}^{2} \left(\frac{\alpha_{\text{S}}(\mu_R)}{\pi}\right)^{i+2} \bar{\sigma}^{(i)}(m_t(\mu_m); \mu_m, \mu_R, \mu_F; X)$$

Explicit expressions* at LO, NLO and NNLO

$$\bar{\sigma}^{(0)}(m_t(\mu_m);\mu_F;X) = \Big[\sigma^{(0)}(m;\mu_F;X)\Big]_{m=m_t(\mu_m)}$$

LO

At NLO: 1st derivative of the LO

$$\bar{\sigma}^{(1)}(m_t(\mu_m);\mu_m,\mu_R,\mu_F;X) = \Big[\sigma^{(1)}(m;\mu_R,\mu_F;X) + d^{(1)}(\mu_m)\, m\,\partial_m\sigma^{(0)}(m;\mu_F;X)\Big]_{m=m_t(\mu_m)}$$

NLO

At NNLO: 1st derivative of the NLO and 2nd derivative of the LO

$$\bar{\sigma}^{(2)}(m_t(\mu_m);\mu_m,\mu_R,\mu_F;X) = \Bigg[\sigma^{(2)}(m;\mu_R,\mu_F;X) + m\Bigg(d^{(1)}(\mu_m)\,\partial_m\sigma^{(1)}(m;\mu_R,\mu_F;X) + \frac{1}{2}\left(d^{(1)}(\mu_m)\right)^2 m\,\partial_m^2\sigma^{(0)}(m;\mu_F;X) + d^{(2)}(\mu_m)\,\partial_m\sigma^{(0)}(m;\mu_F;X) + \beta_0\, d^{(1)}(\mu_m)\ln\left(\frac{\mu_R^2}{\mu_m^2}\right)\partial_m\sigma^{(0)}(m;\mu_F;X)\Bigg)\Bigg]_{m=m_t(\mu_m)}$$

NNLO

- result depends on renormalization coefficients $d^{(k)}$, pertubative terms $\sigma^{(k)}$ of on-shell cross section and their mass derivatives $\partial_m^n\sigma^{(k)}$

- WARNING : mass derivatives can be very sizeable thus spoiling the perturbative convergence of $\overline{\text{MS}}$ cross section $\bar{\sigma}$ (e.g., invariant mass of $t\bar{t}$ pair close to its threshold region)

see backup slides

* same perturbative formulae used by Langenfeld-Moch-Uwer (2009), Dowling-Moch (2014) and applied to total cross section up to NNLO and single-differential distributions up to NLO

within this formulation, pole scheme and $\overline{\text{MS}}$ scheme results are
formally equivalent to all orders in α_S
but different if expanded * at fixed orders

* α_S expansion at fixed M_t (in σ) or $m_t(\mu_m)$ (in $\bar{\sigma}$)

- our general expectations

  - at low orders, σ and $\bar{\sigma}$ can give consistent (within scale uncertainties) results
    [differences can be larger for observables close to kinematical thresholds for $t\bar{t}$ on-shell production]

  - at higher orders, σ and $\bar{\sigma}$ can be quantitatively very similar
    → equivalent perturbative description

then

  - for observables at high scales $X \gg m_{\text{top}}$
    (e.g., top quark at large p_T or $t\bar{t}$ pair at high invariant mass)
    → investigate effects of running $\overline{\text{MS}}$ mass
    $m_t(\mu_m)$ with $\mu_m \sim X$

    Note: at such scales μ_m the coefficients $d^{(k)}(\mu_m)$ are sizeable

our main motivation for using $\overline{\text{MS}}$ mass

LHC RESULTS up to NNLO

- two independent NNLO fully differential calculations of $t\bar{t}$ on-shell production with pole mass

  Czakon, Fiedler, Mitov (2016)
  Devoto, Grazzini,Kallweit, Mazzitelli + S.C. (2019)

- we use our calculation by numerically computing mass derivatives $\partial_m^n \hat{\sigma}^{(k)}(m)$ on a bin-by-bin basis (X bins)

- 3 auxiliary scales $\mu_i = \{\mu_R, \mu_F, \mu_m\}$ and independent scale variations by a factor of two around central μ_0 :

  $\mu_i = \xi_i \mu_0$, $\xi_i = \{1/2, 1, 2\}$ with constraints $\mu_i/\mu_j \leq 2$

  ➡ 15-point scale variation in $\overline{\text{MS}}$ scheme
  (customary 7-point in pole scheme with 2 auxiliary scales)

- we compare pole scheme and $\overline{\text{MS}}$ scheme by setting

  pole scheme: $M_t = 173.3$ GeV and use $\mu_0 = M_t$

  $\overline{\text{MS}}$ scheme: $\overline{m}_t = 163.7$ GeV and use $\mu_0 = \overline{m}_t$
  (varying μ_m with $0.5 < \mu_m/\mu_0 < 2 \longrightarrow 155$ GeV $< m_t(\mu_m) < 173$ GeV)

- we use NNPDF31 and $\sqrt{s} = 13$ TeV

TOTAL CROSS SECTION

scheme	pole	$\overline{\text{MS}}$			
variation	7-point	15-point	$\mu_m = \mu_0$	$\mu_{R/F} = \mu_0$	$\mu_{R/F} = \mu_m$
LO (pb)	$478.9\ ^{+29.6\%}_{-21.4\%}$	$625.7\ ^{+29.4\%}_{-21.9\%}$	$^{+29.4\%}_{-21.3\%}$	$^{+24.7\%}_{-21.9\%}$	$^{+1.5\%}_{-1.5\%}$
NLO (pb)	$726.9\ ^{+11.7\%}_{-11.9\%}$	$826.4\ ^{+7.6\%}_{-9.7\%}$	$^{+7.6\%}_{-9.6\%}$	$^{+5.6\%}_{-9.7\%}$	$^{+1.2\%}_{-1.2\%}$
NNLO (pb)	$794.0\ ^{+3.5\%}_{-5.7\%}$	$833.8\ ^{+0.5\%}_{-3.1\%}$	$^{+0.4\%}_{-2.9\%}$	$^{+0.3\%}_{-3.1\%}$	$^{+0.0\%}_{-0.3\%}$
			(a)	(b)	(c)

- comparison pole scheme ($\mu_0 = M_t$) and $\overline{\text{MS}}$ scheme ($\mu_0 = \overline{m}_t$)
  - order-by-order consistency of the results and very similar at NNLO
  - $\overline{\text{MS}}$ typically higher at central scale and with smaller uncertainties at NLO and NNLO [μ_R (a) and μ_m (b) dependences have similar size but opposite sign (cancellations (c))]
  - $\overline{\text{MS}}$ results have faster apparent convergence *

$$\frac{\text{NLO}}{\text{LO}} = 1.52\ (\text{pole}),\ 1.32\ (\overline{\text{MS}})$$

$$\frac{\text{NNLO}}{\text{NLO}} = 1.09\ (\text{pole}),\ 1.01\ (\overline{\text{MS}})$$

* first noticed by Langenfeld-Moch-Uwer (2009)

pole vs. $\overline{\text{MS}}$ scheme: slower/faster apparent convergence

central scales

$\mu_0 = M_t$ vs. $\mu_0 = \overline{m}_t$: we do not have a physical interpretation
but we do have a technical explanation
(valid in any scheme with renormalized mass $m_{\text{ren.}} < M_t$)

moreover :

the apparent convergence strongly depends on the choice of central value μ_0 of auxiliary scales

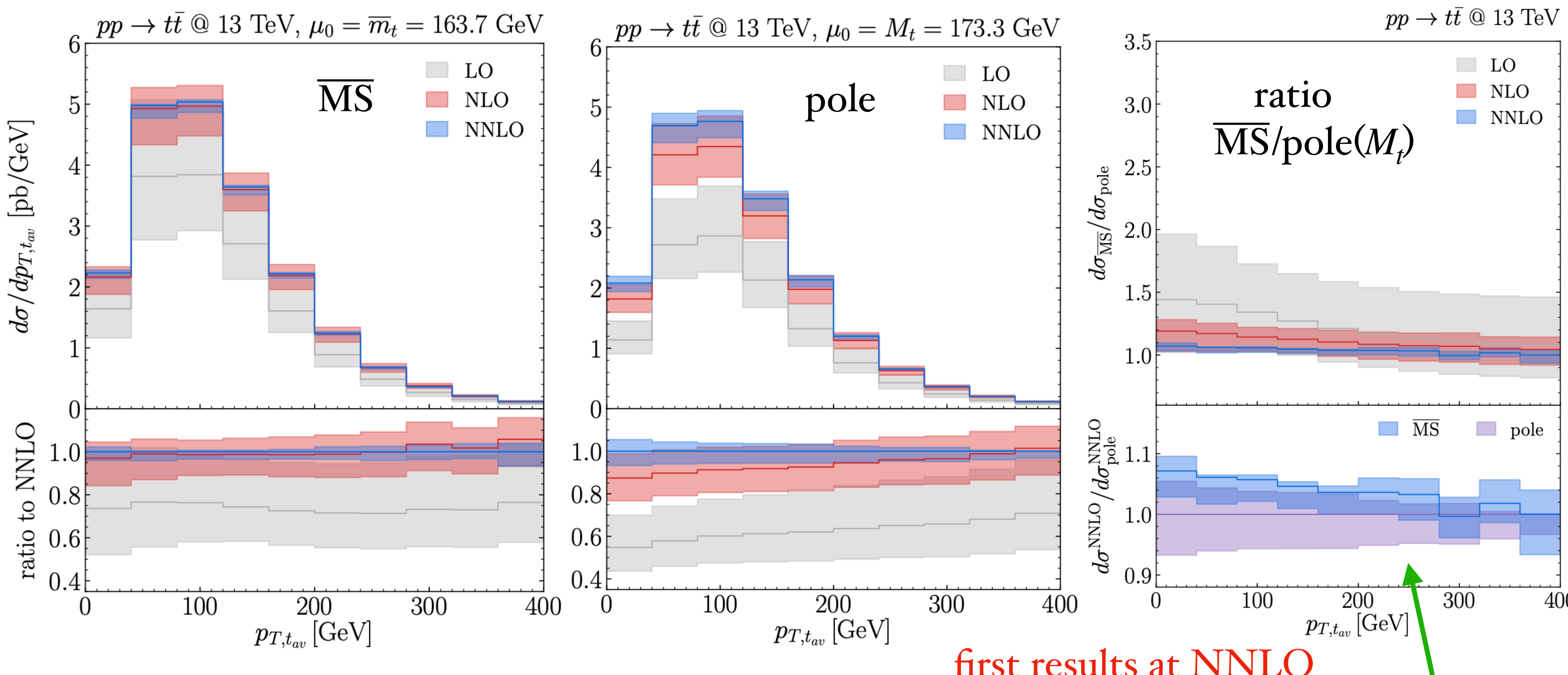
scheme	pole	$\overline{\text{MS}}$	$\overline{\text{MS}}$	pole
central scale choice	$\mu_{R/F} = M_t$	$\mu_{R/F} = \overline{m}_t$ $\mu_m = \overline{m}_t/2$	$\mu_{R/F} = \overline{m}_t$ $\mu_m = \overline{m}_t$	$\mu_{R/F} = M_t/2$
LO (pb)	478.9	488.9	625.7	619.8
NLO (pb)	726.9	746.4	826.4	811.4
NNLO (pb)	794.0	808.0	833.8	822.4

Slower: $\overline{\text{MS}}$ scheme ($\mu_{0,m} = \overline{m}_t/2$) and pole scheme ($\mu_0 = M_t$) behave similarly

Faster: $\overline{\text{MS}}$ scheme ($\mu_{0,m} = \overline{m}_t$) and pole scheme ($\mu_0 = M_t/2$)* behave similarly

* scale suggested by Czakon-Deymes-Mitov (2017)

DIFFERENTIAL CROSS SECTIONS



- transverse-momentum p_T distribution of t quark :

  comparison pole scheme ($\mu_0 = M_t$) vs. $\overline{\mathrm{MS}}$ scheme ($\mu_0 = \overline{m}_t$)

- overall features similar to those for total cross sections
- at NNLO (see ratio $\overline{\mathrm{MS}}$/pole): shape differences are quite small and within scale uncert.

→ the results in the two schemes behave similarly at (sufficiently) high order

- similar comments apply to other differential cross sections :

  rapidity of t quark or $t\bar{t}$ pair

  invariant-mass $m_{t\bar{t}}$ distribution of $t\bar{t}$ pair at high $m_{t\bar{t}}$

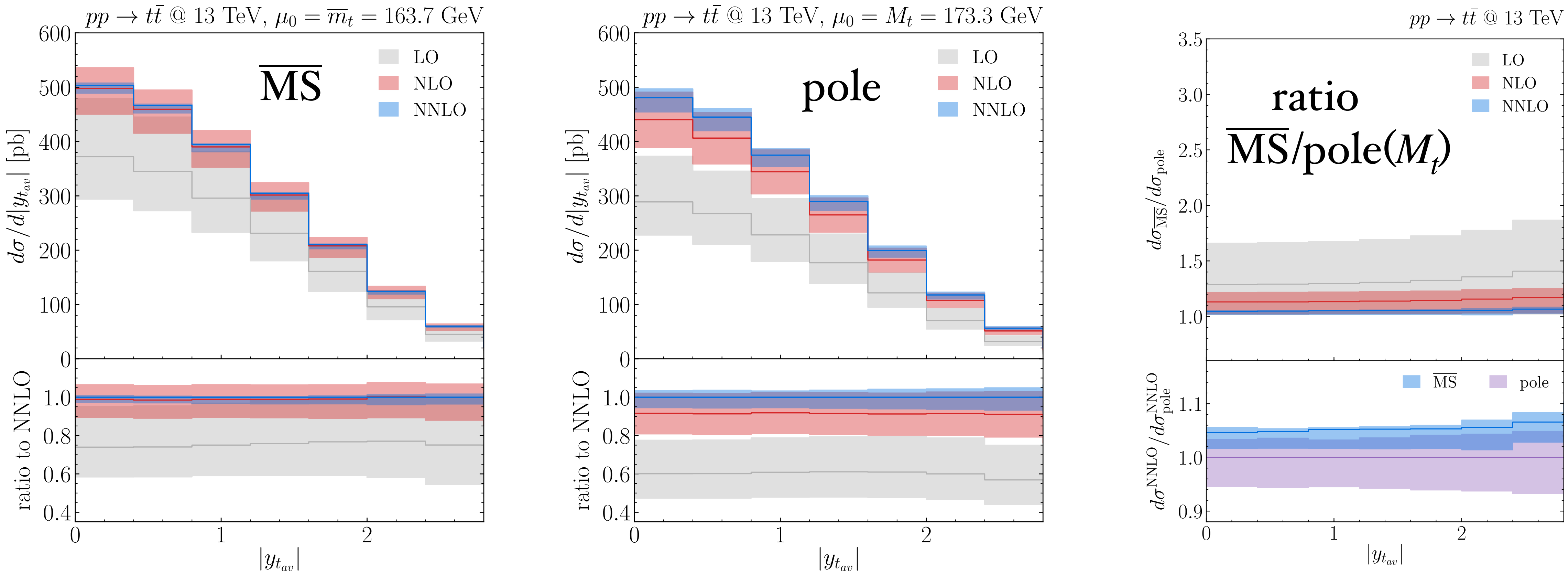
  - exception :

    invariant-mass distribution of $t\bar{t}$ pair close to its threshold region

- overall observations

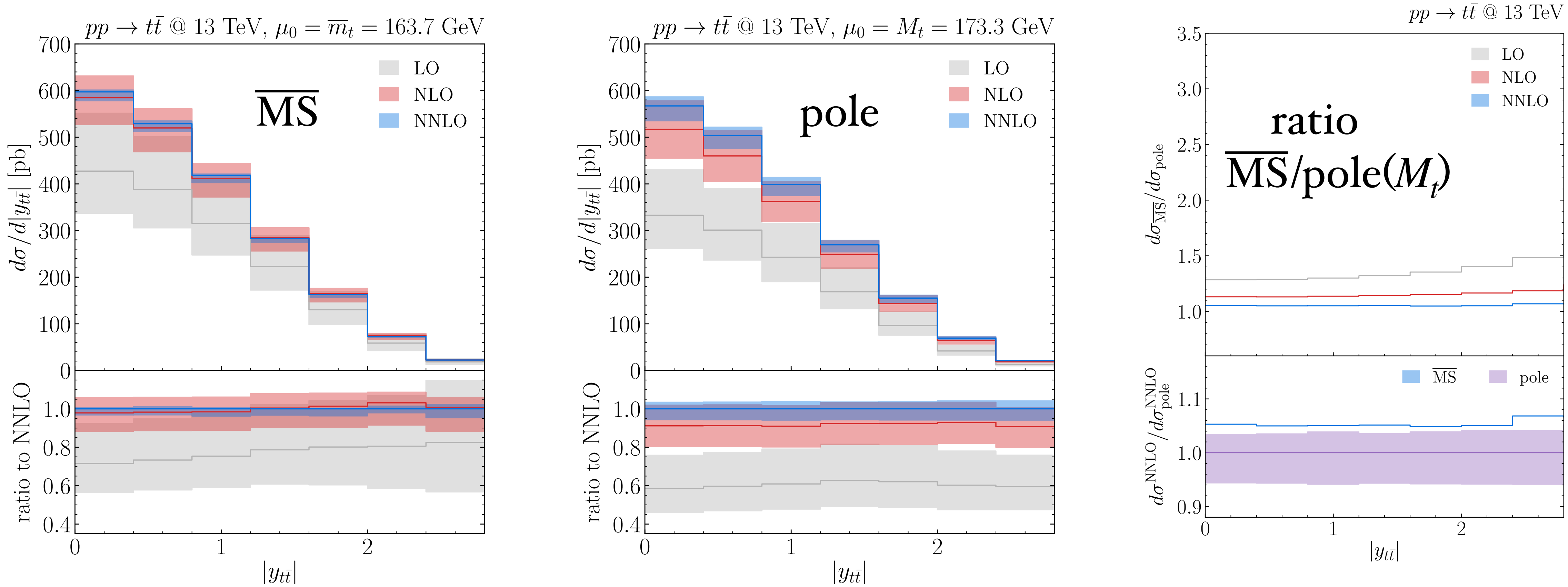
  - results in pole and $\overline{\text{MS}}$ schemes become increasingly similar at high orders
  - NNLO results: precise QCD predictions in both schemes

rapidity of t quark (antiquark)



NNLO ratio

rapidity of top-quark pair

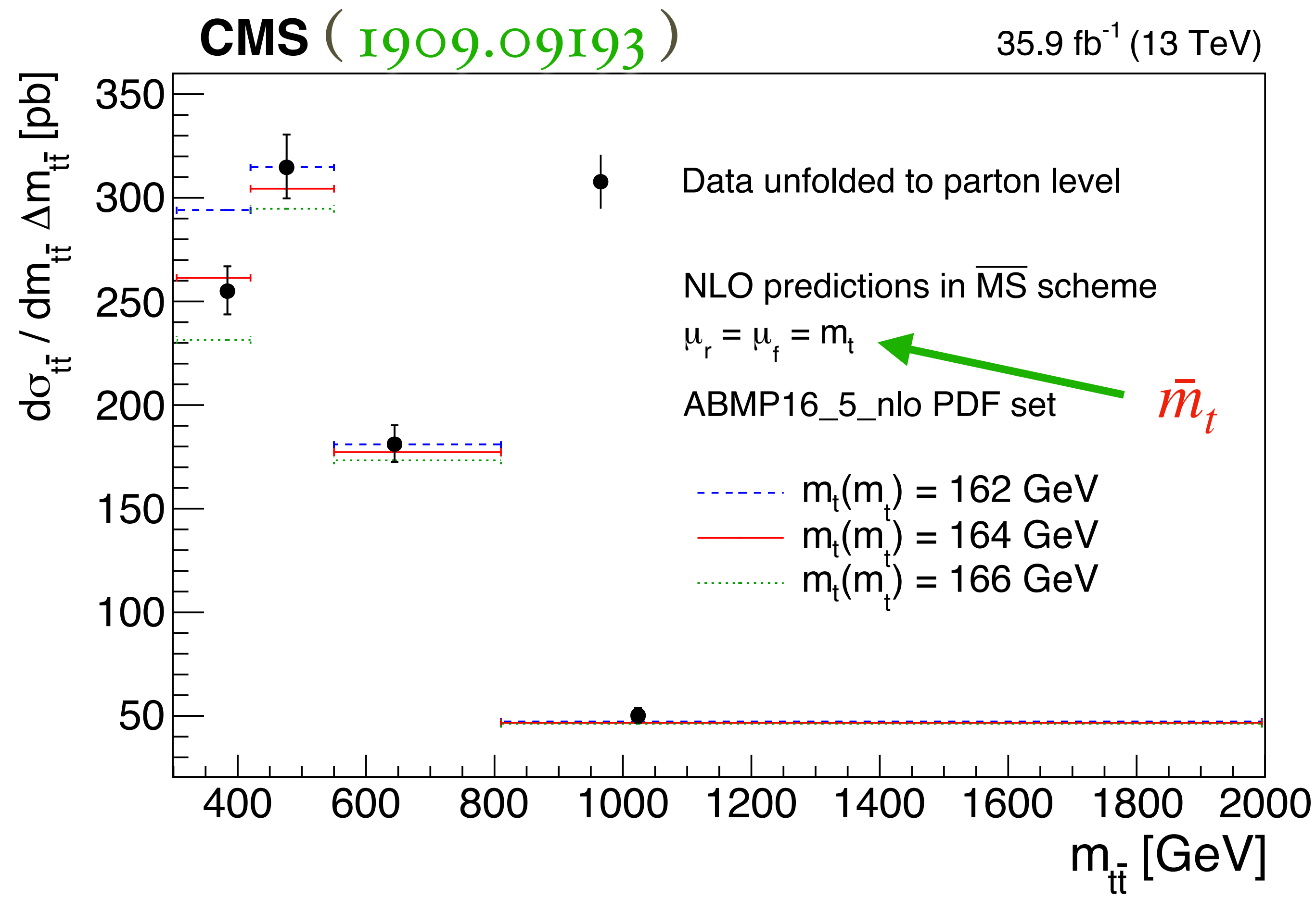


INVARIANT-MASS DISTRIBUTION of $t\bar{t}$ PAIR

recent CMS study (2020)

- precise measurement of $m_{t\bar{t}}$ cross section: 4 bins over region $\sim 380 - 1000$ GeV

- use NLO calculation with FIXED $\overline{\text{MS}}$ mass $\bar{m}_t$ (i.e. $\mu_m = \bar{m}_t$ in all bins) and fit value of $\bar{m}_t$ to data in each bin



our conclusions :

data/NLO consistency with a single common (i.e., bin-independent within errors) value of $\bar{m}_t$

- can we study effects due to running $\overline{\text{MS}}$ mass $m_t(\mu)$?
  this unavoidably requires calculation with RUNNING (bin-dependent) value of μ_m (i.e., $m_t(\mu_m)$)

$m_{t\bar{t}}$ DISTRIBUTION: EFFECTS OF RUNNING $\overline{\text{MS}}$ MASS

- we investigate QCD results in $\overline{\text{MS}}$ scheme with two different options for central scale μ_0

  (i) FIXED mass : set $\mu_0 = \bar{m}_t$ (for μ_m, μ_R, μ_F)
  [NNLO extension of CMS NLO calculation]

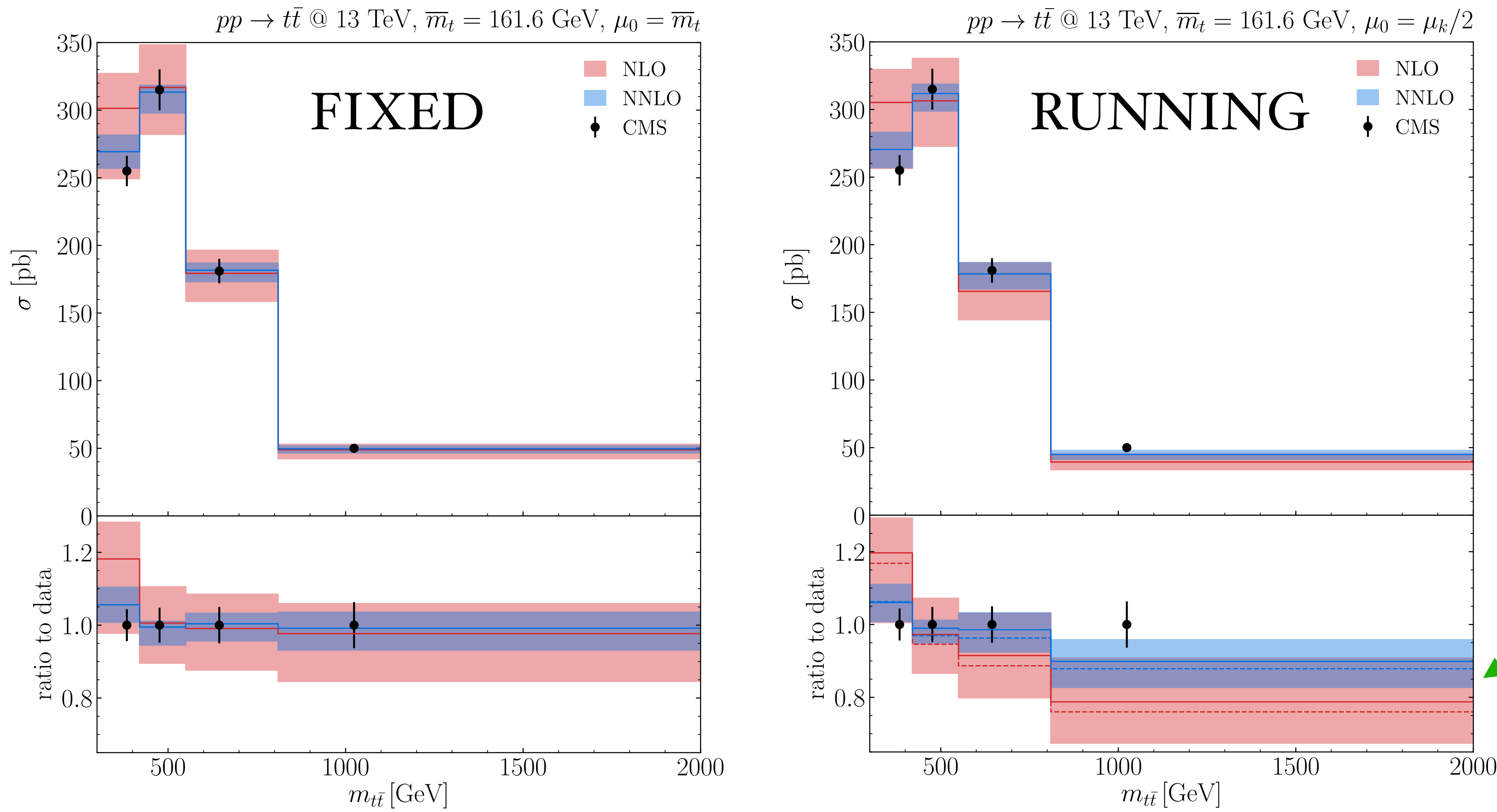
  (ii) RUNNING mass : set $\mu_0 \simeq m_{t\bar{t}}/2$ (for μ_m, μ_R, μ_F)
  (i.e. $m_t(m_{t\bar{t}}/2)$ is bin-dependent and it varies by about 10 GeV :
  from $m_t \sim 160$ GeV in 1-st. bin $\rightarrow$ to $m_t \sim 150$ GeV in 4-th. bin)

- set up : ABMP16 PDFs (as in CMS study of $m_{t\bar{t}}$ distribution) ;
  $\bar{m}_t = 161.6$ GeV (as measured at NNLO by CMS study of total cross section)
  [it corresponds to $M_t = 170.8$ GeV]

* Aside comment

high (multi TeV) $m_{t\bar{t}}$ region : two very different scales, M_t and $m_{t\bar{t}}$

$\rightarrow$ resummation of soft/collinear effects
[e.g., Ahrens et al. (2010), Ferroglia et al. (2012), Czakon et al. (2018)]
could be combined with running mass effects



$\mu_0 \simeq m_{t\bar{t}}/2$

dashed lines: results with
$\mu_R = \mu_F \simeq m_{t\bar{t}}/2$
but keeping $\mu_m = \bar{m}_t$

- comparison FIXED vs. RUNNING (including 15-point scale variations)
  - practically ("by definition") no theory differences at low $m_{t\bar{t}}$
  - differences at high $m_{t\bar{t}}$ are "small" and mainly driven by running of α_S and PDFs
  - very similar/consistent (within scale uncertainties) results at NNLO

- our conclusions :
  - NNLO corrections lead to reduced th. uncert. and to improved agreement with data
    [moreover : pole scheme calculation with $M_t = 170.8$ GeV can do a similar job]
  - no significant sensitivity to running mass effects

Summary

- On-shell top-quark production:
  reformulation of QCD calculation from pole to $\overline{\text{MS}}$ mass

- $t\bar{t}$ production at the LHC:
  first NNLO results for single-differential cross sections by using $\overline{\text{MS}}$ mass
  [extension to multi-differential and/or fiducial cross section is straightforward (feasible)]

  QCD comparison pole vs. $\overline{\text{MS}}$ schemes (at fixed $\overline{\text{MS}}$ mass: $m_t(\mu_m)$ with $\mu_m \sim \bar{m}_t$)
  including perturbative uncertainties (15-point scale variations in $\overline{\text{MS}}$ scheme)

  - consistent order-by-order results and increasingly similar results at high order
  - at NNLO: precise QCD predictions in terms of $\overline{\text{MS}}$ mass
    → relevant for ensuing studies with $\overline{\text{MS}}$ mass

- Effects due to the running of $\overline{\text{MS}}$ mass
  first study of running mass effects ($m_t(\mu_m)$ with $\mu_m \sim m_{t\bar{t}}/2$)
  for invariant-mass distribution of $t\bar{t}$ pair in region up to $m_{t\bar{t}} \sim 1$ TeV

  - no significant sensitivity to running mass effects
  - further studies of running mass effects feasible and warranted

Backup

ON-SHELL CROSS SECTION for $t\bar{t}$ PRODUCTION: from pole to $\overline{\text{MS}}$ mass

- some obvious unphysical features

definitely unphysical
if $M_t - m_t(\mu_m)$ is large w.r.t. Γ_t

e.g., consider invariant-mass $m_{t\bar{t}}$ of $t\bar{t}$ pair

it has a physical threshold at minimum value $m_{t\bar{t}}^{\text{min.}} = 2M_t$

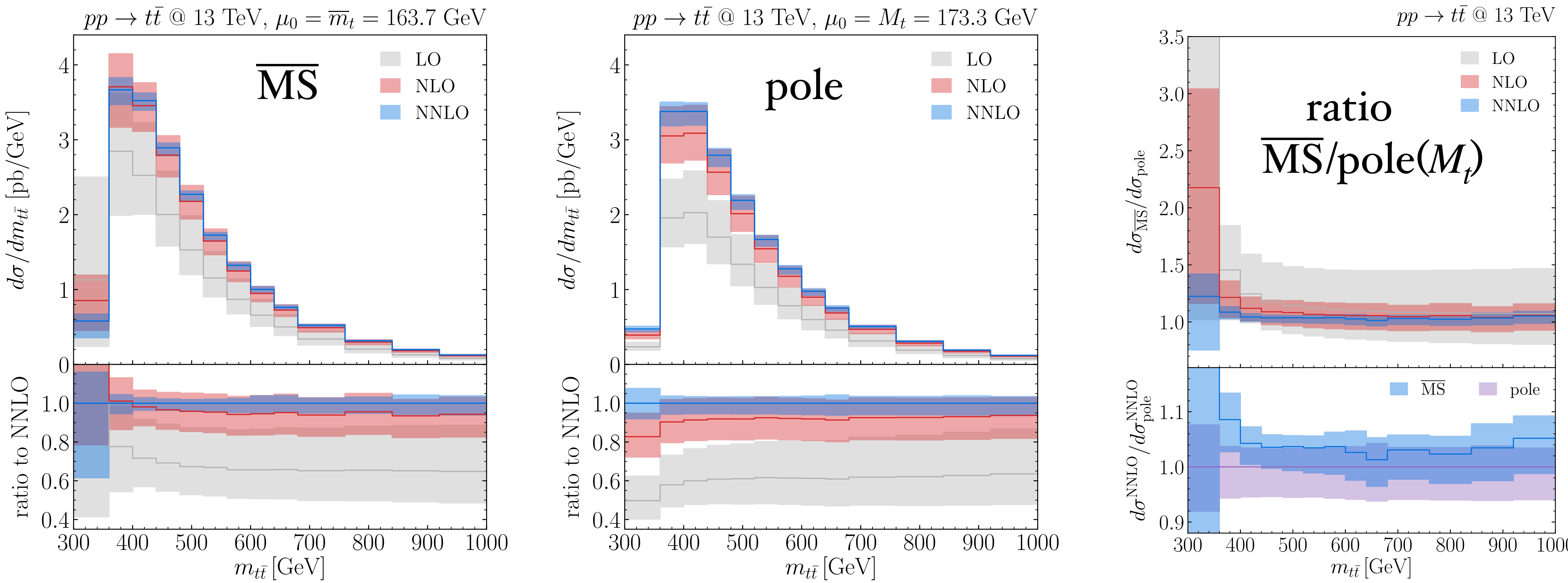
- physical threshold fulfilled order-by-order in α_S within pole scheme

within $\overline{\text{MS}}$ scheme :

- $m_{t\bar{t}}^{\text{min.}} = 2m_t(\mu_m)$ at LO → arbitrary dependence on μ_m ; definitely unphysical if $M_t - m_t(\mu_m)$ is large w.r.t. Γ_t

- near threshold: $\partial_m^n \hat{\sigma}^{(l)}$ very large → very large N^{k}LO corrections (badly convergent α_S expansion)

- invariant-mass distribution of $t\bar{t}$ pair



NNLO ratio

- comparison pole ($\mu_0 = M_t$) vs. $\overline{\text{MS}}$ ($\mu_0 = \overline{m}_t$) : similar to other distributions but

  exception ➡ region close to threshold (1st. bin: 300-360 GeV, 2nd. bin: 360-400 GeV)

  - low-$m_{t\bar{t}}$ region : $\overline{\text{MS}}$ results have larger uncertainty (dominated by μ_m variations) and larger radiative corrections

    consequence of unphysical order-by-order “identification” $M_t \to m_t(\mu_m)$

    [mis-behaviour partly alleviated at high orders and/or using wide bin size]

- sufficiently close to threshold : no point in using $\overline{\text{MS}}$ mass

  ➡ use pole scheme (possibly refined by resummation of Coulomb-type effects *)

* see talk by Li Lin Yang

CMS Coll. (2020)
1909.09193